Light meson correlation functions near the deconfining transition on anisotropic lattices

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Contents

Introduction I

QCD at high density and high temperature

$\Longrightarrow$ Quark-Gluon Plasma phase

- deconfinement
- chiral symmetry restoration

Study of QGP

- NJL model   Hatsuda&Kunihiro(1994)
  soft mode : fluctuation of $\langle q\bar{q}\rangle \rightarrow \pi\ \rho$ mode
- DeTar's conjecture   DeTar(1985)
  confining feature for scale $\geq 1/g^2T$
- QCD sum rule   Hatsuda,Koike and Lee (1993)

fundamental degree of freedom

quark & gluon $\Longleftrightarrow$ hadron

$\Longrightarrow$ Non-perturbative treatment of QCD

Lattice QCD at $T > 0$

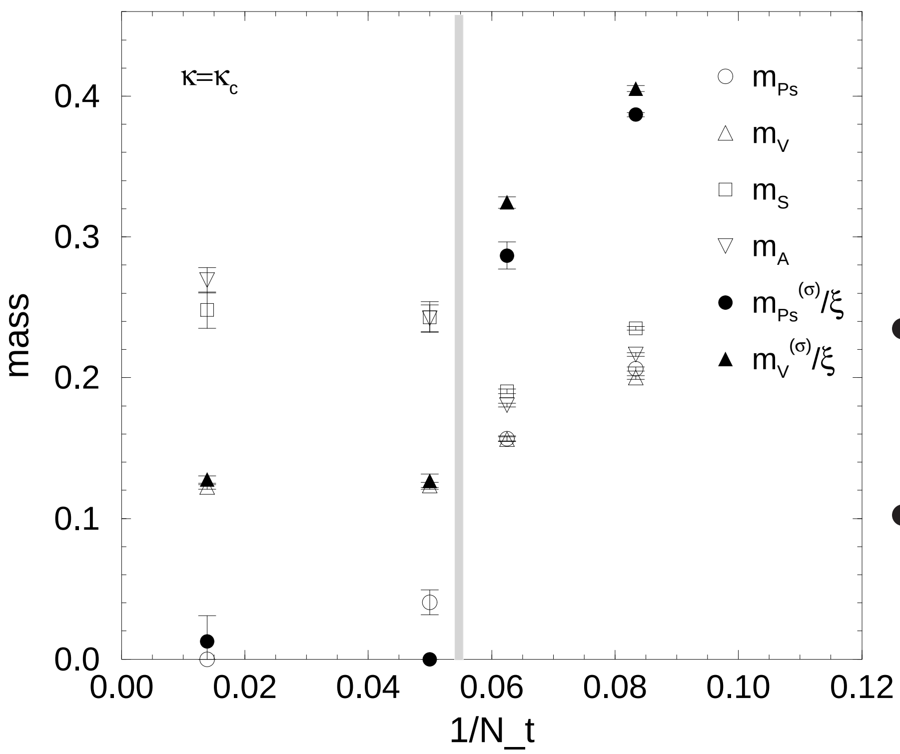
Introduction II

- Screening mass at $T > 0$

  DeTar and Kogut Phys. Rev. Lett. 59(1987)399
  Born et al., Phys. Rev. Lett. 67 (1991) 302
  S.Gupta Phys. Lett. B 288 (1992) 171

- Temporal meson correlator at $T > 0$

  QCD-TARO Phys. Rev. D63 (2001) 054501



assuming hadronic states at all T

- chiral symmetry restoration
- consistent with the NJL model

$\Longrightarrow$ Thermal effects for hadronic states below & above T_c ?

We study a spectral function of light mesons at $T > 0$

Our approach I

Temporal correlators in lattice simulations

$$C(t) = \sum_{\vec{x}} \langle O(\vec{x},t) O^\dagger(0) \rangle \qquad [O(\vec{x},t) : \text{meson op.}]$$

$\Longrightarrow$ Spectral function $A(\omega)$

Abrikosov et al. Sov. Phys. J. 36 (1959) 636

$$C(t) = \int d\omega K(t,\omega) A(\omega) \qquad K(t,\omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{-N_t \omega}}$$

- Anisotropic lattice
- Our analysis procedure

  We use the method applied in a study of charmonium

  Umeda et al.,hep-lat/0211003 $\Rightarrow$ 8 page.

