O(a) improved Wilson quark action on anisotropic lattice (I)

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Lattice2001
19–24 August 2001, Berlin, Germany

c.f. hep-lat/0107001 to appear in Phys.Rev.D

Introduction

Fine resolution in temporal direction $a_\tau < a_\sigma$

Advantages

- Finite temperature

  c.f. QCD-TARO (2000), Umeda et al.(2001)

- Heavy quark systems c.f. Harada et al.(2001)

- Better signal to noise ratio

  c.f. Ishii's talk glueball at $T > 0$

Disadvantages

- Tuning of anisotropy parameter ($\xi \equiv a_\sigma / a_\tau$)

  $\Longrightarrow$ High cost for light quark region

Goal of this work

- Tuning of anisotropy parameter at each β

  from chiral limit to c-quark mass region

Contents

1. Form of action
   i. Choice of spatial Wilson parameter
   ii. Dispersion relation for free quark
2. Calibration procedure
3. Calibration results
4. Conclusion

In the next talk

1. Systematic error of calibration
2. Light hadron spectroscopy
3. Conclusion & Outlook

We focus on quenched QCD in this work

Form of action

O(a) improved Wilson action on Anisotropic lattice

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$K(x,y) = \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4)\, U_4(x)\delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4})\delta_{x-\hat{4},y} \right]$$
$$- \kappa_\sigma \sum_i \left[(r-\gamma_i)\, U_i(x)\delta_{x+\hat{i},y} + (r+\gamma_i)\, U_i^\dagger(x-\hat{i})\delta_{x-\hat{i},y} \right]$$
$$- \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r\kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y}$$

with tadpole improvement $\kappa_\sigma = \dfrac{\tilde{\kappa}_\sigma}{u_\sigma}, \ \ \kappa_\tau = \dfrac{\tilde{\kappa}_\tau}{u_\tau}$

$u_\tau, \ u_\sigma$: mean-field values
in Landau gauge (self-consistent)

$$\gamma_F \equiv \frac{\tilde{\kappa}_\tau}{\tilde{\kappa}_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\tilde{\kappa}_\sigma} - 2(\gamma_F + 3r - 4) \ \ (= 2(m_0\gamma_F + 4))$$

This action follows the Fermilab approach

El-Khadra et al.(1997)

Choice of spatial Wilson parameter r

Small m_0 expansion of γ_F
from the dispersion relation of free quark

$$\frac{1}{\gamma_F} = \frac{1}{\xi}[1 + \frac{1}{2}(1 - r\xi)m_0 + \frac{1}{24}(-1 + 6r\xi + 3r^2\xi^2)m_0^2]$$

- $r = 1$

  Klassen (1998), Chen (2000), CP-PACS (2000)

  – Strong m_0 dependence of γ_F

- $r = 1/\xi$ (Our Choice)

  Umeda et al. (2001), Harada et al (2001)

