Heavy quark systems in anisotropic lattice QCD

— Toward precision computation of heavy-light matrix elements —

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PANIC 2002
30 Sep – 4 Oct 2002, Osaka, Japan

Contents:

Refs.:

J. Harada et al., Phys. Rev. D 64 (2001) 074501.
H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D 64 (2001) 114503.
J. Harada et al., Phys. Rev. D 66 (2002) 014509.
J. Harada, H. Matsufuru, T. Onogi and A. Sugita, hep-lat/0209090.

This copy: `http://www.rcnp.osaka-u.ac.jp/~matufuru/`

Introduction

Recent experimental developments in flavor physics
— B factories, Charm factory, ...
⇒ need precise theoretical predictions ($\simeq 2\%$)
of hadron matrix elements

Lattice QCD calculations:
for heavy quarks (c and b), large $O(am_Q)$ error

We need a framework which has
(i) continuum limit
(ii) systematic improvement
such as Nonperturbative renormalization technique (for $m_q \simeq 0$)
(iii) modest size of computation

Previous approaches (∼10% systematic accuracy):
a). Effective theories (NRQCD, etc) — (i)×
Thacker and Lepage, Phys. Rev. D 43 (1991) 196.
b). Relativistic framework — (iii)×
c). Fermilab approach (with/without mass dependent tuning)
El-Khadra et al., Phys. Rev. D 55 (1997) 3933.
Sroczynski, et al., Nucl. Phys. B (PS) 83 (2000) 971.
— (ii)×

To achieve calculations with ∼2% precision, we need yet another approach which satisfies above condition (i)–(iii).
⇒ our proposal: *Anisotropic lattice*

Anisotropic lattice QCD

Anisotropic lattice: $a_\tau < a_\sigma$ $\Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Quark action: $O(a)$ improved, along with Fermilab approach

□ bare anisotropy parameter γ_F:

in general, to be tuned mass dependently

e.g., using meson dispersion relation

∘ Continuum limit ? yes ! ☺

∘ Modest computational cost ? yes ! ☺

∘ Systematic improvement ?

If quark mass is sufficiently less than a^{-1},

tuned paramerers for massless quark are applicable.

→nonperturbative renormalziation technique

Lüscher et al., Nucl.Phys. B 491 (1997) 323.

Our expectation:

For $m_Q \ll a_\tau^{-1}$, (not necessarily $m_Q \ll a_\sigma^{-1}$),
mass dependences of parameters in the action are so small that the tuned parameters for massless quark are also applicable in such a quark mass region.

Then, systematic improvement is possible. ☺

(performed at $m_q \simeq 0$)

$\Rightarrow$ To be justified numerically, and in perturbation theory.

Anisotropic quark action

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) &= \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r\kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215
J. Harada et al., Phys. Rev. D 64 (2001) 074501

- Constructed following the Fermilab approach.
  El-Khadra et al., Phys. Rev. D 55 (1997) 3933

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted in several works.)

- Tadpole improvement: $c_E = 1/u_\sigma^2 u_\tau$, $c_B = 1/u_{\sigma^3}$
  u_σ, u_τ: mean-field values of spatial and temporal link variables

- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0\gamma_F + 4))$$

γ_F: bare anisotropy parameter
m_0: bare quark mass (in units of a_τ^{-1})