  - Maximum entropy method (MEM) $\Rightarrow$ 6 page.

    — no assumption for the form of $A(\omega)$
  - Fit with ansatz for spectral function

    Constrained curve fitting $\Rightarrow$ 7 page.

    — need information on the form of $A(\omega)$

    — with given form $A(\omega)$, more quantitative
- smearing operator

  enhancement of low frequency modes

Our approach II

☐ Reliability check for an analysis at $T > 0$

(1) Check of the stability;

stability against changes of input parameters
or model functions

(2) Check with T=0 correlators;

stability against restriction of d.o.f. used in analysis

(We use the data at $T = 0.9T_c$ for the latter check,
but it should be done with the data at $T = 0$.)

☐ Validity of smeared correlators

possibility of an artificial peak

I.Wetzorke et al., Nucl.Phys.B(PS)106(2002)513

$\Longrightarrow$ check with various smearing functions

Maximum Entropy Method (MEM)

Reconstruction of a spectral function
with Maximum Entropy Method

Y. Nakahara, et al., Phys. Rev. D60 (1999) 091503

$$C(\tau) = \int d\omega K(\tau,\omega)A(\omega)$$

kernel :

$$K(t,\omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{-N_t \omega}}$$

Standard χ^2-fit $\longrightarrow$ ill-posed problem

$\Longrightarrow$ MEM (based on Bayes' theorem)

Maximization of $Q = \alpha S - L$

$$S = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right]$$

$m(\omega) = m_0 \omega^2$: model function

α : constant $\rightarrow$to be integrated out

L : Likelihood function (χ^2-term)

Constrained Curve Fit (CCF)

A simple modification of standard χ^2 fitting
based on the Bayesian statistics

G.Lepage et al.,Nucl. Phys.B(PS)106(2002)12

$$\chi^2 \to \chi^2_{arg} \equiv \chi^2 + \chi^2_{prior}$$

where

$$\chi^2_{prior} \equiv \sum_i \frac{(c_i - \tilde{c}_i)^2}{\tilde{\sigma}^2_{c_i}}$$

c_i : fit parameters,

$\tilde{c}_i$, $\tilde{\sigma}_{c_i}$: input parameters as prior knowledge

- many parameters fitting become stable
- bias from the input parameters

$\Longrightarrow$ MEM results are suitable for prior knowledge

Function form

We suppose the shape of spectral function.

(MEM gives rough estimate of shape of spectral function.)

◦ Breit-Wigner (BW) type :

$$\rho(\omega) = \frac{C\Gamma m}{(\omega^2 - m^2)^2 - \Gamma^2 m^2}$$

$$A(\omega) = \omega^2 \rho(\omega)$$

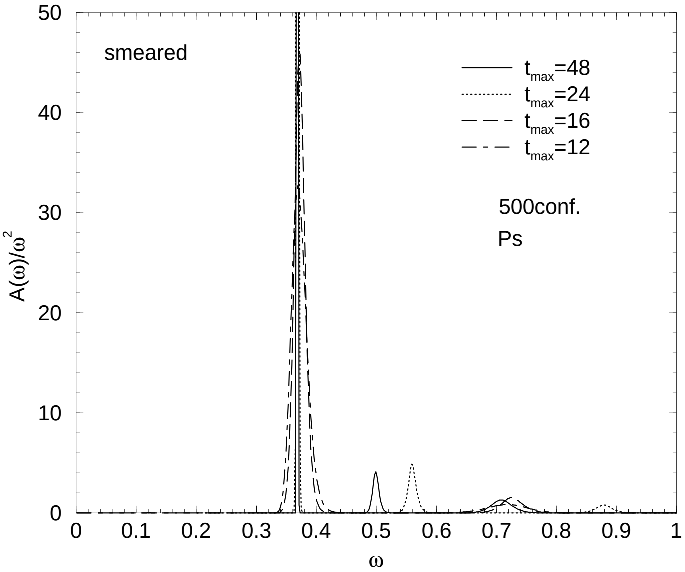
C : overlap, m : mass, Γ : width

$\Longrightarrow$ We apply multi-BW fits
and confirm number of terms (N_{term}) dependences.