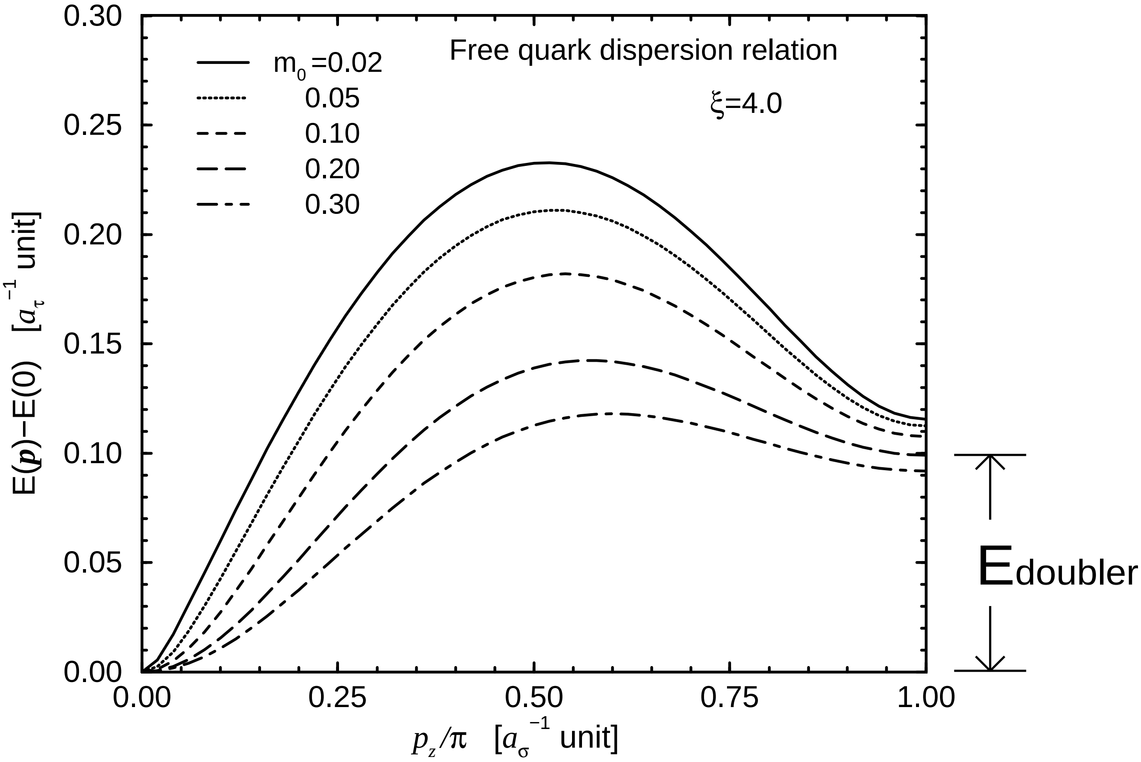
  – Small m_0 dependence of γ_F

$$\frac{1}{\gamma_F} = \frac{1}{\xi}[1 + \frac{1}{3}m_0^2]$$

$m_0 \simeq 0.3$ (for charm-quark) at $a_\tau^{-1} = 4\text{GeV}$

  – Smaller spatial Wilson term due to ξ
  $\Longrightarrow$ Need to check doubler effects

The dispersion relation of free quark



Typical momentum scale inside hadron (∼c-quark)

- light, heavy-light hadrons $\sim \Lambda_{QCD}$
- heavy quarkonium $\sim \frac{1}{2}mv^2 \sim$ 200MeV (for $c\bar{c}$)

β	5.75	5.95	6.10
$E_{doubler}$	∼400MeV	∼650MeV	∼800MeV
Safety	△	○	◎

Simulation parameters

Gauge parameters

Anisotropic plaquette action $(\xi = 4)$

β	γ_G	size	a_σ^{-1} (GeV)
5.75	3.072	$12^3 \times 96$	1.100(6)
5.95	3.1586	$16^3 \times 128$	1.623(9)
6.10	3.2108	$20^3 \times 160$	2.030(13)

from Sommer scale r_0

Quark parameters

$$\kappa = 0.124 \sim 0.093 \quad \text{(13 points)}$$
$$\text{(strange} \sim \text{charm quark mass)}$$
$$N_{conf} = 160 \text{ (heavy)} \sim 600 \text{ (light)}$$
$$\text{input } \gamma_F \; : \; 2 \sim 4 \text{ points } (\Rightarrow \text{interpolation})$$

at each β

Clover coefficients

Tadpole improved tree level

Calibration procedure

We search for a γ_F^* which satisfies

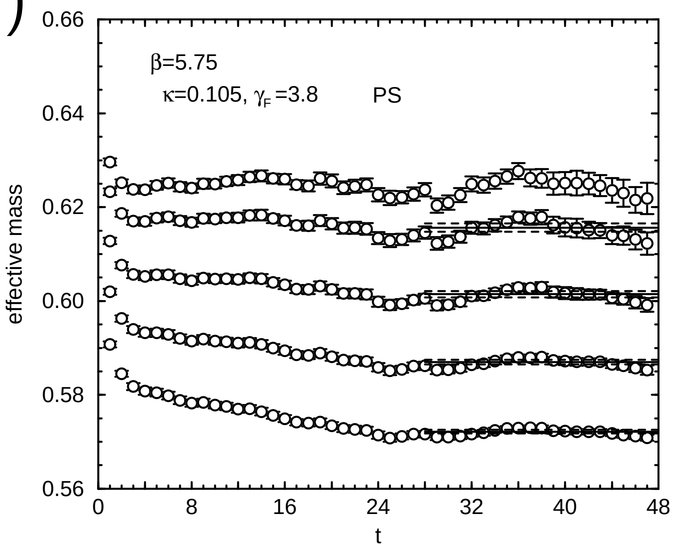
$$\xi_F(\beta,\gamma_G;\kappa,\gamma_F) = \xi_G(\beta,\gamma_G;\kappa,\gamma_F) = \xi$$

using the relativistic dispersion relation of meson

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$$

Carrying out the calibration with following 3 steps

(1)



