Meson correlators in lattice QCD at finite temperature

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Lattice QCD at Finite Temperature and Density

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Contents

— Charmonium systems at finite temperature in lattice QCD —

Introduction

Motivations: to understand properties and roles of hadronic correlators at finite temperature (and density).

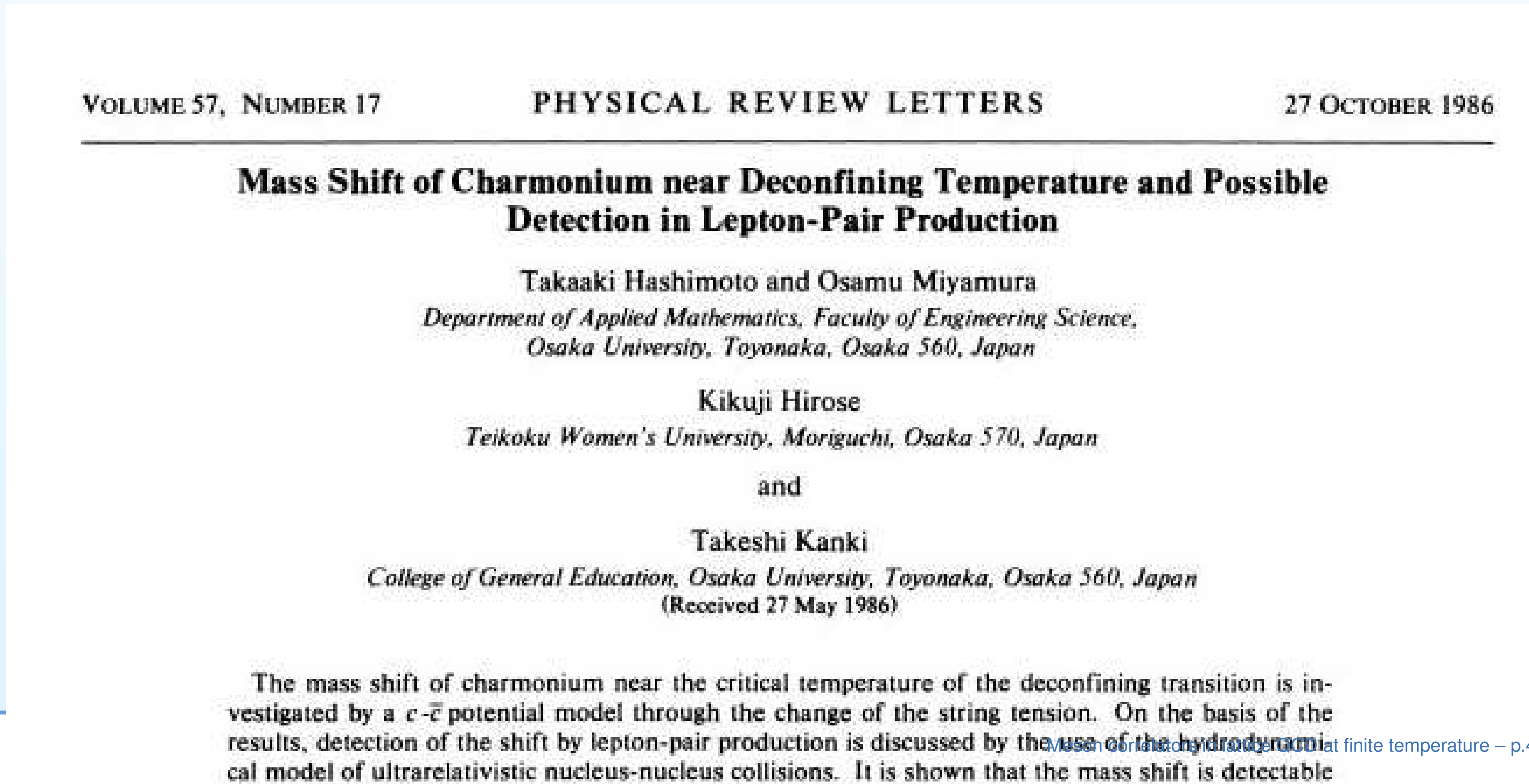
- Space-time structure of the hadronic correlators
- Spectral function
- Confinement mechanism
- Chiral symmetry breaking/restoration
- Relations to effective models
  (potential model, etc.)

In this talk, we focus on the charmonium correlators.

Ref: T.Umeda, K. Nomura and H. Matsufuru, hep-lat/0211003
(to appear in Eur. Phys. J. C)

Introduction

Hashimoto, Miyamura, Hirose and Kanki, Phys. Rev. Lett. 57 (1986) 2123.
— Mass shift of charmonium near T_c.
Also pointed out *"A related question is whether charmoniumlike clusters may still exist in a quark-gluon plasma. We have made tentative calculations by screened Coulombic potential and found that possibly small."*



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Mass Shift of Charmonium near Deconfining Temperature and Possible Detection in Lepton-Pair Production

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(Received 27 May 1986)

The mass shift of charmonium near the critical temperature of the deconfining transition is investigated by a c-$\bar{c}$ potential model through the change of the string tension. On the basis of the results, detection of the shift by lepton-pair production is discussed by the use of the hydrodynamical model of ultrarelativistic nucleus-nucleus collisions. It is shown that the mass shift is detectable

Introduction

Charmonium is an important signal for formation of QGP in heavy ion collision experiments.

Theoretical understanding

- Potential model
  - Mass shift near T_c
    T.Hashimoto et al., Phys. Rev. Lett. 57 (1986) 2123.
  - J/ψ suppression above T_c
    T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
- Lattice QCD
  - Meson correlators
    T. Umeda et al., Int. J. Mod. Phys. A 16 (2000) 2215
    $\longrightarrow$ strong spatial correlation even at $T \simeq 1.5T_c$
  - Spectral function analyses

q-$\bar{q}$ correlation

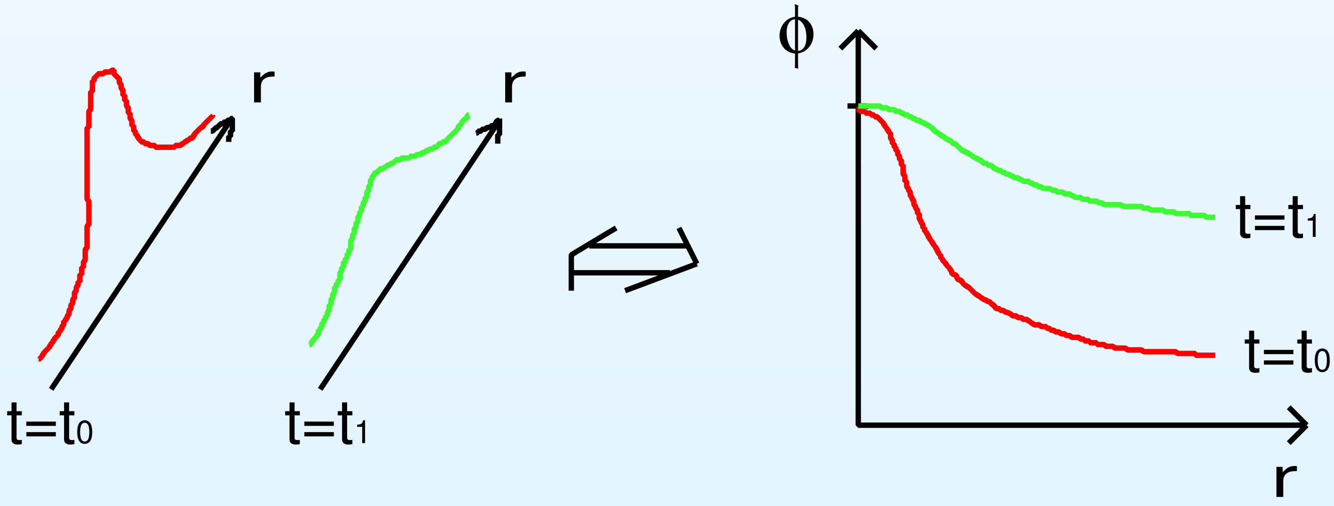
q-$\bar{q}$ correlation (“wave function”) in Coulomb gauge

