# *Heavy-light meson in anisotropic lattice QCD*

## *— Toward precision computation of heavy-light matrix elements —*

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*Contents:*



*Refs.:*
*J. Harada et al., Phys. Rev. D 64 (2001) 074501.*
*H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D 64 (2001) 114503.*
*J. Harada, H. Matsufuru, T. Onogi and A. Sugita, hep-lat/0202004.*
*J. Harada, H. Matsufuru, T. Onogi and A. Sugita, hep-lat/0203025.*

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## *Introduction*

Recent experimental developments
- ◦ B factories (KEK/Belle, SLAC/BaBar, ...)
- ◦ Charm factory (CLEO-c project)

⇒need precise theoretical predictions ($\simeq 2\%$)

Lattice QCD calculation of matrix elements
- nonperturbative
- errors can be systematically removed

However, for heavy quarks (charm and bottom),
$m_Q \gtrsim a^{-1}$ ⇒large $O(am_Q)$ error

We need a framework which has
(i) continuum limit
(ii) systematic improvement
such as Nonperturbative renormalization technique (for $m_q \simeq 0$)
(iii) modest size of computation

Previous approaches (∼10% systematic accuracy):
a). Effective theories (NRQCD, etc) — (i)×
*Thacker and Lepage, Phys. Rev. D 43 (1991) 196.*
b). Relativistic framework — (iii)×
c). Fermilab approach (with/without mass dependent tuning)
*El-Khadra et al., Phys. Rev. D 55 (1997) 3933.*
*Sroczynski, et al., Nucl. Phys. B (PS) 83 (2000) 971.*
— (ii)×

To achieve calculations with ∼2% precision, we need
yet another approach which satisfies above condition (i)–(iii).
⇒ our proposal: *Anisotropic lattice*

# *Anisotropic lattice QCD*

Anisotropic lattice: $a_\tau < a_\sigma$ $\Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Quark action: $O(a)$ improved, along with Fermilab approach

□ bare anisotropy parameter $\gamma_F$:

in general, to be tuned mass dependently

e.g., using meson dispersion relation

∘ Continuum limit ? yes ! ☺

∘ Modest computational cost ? yes ! ☺

∘ Systematic improvement ?

If quark mass is sufficiently less than $a^{-1}$,

tuned paramerers for massless quark are applicable.

$\rightarrow$nonperturbative renormalziation technique

*Lüscher et al., Nucl.Phys. B 491 (1997) 323.*

Our expectation:

For $m_Q \ll a_\tau^{-1}$, (not necessarily $m_Q \ll a_\sigma^{-1}$),
mass dependences of parameters in the action is so small that the tuned parameters for massless quark are also applicable in such a quark mass region.

Then, systematic improvement is possible. ☺

$\Rightarrow$ To be justified numerically, and in perturbation theory.

If this is the case, mass dependence of $\gamma_F$ must be small.

( Without tuning, i.e. setting to $\gamma_F(m_q = 0)$,
heavy quark can be correctly described.)

For heavy-light systems: really hold ?

∘ Tree-level analysis — yes

∘ In numerical simulation — yes, as shown below

# *Anisotropic quark action*

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_\tau \left[ (1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_\sigma \sum_i \left[ (r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

*T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215*
*J. Harada et al., Phys. Rev. D 64 (2001) 074501*

- Constructed following the Fermilab approach.
  *El-Khadra et al., Phys. Rev. D 55 (1997) 3933*
- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted in several works.)
- Tadpole improvement: $c_E = 1/u_\sigma^2 u_\tau$, $c_B = 1/u_{\sigma^3}$
  $u_\sigma$, $u_\tau$: mean-field values of spatial and temporal link variables
- Parameters varied in simulations: $(\kappa, \gamma_F)$

$$\gamma_F \equiv \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0 \gamma_F + 4))$$

$\gamma_F$: bare anisotropy parameter
$m_0$: bare quark mass (in temporal lattice units)

At the tree level, with $r = 1/\xi$ and for small quark mass,

$$\frac{1}{\gamma_F} = \frac{1}{\xi}\left(1 + \frac{1}{3} m_0^2\right).$$

— no linear term in $m_0$ for $m_0 \ll a_\tau^{-1}$.

