Heavy quark physics on anisotropic lattices

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Contents:

Refs.:

J. Harada et al., Phys. Rev. D 64 (2001) 074501
H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D 64 (2001) 114503
J. Harada, H. Matsufuru, T. Onogi and A. Sugita, Phys. Rev. D 66 (2002) 014509
H. Matsufuru, M. Okawa, T. Onogi, and T. Umeda, hep-lat/0312009.

Introduction

"Hadron Physics and Lattice QCD"

□ To study nonperturbative QCD structure
Confinement mechanism, chiral symmetry, phase structure of QCD, topological structure, etc.

□ To provide fundamental quantities for ...
elementary particle physics, nuclear physics, etc.
— high precision is required
(or, systematic error must be controlled)

Our motivation:
to provide hadoronic matrix elements for flavor physics with high precision

Recent experimental developments in flavor physics
— B factories, Charm factory, ...

$\Rightarrow$ need precise theoretical predictions ($\simeq 2\%$) of hadron matrix elements

decay constant, bag parameter, form factor,
quark mass, light-cone wave function, etc.

Why anisotropic lattice ?

Problem in lattice QCD calculations:
for heavy quarks (c and b), large $O(am_Q)$ error

We need a framework which has
(i) continuum limit
(ii) systematic improvement
such as Nonperturbative renormalization technique (for $m_q \simeq 0$)
(iii) modest size of computation

Previous approaches (∼10% systematic accuracy):
a). Effective theories (NRQCD, etc) — (i)×
Thacker and Lepage, Phys. Rev. D 43 (1991) 196.
b). Relativistic framework — (iii)×
c). Fermilab approach (with/without mass dependent tuning)
El-Khadra et al., Phys. Rev. D 55 (1997) 3933.
Sroczynski, et al., Nucl. Phys. B (PS) 83 (2000) 971.
— (ii)×
d). AKT action (Kayaba's talk)
Aoki, Kuramashi, Tominaga, Prog.Theor.Phys.109 (2003) 383.
Mass dependent renormalization is necessary — (ii)?

To achieve calculations with ∼2% precision, we need
yet another approach which satisfies above condition (i)–(iii).
⇒ our proposal: *Anisotropic lattice*

Why anisotropic lattice ?

Anisotropic lattice: $a_\tau < a_\sigma$ $\Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Quark action: $O(a)$ improved, along with Fermilab approach
- □ bare anisotropy parameter γ_F:
  - in general, to be tuned mass dependently
    - e.g., using meson dispersion relation

- ∘ Continuum limit ☺
- ∘ Modest computational cost ☺
- ∘ Systematic improvement ?
  - If quark mass is sufficiently less than a^{-1},
  - tuned parameters for massless quark are applicable.
    - →nonperturbative renormalization technique
      - *Lüscher et al., Nucl.Phys. B 491 (1997) 323.*

Our expectation:

For $m_Q \ll a_\tau^{-1}$, (not necessarily $m_Q \ll a_\sigma^{-1}$),
mass dependences of parameters in the action are so small
that the tuned parameters for massless quark are also
applicable in such a quark mass region.

Then, systematic improvement is possible. ☺
(performed at $m_q \simeq 0$)

$\Rightarrow$ To be justified numerically, and in perturbation theory.
- ∘ Tree level: OK
- ∘ $O(a^0)$ improved version: OK (2% accuracy)
- ∘ High precision: in progress

Formulation

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & - \kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & - \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215

Harada et al., Phys. Rev. D 64 (2001) 074501

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)

For a few percent level calculation,
parameters γ_F, c_E, c_B should be tuned nonperturbatively

Present results:
parameters at tadpole improved tree level:
$\rightarrow$ applicable to $O(10\%)$ simulations

- Clover coefficients: $c_E = 1/u_\sigma^2 u_\tau$, $c_B = 1/u_{\sigma^3}$
  u_σ, u_τ: mean-field values of spatial and temporal link variables

- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0 \gamma_F + 4))$$

γ_F: bare anisotropy parameter
m_0: bare quark mass (in units of a_τ^{-1})

Preceding results

☐ One-loop perturbative calculation:

Harada et al., Phys. Rev. D 64 (2001) 074501

Renormalization factors of heavy-light bilinears and quark rest mass at $m_Q a_\sigma \sim 1$, $m_Q \ll a_\tau^{-1}$

