Numerical study of staggered fermion on anisotropic lattices

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Contents:

Introduction(1)

Anisotropic lattice:

Fine resolution in temporal direction $a_\tau < a_\sigma$

Advantages

□ Large number of d.o.f. in t direction

$\Longrightarrow$ Large number of Matsubara frequencies

∗ Finite temperature

Equation of state

cf. Y.Namekawa et al., Phys. Rev. D64 (2001) 074507

Analysis of correlation functions

cf. QCD-TARO Collaboration, Phys. Rev. D63 (2001) 054501

cf. T.Umeda, K.Nomura, H.Matsufuru, hep-lat/0211003

□ Large cutoff in t direction

∗ Heavy quark systems

cf. J.Harada et al., Phys. Rev. D64 (2001) 074501

□ Easy to follow the change of correlators

∗ Glueball spectrum

cf. C.J.Morningstar et al., Phys. Rev. D60 (1999) 034509

cf. N.Ishii et al., Phys. Rev. D66 (2002) 014507

∗ Negative parity baryon spectrum

cf. Y.Nemoto et al., hep-lat/0302013

Introduction(2)

Disadvantages

- Need to tune anisotropy parameters nonperturbatively $\quad (\xi \equiv a_\sigma / a_\tau)$

Observable suffer from calibration uncertainties.

$\Longrightarrow$ Must be tuned with high precision

We investigate which calibration procedure provides good precision and good scaling behavior.

Our goal is to tune anisotropy parameters within 1 % statistical error on dynamical lattices.

Target anisotropy: $\xi = 4$

Form of action

☐ Wilson gauge + staggered fermion actions

Previous work ($\gamma_G = \gamma_F$)

L.Levkova et al., Nucl.Phys.Proc.Suppl.106 (2002)218

$\Longrightarrow$ Precision is not enough

Gauge sector:

$$S_G = \beta \sum_x \left[\sum_{i<j}^{3} \frac{1}{\gamma_G} (1 - \frac{1}{3} \mathrm{Re}\ \mathrm{tr} W_{ij}(x)) + \sum_i^3 \gamma_G (1 - \frac{1}{3} \mathrm{Re}\ \mathrm{tr} W_{i4}(x)) \right]$$

γ_G: bare gauge anisotropy parameter

Quark sector:

$$S_F = \sum_{x,y} \bar{\chi}(x) K(x,y) \chi(y)$$

$$\begin{aligned} K(x,y) &= \delta_{x,y} - \gamma_F \kappa_\sigma \eta_{x4} \left[U_4(x) \delta_{x,y-\hat{4}} - U_4^\dagger(x-\hat{4}) \delta_{x,y+\hat{4}} \right] \\ &\quad - \kappa_\sigma \sum_i \eta_{x\mu} \left[U_i(x) \delta_{x,y-\hat{i}} - U_i^\dagger(x-\hat{i}) \delta_{x,y+\hat{i}} \right] \end{aligned}$$

$$\kappa_\sigma = 1/2m_0$$

γ_F: bare fermionic anisotropy parameter
m_0: bare quark mass (in spacial lattice units)

Tuning of anisotropy parameters (1)

In a simulation on the anisotropic lattice,
we need to find $\gamma_G = \gamma_G^*$ and $\gamma_F = \gamma_F^*$ which satisfy

$$\xi_G(\gamma_G^*, \gamma_F^*) = \xi_F(\gamma_G^*, \gamma_F^*) = \xi$$

for each (β, m_0).

Renormalized anisotropies
- ξ_G : defined with gauge observable
- ξ_F : defined with fermion observable

ξ_G, ξ_F must be determined in good accuracy.

□ In quenched simulation
  One can first tune γ_G independently of γ_F.

□ In dynamical simulation
  We have to tune γ_G and γ_F simultaneously.