QCD-TARO Collab. Phys. Rev. D 63 (2001) 054501;

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001)2215

$$\phi(r,t) = \frac{w(r,t)}{w(0,t)}, \qquad w(r,t) = \sum_{\vec{y}} \langle \bar{q}(\vec{y}+\vec{r},t)\Gamma q(\vec{x},t) O^\dagger(0)\rangle$$

In the case of unbound quarks,

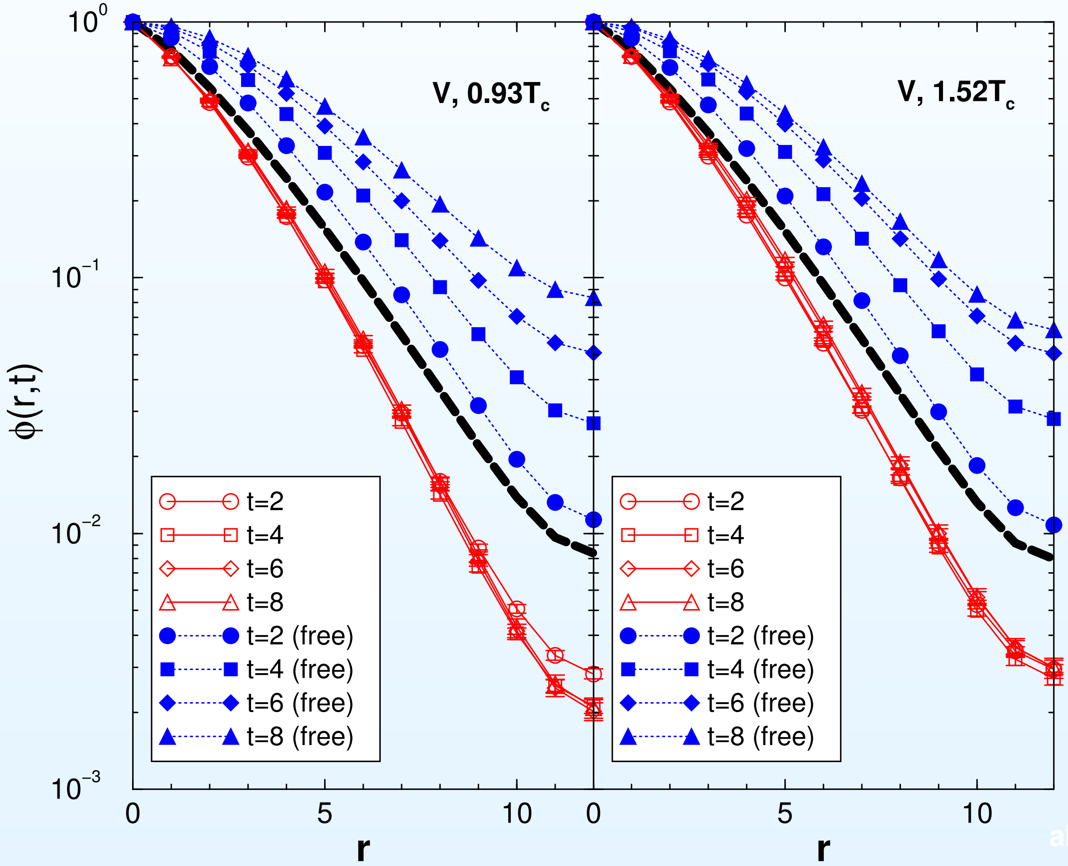


q-$\bar{q}$ correlation

q-$\bar{q}$ correlation (quenched, charmonium)

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001)2215

c-$\bar{c}$ strongly correlate even at $T \simeq 1.5 T_c$.



Similar tendency also found in light meson channels.

Spectral function

Temporal correlators in lattice simulations

$$C(t) = \sum_{\vec{x}} \langle O(\vec{x},t) O^\dagger(0) \rangle \qquad O(\vec{x},t)\text{: hadronic operator}$$

$\longrightarrow$ spectral function $A(\omega)$: inverse problem

$$C(t) = \int d\omega K(t,\omega) A(\omega), \qquad K(t,\omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{-N_t \omega}}$$

A.A.Abrikosov et al., Sov. Phys. J. 36 (1959) 635;
E.S.Fradkin, ibid. 36 (1959) 912.

In lattice QCD,

T.Hashimoto, A.Nakamura and I.-O.Stamatescu, Nucl. Phys. B400 (1993) 267, B406 (1993) 325.
QCD-TARO Collab., Nucl. Phys. B (PS) 63 (1998) 460
E.G.Klepfish, Creffield, Pike, Nucl. Phys. B (PS) 63 (1998) 460.

Analysis of temporal correlators

- Light mesons

  QCD-TARO Collab. Phys. Rev. D 63 (2001) 054501

  Wave function $\rightarrow$ bound state-like structure at $T \simeq 1.5T_c$

  chiral restoration above T_c

- Charmonium

  T.Umeda, R.Katayama, O.Miyamura and H.M., Int. J. Mod. Phys. A 16 (2001)2215

  Wave function $\rightarrow$ bound state-like structure at $T \simeq 1.5T_c$

- Glueballs

  N.Ishii, H.Suganuma and H.M., Phys. Rev. D 66 (2002) 014507, 66 (2002) 094506.

  30 % mass reduction below T_c (or broadening width)

  Peak with finite width above T_c

Our approach

Properties of mesonic modes through the spectral function.

- Apply as many procedures as possible.
  - Maximum entropy method (MEM)
  - Fit analysis assuming presumable forms($\leftarrow$ MEM) (constrained curve fitting)
- Semi-quantitative analysis.
  - evaluation of mass shift, width
- Smearing — enhancement of low energy part
  - $\leftarrow$ Problem of point correlators
  - $\rightarrow$ Uncertainty from smearing functions
- Anisotropic lattice: finer temporal lattice spacing a_τ than a_σ
  - Sufficient number of d.o.f. in temporal direction
  - Toward dynamical simulations

Maximum entropy method (MEM)

Reconstruction of spectral function from lattice data with MEM

Y.Nakahara, M.Asakawa and T.Hatsuda, Phys. Rev. D60 (1999) 091503

Maximize $Q = \alpha S - L$

$$S = \int d\omega \left\{ A(\omega) - m(\omega) - A(\omega) \ln \left[\frac{A(\omega)}{m(\omega)} \right] \right\}$$

- $m(\omega)$: model function ($m(\omega) = m_{DM}\omega^2$ is used)
- α — to be integrated out
- L: standard likelihood function

As functional basis, singular value decomposition is used (cf. *H.R.Fiebig, Phys. Rev. D 65 (2002) 094512.*)

MEM in lattice QCD at $T > 0$

MEM applied to lattice QCD at finite temperature

- Bielefeld group (light mesons, diquark, charmonium)

  F.Karsch et al., Phys. Lett. B530 (2002) 147;

  I.Wotzorke et al., Nucl. Phys. B (PS) 106 (2002) 513;