$\Rightarrow$ Well approximated with linear form in $m_Q a_\tau$

— quark mass dependence can be controlled

☐ Numerical simulation:

In quenched approximation, tadpole improved c_E and c_B

- Mass dependent tuning

Matsufuru, Onogi and Umeda, Phys. Rev. D 64 (2001) 114503

γ_F is tuned with meson dispersion relation

Quark mass dependence is small for $m_q a_\tau \ll 1$

- Test of relativity relation

Harada et al., Phys. Rev. D 66 (2002) 014509

Heavy-light meson dispersion relation for $\gamma_F = \gamma_F(m_q = 0)$

Relativity relation well holds for $m_q a_\tau \ll 1$ (while $m_q a_\sigma \gtrsim 1$)

- Application to decay constant

Matsufuru, Harada, Onogi and Sugita, hep-lat/0209090

Around charm quark mass, with $O(10\%)$ accuracy

Result consistent with previous works

— Encouraging results for further development

Results: mass dependent tuning

Mass dependent calibration of γ_F

using meson dispersion relation

Simulation: quenched lattices with $\xi = 4$

β	γ_G	size	$a_\sigma^{-1}(r_0)$ [GeV]
5.75	3.072	$12^3 \times 96$	1.100(6)
5.95	3.1586	$16^3 \times 128$	1.623(9)
6.10	3.2108	$20^3 \times 160$	2.030(13)

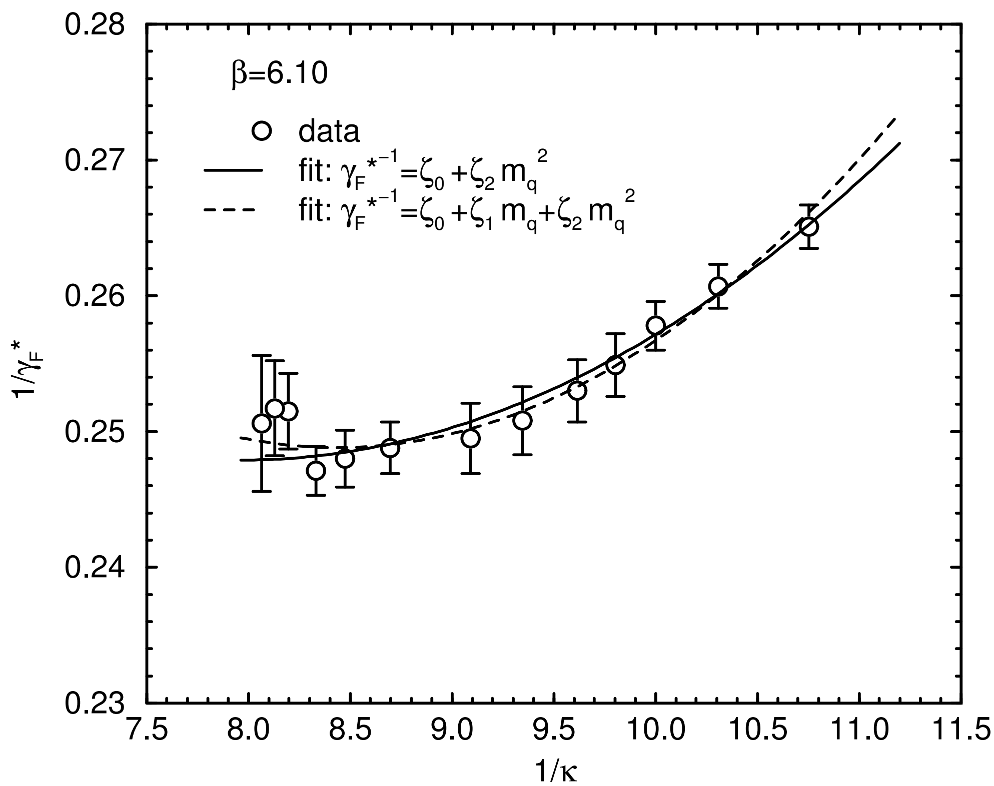
Calibration result: γ_F^*

— well fitted to linear form in m_q^2:

$$\frac{1}{\gamma_F^*} = \zeta_0 + \zeta_1 m_q + \zeta_2 m_q^2 \qquad m_q = \frac{1}{2\xi}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

$\Rightarrow$ For $m_q a_\tau \ll 1$, γ_F^* is well approximated with $\gamma_F^*(m_q\!=\!0)$

$\gamma_F^*(m_q = 0)$ is determined with $O(2\%)$ accuracy



Results: Relativity relation

Simulation: on quenched anisotropic lattice of
size $16^3 \times 128$, $a_\sigma^{-1} \simeq 1.6$ GeV and anisotropy $\xi = 4$.

