Anisotropic lattices for precision computations in heavy flavor physics

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Contents:

Refs.:

J. Harada et al., Phys. Rev. D 64 (2001) 074501
H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D 64 (2001) 114503
J. Harada, H. Matsufuru, T. Onogi and A. Sugita, Phys. Rev. D 66 (2002) 014509
H. Matsufuru, J. Harada, T. Onogi and A. Sugita, hep-lat/0209090

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Introduction

Recent experimental developments in flavor physics
— B factories, Charm factory, ...
⇒ need precise theoretical predictions ($\simeq 2\%$)
of hadron matrix elements

Problem in lattice QCD calculations:
for heavy quarks (c and b), large $O(am_Q)$ error

We need a framework which has
(i) continuum limit
(ii) systematic improvement
such as Nonperturbative renormalization technique (for $m_q \simeq 0$)
(iii) modest size of computation

Previous approaches (∼10% systematic accuracy):
a). Effective theories (NRQCD, etc) — (i)×
Thacker and Lepage, Phys. Rev. D 43 (1991) 196.
b). Relativistic framework — (iii)×
c). Fermilab approach (with/without mass dependent tuning)
El-Khadra et al., Phys. Rev. D 55 (1997) 3933.
Sroczynski, et al., Nucl. Phys. B (PS) 83 (2000) 971.
— (ii)×

To achieve calculations with ∼2% precision, we need
yet another approach which satisfies above condition (i)–(iii).
⇒ our proposal: *Anisotropic lattice*

Anisotropic lattice QCD (1)

Anisotropic lattice: $a_\tau < a_\sigma$ $\Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Quark action: $O(a)$ improved, along with Fermilab approach

□ bare anisotropy parameter γ_F:

in general, to be tuned mass dependently

e.g., using meson dispersion relation

◦ Continuum limit ☺

◦ Modest computational cost ☺

◦ Systematic improvement ?

If quark mass is sufficiently less than a^{-1},
tuned parameters for massless quark are applicable.

$\rightarrow$nonperturbative renormalization technique

Lüscher et al., Nucl.Phys. B 491 (1997) 323.

Our expectation:

For $m_Q \ll a_\tau^{-1}$, (not necessarily $m_Q \ll a_\sigma^{-1}$),
mass dependences of parameters in the action are so small that the tuned parameters for massless quark are also applicable in such a quark mass region.

Then, systematic improvement is possible. ☺

(performed at $m_q \simeq 0$)

$\Rightarrow$ To be justified numerically, and in perturbation theory.

□ 1st stage: $O(a^0)$ calibration $\rightarrow$ physics results

□ 2nd stage: high precision calibration

Anisotropic lattice QCD (2)

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215
Harada et al., Phys. Rev. D 64 (2001) 074501

- Constructed following the Fermilab approach.
  El-Khadra et al., Phys. Rev. D 55 (1997) 3933
- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted in several works.)

Parameters in tadpole improved tree level:
$\rightarrow$ applicable to $O(10\%)$ simulations

- Clover coefficients: $c_E = 1/u_\sigma^2 u_\tau$, $c_B = 1/u_{\sigma^3}$
  u_σ, u_τ: mean-field values of spatial and temporal link variables
- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0 \gamma_F + 4))$$

γ_F: bare anisotropy parameter
m_0: bare quark mass (in units of a_τ^{-1})

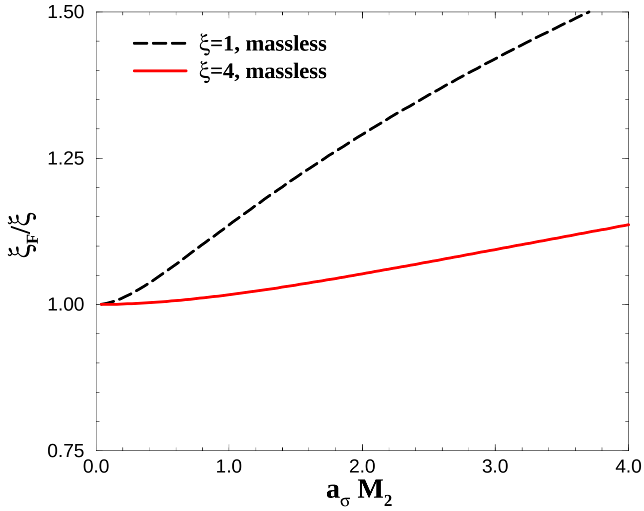
For a few percent level calculation,
parameters γ_F, c_E, c_B should be tuned nonperturbatively