Tuning of anisotropy parameters (2)

We adopt following definitions of ξ_G and ξ_F

☐ ξ_G: defined with hadronic radius r_0

$\Rightarrow$Good scaling behavior

Precise static potentials in x and t(fine) directions

- Quench: Lüscher-Weisz noise reduction technique

cf. Lüscher and Weisz JHEP 0109 (2001) 010

- Dynamical: smearing technique in (x, t) plane

$\Rightarrow r_0 : r_0^2 F(r_0) = 1.65 \quad \Rightarrow \quad \xi_G = r_0^{(t)} / r_0^{(x)}$

☐ ξ_F: defined with ratio of meson masses

in x and t directions $\xi_F = m_h^{(z)} / m_h^{(t)}$

— Less statistical error

Dispersion relation: $E^2(\vec{p}) = m^2 + \vec{p}^2 / \xi_F^2$

— used to check the systematic uncertainties

Quenched lattice simulation

☐ Gauge parameters ($\xi = 4$)

β	γ_G	size	a_σ^{-1} (GeV)	N_{conf}
5.75	3.136	$12^2 \times 24 \times 96$(fine)	1.100(6)	224

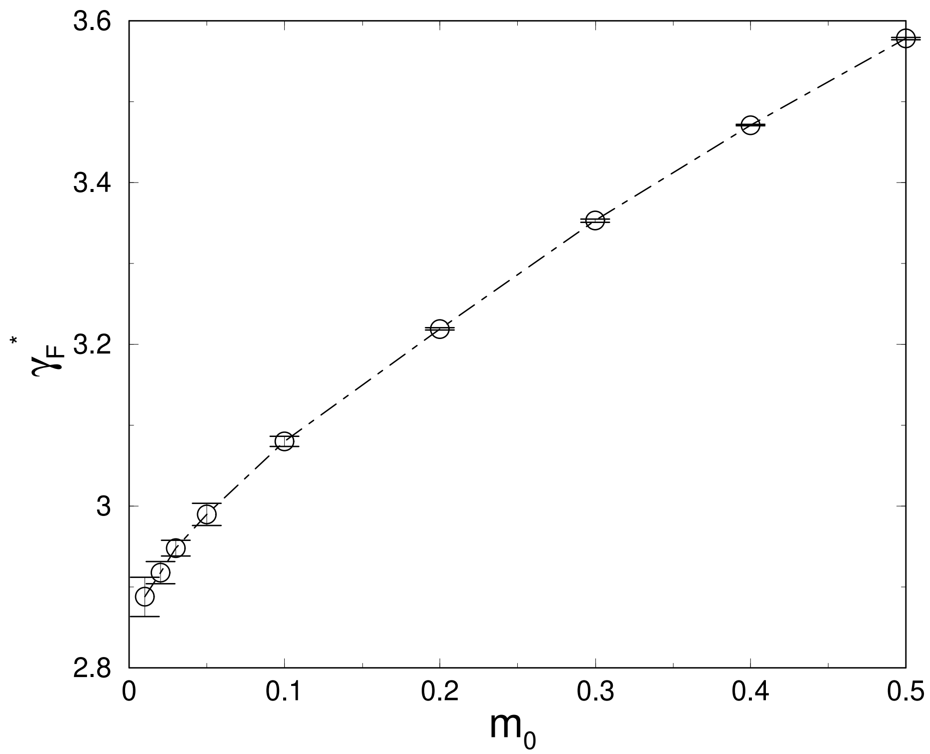
γ_G determined with 0.3% error from hadronic radius r_0

☐ Quark parameters

m_0=0.01∼0.5 ($m_{PS}/m_V = 0.33 \sim 0.90$)

input γ_F : $2 \sim 4$ points ⇒interpolation to γ_F^*

☐ Simulation result



- Need to compare with free quark analysis
- Need to compare with result of dispersion relation

$N_f = 2$ *full QCD simulation*

Lattice: $10^2 \times 20 \times 80$(fine), $\xi = 4$, $N_f = 2$

$\beta = 5.3 \sim 5.45$ ($a_\sigma^{-1} = 0.69 \sim 0.85$ GeV)

Bare quark mass: $m_0 = 0.1$ ($m_{PS}/m_V \sim 0.6$)

N_{conf}: $300 \sim 600$ trajectories with Hybrid R algorithm

Determination of anisotropy parameters (γ_G^*, γ_F^*):
4∼6 points of input (γ_G, γ_F) for each (β, m_0)
tuning to $\Rightarrow \xi = 4$