Small mass dependence was also numerically found.

*Matsufuru, Onogi and Umeda, Phys. Rev. D 64 (2001) 114503.*

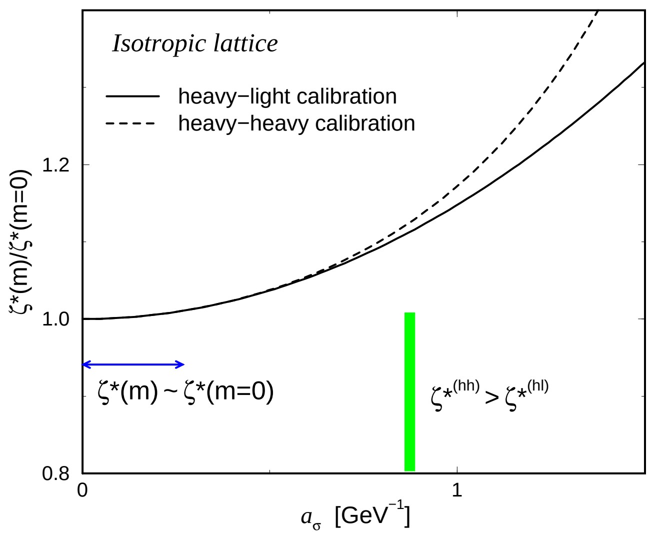
# *Conjecture of anisotropy parameter*

Conjecture of $a_\sigma$ dependence of tuned anisotropy parameter (figures roughly at charm quark mass)

- $\circ$ $a_\sigma^{hh\text{-}hl}$ : above which heavy-heavy and heavy-light systems are inconsistent for single $\gamma_F$ value.
  $\leftarrow O((a_\sigma p)^2)$ error in quarkonia, not improved by anisotropy
- $\circ$ $a_\sigma^{hl\text{-}ll}$ : below which $\gamma_F(m_q) \simeq \gamma_F(m_q = 0)$ holds, within certain (say, 2%) accuracy.

Isotropic lattice

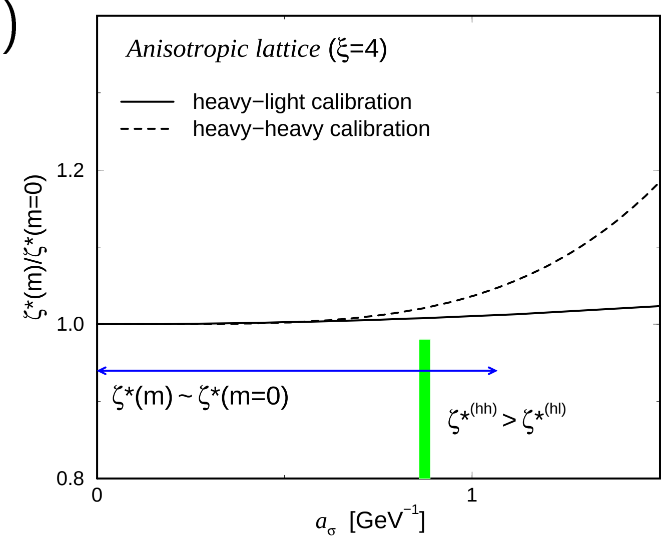
$a_\sigma^{hl\text{-}ll} < a_\sigma^{hh\text{-}hl}$




Anisotropic lattice ($\xi = 4$)

$a_\sigma^{hl\text{-}ll} \gtrsim a_\sigma^{hh\text{-}hl}$

$a_\sigma^{hl\text{-}ll}$ is extended by a factor $\xi$.