  S.Datta et al., hep-lat/0208012 (Lattice 2002), hep-lat/0312037

- Asakawa, Hatsuda and Nakahara (light mesons, charmonium)

  M.Asakawa et al., hep-lat/0208059 (Lattice 2002)

  M.Asakawa and Hatsuda, Phys. Rev. Lett. 92 (2004) 012001

- Our works (charmonium, light mesons)

  K.Nomura et al., hep-lat/0209139 (Lattice 2002)

  T.Umeda et al., hep-lat/0211003 (to appear in Eur. Phys. J. C)

  T.Umeda et al., hep-lat/0309178 (Lattice 2003)

- Ishii and Suganuma (glueballs)

  N.Ishii and H.Suganuma, hep-lat/0309102 (Lattice 2003)

Fit analysis

Fit analysis assuming presumable forms for spectral function

Constrained curve fitting

P. Lepage et al., Nucl. Phys. B (PS) 106 (2002) 12)

— simple modification of standard χ^2 fit
based on Bayes' theorem

$$\chi^2 \;\rightarrow\; \chi^2_{arg} = \chi^2 + \chi^2_{prior}, \qquad \chi^2_{prior} \equiv \sum_i \frac{(c_i - \tilde{c}_i)^2}{\tilde{\sigma}^2_{c_i}}$$

c_i: fit parameters, $\tilde{c}_i$, $\tilde{\sigma}_{c_i}$: input parameters as prior knowledge

- fitting with many parameters become stable
- bias from the input parameters (prior knowledge)
  $\rightarrow$ MEM results are suitable for prior knowledge

Fit analysis

Ansätze for the form of spectral function

- Breit-Wigner type

$$A(\omega) = \omega^2 \rho(\omega), \qquad \rho(\omega) = \frac{C\Gamma m}{(\omega^2 - m^2)^2 - \Gamma^2 m^2}$$

  C: overlap, m: mass, Γ: width

- Pole form

$$A(\omega) = C\delta(\omega - m)$$

We apply multi-Breit-Wigner fit using the constrained curve fitting.

If the width is enough small, pole-type fit is also applied.

Lattice setup

Anisotropic quenched lattices: $20^3 \times N_t$

$\xi = a_\sigma / a_\tau = 4$ (*T.R.Klassen, Nucl. Phys. B533 (1998) 557*)

$a_\sigma^{-1} = 2.030(13)$ GeV (set by hadronic radius r_0)

N_t	T/T_c	$N_{conf} \times N_{src}$	smearing
160	$\simeq 0$	500 $\times$ 16	point, smeared, (half smeared)
32	0.88	1000 $\times$ 16	smeared
30	0.93	1000 $\times$ 16	smeared
29	0.97	680 $\times$ 16	smeared
27	1.04	300 $\times$ 16	smeared
26	1.08	1000 $\times$ 16	smeared, half smeared
24	1.17	1000 $\times$ 16	smeared
20	1.40	1000 $\times$ 16	smeared, half smeared
16	1.75	1000 $\times$ 16	smeared, half smeared

Quark action

Quark action: $O(a)$ improved Wilson action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) &= \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ &\quad - \kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ &\quad - \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

$\kappa_\tau / \kappa_\sigma$ nonperturbatively tuned
$r = 1/\xi$, c_E and c_B: mean-field improved tree level

H. Matsufuru et al., Phys. Rev. D 64 (2001) 114503

Analysis at $T = 0$

Why $T = 0$ analysis ?

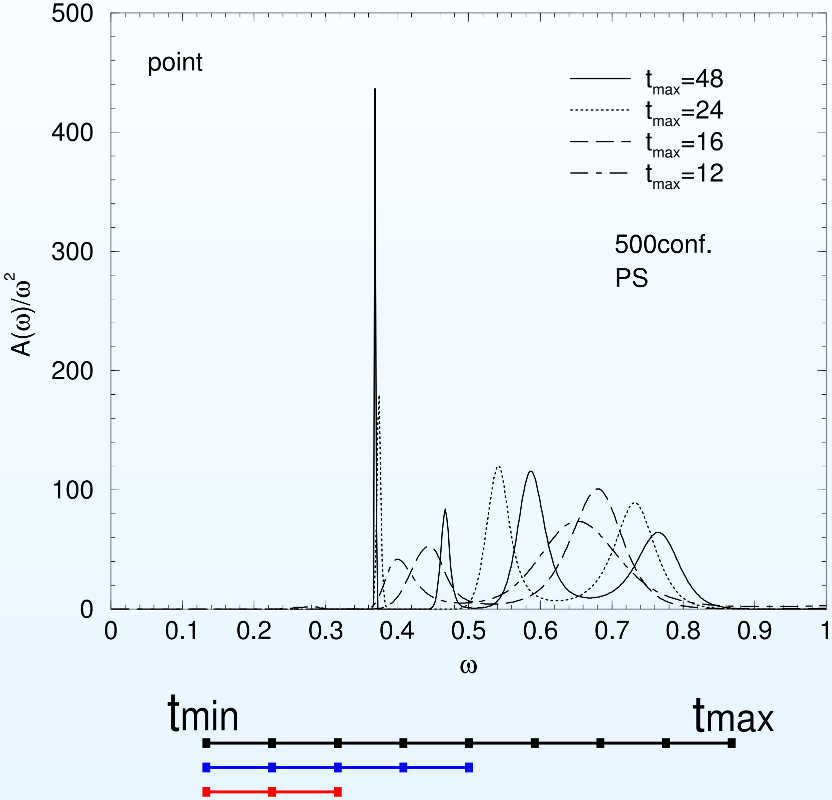
We know what must be obtained by applying analysis methods.

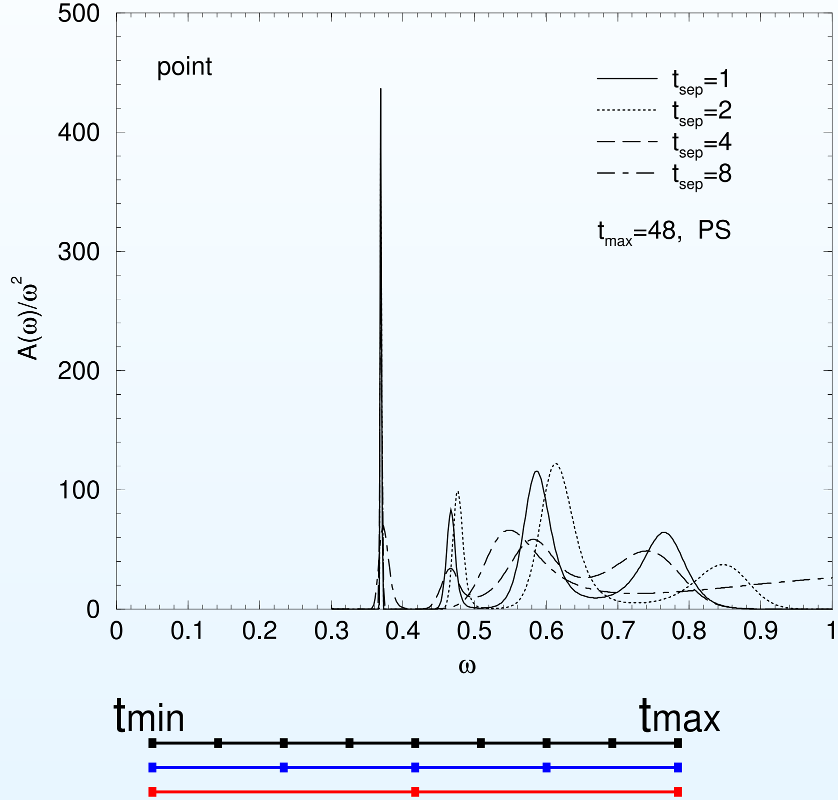
$\Rightarrow$

- reliability check of procedures
- numbers of data points required
- range of correlators required

Point correlators at $T = 0$

Restriction of data points used in MEM analysis





(N_{dof}=48,24,16,12) (N_{dof}=48,24,12,6)

- $O(10)$ d.o.f. is sufficient
- $N_t a_\tau \sim 1$ fm is necessary

Smearing

Point correlators a $T = 0$: $N_t a_\tau \sim 1$ fm is necessary
$\Rightarrow$ near T_c ($N_t a_\tau < 1$ fm), result is not reliable.