Anisotropic lattice QCD (3)

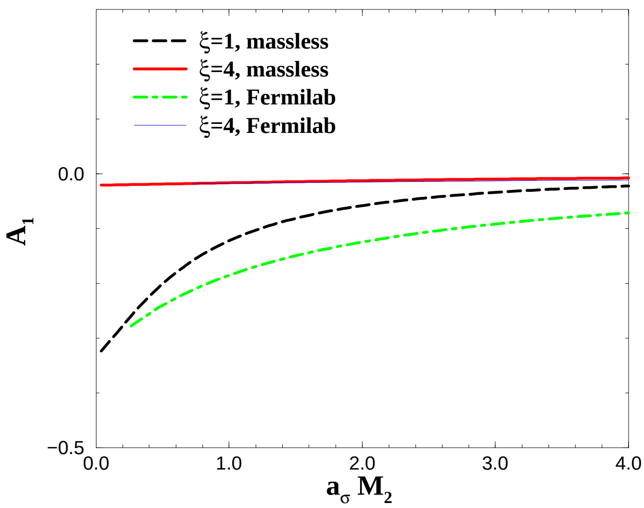
Tree level analysis of quark dispersion relation (γ_F set to ξ)

$$E^2 = M_1^2 + \left(\frac{\xi^{\rm tree}}{\xi_F^{\rm tree}}\right)^2 \vec{p}^2 + A_1 a_\sigma^2 (\vec{p}^2)^2 + A_2 a_\sigma^2 \sum_i p_i^4 + \cdots,$$

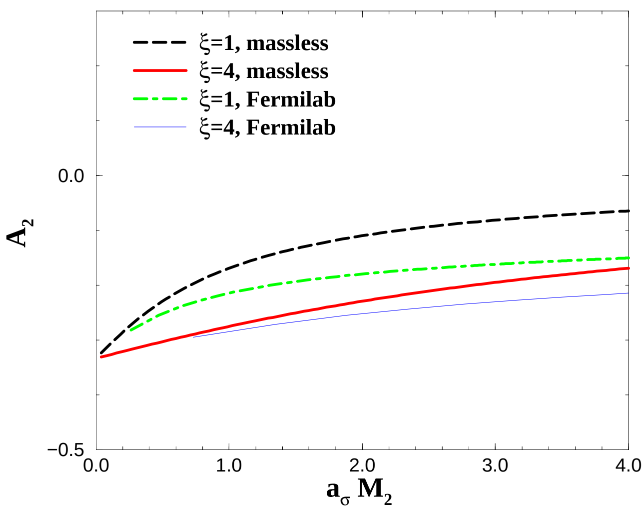
- ξ_F/ξ : reduced by a factor of anisotropy (M_2: kinetic mass)



- A_1: reduced by a factor of anisotropy



- A_2: not reduced, same size as massless quark — severe in heavy quarkonium ($p \sim \alpha m_Q$)



Anisotropic lattice QCD (4)

Conjecture of a_σ dependence of tuned anisotropy parameter
(figures roughly at charm quark mass)

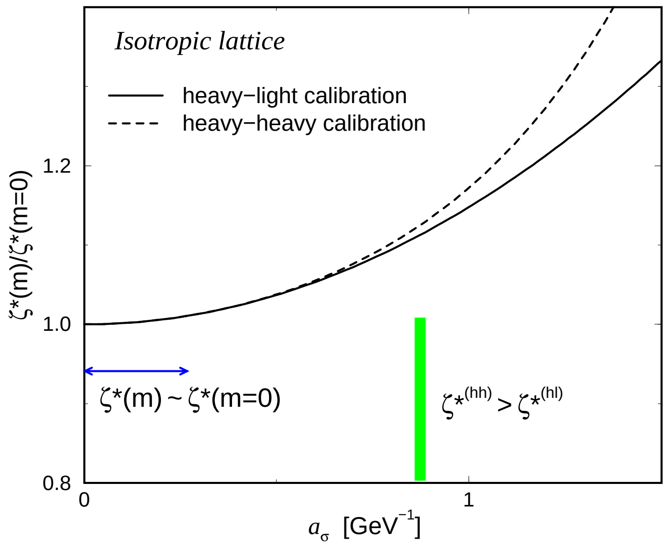
∘ $a_\sigma^{hh\text{-}hl}$: above which heavy-heavy and heavy-light systems are inconsistent for single γ_F value.
$\leftarrow O((a_\sigma p)^2)$ error in quarkonia, not improved by anisotropy