Previous study for the charmonium

Smeared operators + MEM & Fit analysis

MEM at T=0 (No assumption)



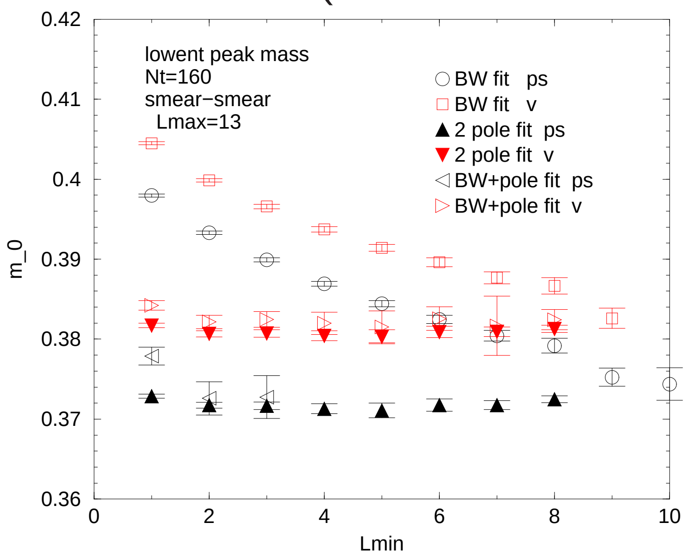
For the lowest peak

- peak position

  ⇒ consistent with that of point correlator
- width

  ⇒ sensitive to a model function

Fit at T=0 (based on MEM results)



For the lowest peak

- peak position is consistent with that of point correlator
- width

  consistent with zero

Our analysis procedure satisfies the stability conditions and are applied to $T > 0$ systems.

$\Longrightarrow$ $T \simeq 0.9T_c$: almost same results as T=0

$T \simeq 1.1T_c$: almost same peak position and finite width

Lattice setup

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_t \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_s \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_s c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_s c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215

J. Harada et al., Phys. Rev. D 64 (2001) 074501

- Constructed following the Fermilab approach.

  El-Khadra et al., Phys. Rev. D 55 (1997) 3933

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)

  (cf. another choice $r = 1$ was adopted in several works.)

- Tadpole improvement: $c_E = 1/u_s^2 u_t$, $c_B = 1/u_s^3$

  u_s, u_t: mean-field values of link variables

- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_t u_t}{\kappa_s u_s}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_s u_s} - 2(\gamma_F + 3r - 4) \quad (= 2(m_0 \gamma_F + 4))$$

  γ_F: bare anisotropy parameter

  m_0: bare quark mass (in temporal lattice units)

Simulation parameters

Gauge parameters:

— Anisotropic plaquette action in quenched approximation

□ $(\beta, \gamma_G) = (6.10, 3.2108)$

- $\xi = a_s/a_t = 4$

  Klassen, Nucl. Phys. B 533 (1998) 557

- $a_s^{-1} = 2.030(13)$ GeV

  Scale is set by hadronic radius r_0

□ Size : $20^3 \times N_t$

N_t	T/T_c	$N_{conf} \times N_{source}$	smearing function
160	~ 0	500×4	point
32	~ 0.9	1000×16	point, half-smeared,smeared
26	~ 1.1	1000×16	point, smeared

Quark parameters :

— $O(a)$ improved Wilson quark action

□ $(\kappa, \gamma_F) = (0.1235, 4.034)$

$\longrightarrow$ roughly correspond to strange quark mass

H. Matsufuru et al., Phys. Rev. D 64 (2001) 114503

ground state & 1st excited state mass m_0 & m_1 in a_t units

m_0(PS)$=$ 0.1160(3), m_1(PS)$\sim$ 0.27

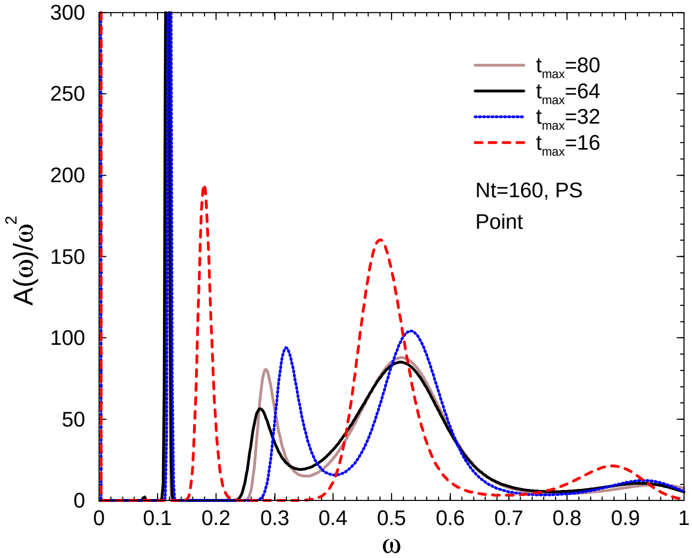
m_0(V) $=$ 0.1489(7), m_1(V) $\sim$ 0.29