We focus on the low energy properties of spectral function.
$\rightarrow$ smearing technique

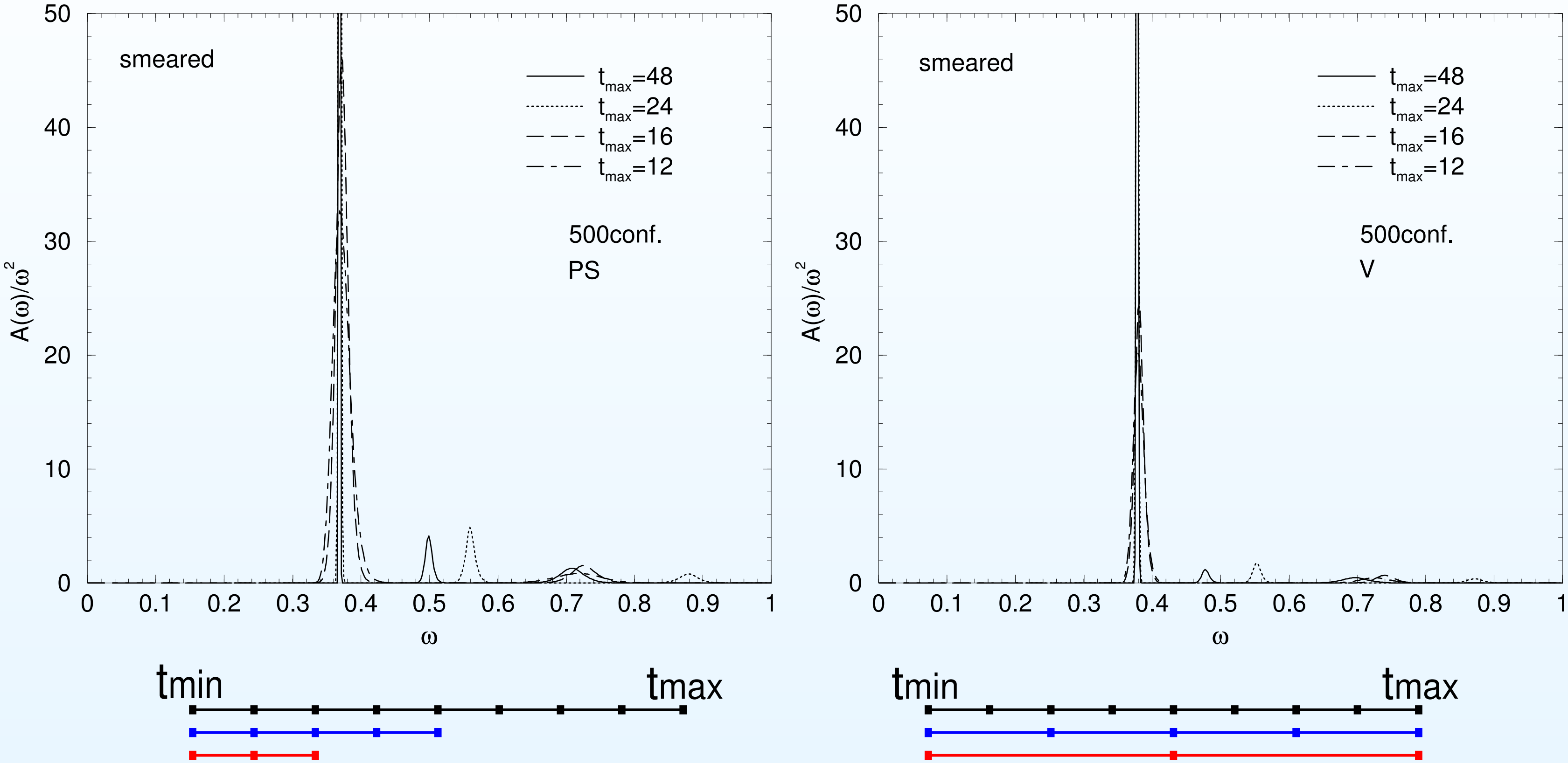
$$O(x,t) = \sum_{\vec{y}} \bar{q}(\vec{x}+\vec{y},t)\phi(\vec{y})q(\vec{x},t)$$

- in the Coulomb gauge
- $\phi(\vec{y})$: the wave function obtained at $T = 0$

The smearing changes the shape of spectral function.
$\rightarrow$ fictitious peak structure (even for free quarks)

— Comparison with two smearing functions;
smeared / half-smeared (with narrower function) correlators

Smeared correlators at $T = 0$



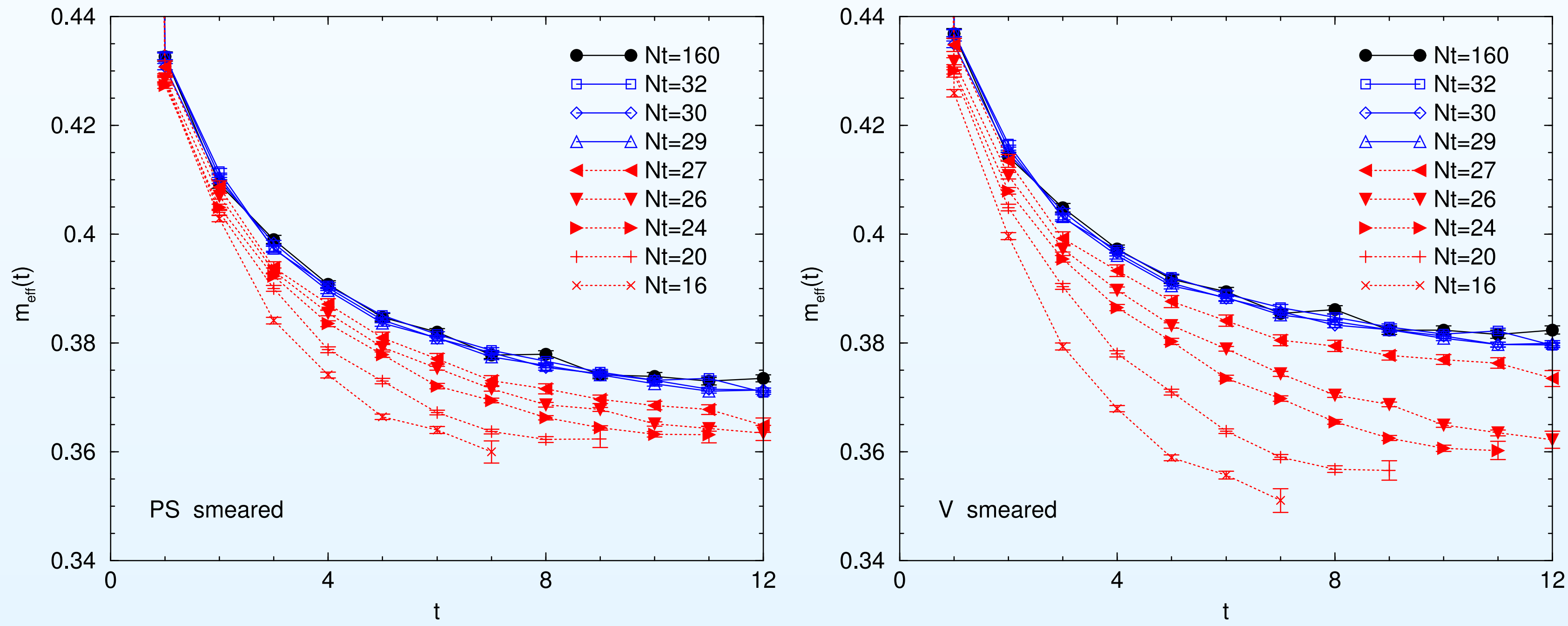
(N_{dof}=48,24,16,12) (N_{dof}=48,24,12,6)