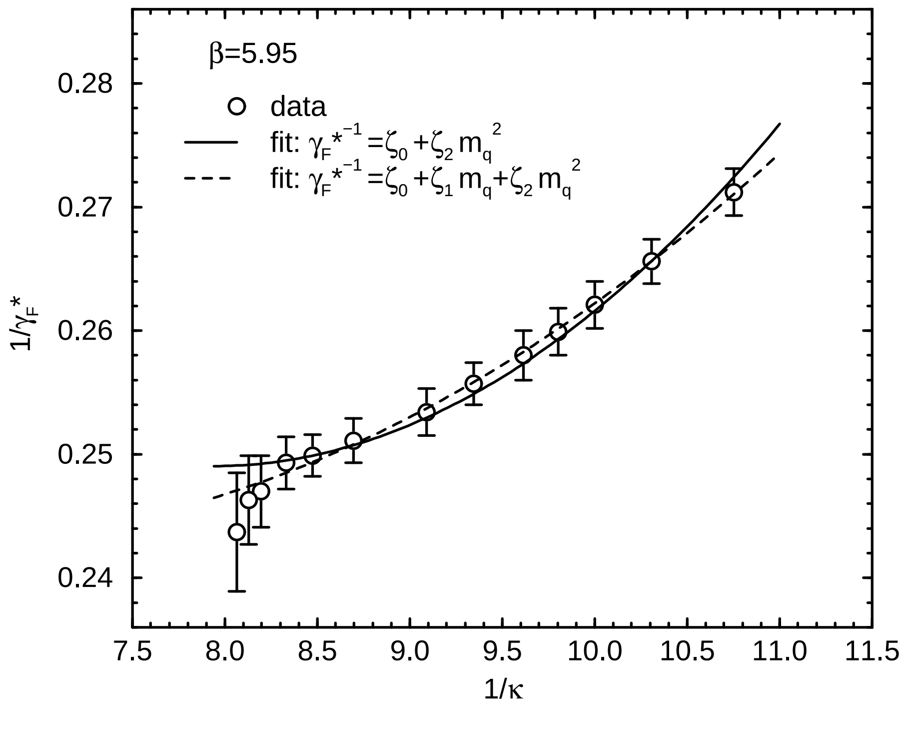
Anisotropic quark action

Expectation: for $m_Q \ll a_\tau^{-1}$,
mass dependences of parameters are sufficiently small.

If this is the case, mass dependence of γ_F must be small.
γ_F: *most significant parameter for relativity condition*
$\Rightarrow$Without tuning, i.e. setting to $\gamma_F(m_q = 0)$,
heavy quark can be correctly described.

For heavy-light systems: actually hold ?
- ∘ Tree-level analysis — yes
- ∘ In numerical simulation — yes, as shown below

Calibration of γ_F:
with meson dispersion relation



γ_F at massless limit is used in the following simulation.

Numerical simulation

Simulation: on quenched anisotropic lattice of size $16^3 \times 128$, $a_\sigma^{-1} \simeq$ 1.6 GeV and anisotropy $\xi = 4$.

Quark parameters

light quark: mass $\sim 1.5\, m_s$

heavy quark: 7 values with $m_Q =$ 1 – 6 GeV

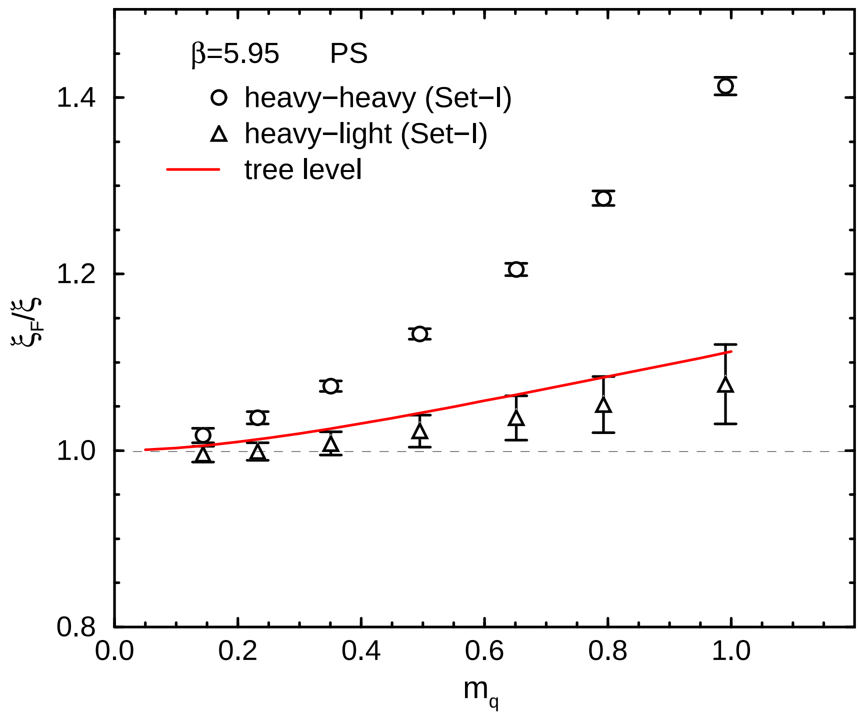
bare anisotropy: tuned value at massless limit ($\gamma_F = 4.016$)