∘ $a_\sigma^{hl\text{-}ll}$: below which $\gamma_F(m_q) \simeq \gamma_F(m_q = 0)$ holds, within certain (say, 2%) accuracy.

Below $a_\sigma^{hl\text{-}ll}$, $\gamma_F(m_q = 0)$ correctly describes heavy-light systems.

- Isotropic lattice

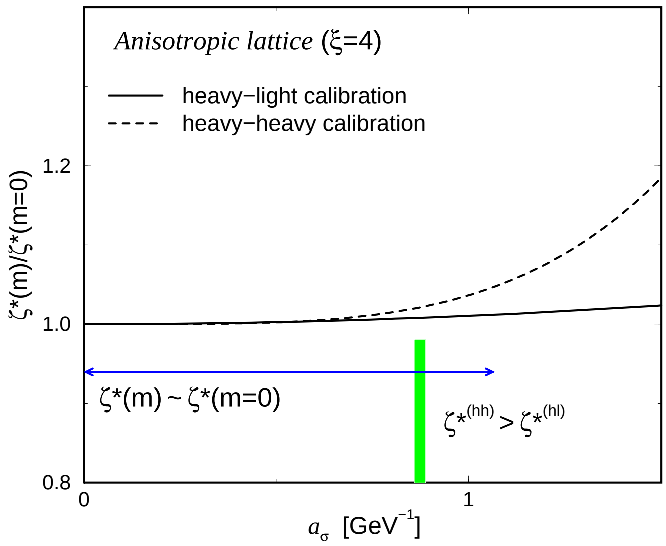
  $a_\sigma^{hl\text{-}ll} < a_\sigma^{hh\text{-}hl}$



- Anisotropic lattice ($\xi = 4$)

  $a_\sigma^{hl\text{-}ll} \gtrsim a_\sigma^{hh\text{-}hl}$

  $\Rightarrow a_\sigma^{hl\text{-}ll}$ is extended by a factor ξ.



1st stage results (1): summary

We have obtained the following results.

□ One-loop perturbative calculation:

Harada et al., Phys. Rev. D 64 (2001) 074501

Renormalization factors of heavy-light bilinears and quark rest mass at $m_Q a_\sigma \sim 1$, $m_Q \ll a_\tau^{-1}$

$\Rightarrow$ Well approximated with linear form in $m_Q a_\tau$

— quark mass dependence can be controlled

□ Numerical simulation:

In quenched approximation, tadpole improved c_E and c_B

- Mass dependent tuning

Matsufuru, Onogi and Umeda, Phys. Rev. D 64 (2001) 114503

γ_F is tuned with meson dispersion relation
Quark mass dependence is small for $m_q a_\tau \ll 1$

- Test of relativity relation

Harada et al., Phys. Rev. D 66 (2002) 014509

Heavy-light meson dispersion relation for $\gamma_F = \gamma_F(m_q = 0)$
Relativity relation well holds for $m_q a_\tau \ll 1$ (while $m_q a_\sigma \gtrsim 1$)

- Application to decay constant

Matsufuru, Harada, Onogi and Sugita, hep-lat/0209090

Around charm quark mass, with $O(10\%)$ accuracy
Result consistent with previous works