- Ground state peak appears at correct position even for $t_{max} = 12$ ($\rightarrow N_t a_\tau \simeq 0.6$ fm, $T \simeq 1.17$)

At $T > 0$: effective mass plot

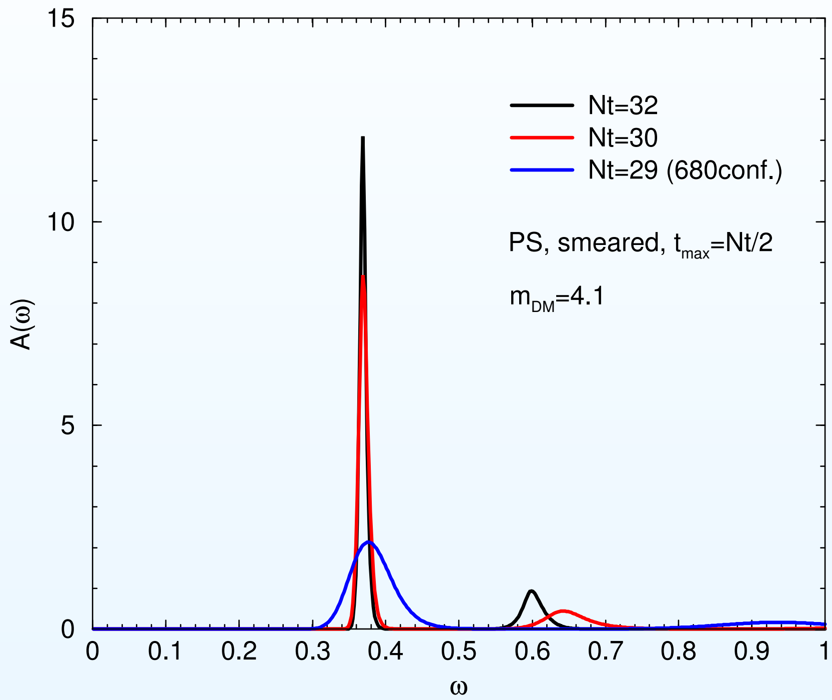
Effective mass plots for smeared correlators

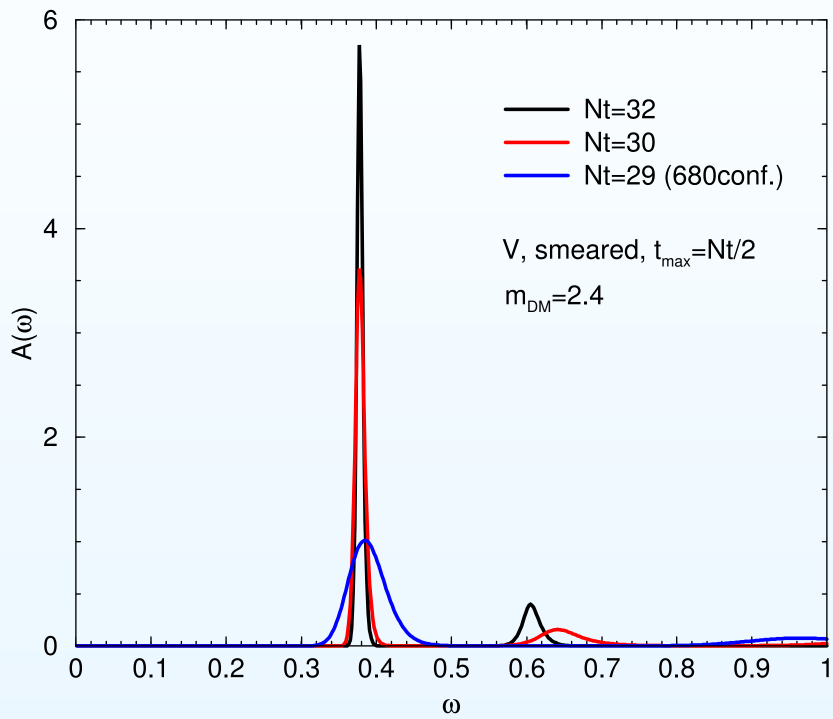
$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2 - t)]}{\cosh[m_{eff}(t)(N_t/2 - t - 1)]}$$



- Correlators below T_c are similar to those at $T = 0$
- Above T_c, correlators change as temperature

Results below T_c: MEM results

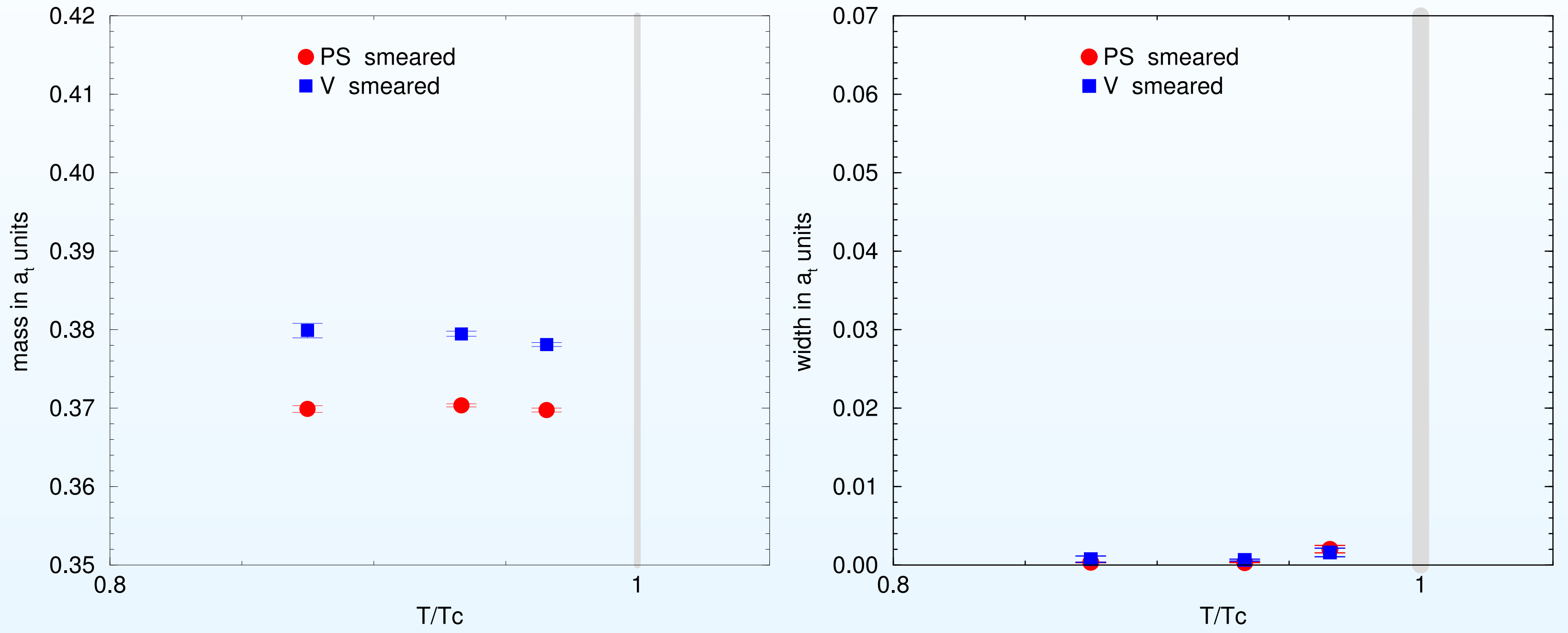




- Peak position is almost same as at $T = 0$
- Width ?