Light quark: mass $\sim 1.5\, m_s$

Heavy quark: 7 values with $m_Q = 1$ – 6 GeV

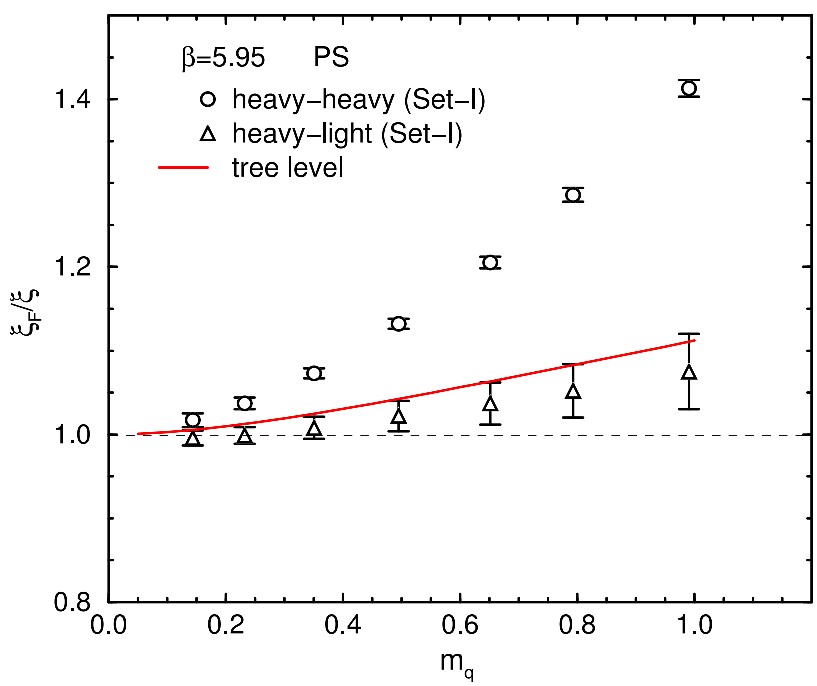
Bare anisotropy: tuned value at massless limit ($\gamma_F = 4.016$)

Heavy-heavy, heavy-light meson dispersion relations

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$$

$\rightarrow$ fermionic anisotropy ξ_F

$\xi_F \neq \xi$ signals breaking of relativity relation



- $m_q < 0.3$: for heavy-light mesons, γ_F tuned for massless quark can be applied within 2% accuracy
- $m_q < 0.2$: both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy
- $0.2 < m_q$: heavy quarkonia suffer from larger $O((ap)^2)$ error

Heavy-light decay constant (1)