— Encouraging results for further development

1st stage results (2): mass dependent tuning

Mass dependent calibration of γ_F
using meson dispersion relation

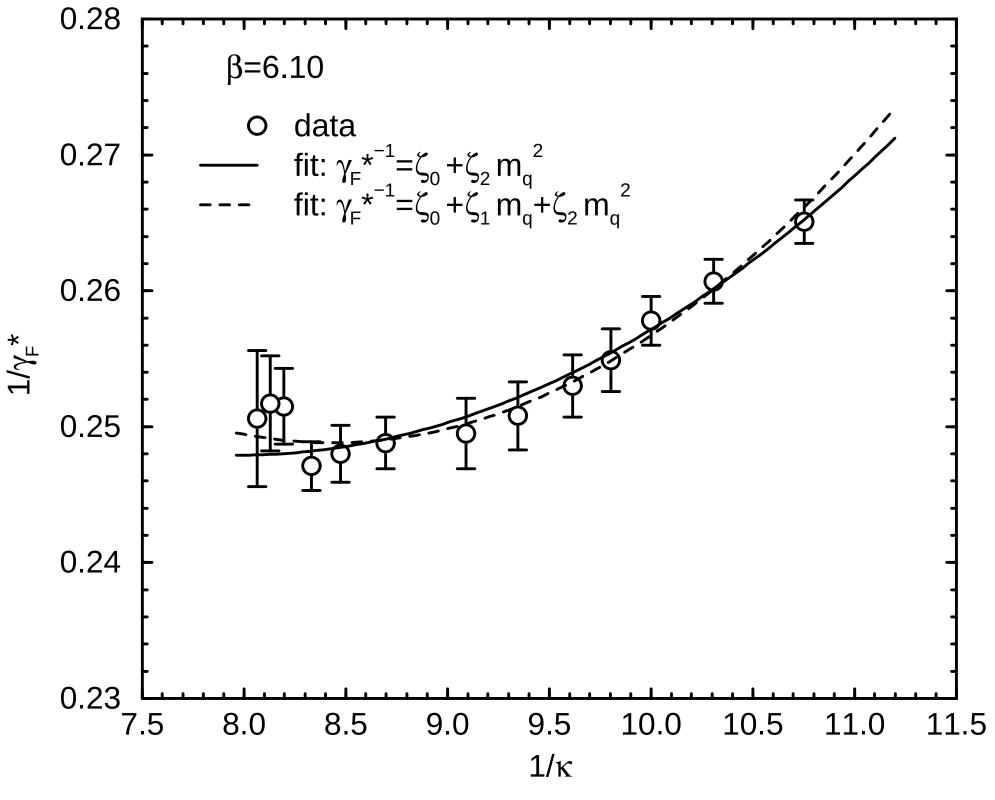
Simulation: quenched lattices with $\xi = 4$

β	γ_G	size	$a_\sigma^{-1}(r_0)$ [GeV]
5.75	3.072	$12^3 \times 96$	1.100(6)
5.95	3.1586	$16^3 \times 128$	1.623(9)
6.10	3.2108	$20^3 \times 160$	2.030(13)

Calibration result: γ_F^*
— well fitted to linear form in m_q^2:

$$\frac{1}{\gamma_F^*} = \zeta_0 + \zeta_1 m_q + \zeta_2 m_q^2 \qquad m_q = \frac{1}{2\xi}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

$\Rightarrow$ For $m_q a_\tau \ll 1$, γ_F^* is well approximated with $\gamma_F^*(m_q\!=\!0)$
$\gamma_F^*(m_q = 0)$ is determined with $O(2\%)$ accuracy



1st stage results (3): Relativity relation

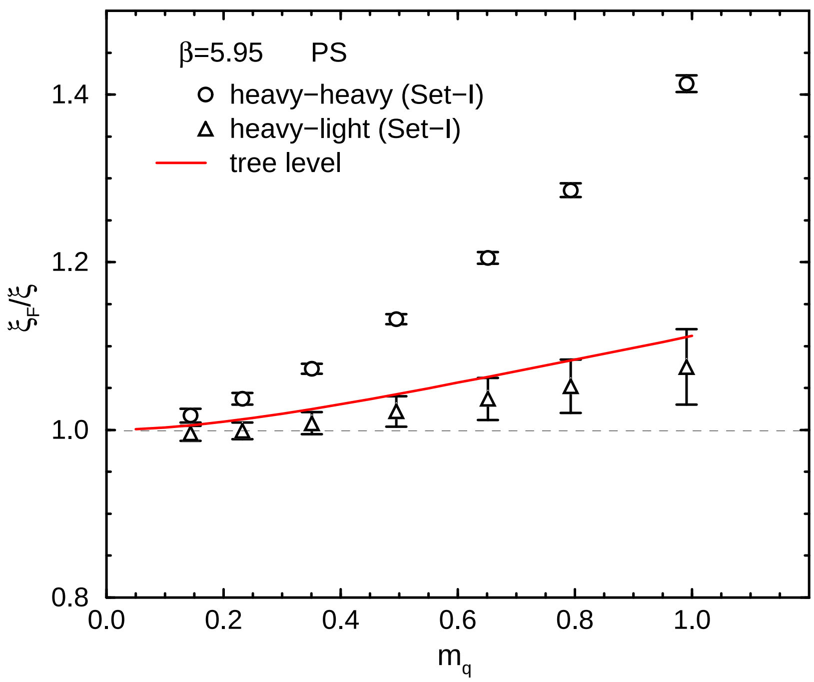
Simulation: on quenched anisotropic lattice of
size $16^3 \times 128$, $a_\sigma^{-1} \simeq$ 1.6 GeV and anisotropy $\xi = 4$.
Light quark: mass $\sim 1.5\, m_s$
Heavy quark: 7 values with $m_Q = 1$ – 6 GeV
Bare anisotropy: tuned value at massless limit ($\gamma_F = 4.016$)