Results below T_c: Fit results

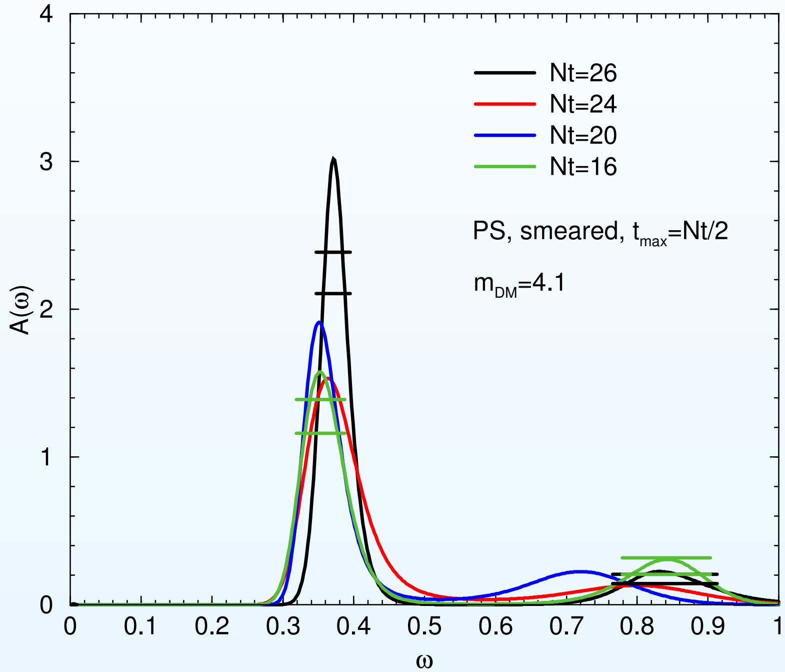
Fit results (constrained curve fitting).

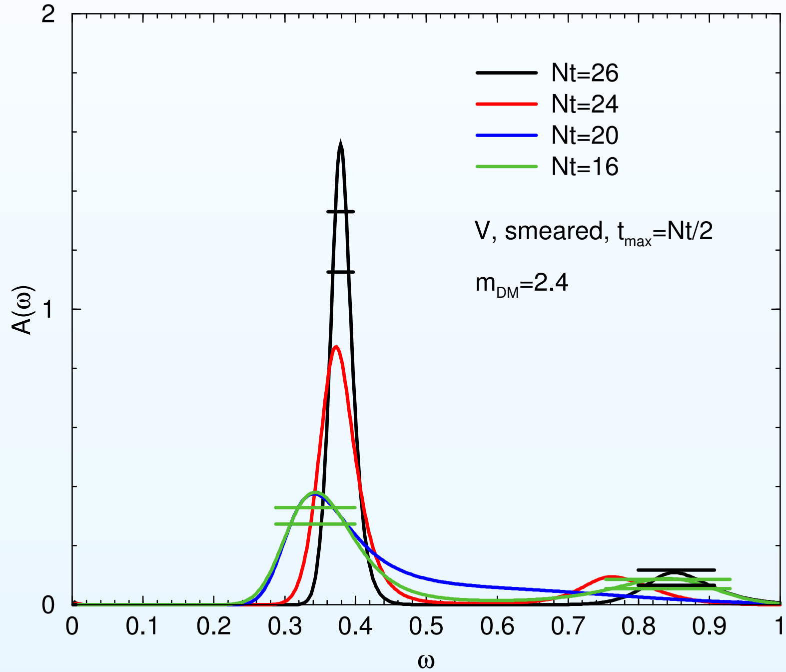


- No significant mass shift is observed.
- No sizable width is observed.

Results above T_c: MEM results $T \simeq$ 1.08–1.75

MEM results ($N_t \leq 20$ is less reliable).

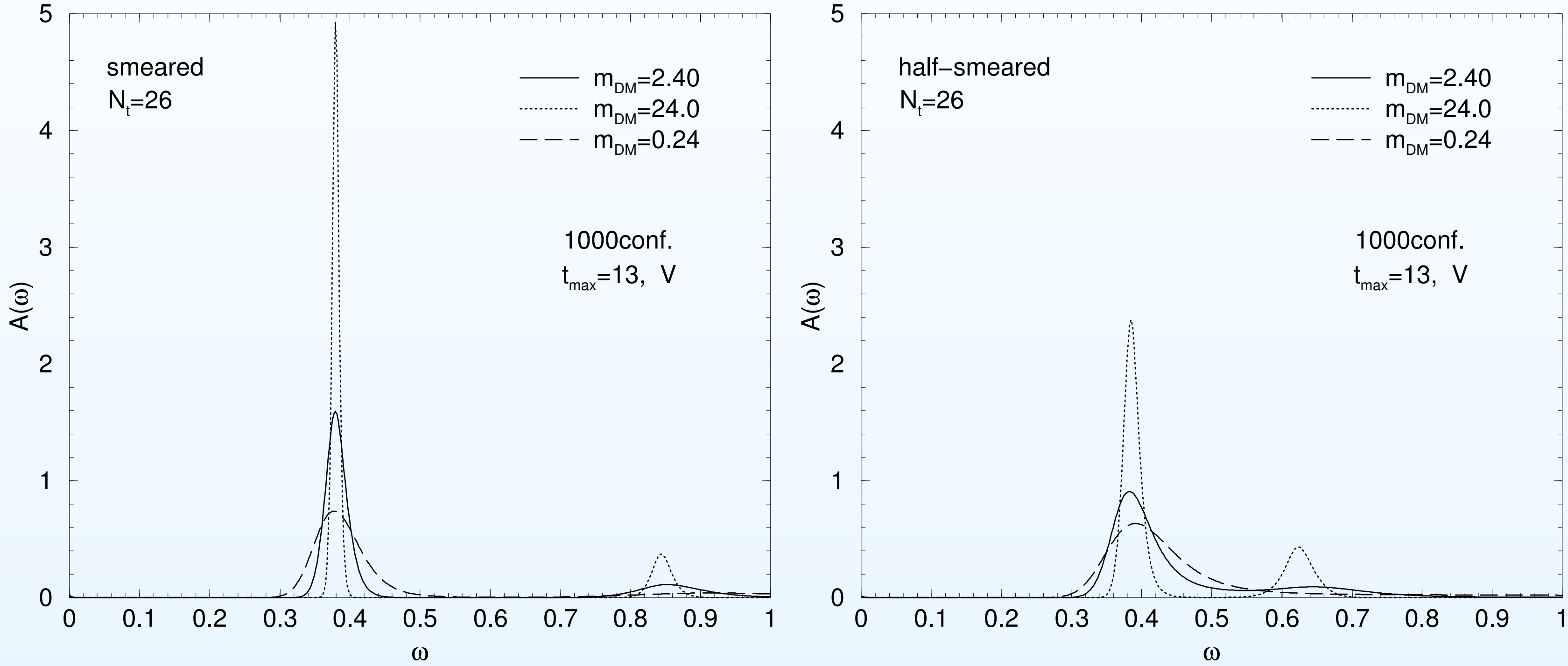




- Correlators change gradually as T increases
- Small or no shift of peak position
- Vector channel shows large change at high T