Simulation: quenched anisotropic lattices with $\xi = 4$,

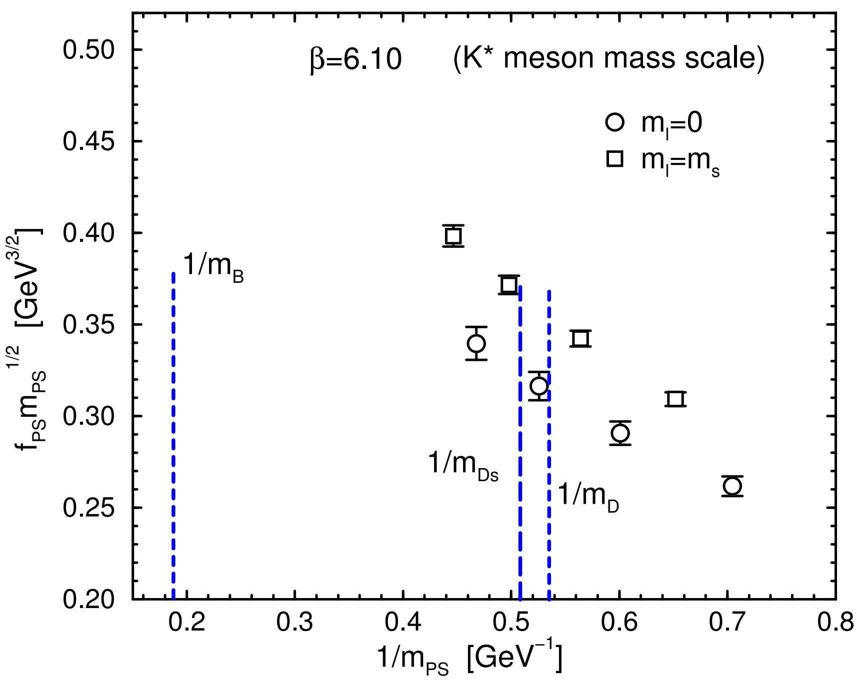
- $\circ$ $16^3 \times 128$, $\beta = 5.95$, $a_\sigma^{-1} \simeq$ 1.6 GeV
- $\circ$ $20^3 \times 160$, $\beta = 6.10$, $a_\sigma^{-1} \simeq$ 2.0 GeV

Light quark: 3 values with masses 1–$1.5\ m_s$
$\rightarrow$ chiral extrapolation

Heavy quark: 4 values with masses 0.7–1.5 GeV
(target mass: charm quark)

Heavy-light pseudoscalar meson decay constant:

- $\circ$ mean-field improved tree level matching



	$\beta = 5.95$	$\beta = 6.10$	previous works*
f_D/f_π	1.566(43)	1.515(43)	1.55(11)
f_{D_s}/f_D	1.140(14)	1.142(14)	1.12(2)

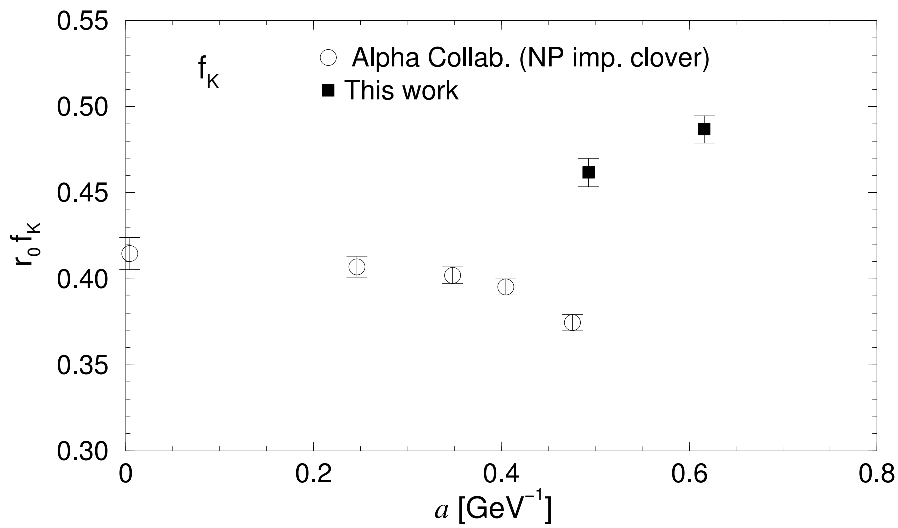
*Average of quenched works,
S.Ryan, Nucl. Phys. B (PS) 106 (2002) 86.

