Temporal meson correlators at finite temperature on quenched anisotropic lattice

— Charmonium correlators below and above T_c —

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Introduction

In this work,
we focus on charmonium correlators at finite temperature.

— Mass shift near T_c

T. Hashimoto et al., Phys. Rev. Lett. 57 (86) 2123

— J/ψ suppression above T_c

T. Matsui and H. Satz, Phys. Lett. B 178 (86) 416

$\longrightarrow$ Signal of QCD phase transition.

Our goal is to investigate nature of charmonium correlators at $T > 0$ using lattice QCD simulation.

— Mass shift near T_c ?

— No collective excitation mode above T_c ?

$\longrightarrow$ Analysis of correlators in Euclidean temporal direction.

Our Approach (1)

Detailed analysis of correlators in t-direction at $T > 0$.

— Need sufficient number of d.o.f.

$\longleftarrow$ restricted by short temporal extent.

$\Longrightarrow$ *Anisotropic lattice*

$a_\sigma > a_\tau \quad (\xi = a_\sigma / a_\tau$: anisotropy)

QCD-TARO, Phys. Rev. D63 (01) 054501

We use $O(a)$ improved Wilson quark action.

T. Umeda et al., Int. J. Mod. Phys. A16 (01) 2215

J. Harada et al., Phys. Rev D64 (01) 074501

We focus on low energy structure of spectral function.

— Smearing technique.

(with wave function at $T = 0$)

$\longrightarrow$ enhances low energy part

Our Approach (2)

Correlators measured in lattice simulation
$\Longrightarrow$ Spectral function

$$C(t) = \sum_{\vec{x}} \langle O(\vec{x}, t), O^\dagger(0) \rangle$$

$$\Downarrow$$

$$C(t) = \int d\omega K(t, \omega) A(\omega)$$

$$K(t, \omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{N_t \omega}}$$

We apply two analysis procedures.

(1) Maximum Entropy Method (MEM)

Y.Nakahara. et al., Phys. Rev. D60 (99) 091503

∘ direct determination of $A(\omega)$

— successful at $T = 0$

— application to $T > 0$ may be not straigftforward

(2) Fit with ansatz for spectral function

– need information on the form of $A(\omega)$

– with given form $A(\omega)$, more quantitative

These procedures are complementary.

With estimate of spectral function from MEM, quantitative evaluation is given by fit.

Maximum Entropy Method (MEM)

Reconstruction of a spectral function
with Maximum Entropy Method

Y. Nakahara, et al., Phys. Rev. D60 (1999) 091503

$$C(\tau) = \int d\omega K(\tau,\omega) A(\omega)$$

kernel :

$$K(t,\omega) = \frac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{N_t \omega}}$$

Standard χ^2-fit $\longrightarrow$ ill-posed problem

$\Longrightarrow$MEM (based on Bayes' theorem)

Maximization of $Q = \alpha S - L$

$$S = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right]$$

$m(\omega) = m_0 \omega^2$: default model

α : constant $\rightarrow$to be integrated out

L : Likelihood function (χ^2-term)

Fit with ansatz

We suppose the shape of spectral function.

(MEM gives rough estimate of shape of spectral function.)

∘ Pole type :

$$\rho(\omega) = C\delta(m - \omega)$$

∘ Breit-Wigner type :

$$\rho(\omega) = \frac{C\Gamma m}{(\omega^2 - m^2)^2 - \Gamma^2 m^2}$$

$$A(\omega) = \omega^2 \rho(\omega)$$

C : overlap, m : mass, Γ : width

	lowest peak	second peak
2-pole fit	pole	pole
1-BW fit	BW	
BW+pole fit	BW	pole

∗2-pole : expected to suffice below T_c

∗1-BW : workable for large L_{min}

∗BW+pole : for subtraction of contribution from large ω part

Fit range : L_{min} – L_{max}

We study L_{min} dependencies of the fitting parameters.

The region in which there value are constant

$\Longrightarrow$ ansatz is successfully applicable.

Anisotropic quark action

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) &= \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & \quad - \kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & \quad - \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215
J. Harada et al., Phys. Rev. D 64 (2001) 074501

- Constructed following the Fermilab approach.
  El-Khadra et al., Phys. Rev. D 55 (1997) 3933

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted in several works.)