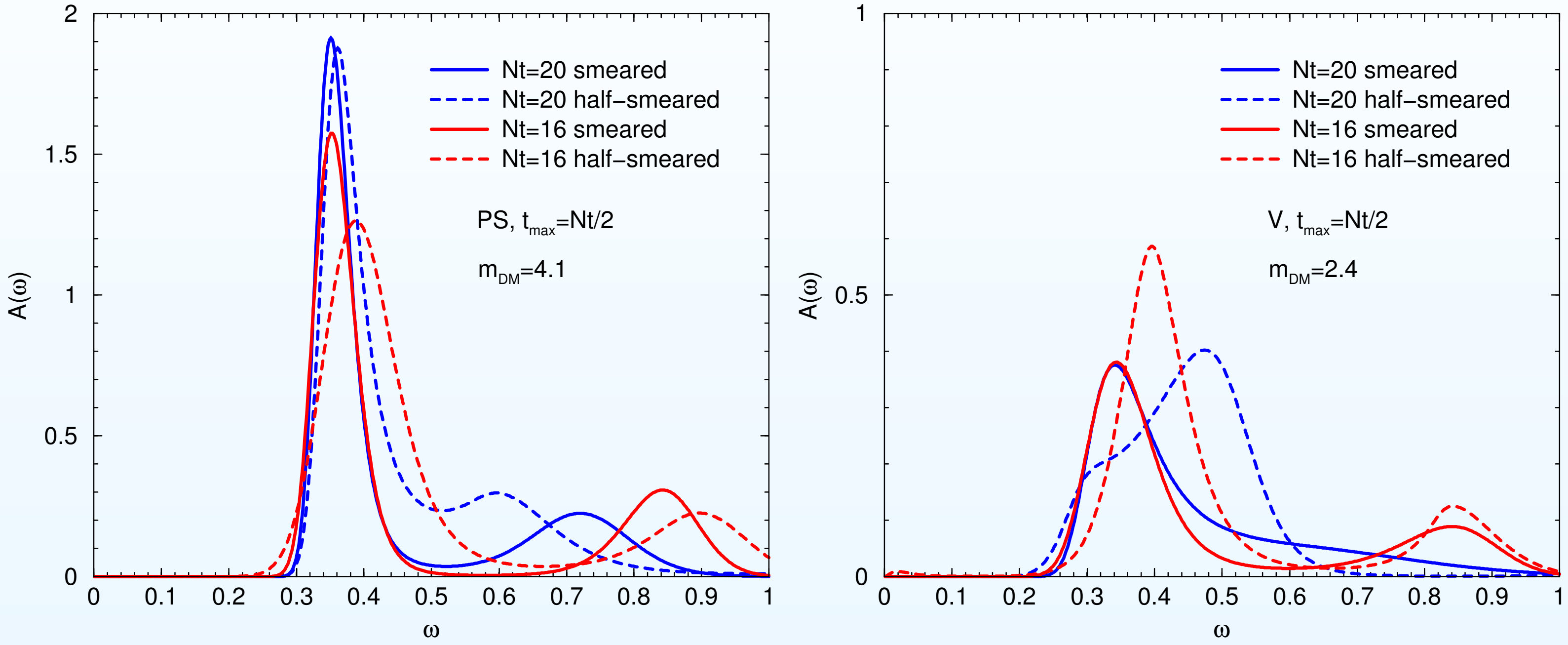
At $T > T_c$: Smearing/model function dependences

Smeared/half-smeared correlators at $N_t = 26$ ($T \simeq 1.08T_c$)



- Comparison with $m_{DM} \times$ 10, 0.1
  — peak position is stable, width much depends on m_{DM}.
- Smearing function dependence
  — peak position is stable, width depends on the smearing.

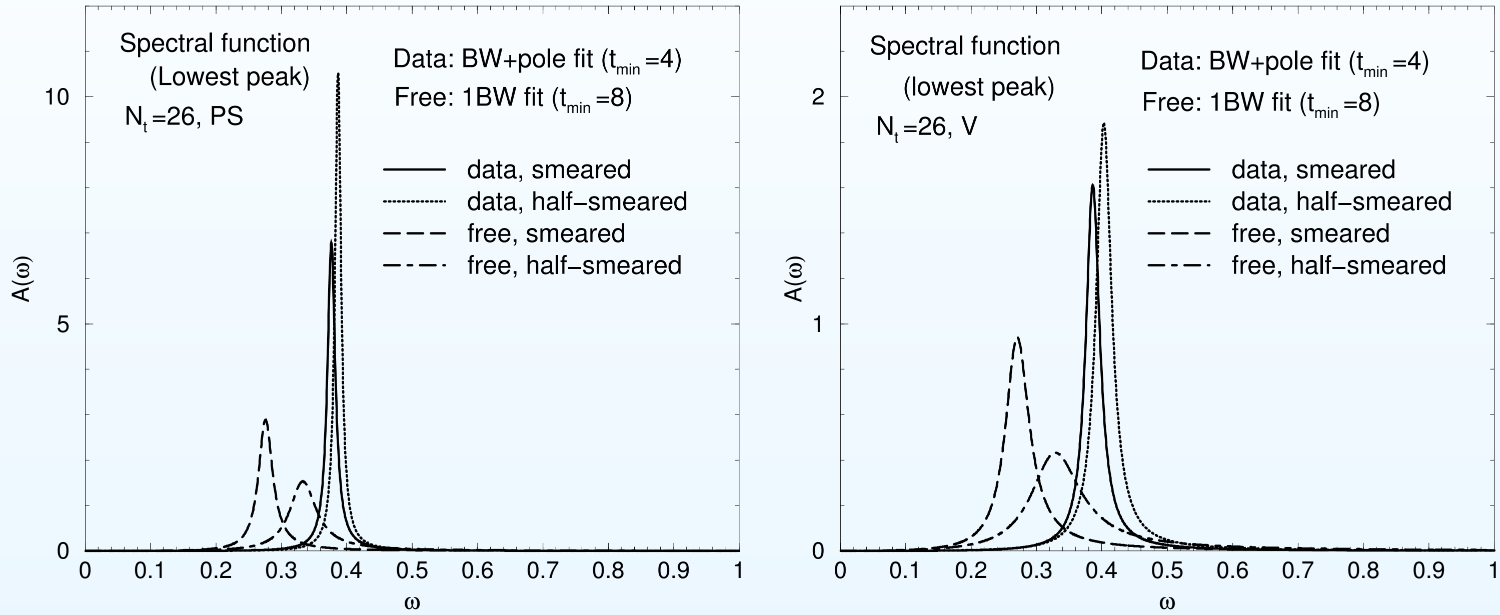
At $T > T_c$: smeared and half-smeared correlators



- Pseudoscalar channel: peaks of smeared and half-smeared correlators are similar
- In vector channel, differences are large