Heavy-heavy, heavy-light meson dispersion relations

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$$

$\rightarrow$ fermionic anisotropy ξ_F

$\xi_F \neq \xi$ signals breaking of relativity relation



∘ $m_q < 0.3$: for heavy-light mesons, γ_F tuned for massless qaurk can be applied within 2% accuracy

∘ $m_q < 0.2$: both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy

$0.2 < m_q$: heavy quarkonia suffer from larger $O((ap)^2)$ error

Heavy-light meson decay constant (preliminary)

Simulation: quenched anisotropic lattices with $\xi = 4$,

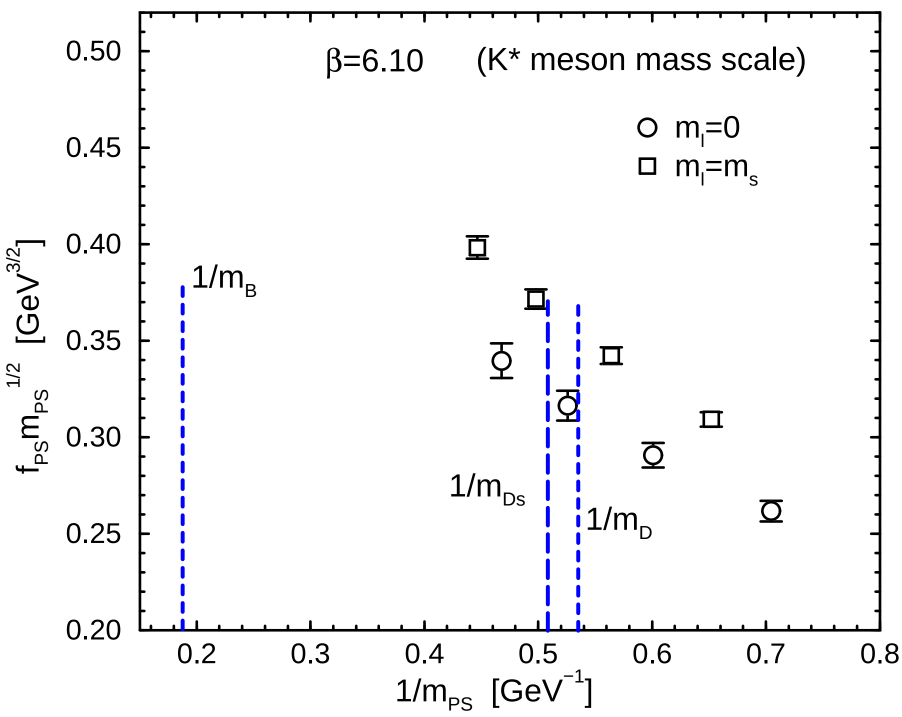
- $\circ$ $16^3 \times 128$, $\beta = 5.95$, $a_\sigma^{-1} \simeq$ 1.6 GeV
- $\circ$ $20^3 \times 160$, $\beta = 6.10$, $a_\sigma^{-1} \simeq$ 2.0 GeV

Light quark: 3 values with masses 1–$1.5\ m_s$
$\rightarrow$ chiral extrapolation

Heavy quark: 4 values with masses 0.7–1.5 GeV
(target mass: charm quark)

Heavy-light pseudoscalar meson decay constant:

- $\circ$ mean-field improved tree level matching



	$\beta = 5.95$	$\beta = 6.10$	previous works*
f_D/f_π	1.566(43)	1.515(43)	1.55(11)
f_{D_s}/f_D	1.140(14)	1.142(14)	1.12(2)

*Average of quenched works,
S.Ryan, Nucl. Phys. B (PS) 106 (2002) 86.

Conclusion and outlook

□ Test of relativity relation:

For $a_\sigma m_Q < 1.2$, heavy-light systems are correctly described by $\gamma_F = \gamma_F(m_q = 0)$.

— mass dependence is actually small.

⇒ Tuned parameters for massless quark are expected to be applicable.

□ Decay constant of D meson:

— present calculation is for consistency check with other approaches at $O(10\%)$ level.

Outlook:

□ Improvement conditions for other parameters (c_B and c_E)

e.g., consistency of dispersion relations, masses and splittings of other hadrons

→ verify that $c(m_q{=}0)$ suffices for heavy quark mass

□ Further development with $\gamma_F = \gamma_F(m_q = 0)$.

∘ $O(a)$ improvement by nonperturbative renormalization

□ Mass dependent tuning for bottom quark region

∘ Tree level γ_F may be a good approximation.