- Tadpole improvement: $c_E = 1/u_\sigma^2 u_\tau$, $c_B = 1/u_{\sigma^3}$
  u_σ, u_τ: mean-field values of spatial and temporal link variables

- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \quad (= 2(m_0 \gamma_F + 4))$$

γ_F: bare anisotropy parameter
m_0: bare quark mass (in temporal lattice units)

Simulation parameters

Gauge parameters:
— Anisotropic plaquette action in quenched approximation

☐ $(\beta, \gamma_G) = (6.10, 3.2108)$
- $\xi = a_\sigma / a_\tau = 4$

  Klassen, Nucl. Phys. B 533 (1998) 557
- $a_\sigma^{-1} = 2.030(13)$ GeV

  Scale is set by hadronic radius r_0

☐ Size : $20^3 \times N_t$

N_t	T/T_c	$N_{conf} \times N_{source}$
160	$\sim$ 0	500 $\times$ 16
32	$\sim$ 0.9	1000 $\times$ 16
26	$\sim$ 1.1	1000 $\times$ 16

Quark parameters :
— $O(a)$ improved Wilson quark action

☐ $(\kappa, \gamma_F) = (0.1120, 4.000)$
$\rightarrow$ roughly correspond to charm quark mass

H. Matsufuru et al., Phys. Rev. D 64 (2001) 114503

☐ Smeared-smeared correlators in PS and V channels

☐ Smearing function : wave function V meson measured at $T = 0$ in Coulomb gauge

☐ Source points: N_{source} points on each configuration
$\rightarrow$ reduce statistical fluctuation

Numerical results

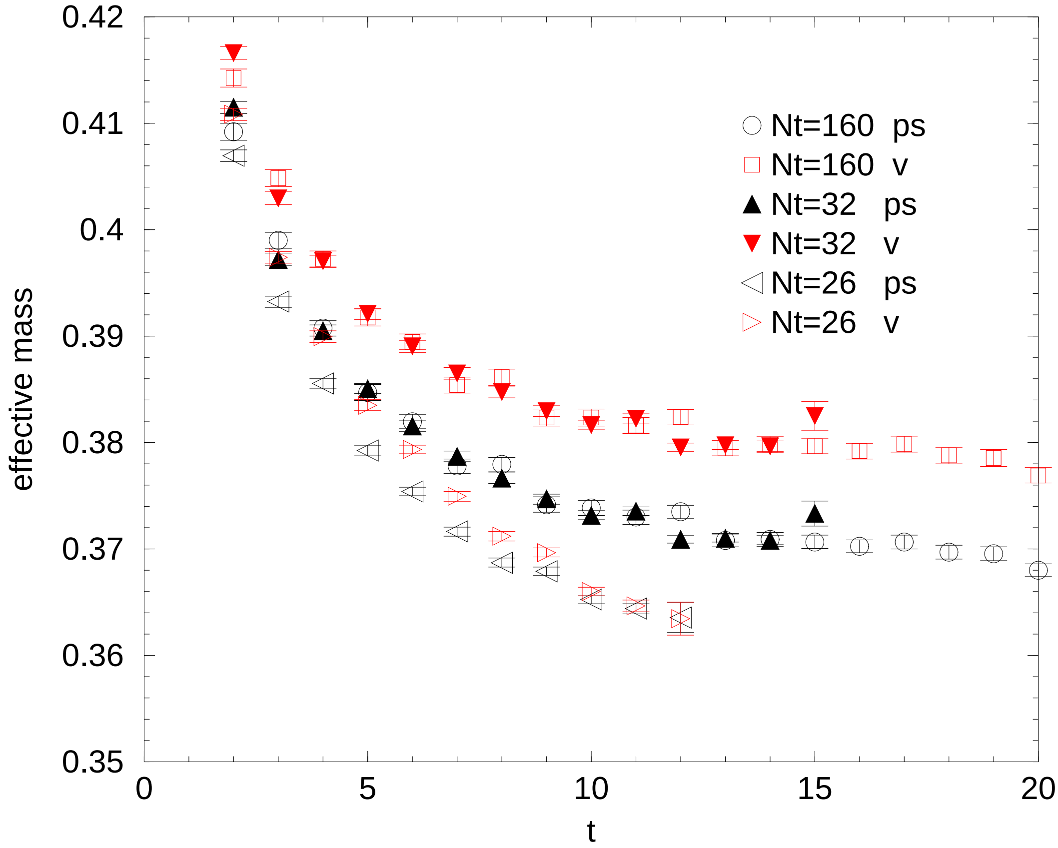
In the following, we focus on the low frequency part (lowest peak) of $\rho(\omega)$.