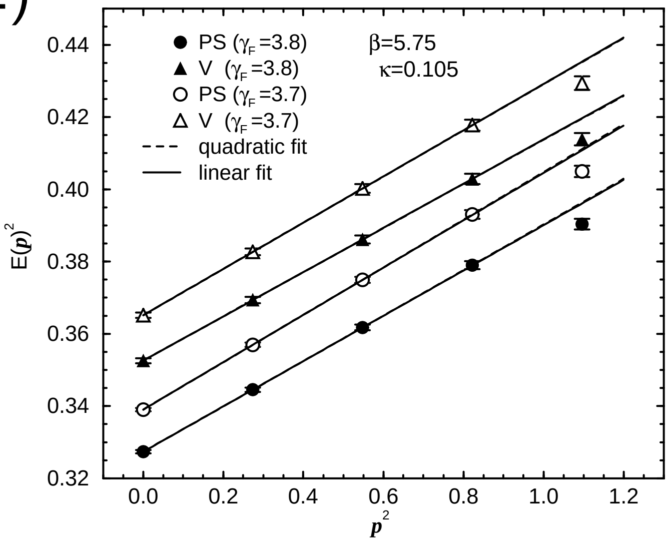
$E(\vec{p})$: meson mass with momentum $\vec{p}$

we use $\vec{p} = \vec{n}(2\pi/L)$

$\vec{n} = (0,0,0), (1,0,0), (1,1,0), (1,1,1), (2,0,0)$

Calibration procedure

(2)

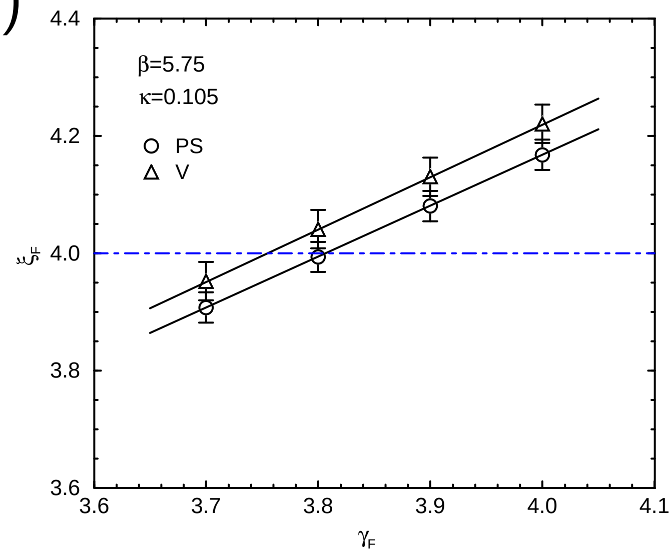


Determination of ξ_F from dispersion relation

- linear fit
- quadratic fit

(lowest 3 and 4 states at this β)

(3)

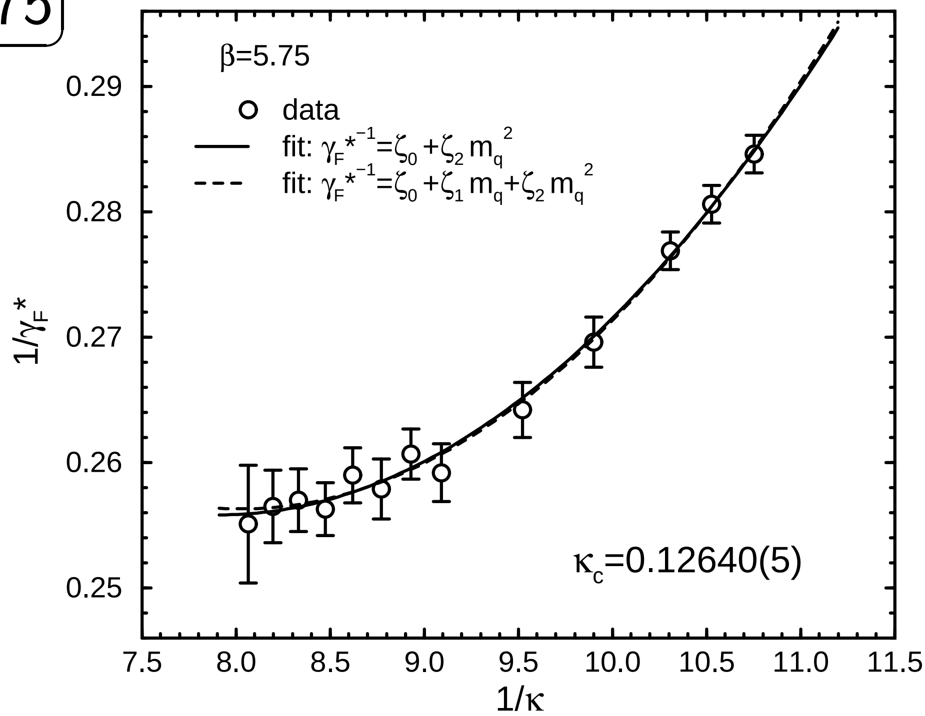


Linear interpolation to $\xi_F = \xi$ from several γ_F points $\Longrightarrow \gamma_F^*$