□ Applications to heavy-light matrix elements

→ High precision computation actually possible ?

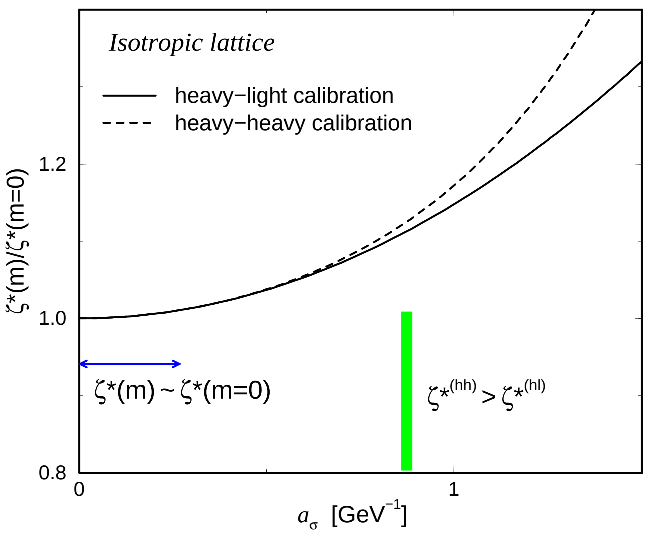
Conjecture of anisotropy parameter (appendix)

Conjecture of a_σ dependence of tuned anisotropy parameter
(figures roughly at charm quark mass)

- $a_\sigma^{hh\text{-}hl}$: above which heavy-heavy and heavy-light systems are inconsistent for single γ_F value.
  $\leftarrow O((a_\sigma p)^2)$ error in quarkonia, not improved by anisotropy
- $a_\sigma^{hl\text{-}ll}$: below which $\gamma_F(m_q) \simeq \gamma_F(m_q = 0)$ holds, within certain (say, 2%) accuracy.

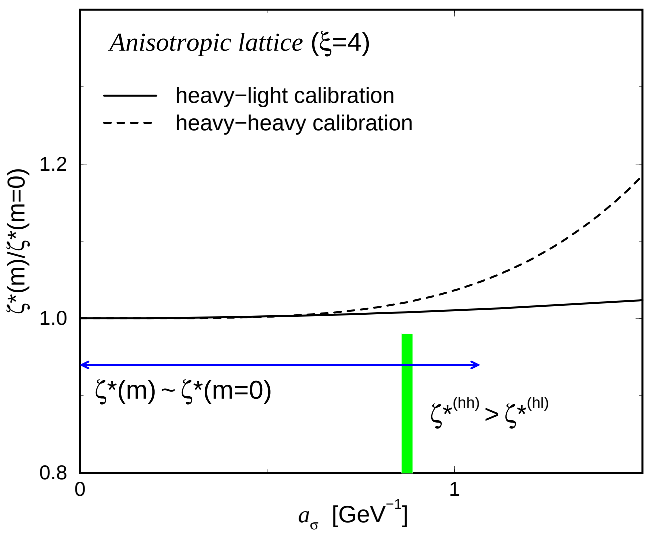
Below $a_\sigma^{hl\text{-}ll}$, $\gamma_F(m_q = 0)$ correctly describes heavy-light systems.

Isotropic lattice



$a_\sigma^{hl\text{-}ll} < a_\sigma^{hh\text{-}hl}$

Anisotropic lattice ($\xi = 4$)



$a_\sigma^{hl\text{-}ll} \gtrsim a_\sigma^{hh\text{-}hl}$

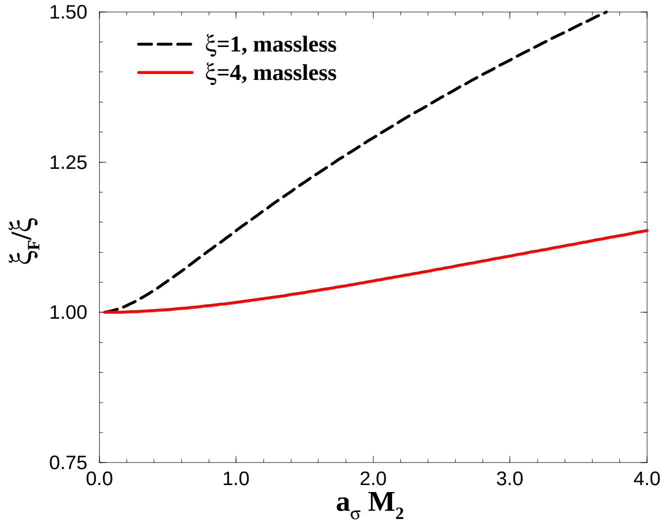
— $a_\sigma^{hl\text{-}ll}$ is extended by a factor ξ.