Correlators

□ Effective mass plot

m_{eff} defined through

$$\frac{C(t)}{C(t+1)} = \frac{\cosh\left[m_{eff}(t)(N_t/2-t)\right]}{\cosh\left[m_{eff}(t)(N_t/2-t-1)\right]}$$



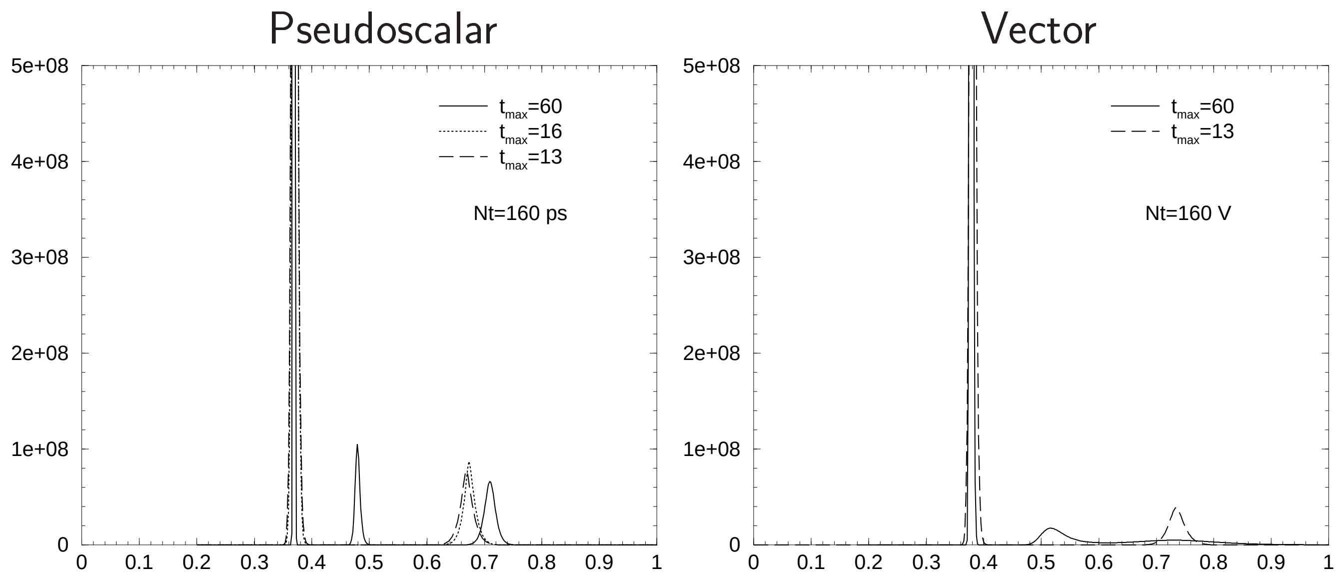
□ Spectrum at $T = 0$

— determined by two-pole fit

state	m_{PS}	m_V
ground	0.36856(9)	0.37769(12)
first exc.	0.500(22)	0.479(23)
fit range	17–80	15–80