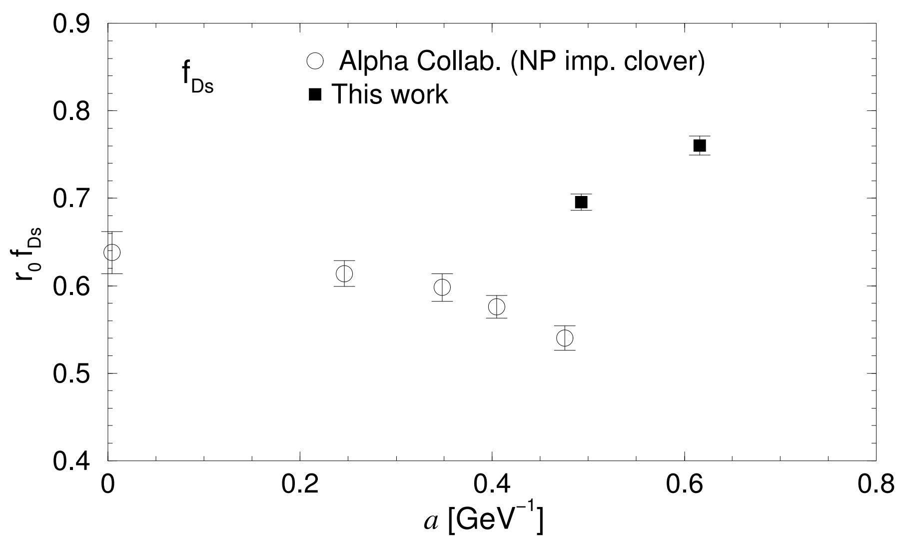
Heavy-light decay constant (2)

Scaling violation large ?

Comparison with the result of Alpha Collab.

(fully nonperturbative $O(a)$ improved)





	This work		Alpha Collab.	
	$\beta = 5.95$	$\beta = 6.10$	$\beta = 6.0$	cont. limit
f_{D_s}/f_K	1.562(36)	1.507(37)	1.442(41)	1.539(44)

— Scaling violation comparable with Alpha's results.

Precise calibration: strategy

Calibrations of parameters to 0.2% level of accuracy
in quenched approximation

⇒Applicable to a few percent calculation of matrix elements

☐ Gauge field:

- Precise computation of static quark potential
- Renormalized anisotropy defined with r_0

⇒ $O(0.2\%)$ calibration is possible

☐ Quark field:

We need to calibrate γ_F, c_E, c_B, and c_A (for axial current) in massless limit.

— γ_F must be tuned precisely (to $O(0.2\%)$ level)

Combine two procedures:

- Nonperturbative renormalization technique
- Spectroscopy in fine and coarse directions

Numerical simulation is in progress

☐ Test in heavy quark region:

Use the parameters tuned in massless limit

- Test of relativity relation
- Check of $O(a)$ improvement

⇒ Applications to heavy-light matrix elements

Precise calibration: gauge field

Quenched gauge field calibration

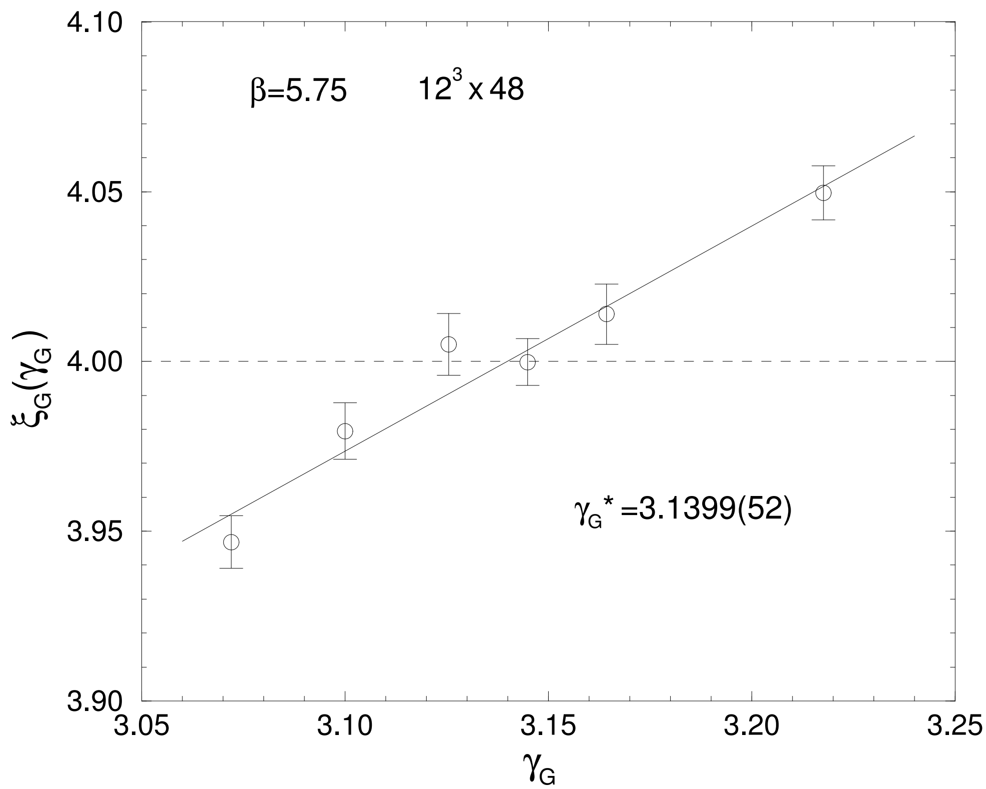
Renormalized anisotropy $\xi_G \Leftarrow$ static potential
We define ξ_G through hadronic radius r_0