$\delta\gamma_F^* = \gamma_F^*(\mathrm{v}) - \gamma_F^*(\mathrm{ps})$ is discussed in next talk

Calibration results

$\beta = 5.75$



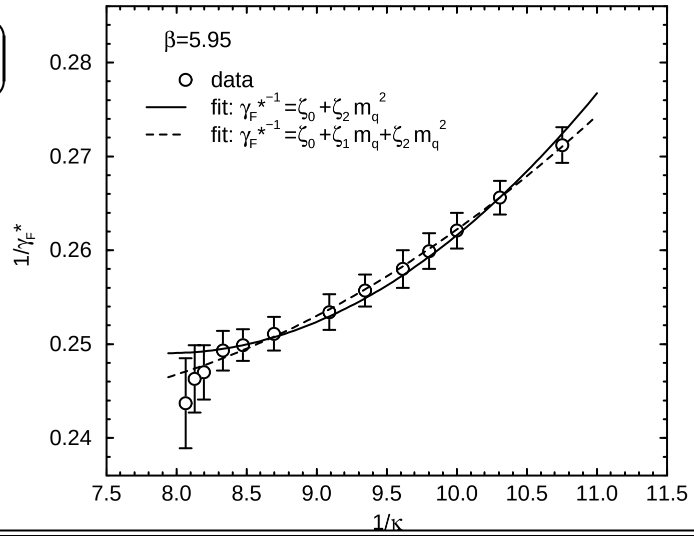
$$\frac{1}{\gamma_F^*} = \zeta_0 + \zeta_1 m_q + \zeta_2 m_q^2 \qquad m_q = \frac{1}{2\xi}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

β	fit-type	ζ_0	ζ_1	ζ_2	$\gamma_F^*\ (m_q = 0)$
5.75	—— fit	0.2558(9)	—	0.230(12)	3.909(14)
	- - - fit	0.2564(23)	–0.007(28)	0.247(68)	3.901(34)

This function is applicable to

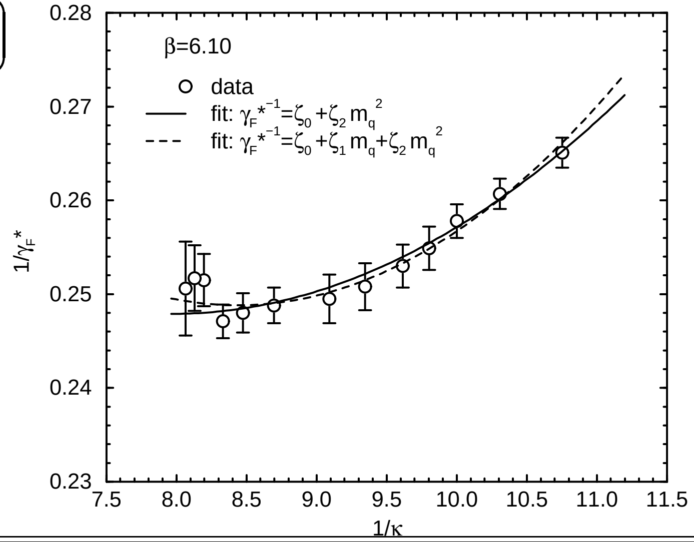
chiral limit $\sim$ charm-quark region

$\beta = 5.95$



β	fit-type	ζ_0	ζ_1	ζ_2	$\gamma_F^* \ (\mathrm{m_q} = 0)$
5.95	—— fit	0.2490(8)	—	0.189(15)	4.016(13)
	- - - fit	0.2465(18)	0.036(23)	0.095(61)	4.057(30)

$\beta = 6.10$



β	fit-type	ζ_0	ζ_1	ζ_2	$\gamma_F^* \ (\mathrm{m_q} = 0)$
6.10	—— fit	0.2479(9)	—	0.143(14)	4.034(14)
	- - - fit	0.2493(18)	–0.022(24)	0.200(63)	4.011(28)

Conclusion

Systematic study of
anisotropic O(a) improved Wilson action

Spatial Wilson parameter $r = 1/\xi$
$\Longrightarrow$ small m_0 dependence of γ_F

Determination of
γ_F^* as a function of m_q nonperturbatively

$$\frac{1}{\gamma_F^*} = \zeta_0 + \zeta_1 m_q + \zeta_2 m_q^2 \qquad m_q = \frac{1}{2\xi}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

applicable to the region
from chiral limit to charm quark mass

Systematic error & application $\Longrightarrow$ next talk