Heavy-heavy, heavy-light meson dispersion relations

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$$

$\longrightarrow$ fermionic anisotropy ξ_F

$\xi_F \neq \xi$ signals breaking of relativity relation



- $m_q < 0.3$: for heavy-light mesons, γ_F tuned for massless quark can be applied within 2% accuracy
- $m_q < 0.2$: both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy
- $0.2 < m_q$: heavy quarkonia suffer from larger $O((ap)^2)$ error

1st stage results (4): Heavy-light decay constant

Simulation: quenched anisotropic lattices with $\xi = 4$,

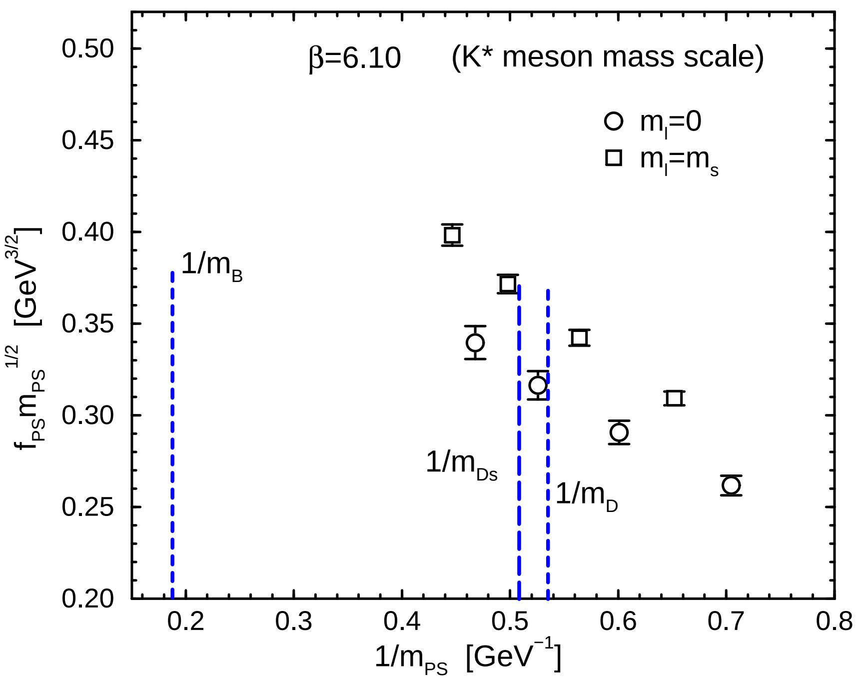
- ∘ $16^3 \times 128$, $\beta = 5.95$, $a_\sigma^{-1} \simeq$ 1.6 GeV
- ∘ $20^3 \times 160$, $\beta = 6.10$, $a_\sigma^{-1} \simeq$ 2.0 GeV

Light quark: 3 values with masses 1–$1.5\ m_s$
$\longrightarrow$ chiral extrapolation

Heavy quark: 4 values with masses 0.7–1.5 GeV
(target mass: charm quark)

Heavy-light pseudoscalar meson decay constant:

- ∘ mean-field improved tree level matching



	$\beta = 5.95$	$\beta = 6.10$	previous works*
f_D/f_π	1.566(43)	1.515(43)	1.55(11)
f_{D_s}/f_D	1.140(14)	1.142(14)	1.12(2)

*Average of quenched works,
S.Ryan, Nucl. Phys. B (PS) 106 (2002) 86.