r_0: $r_0^2 F(r_0) = 1.65$ ($F(r)$: force)

Sommer, Nucl. Phys. B411 (1994) 839

- Precise computation possible
- Good scaling behavior (if lattice scale is set by r_0)

Example at $\beta = 5.75$ ($\xi = 4$):



γ_G is tuned with 0.2% accuracy

Precise calibration: quark field (1)

Five parameters (γ_F, c_E, c_B, c_A, κ_c) should be determined for each quark mass.

Calibration steps:

(1) Schrödinger functional method
$\Rightarrow$ tuning of c_E, (γ_F)

(2) Spectroscopy in coarse and fine directions on lattices with T,$L \gtrsim$ 2 fm
Physical isotropy conditions for m_{PS}, m_V
$\Rightarrow$ tuning of γ_F, c_B

(3) Schrödinger functional method
$\Rightarrow$ determine c_A, κ_c

(4) Check of systematic errors
Light hadron spectrum and dispersion relation
Taking the continuum limit

Precise calibration: quark field (2)

Nonperturbative improvement technique

Lüscher et al., Nucl. Phys. B478 (1996) 365

Lüscher et al., Nucl. Phys. B491 (1997) 323

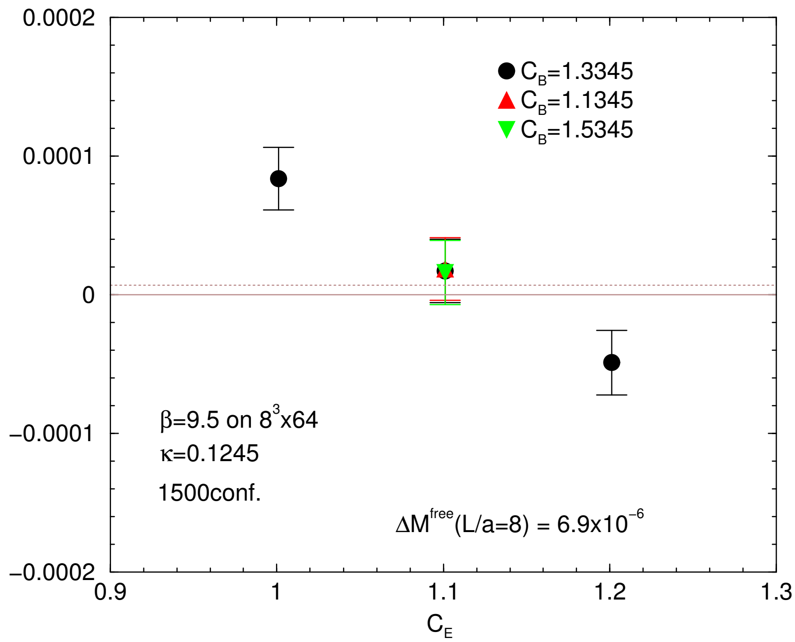
Schrödinger functional method:

Implementation for anisotropic lattice is straightforward.

