Heavy-light meson in anisotropic lattice QCD

— Toward precision computation of heavy-light matrix elements —

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Refs.:

J. Harada et al., Phys. Rev. D 64 (2001) 074501.
H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D 64 (2001) 114503.
J. Harada, H. Matsufuru, T. Onogi and A. Sugita, hep-lat/0202004.
J. Harada, H. Matsufuru, T. Onogi and A. Sugita, hep-lat/0203025.

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Introduction

Recent experimental developments
- ◦ B factories (KEK/Belle, SLAC/BaBar, ...)
- ◦ Charm factory (CLEO-c project)

⇒need precise theoretical predictions ($\simeq 2\%$)

Lattice QCD calculation of matrix elements
- nonperturbative
- errors can be systematically removed

However, for heavy quarks (charm and bottom),
$m_Q \gtrsim a^{-1}$ ⇒large $O(am_Q)$ error

We need a framework which has
(i) continuum limit
(ii) systematic improvement
such as Nonperturbative renormalization technique (for $m_q \simeq 0$)
(iii) modest size of computation

Previous approaches (∼10% systematic accuracy):
a). Effective theories (NRQCD, etc) — (i)×
Thacker and Lepage, Phys. Rev. D 43 (1991) 196.
b). Relativistic framework — (iii)×
c). Fermilab approach (with/without mass dependent tuning)
El-Khadra et al., Phys. Rev. D 55 (1997) 3933.
Sroczynski, et al., Nucl. Phys. B (PS) 83 (2000) 971.
— (ii)×

To achieve calculations with ∼2% precision, we need
yet another approach which satisfies above condition (i)–(iii).
⇒ our proposal: *Anisotropic lattice*

Anisotropic lattice QCD

Anisotropic lattice: $a_\tau < a_\sigma \Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Quark action: $O(a)$ improved, along with Fermilab approach

- □ bare anisotropy parameter γ_F:
  in general, to be tuned mass dependently
  e.g., using meson dispersion relation

- ○ Continuum limit ? yes ! ☺
- ○ Modest computational cost ? yes ! ☺
- ○ Systematic improvement ?
  If quark mass is sufficiently less than a^{-1},
  tuned paramerers for massless quark are applicable.
  →nonperturbative renormalziation technique
  Lüscher et al., Nucl.Phys. B 491 (1997) 323.

Our expectation:
For $m_Q \ll a_\tau^{-1}$, (not necessarily $m_Q \ll a_\sigma^{-1}$),
mass dependences of parameters in the action is so small
that the tuned parameters for massless quark are also
applicable in such a quark mass region.

Then, systematic improvement is possible. ☺

$\Rightarrow$ To be justified numerically, and in perturbation theory.

If this is the case, mass dependence of γ_F must be small.

(Without tuning, i.e. setting to $\gamma_F(m_q = 0)$,
heavy quark can be correctly described.)

For heavy-light systems: really hold ?

- ○ Tree-level analysis — yes
- ○ In numerical simulation — yes, as shown below

Anisotropic quark action

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & - \kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & - \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215
J. Harada et al., Phys. Rev. D 64 (2001) 074501

- Constructed following the Fermilab approach.
  El-Khadra et al., Phys. Rev. D 55 (1997) 3933
- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted in several works.)
- Tadpole improvement: $c_E = 1/u_\sigma^2 u_\tau, \quad c_B = 1/u_{\sigma^3}$
  u_σ, u_τ: mean-field values of spatial and temporal link variables
- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0 \gamma_F + 4))$$

γ_F: bare anisotropy parameter
m_0: bare quark mass (in temporal lattice units)

At the tree level, with $r = 1/\xi$ and for small quark mass,

$$\frac{1}{\gamma_F} = \frac{1}{\xi}\left(1 + \frac{1}{3} m_0^2\right).$$

— no linear term in m_0 for $m_0 \ll a_\tau^{-1}$.

Small mass dependence was also numerically found.

Matsufuru, Onogi and Umeda, Phys. Rev. D 64 (2001) 114503.

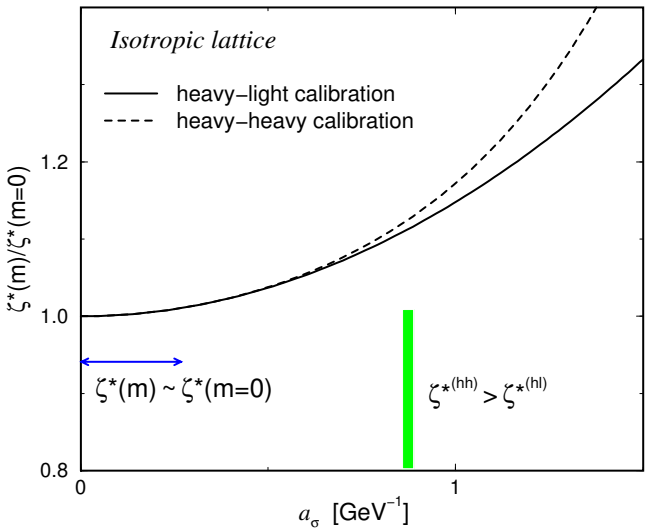
Conjecture of anisotropy parameter

Conjecture of a_σ dependence of tuned anisotropy parameter
(figures roughly at charm quark mass)