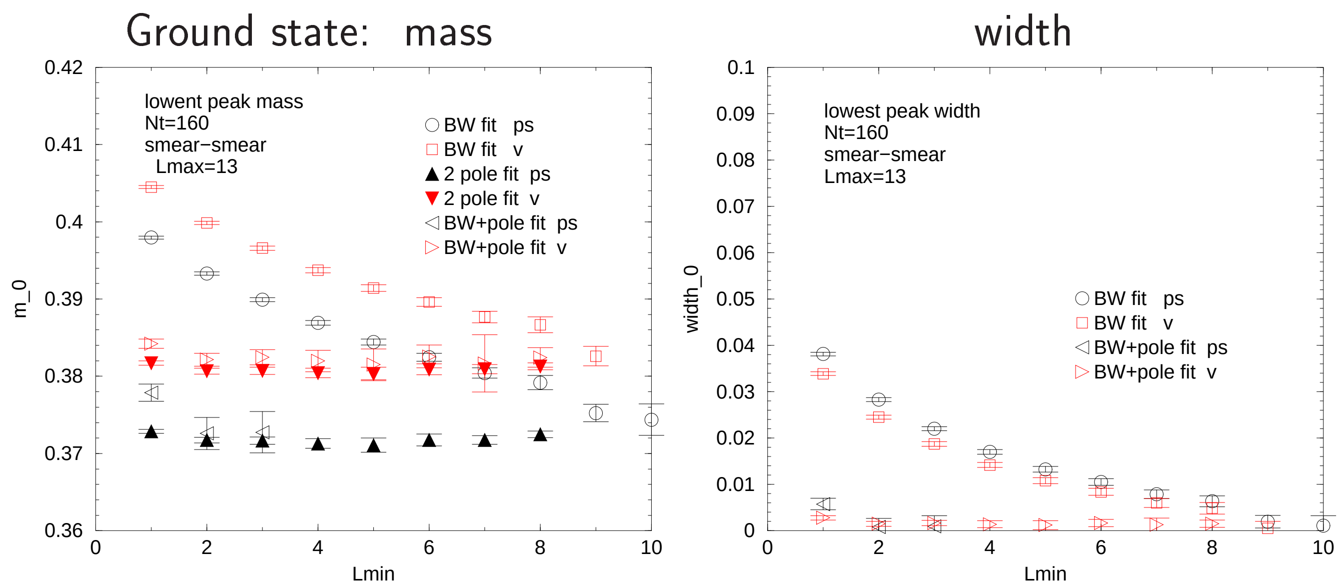
Results at $T=0$

☐ Result of MEM



☐ Result of Fits

— 2-pole, 1-BW, BW+pole forms

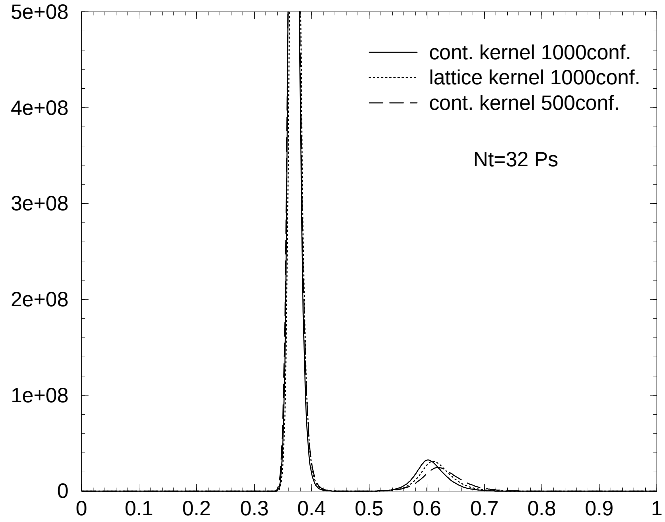


- In MEM, ground state peak is stable with change of t_{max}
- Consistency of methods
- No indication of finite width for ground state, as expected