In the following,
we fix a_σ ($\simeq$ 1.6 GeV), set γ_F to $\gamma_F(m_q = 0)$,
and study mass dependence of fermionic anisotropy ξ_F .

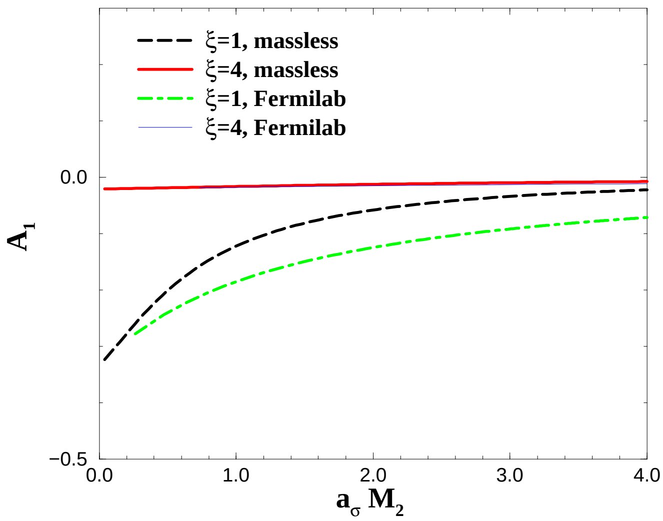
Tree level analysis (appendix)

$$E^2 = M_1^2 + \left(\frac{\xi^{\rm tree}}{\xi_F^{\rm tree}}\right)^2 \vec{p}^2 + A_1 a_\sigma^2 (\vec{p}^2)^2 + A_2 a_\sigma^2 \sum_i p_i^4 + \cdots,$$

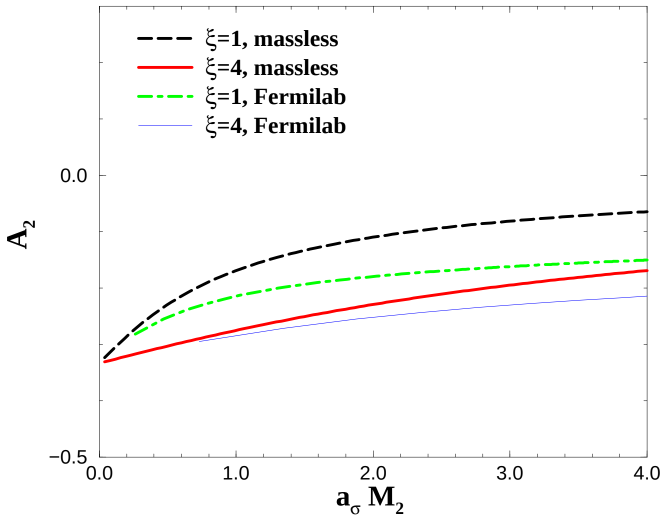
- ξ_F/ξ : reduced by a factor of anisotropy



- A_1: reduced by a factor of anisotropy



- A_2: not reduced, same size as massless quark
  — severe in heavy quarkonium ($p \sim \alpha m_Q$)



Numerical result (appendix)

Inconsistency among binding energies of
heavy-heavy, heavy-light, light-light mesons