- $a_\sigma^{hh\text{-}hl}$: above which heavy-heavy and heavy-light systems are inconsistent for single γ_F value.
  $\leftarrow O((a_\sigma p)^2)$ error in quarkonia, not improved by anisotropy
- $a_\sigma^{hl\text{-}ll}$: below which $\gamma_F(m_q) \simeq \gamma_F(m_q = 0)$ holds, within certain (say, 2%) accuracy.

Isotropic lattice

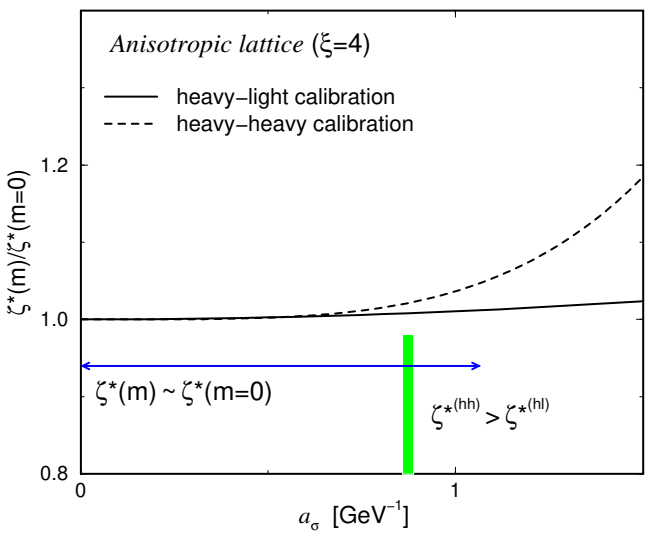
$$a_\sigma^{hl\text{-}ll} < a_\sigma^{hh\text{-}hl}$$



Anisotropic lattice ($\xi = 4$)

$$a_\sigma^{hl\text{-}ll} \gtrsim a_\sigma^{hh\text{-}hl}$$

$a_\sigma^{hl\text{-}ll}$ is extended by a factor ξ.



Below $a_\sigma^{hl\text{-}ll}$, $\gamma_F(m_q = 0)$ correctly describes heavy-light systems.

In the following, we fix a_σ ($\simeq$ 1.6 GeV), set γ_F to $\gamma_F(m_q = 0)$, and study mass dependence of fermionic anisotropy ξ_F .

Numerical simulation

Simulation: on quenched anisotropic lattice of
size $16^3 \times 128$, $a_\sigma^{-1} \simeq$ 1.6 GeV and anisotropy $\xi = 4$.

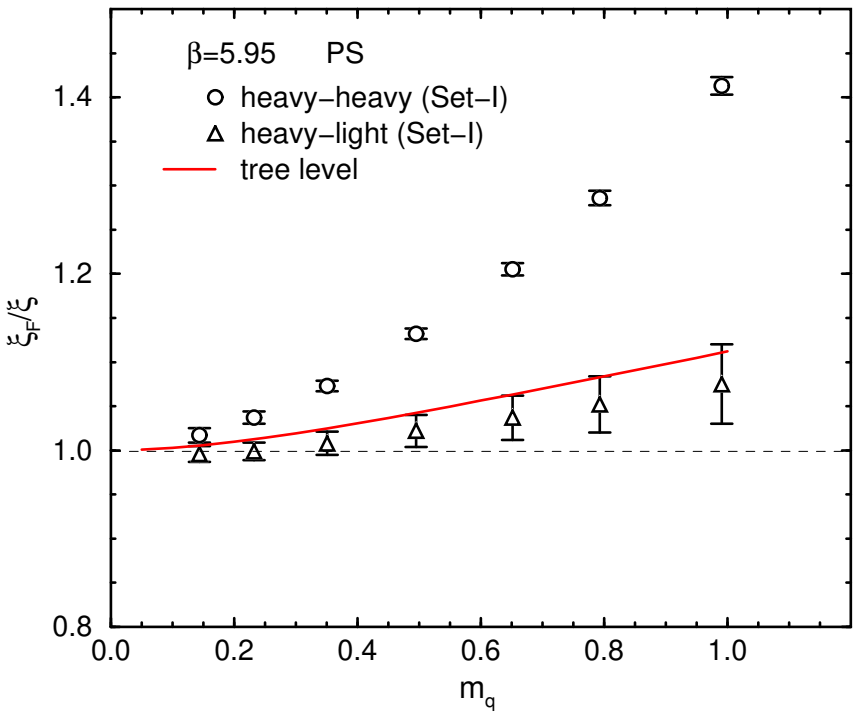
Quark parameters
light quark: mass $\sim 1.5\, m_s$
heavy quark: 7 values with $m_Q =$ 1 – 6 GeV
bare anisotropy: tuned value at massless limit ($\gamma_F = 4.016$)