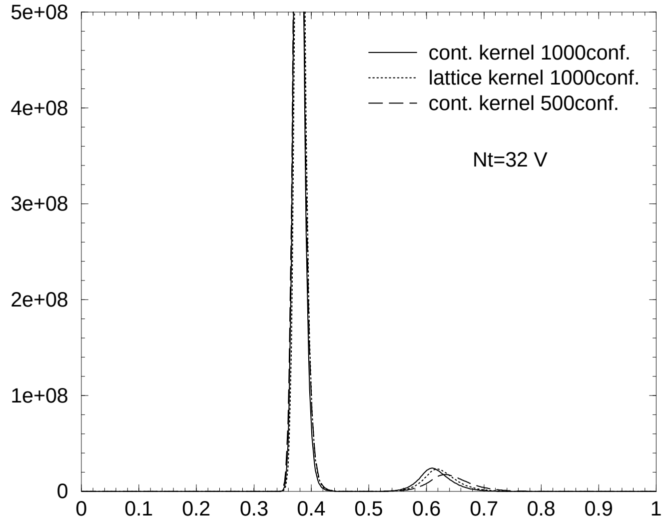
Results at $T = 0.9T_c$

☐ Result of MEM

Pseudoscalar



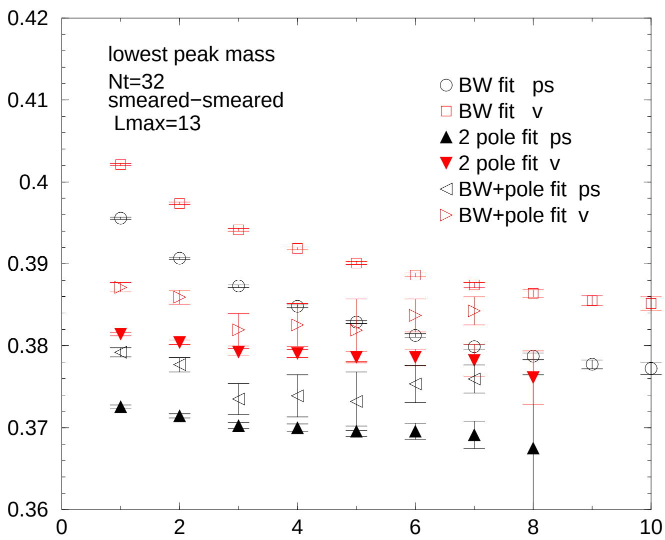
Vector



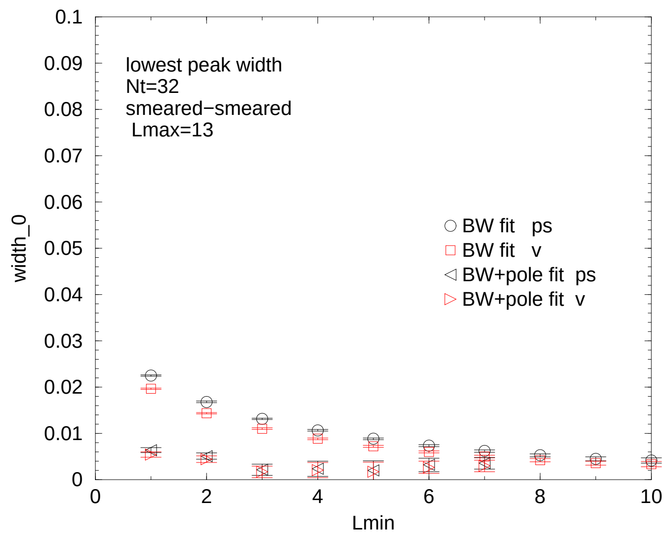
☐ Result of Fits

— 2-pole, 1-BW, BW+pole forms

Ground state: mass



width

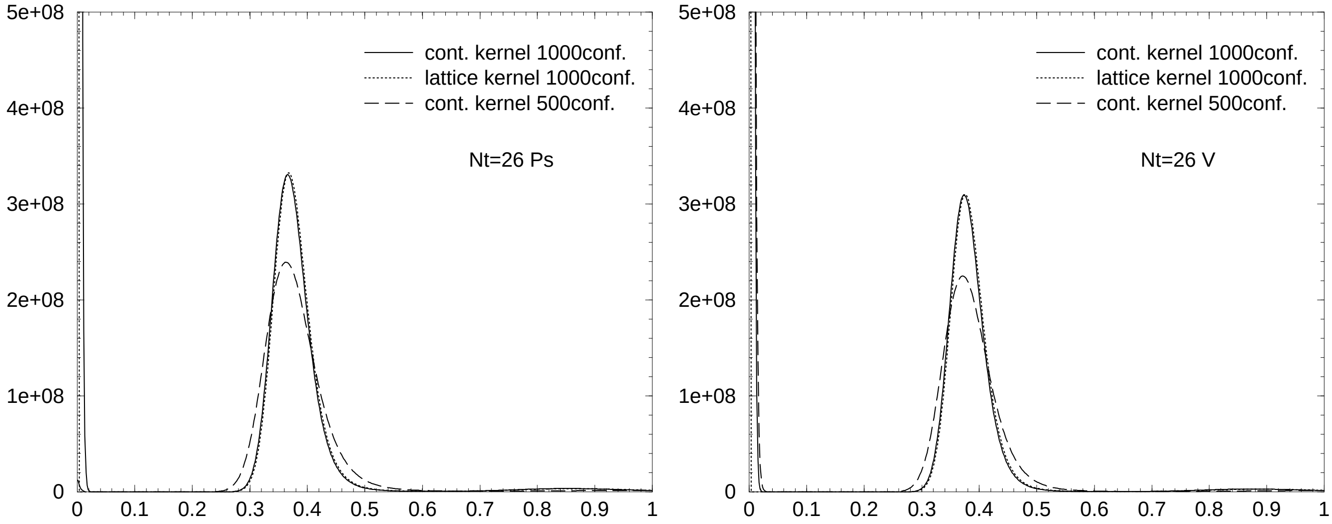


- Ground state peaks locate at almost the same as $T = 0$
- Width from fit is consistent with zero.
  – No indication of finite width for ground state