Numerical results

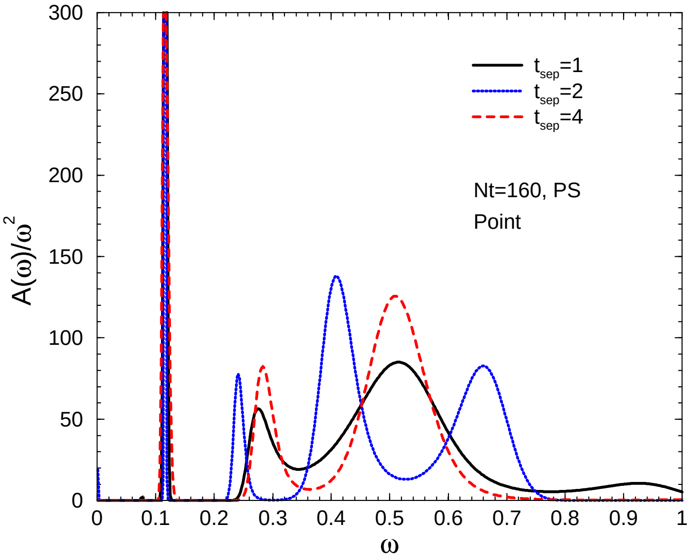
☐ Point-Point correlators

	t_{min}	t_{max}	t_{sep}	N_{DF}
Type-I	1	80	1	80
	1	64	1	64
	1	32	1	32
	1	16	1	16
Type-II	1	64	1	64
	2	64	2	32
	3	64	4	16

Type-I



Type-II



- $t_{max} \simeq 16$ is not acceptable
- lowest peaks are well reproduced at $t_{sep} = 4$ $(N_{DF} = 16)$
- $t_{max}a_t$ of the order of 1 fm is necessary

$\Longrightarrow$ This requirement can not be fulfilled at $T > 0$.