Below $a_\sigma^{hl\text{-}ll}$, $\gamma_F(m_q = 0)$ correctly describes heavy-light systems.

In the following, we fix $a_\sigma$ ($\simeq$ 1.6 GeV), set $\gamma_F$ to $\gamma_F(m_q{=}0)$, and study mass dependence of fermionic anisotropy $\xi_F$ .

## *Numerical simulation*

Simulation: on quenched anisotropic lattice of
size $16^3 \times 128$, $a_\sigma^{-1} \simeq$ 1.6 GeV and anisotropy $\xi = 4$.

Quark parameters

light quark: mass $\sim 1.5\, m_s$

heavy quark: 7 values with $m_Q =$ 1 – 6 GeV

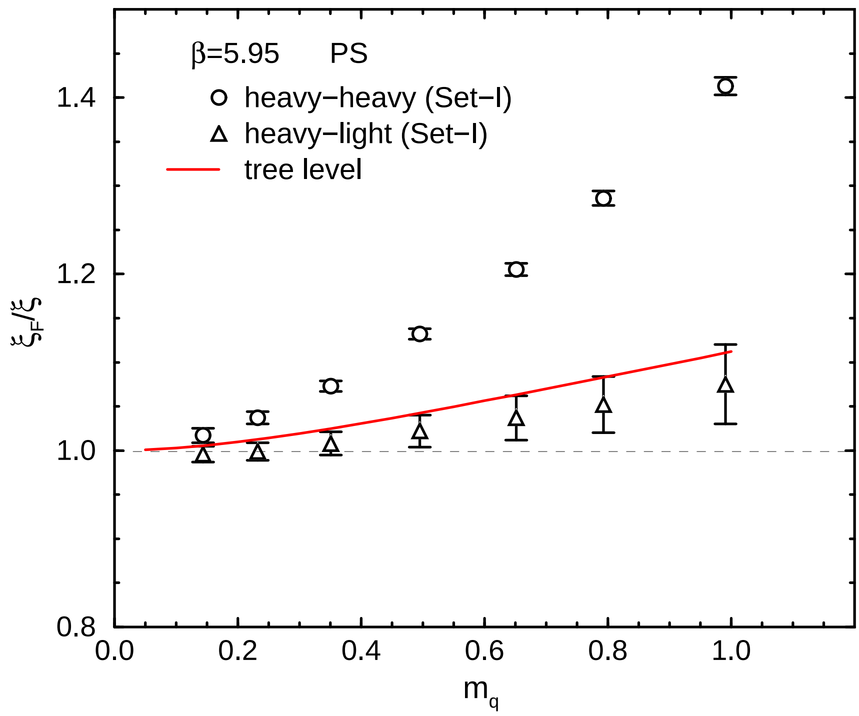
bare anisotropy: tuned value at massless limit ($\gamma_F = 4.016$)

Heavy-heavy, heavy-light meson dispersion relations

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$$

$\rightarrow$ fermionic anisotropy $\xi_F$

$\xi_F \neq \xi$ signals breaking of relativity relation




- $m_q < 0.2$ : both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy
- $m_q < 0.3$ : for heavy-light mesons, $\gamma_F$ tuned for massless qaurk can be applied within 2% accuracy

## *Conclusion and outlook*

Summary of numerical result:

on a lattice of $a_\sigma^{-1} = 1.6$ GeV and $\xi = 4$,

(a) $a_\sigma m_Q < 0.8$ : both heavy-heavy, heavy-light mesons are correctly described within 2% accuracy (charm quark mass is covered ☺)

(b) $a_\sigma m_Q < 1.2$ : for heavy-light mesons, $\gamma_F$ tuned for massless qaurk can be applied within 2% accuracy

(c) $1.2 < a_\sigma m_Q$, $O(a^2)$ systematic error gradually grows.
$\leftarrow$ inconsistency between $\xi_F$'s from PS and V mesons

For $a_\sigma m_Q < 1.2$, heavy-light systems are correctly described by $\gamma_F = \gamma_F(m_q = 0)$.
— mass dependence is actually small.