Results at $T = 1.1$

☐ Result of MEM

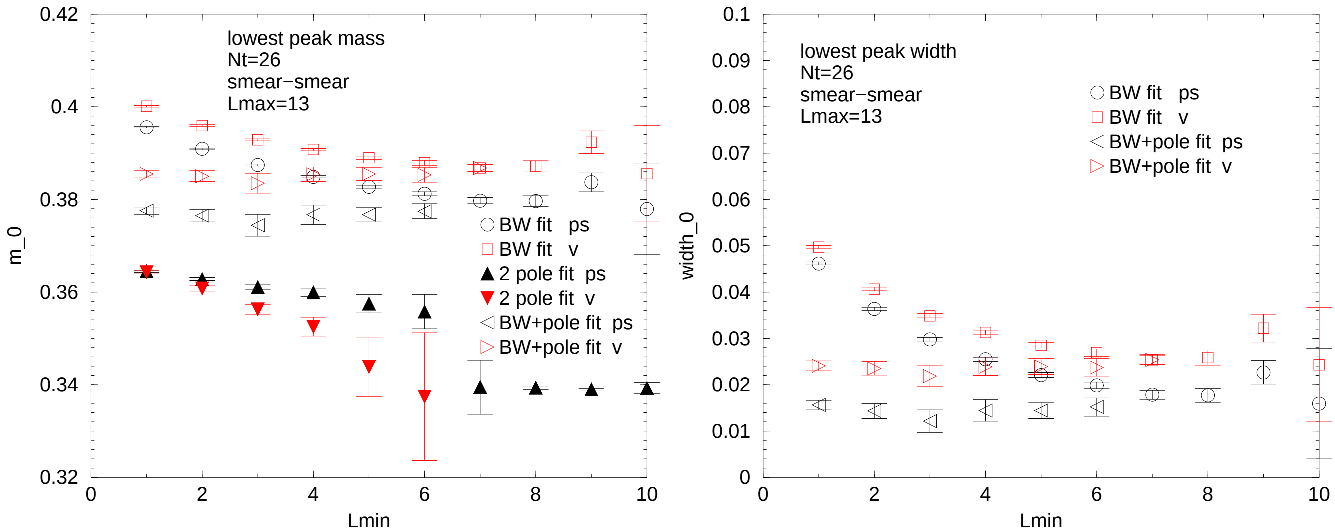
Pseudoscalar Vector



☐ Result of Fits

— 2-pole, 1-BW, BW+pole forms

Ground state: mass width



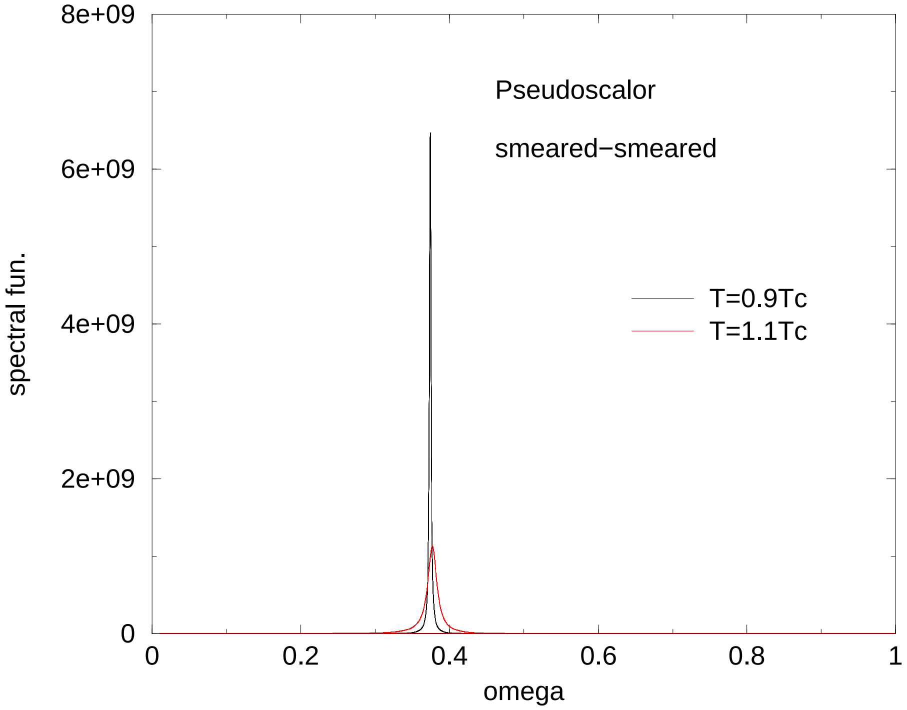
- There observed hadron-like peak.
- Finite width is observed ($\Gamma \sim$ 200 MeV).
- Peak position is almost same as $T < T_c$.

Spectral function

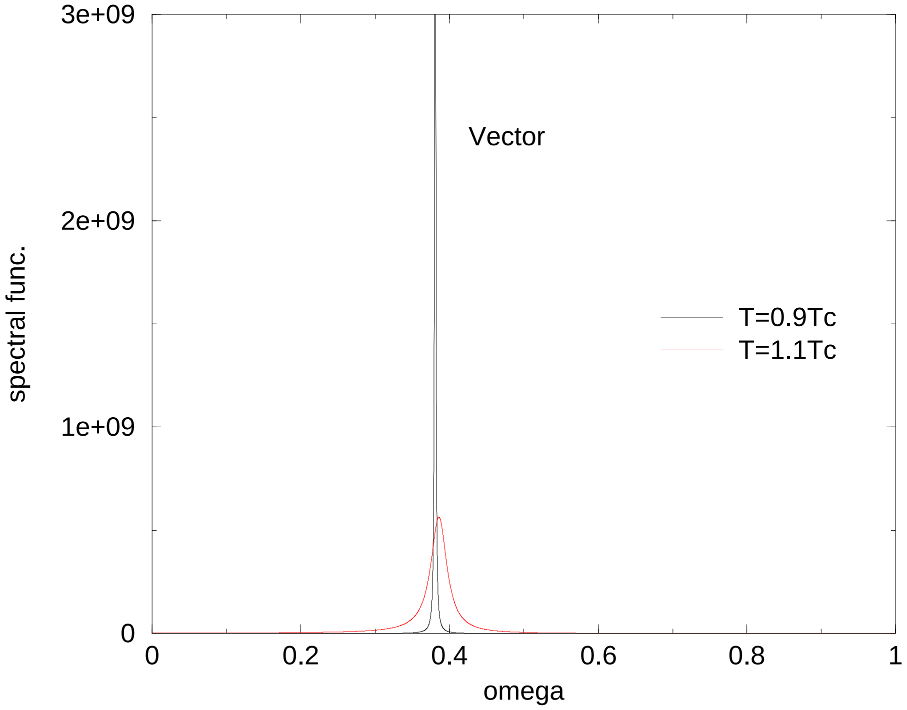
☐ Spectral function determined by fit

(BW+pole fit, $L_{min} = 4$, $L_{max} = 13$)

Pseudoscalar



Vector



Only lowest peak is shown.

Summary

□ We analyzed smeard charmonium correlators measured in quenched lattice QCD with two method.

—results of fit and MEM are qualitatively consistent

○ Below T_c ($T \sim 0.9T_c$)

- No indication of finite width
- Mass is almost the same as $T = 0$

○ Above T_c ($T \sim 1.1T_c$)

- There observed hadron-like peak.
- Finite width is observed ($\Gamma \sim$ 200 MeV).
- Peak position is almost the same as $T = 0$.

Complementary use of these procedures is preferable for reliable analysis.

□ Outlook

- More quantitative study in wide range of T
- effect of dynamical quark