Smeared correlators

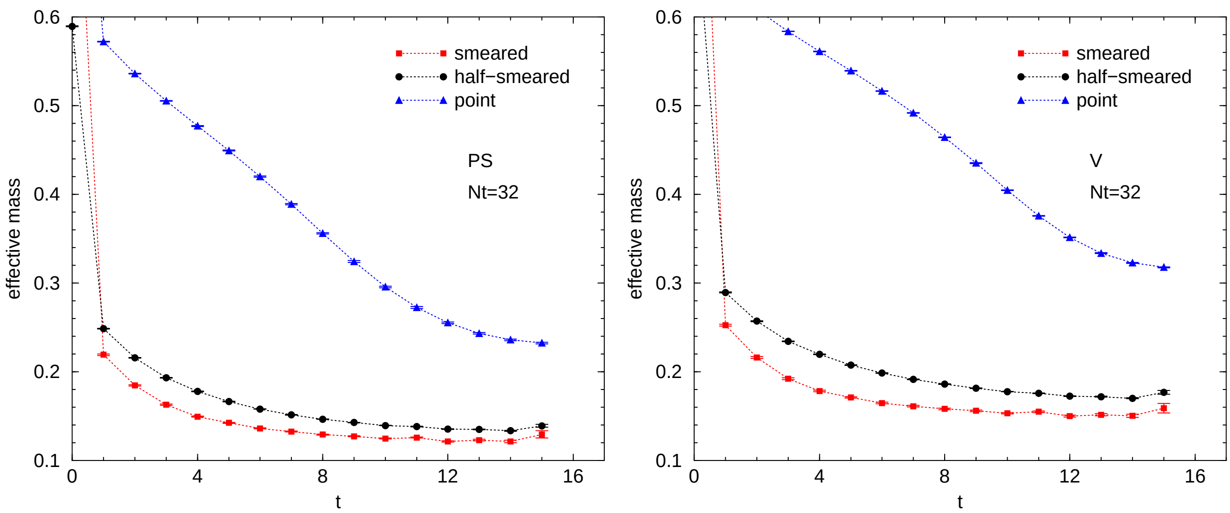
☐ Smeared-Smeared correlators

enhancement of low frequency modes

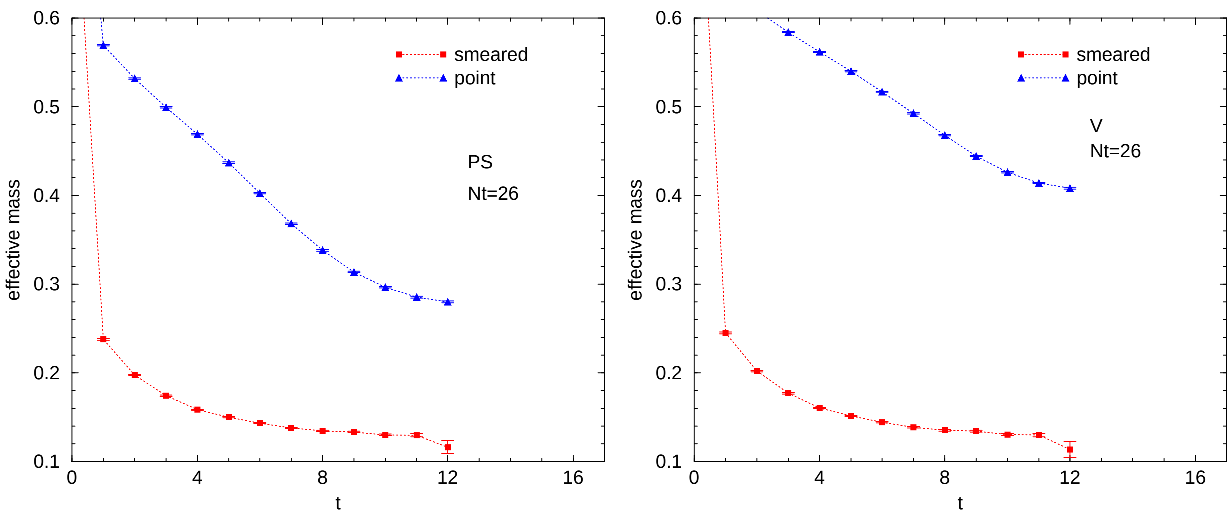
$\Longrightarrow$ efficient to extract the spectral function

☐ Effective mass plot of the correlators

Nt=32 ($T \simeq 0.9T_c$)

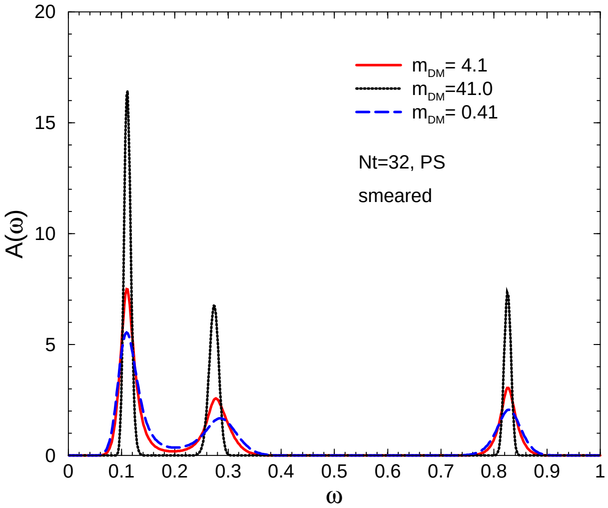


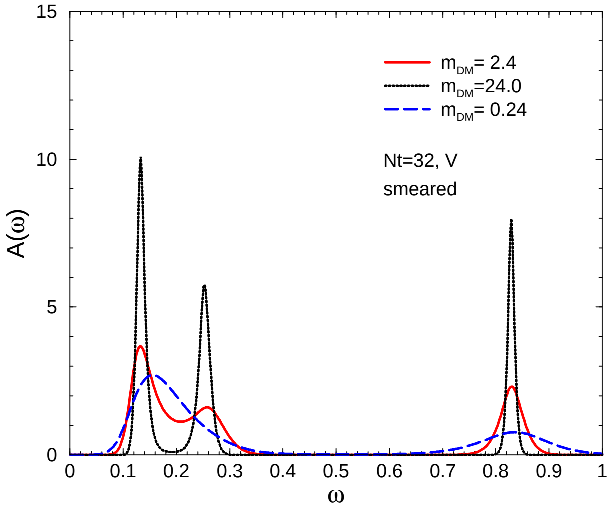
Nt=26 ($T \simeq 1.1T_c$)