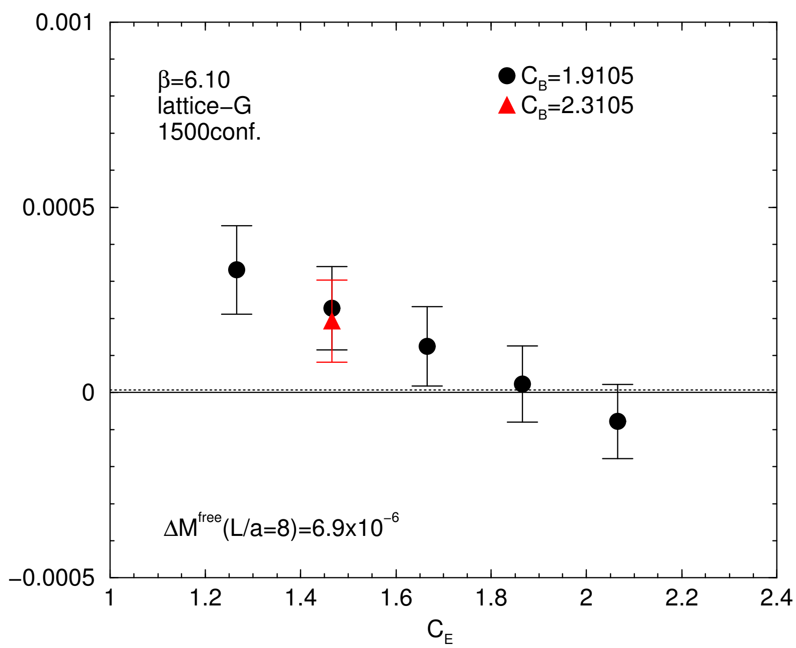
PCAC relation up to $O(a^2)$

$\Rightarrow$ improvement conditions for c_E, c_A (and γ_F)

$\beta = 9.5$



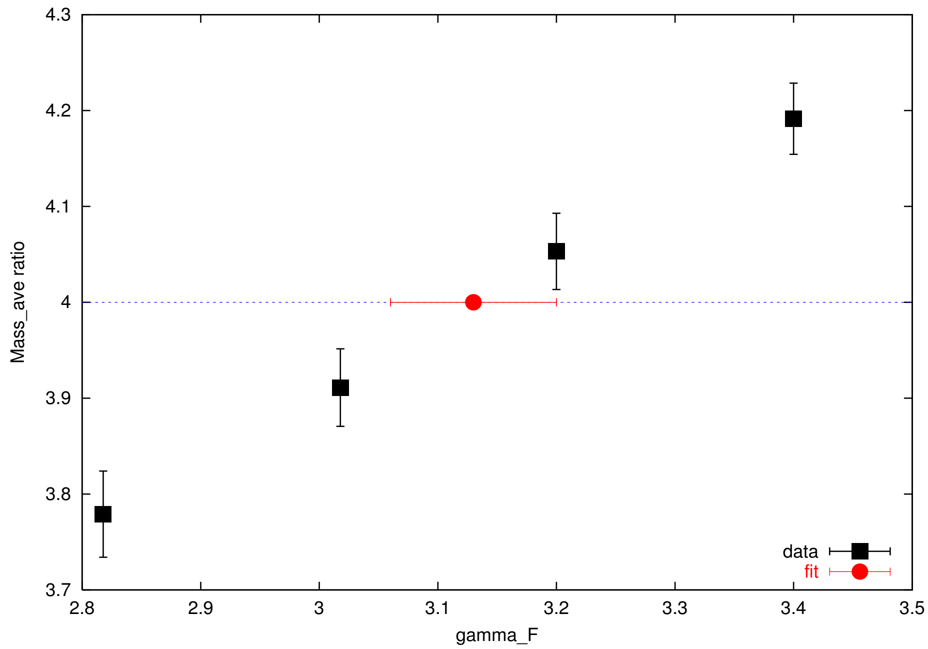
$\beta = 6.1$



Precise calibration: quark field (2)

Determination of γ_F and c_B:

- On large lattice
- Ratio of masses in spatial and temporal directions
- PS and V channels $\rightarrow$ tuning of γ_F and c_B



Tuning of c_B: in progress (need some improvement)

Summary and outlook

We are developing anisotropic lattices for precise computation of heavy-light matrix elements.

Results obtained so far are encouraging for further development.

We have started precise calibrations of gauge and quark fields in quenched approximation.

Outlook:

- ☐ How can we verify that parameters in $m_q = 0$ suffices for heavy quark mass ?
- ☐ Mass dependent tuning for bottom quark region
  - ○ Tree level γ_F may be a good approximation.
- ☐ Applications to heavy-light matrix elements
  $\rightarrow$ High precision computation actually possible ?
- ☐ Extension to dynamical QCD:
  Which calibration procedure is most efficient ?