We suppose the following linear relations

$$\xi_G = a_0 + a_1(\gamma_G - \gamma_G^*) + a_2(\gamma_F - \gamma_F^*)$$

$$\xi_F = b_0 + b_1(\gamma_G - \gamma_G^*) + b_2(\gamma_F - \gamma_F^*)$$

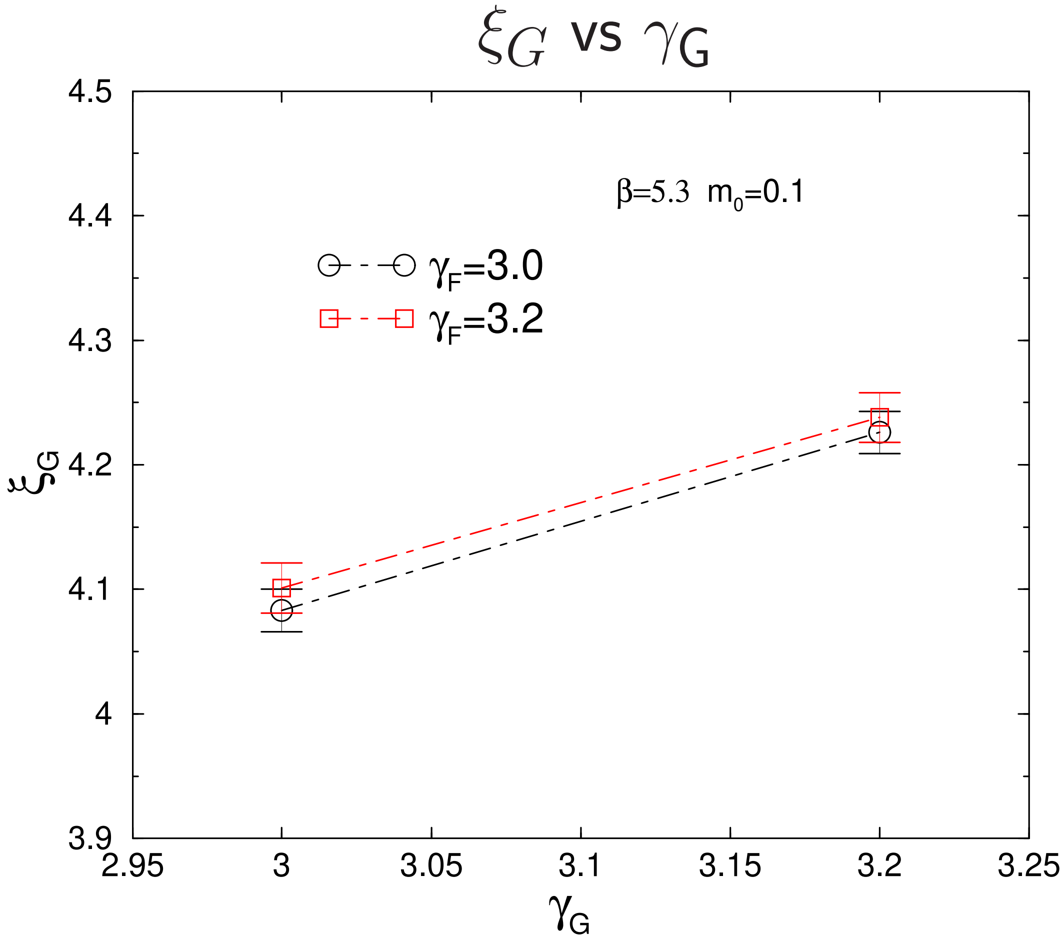
χ^2 fitting $\Rightarrow (\gamma_G^*, \gamma_F^*)$