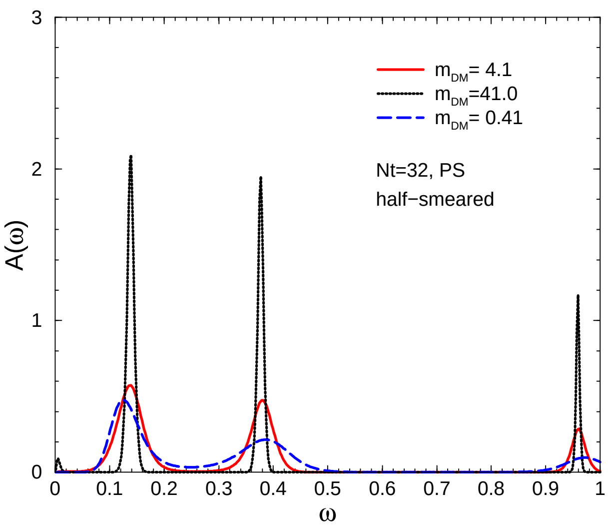
MEM analysis at $T = 0.9T_c$

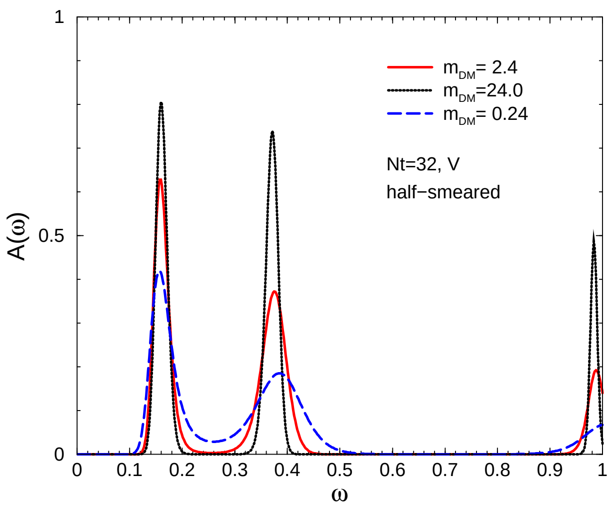
Nt=32 $(T \simeq 0.9T_c)$: smeared





Nt=32 $(T \simeq 0.9T_c)$: half-smeared



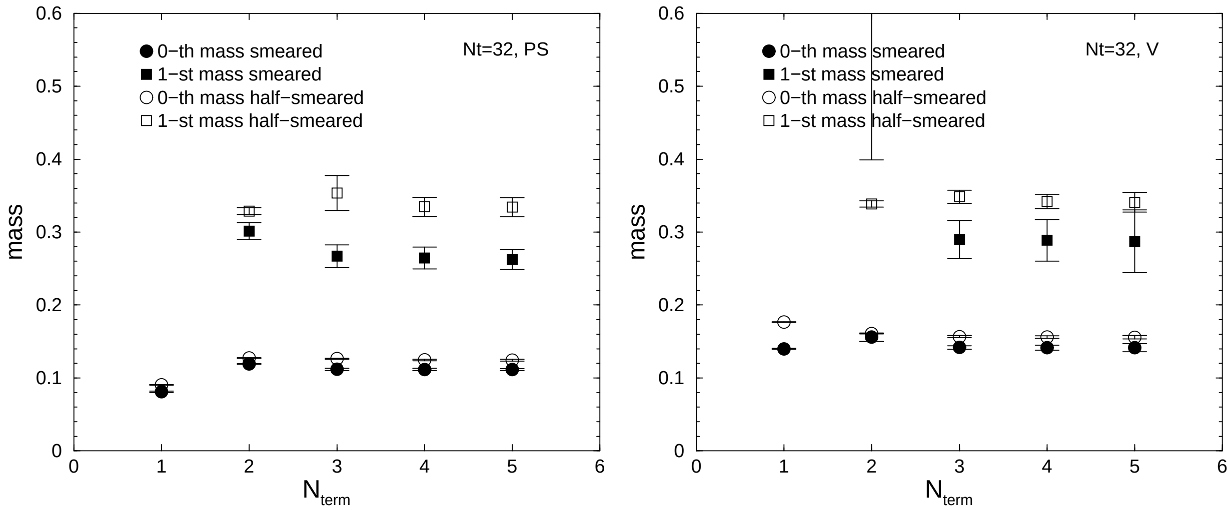


- ☐ most cases indicate pole-like structure
- ☐ lower masses are consistent with T=0 in smeared cases
- ■ masses are slightly different from T=0 in half-smeared case
- ■ width significantly depends on the model function

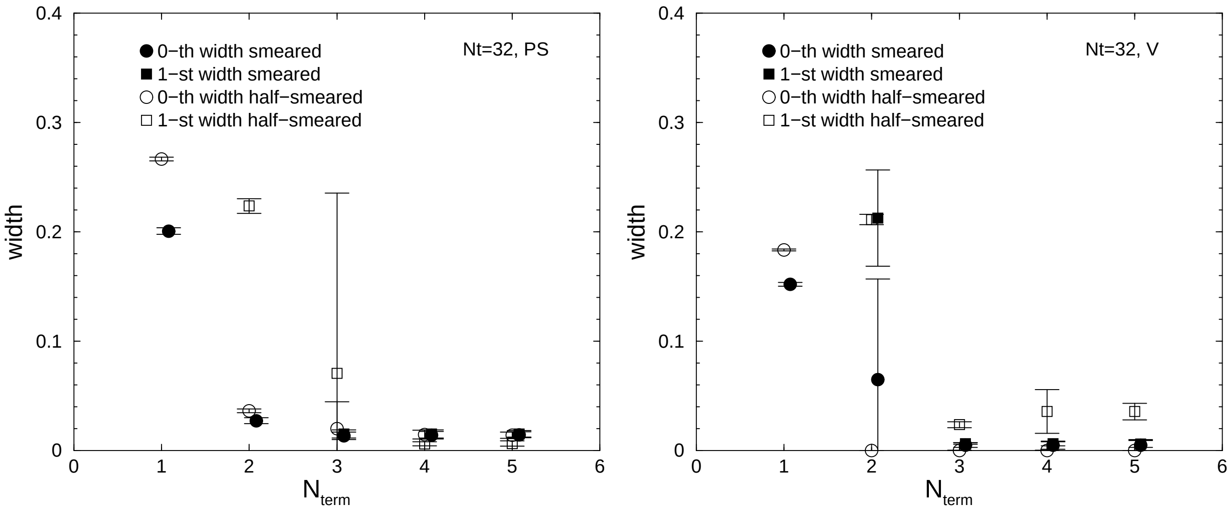
Fit analysis at $T = 0.9T_c$

☐ input parameters are chosen based on the results of MEM

Nt=32 ($T \simeq 0.9T_c$) : ground state & 1st excited state mass



Nt=32 ($T \simeq 0.9T_c$) : ground state & 1st excited state width



☐ fit parameters become stable as N_{term} increases

■ differences between smeared and half-smeared are beyond statistical error

■ result is largely dependent on the input parameters
$\Longrightarrow$ systematic error is larger than statistical one

Discussion

☐ Reliablility check of our procedure

(1) Check of the stability;

— stability against changes of input parameters or medel functions —

MEM and CCF results largely depends on input parameters or model functions.

- MEM can work only for rough estimation of SFs

- Systematic error of CCF results is larger than statistical error

⇒ The results of smeared and half-smeared may be consistent.

(2) Check with T=0 correlators;

—stability against restriction of d.o.f. used in analysis—

(Here we suppose the system at $T = 0.9T_c$ is similar to that at T=0)

Results with smeared operators are consistent with the results at T=0.

⇒ need more checks at T=0

Summary

- ☐ Light quark systems are analyzed with the method used in the study of charmonium.

  (Smeared operator + MEM&Fit analysis)

- ☐ We discuss the reliability conditions as a necessary condition of analysis method.

- ☐ We apply the reliability check to our analysis procedure.

Outlook

- ☐ reliability check with smeared correlators at T=0
- ☐ improvement of smearing functions
- ☐ application to systems at $T >_c$
- ☐ calculations with dynamical quarks