At $T > T_c$: Fit analysis at $T \simeq 1.08 T_c$

Result of standard χ^2 fit analysis with BW+pole form



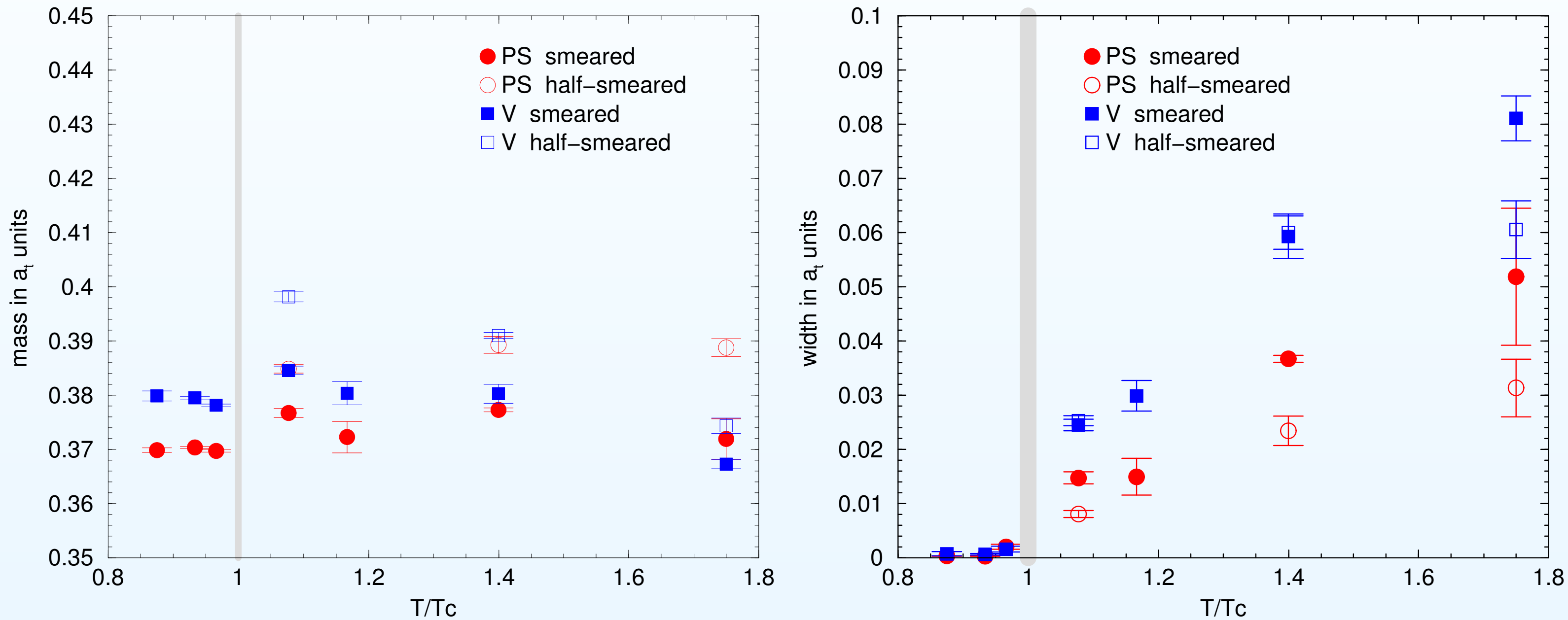
By changing smearing function,

- Free quark case:
  peak shift more than the widths / widths become broader
- Numerical result:
  peak does not shift beyond the widths / stable widths

$\Rightarrow$ We conclude peaks are physical (mesonic modes)

At $T > T_c$: Fit analysis

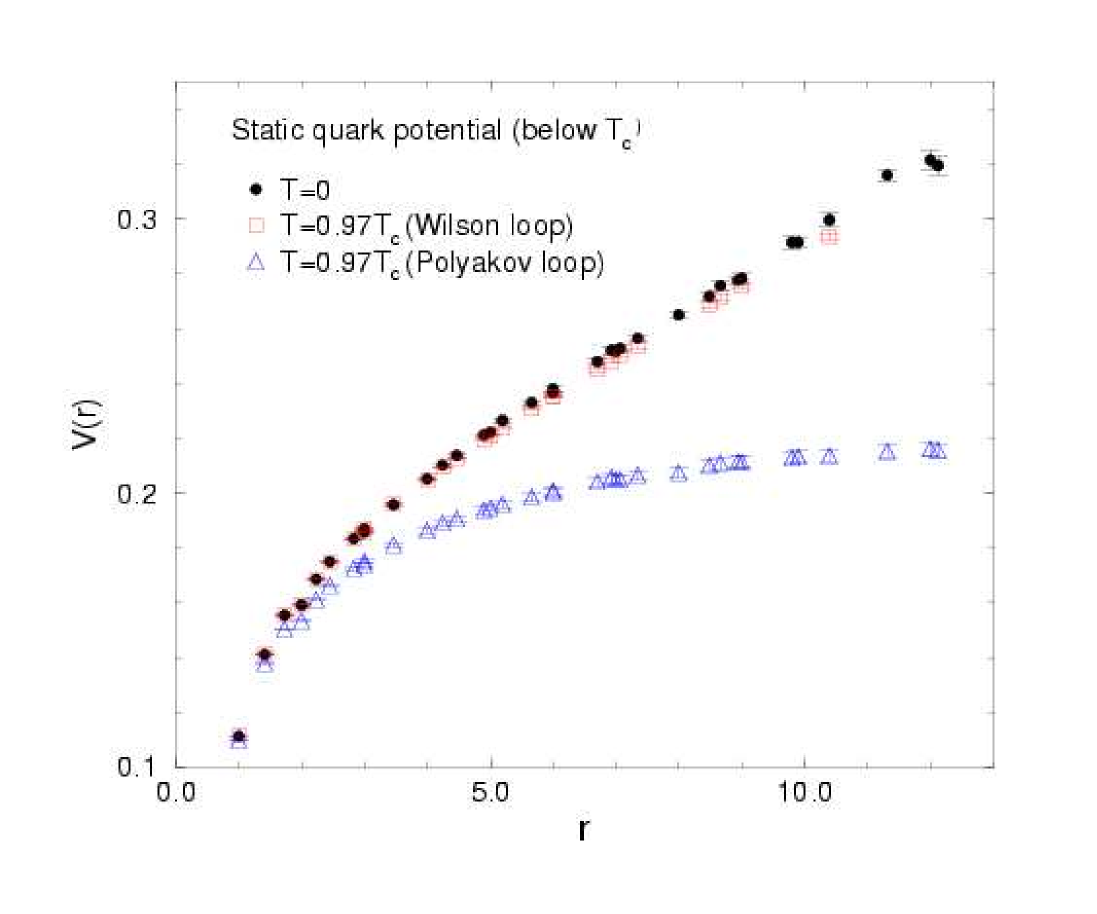
Result of constrained curve fitting (preliminary)



- Differences of masses for smeared and half-smeared correlators are less than widths
  $\longrightarrow$ size of systematic uncertainty
- For $T > 1.4T_c$, rather large ambiguity exists

Discussion: why no mass shift observed below T_c ?

Which static potential should be used in potential model ?
— from Polyakov loop or Wilson loop ?
Wilson loop: meson correlators of two static quarks separated by distance r $\rightarrow$ same analysis can be applied



Summary

We studied the charmonium correlators through the spectral functions.
We proposed an analysis combining MEM and fit analysis.

- Below T_c
  No significant mass shift is observed in PS and V channels
  No sizable width is observed
- Above T_c
  Peak structure up to $T \sim 1.2T_c$ with small mass shift
  (This feature seems to hold up to $T \sim 1.5T_c$)
  Finite width growing as temperature increases

Outlook

- Re-attack the point correlator ?
- Light meson channels
- Larger lattice size (larger cutoff and physical volume) for more quantitative analysis
- Dynamical simulation
  Anisotropic lattice required — development in progress