Heavy-heavy, heavy-light meson dispersion relations

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^{\,2}}{\xi_F^2} + O(\vec{p}^{\,4})$$

$\rightarrow$ fermionic anisotropy ξ_F

$\xi_F \neq \xi$ signals breaking of relativity relation



- $m_q < 0.2$: both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy
- $m_q < 0.3$: for heavy-light mesons, γ_F tuned for massless qaurk can be applied within 2% accuracy

Conclusion and outlook

Summary of numerical result:

on a lattice of $a_\sigma^{-1} = 1.6$ GeV and $\xi = 4$,

(a) $a_\sigma m_Q < 0.8$: both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy
(charm quark mass is covered ☺)

(b) $a_\sigma m_Q < 1.2$: for heavy-light mesons, γ_F tuned for massless qaurk can be applied within 2% accuracy

(c) $1.2 < a_\sigma m_Q$, $O(a^2)$ systematic error gradually grows.
$\leftarrow$ inconsistency between ξ_F's from PS and V mesons

For $a_\sigma m_Q < 1.2$, heavy-light systems are correctly described by $\gamma_F = \gamma_F(m_q = 0)$.
— mass dependence is actually small.

$\Rightarrow$ Tuned parameters for massless quark are expected to be applicable.

Outlook:

- Further development with $\gamma_F = \gamma_F(m_q = 0)$.
  - $O(a)$ improvement by nonperturbative renormalization
- Mass dependent tuning for bottom quark region
  - Tree level γ_F may be a good approximation.
- Applications to heavy-light matrix elements
  $\rightarrow$ High precision computation actually possible ?

Heavy-light meson decay constant (preliminary)

Simulation: quenched anisotropic lattices with $\xi = 4$,

- $16^3 \times 128$, $\beta = 5.95$, $a_\sigma^{-1} \simeq$ 1.6 GeV
- $20^3 \times 160$, $\beta = 6.10$, $a_\sigma^{-1} \simeq$ 2.0 GeV

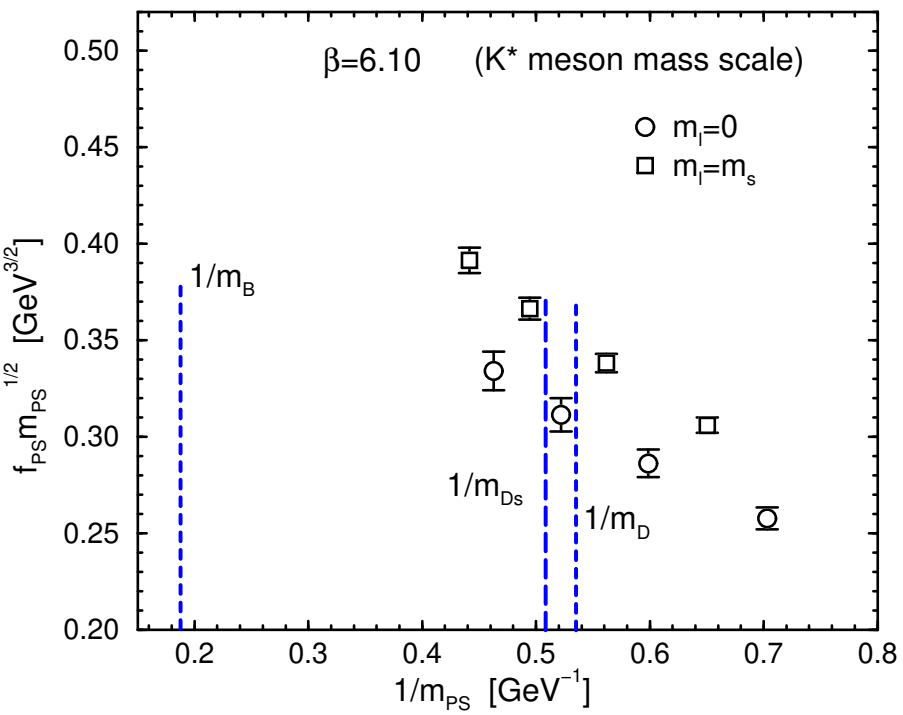
Light quark: 3 values with masses 1–$1.5\ m_s$
$\rightarrow$ chiral extrapolation

Heavy quark: 4 values with masses 0.7–1.5 GeV
(target mass: charm quark)

Heavy-light pseudoscalar meson decay constant:

- mean-field improved tree level matching

J. Harada et al., hep-lat/0202004



	$\beta = 5.95$	$\beta = 6.10$	previous works*
f_D/f_π	1.516(44)	1.536(49)	1.55(11)
f_{D_s}/f_D	1.141(15)	1.145(16)	1.12(2)

*Average of quenched works,
S.Ryan, Nucl. Phys. B (PS) 106 (2002) 86.