2nd stage (1): strategy

Calibrations of parameters to 0.2% level of accuracy
in quenched approximation

⇒Applicable to a few percent calculation of matrix elements

☐ Gauge field:

- Precise computation of static quark potential
- Renormalized anisotropy defined with r_0

⇒ $O(0.2\%)$ calibration is possible

☐ Quark field:

We need to calibrate γ_F, c_E, c_B, and c_A (for axial current) in massless limit.

— γ_F must be tuned precisely (to $O(0.2\%)$ level)

Combine two procedures:

- Nonperturbative renormalization technique
- Spectroscopy in fine and coarse directions

Numerical simulation is in progress

☐ Test in heavy quark region:

Use the parameters tuned in massless limit

- Test of relativity relation
- Check of $O(a)$ improvement

⇒ Applications to heavy-light matrix elements

2nd stage (2): gauge field

Quenched gauge field calibration

Renormalized anisotropy ξ_G: defined with hadronic radius r_0

- Precise computation possible
- Good scaling behavior (if lattice scale is set by r_0)

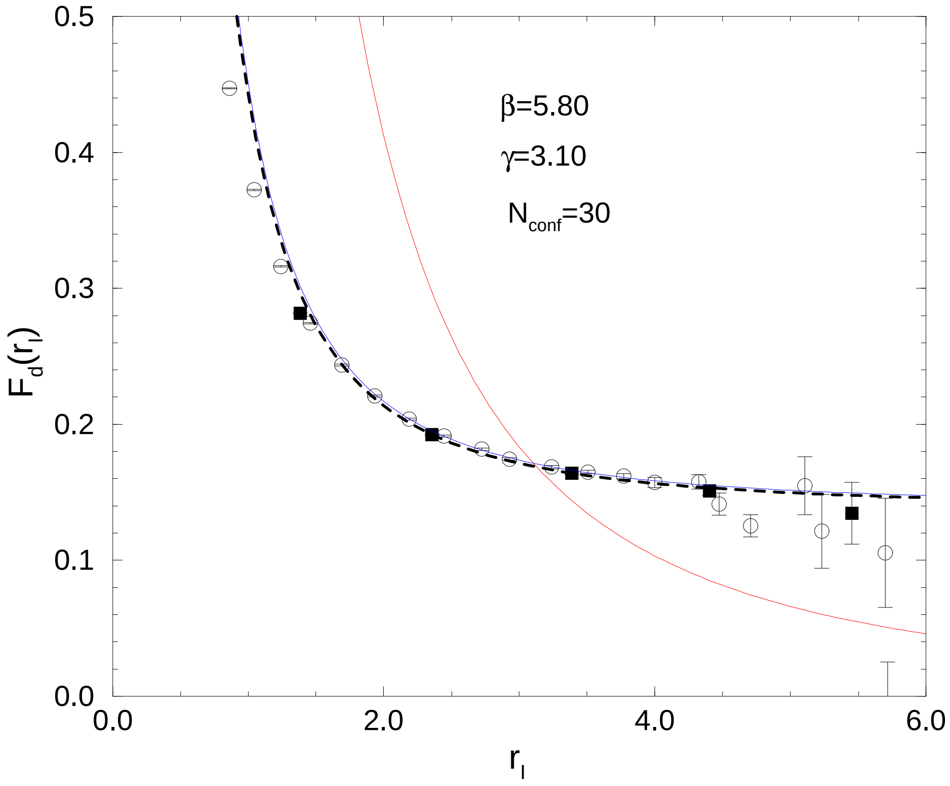
Precise computation of static potentials
in x and t (fine) directions $(V^{(x)}(x), V^{(t)}(t))$

☐ Lüscher-Weisz noise reduction technique
Lüscher and Weisz JHEP 0109 (2001) 010

Hadronic radius r_0: $r_0^2 F(r_0) = 1.65$ ($F(r)$: force)
Sommer, Nucl. Phys. B411 (1994) 839