□ $\beta = 5.3$ result

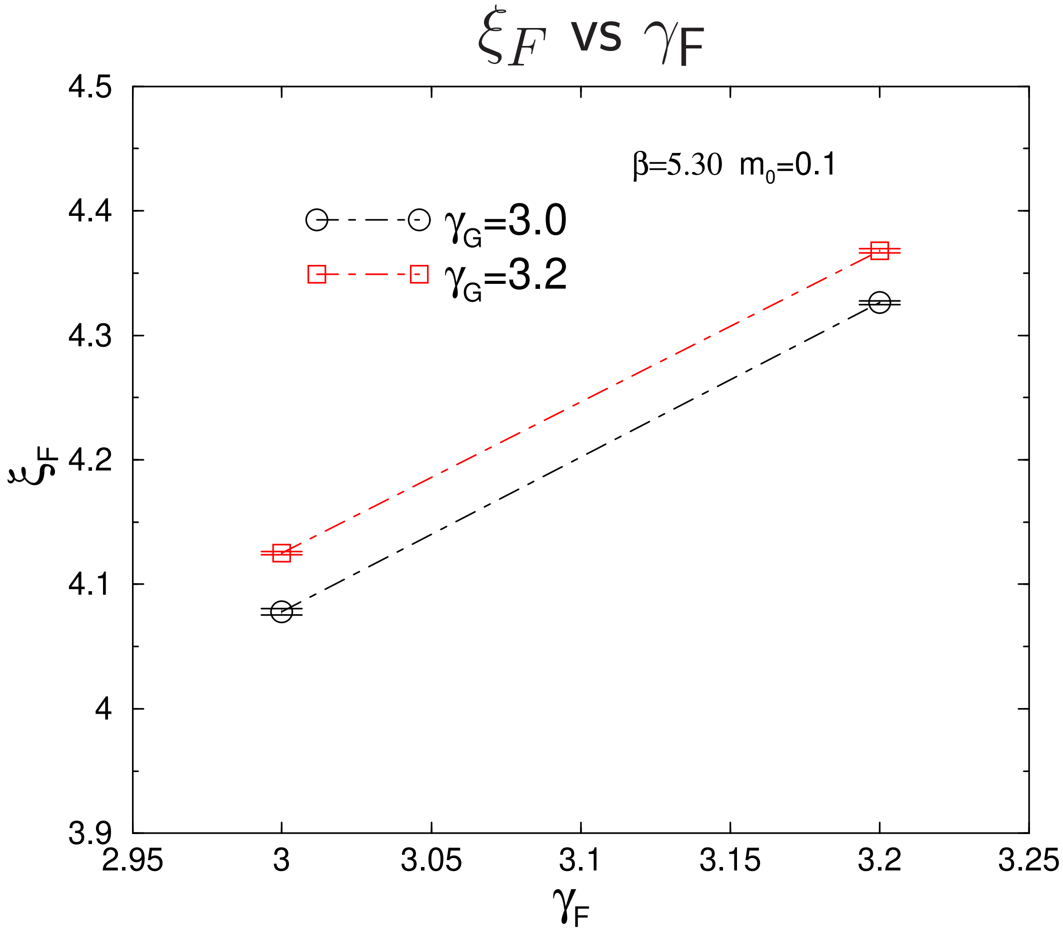
$$[a_0, a_1, a_2] = [4.000(24), 0.702(92), 0.075(92)]$$
$$[b_0, b_1, b_2] = [4.000(28), 0.220(08), 1.225(08)]$$

$$\gamma_G^* = 2.885(35), \quad \gamma_F^* = 2.955(31)$$

- γ_F has small influence on ξ_G
- γ_G has small influence on ξ_F
- Errors of γ_G and γ_F are about 1 %



ξ_G vs γ_G
β=5.3 m_0=0.1
γ_F=3.0
γ_F=3.2
ξ_G
γ_G
4.5
4.4
4.3
4.2
4.1
4
3.9
2.95
3
3.05
3.1
3.15
3.2
3.25



ξ_F vs γ_F
β=5.30 m_0=0.1
γ_G=3.0
γ_G=3.2
ξ_F
γ_F
4.5
4.4
4.3
4.2
4.1
4
3.9
2.95
3
3.05
3.1
3.15
3.2
3.25

Conclusion

□ We carried out the quenched and full QCD simulations on the anisotropic lattices with Wilson gauge + staggered fermion action

$*$ Calibration is carried out using
gauge sector: hadronic radius r_0
quark sector: ratio of masses in x and t directions

∘ Simulation result
–We can perform calibration within 1 % error on both quenched and dynamical lattice.

□ Outlook

- Tuning in wide range of β and m_0
- Comparison with free quark analysis
- Comparison with dispersion relation
- Spectroscopy on anisotropic lattices
- Application to finite temperature