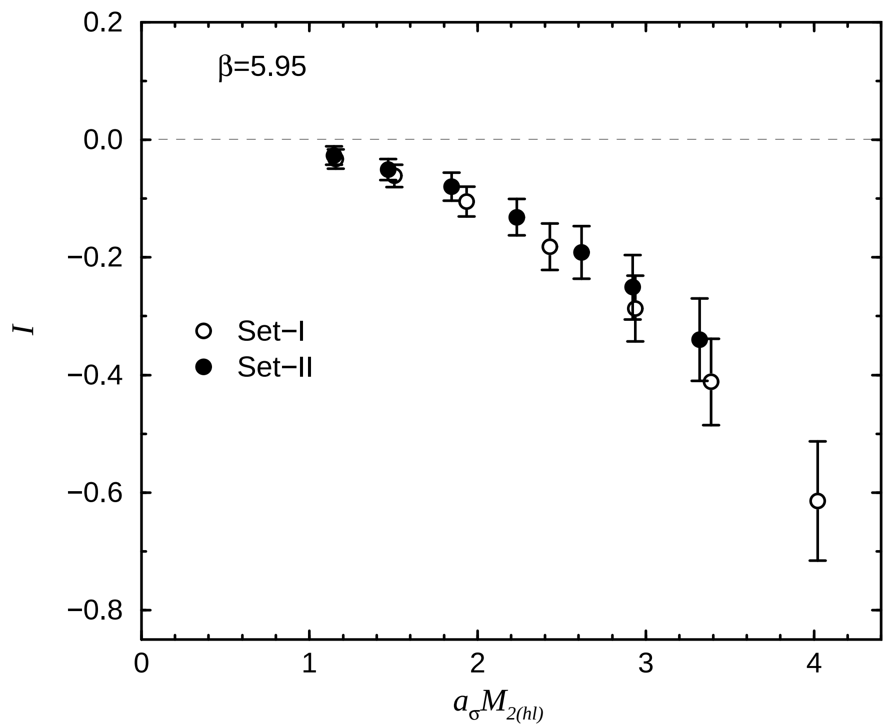
$$I \equiv \frac{2\delta M_{hl} - (\delta M_{hh} + \delta M_{ll})}{2M_{2hl}}$$

$\delta M = M_2 - M_1$ (kinetic mass − rest mass)

$M_2 = \frac{\xi_F^2}{\xi^2} M_1$

S. Collins et al., Nucl. Phys. B (PS) 47 (1996) 455

A. S. Kronfeld, Nucl. Phys. B (PS) 53 (1997) 401



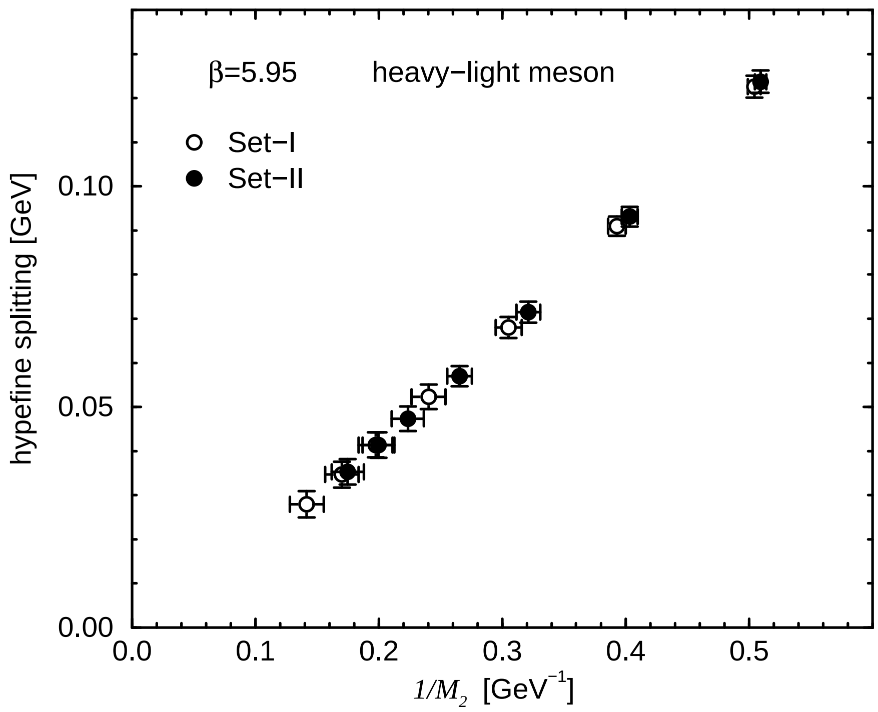
- *Set-I*: γ_F tuned for massless quark
- *Set-II*: mass-dependent tuning of γ_F

For $a_\sigma M_{2hl} > 1$, inconsistency grows:
same behavior as on isotropic lattice

Similar behavior of I for *Set-I* and *II*
⇒mass-dependent tuning does not improve I.

Numerical result (appendix)

Hyperfine splitting of heavy-light meson
vs kinatic meson mass inverse



Similar behavior for Set-I and II

$\Rightarrow$consistent with Fermilab interpretation

Small nonzero intercept: $O(a^2)$ error