$\Rightarrow \xi_G = r_0^{(t)}/r_0^{(x)}$

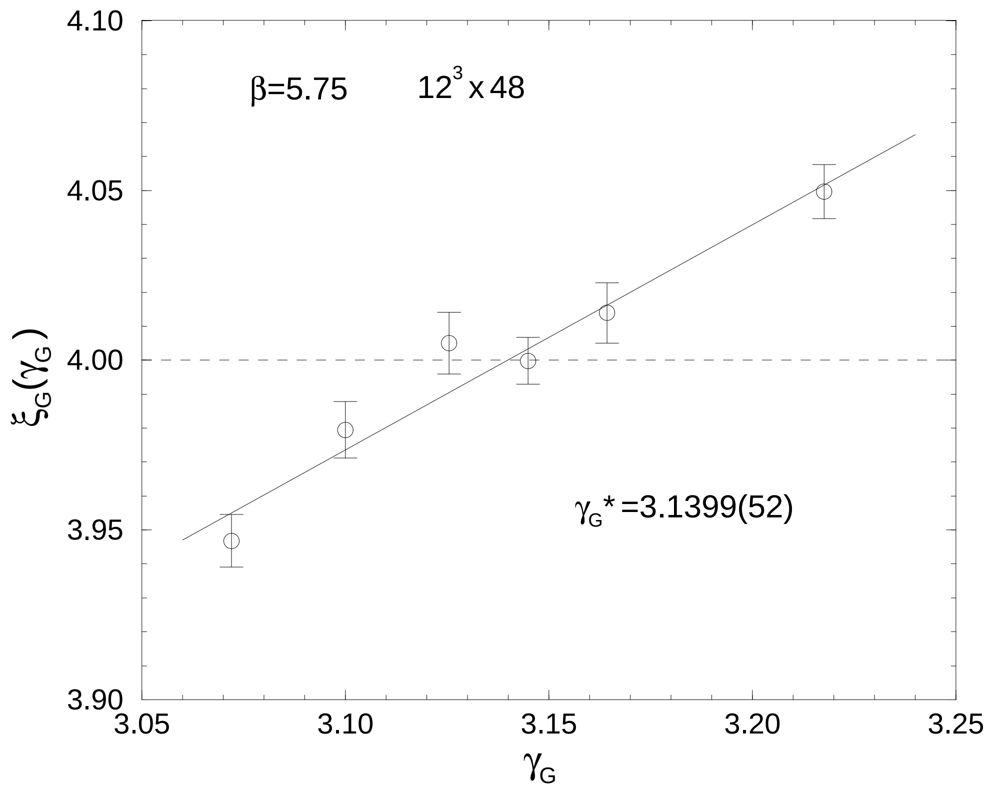
Example in simulation:



2nd stage (3): gauge field

Numerical simulation at $\beta = 5.75$ ($\xi = 4$):

Input γ_G (6 points) $\rightarrow$ $\xi_G(\gamma_G)$ with 0.2%
γ_G^* (s.t. $\xi_G(\gamma_G^*) = \xi$) determined by fit with 0.2% accuracy



Cf. previous work by Klassen:

Klassen, Nucl. Phys. B533 (1998) 557

- ξ_G defined with ratios of Wilson loops
- global fit in β and ξ

$\Rightarrow \gamma_G^*(\beta, \xi)$ with 1% accuracy

We are now performing calibration in wide range of β at $\xi = 4$.

2nd stage (4): quark field

Quenched quark field calibration

Nonperturbative improvement technique

[1] *Lüscher et al., Nucl. Phys. B478 (1996) 365*
[2] *Lüscher and Weisz, Nucl. Phys. B479 (1996) 429*
[3] *Lüscher et al., Nucl. Phys. B491 (1997) 323*

Schrödinger functional method:
Implementation for anisotropic lattice is straightforward.
(t-direction set to the fine direction)

PCAC relation up to $O(a^2)$
$\Rightarrow$ improvement conditions for c_E, c_A (and γ_F)