$\Rightarrow$ Tuned parameters for massless quark are expected to be applicable.

Outlook:

□ Further development with $\gamma_F = \gamma_F(m_q = 0)$.
- ∘ $O(a)$ improvement by nonperturbative renormalization

□ Mass dependent tuning for bottom quark region
- ∘ Tree level $\gamma_F$ may be a good approximation.

□ Applications to heavy-light matrix elements
$\rightarrow$ High precision computation actually possible ?

## *Heavy-light meson decay constant (preliminary)*

Simulation: quenched anisotropic lattices with $\xi = 4$,

- $16^3 \times 128$, $\beta = 5.95$, $a_\sigma^{-1} \simeq$ 1.6 GeV
- $20^3 \times 160$, $\beta = 6.10$, $a_\sigma^{-1} \simeq$ 2.0 GeV

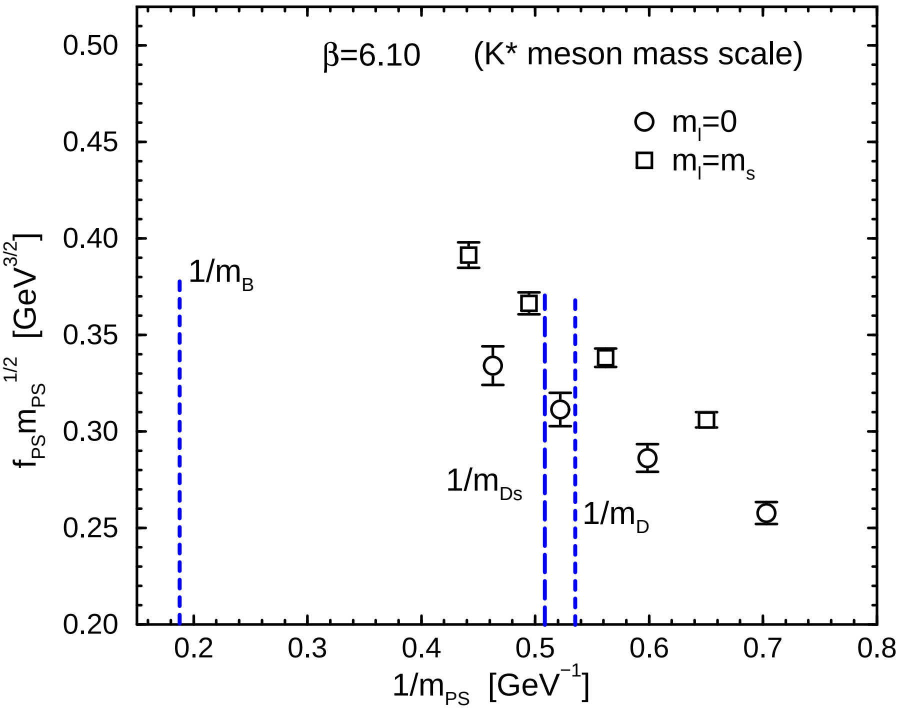
Light quark: 3 values with masses $1$–$1.5\ m_s$
$\rightarrow$ chiral extrapolation

Heavy quark: 4 values with masses $0.7$–$1.5$ GeV
(target mass: charm quark)

Heavy-light pseudoscalar meson decay constant:

- mean-field improved tree level matching

*J. Harada et al., hep-lat/0202004*




| | $\beta = 5.95$ | $\beta = 6.10$ | previous works* |
|---|---|---|---|
| $f_D/f_\pi$ | 1.516(44) | 1.536(49) | 1.55(11) |
| $f_{D_s}/f_D$ | 1.141(15) | 1.145(16) | 1.12(2) |

*Average of quenched works,
*S.Ryan, Nucl. Phys. B (PS) 106 (2002) 86.*