In the same way as Ref. [1,3]:
Unrenormalized current quark mass

$$m = \frac{1}{2}[\frac{1}{2}(\partial_0^* + \partial_0)_0 f_A(t) + c_A a_\tau \partial_0^* \partial_0 f_P(t)]/f_P(t)$$

$$f_A(t) = -a_\sigma^6 \sum_{\vec{y},\vec{z}} \frac{1}{3}\langle A_0^a(x)\bar{\zeta}(\vec{y})\gamma_5 \frac{1}{2}\tau^a \zeta(\vec{z})\rangle$$
$$f_P(t) = -a_\sigma^6 \sum_{\vec{y},\vec{z}} \frac{1}{3}\langle P^a(x)\bar{\zeta}(\vec{y})\gamma_5 \frac{1}{2}\tau^a \zeta(\vec{z})\rangle$$

$t = x_0$, $\bar{\zeta}$, ζ are boundary quark field

Improvement condition: m does not depend on kinematical parameters (such as boundary gauge field)

- c_B may not be tuned with sufficient accuracy
  — Nonzero background field is electric at tree level
- γ_F may be tuned more precisely in spectroscopy method

2nd stage (5): quark field

Tree level analysis of Schrödinger functional

— Details follow Sec. 6 of Ref. [2]

With nonzero background gauge field:

$V_k(x) = \exp(ia_\sigma b(t))$, $b(t)$ linear in t,

$$f_A^{(0)}(t) = \frac{1}{2}\mathrm{tr}\{H^{(0)}(x)^\dagger \gamma_0 H^{(0)}(x)\}$$

$H^{(0)}(x)$: tree level propagator defined through

$$\{P_+\partial_0^* - P_-\partial_0 + \mathcal{A}(t) + i\mathcal{B}(t)\gamma + i\mathcal{C}(t)\gamma_0\gamma\}H^{(0)}(x) = 0, \quad 0 < t < T$$

$$P_+H^{(0)}(x)|_{t=0} = P_+, \qquad P_-H^{(0)}(x)|_{t=T} = 0$$

where $\zeta = \gamma_F^{-1}$, $\gamma = \gamma_1 + \gamma_2 + \gamma_3$,

$$\begin{aligned}
\mathcal{A}(t) &= \frac{6}{a_\sigma}\zeta^{(0)}\sin^2[a_\sigma(b(t)+\theta/L)/2], \\
\mathcal{B}(t) &= \frac{1}{a_\sigma}\xi\zeta^{(0)}\sin[a_\sigma(b(t)+\theta/L)], \\
\mathcal{C}(t) &= -\frac{c_E^{(0)}}{2}\frac{\zeta^{(0)}}{a_\tau}\sin[a_\sigma a_\tau \partial_0 b(t)]
\end{aligned}$$

With ansatz $H^{(0)} = [s_1(t) + is_2(t)\gamma]P_+$,

$$\frac{1}{2}(\partial_0^* + \partial_0)f_A^{(0)}(t) = \mathrm{tr}\{c_1(t)s_1(t)^2 + c_2(t)s_2(t)^2 + c_3(t)s_1(t)s_2(t)\}$$

c_1, c_2, c_3 are explicitly given in terms of $\mathcal{A}(t)$, $\mathcal{B}(t)$, $\mathcal{C}(t)$

PCAC relation up to $O(a^2)$: c_1, c_2, c_3 are $O(a^2)$

$\Rightarrow$ $\xi\zeta^{(0)} = 1$, $c_E^{(0)} = 1$: tree level relations reproduced

2nd stage (6): quark field

Calibration steps:

(1) Schrödinger functional method
$\Rightarrow$ tuning of c_E, (γ_F)

(2) Spectroscopy in coarse and fine directions
on lattices with T,$L \gtrsim 2$ fm
Physical isotropy conditions for m_{PS}, m_V
$\Rightarrow$ tuning of γ_F, c_B

(3) Schrödinger functional method
$\Rightarrow$ determine c_A, κ_c

(4) Check of systematic errors
Light hadron spectrum and dispersion relation
Taking the continuum limit

Summary and outlook

We are developing anisotropic lattices for precise computation of heavy-light matrix elements.

Results obtained so far are encouraging for further development.

We have started precise calibrations of gauge and quark fields in quenched approximation.

Outlook:

□ How can we verify that parameters in $m_q = 0$ suffices for heavy quark mass ?

□ Mass dependent tuning for bottom quark region
  ∘ Tree level γ_F may be a good approximation.

□ Applications to heavy-light matrix elements
  $\rightarrow$ High precision computation actually possible ?

□ Extension to dynamical QCD:
  Which calibration procedure is most efficient ?