Spectral function with maximum entropy method on anisotropic lattice

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Introduction

Hadron properties at Finite Temperature:

What happens on hadrons

or what new effects above T_c ?

Quark Gluon Plasma formation

$\Downarrow$

- J/ψ suppression above T_c

  Matsui and Satz (1986), Hashimoto et al (1986)

- Mass shift near T_c

  Hashimoto et al (1986)

Spectral Function at $T > 0$

from Maximum Entropy Method

Contents

Maximum Entropy Method (MEM)

Reconstruction of a Spectral Function with MEM

Asakawa et al. (1999)

$$C(\tau) = \int d\omega K(\tau,\omega) A(\omega)$$

kernel : $K(\tau,\omega) = e^{-\omega\tau} + e^{-\omega(T-\tau)}$

standard χ^2-fit $\Longrightarrow$ ill-posed problem

$\Longrightarrow$ MEM (based on Bayes' theorem)

Maximization of $Q = \alpha S - L$

$$S = \int d\omega \left(A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right)$$

$m(\omega) = m_0 \omega^2$: default model

α : constant $\Rightarrow$ to be integrated out

L : Likelihood function (χ^2-term)

Singular Value Decomposition (SVD)

$$\begin{aligned} K(\tau,\omega) &= e^{-\omega\tau} + e^{-\omega(T-\tau)} \\ &= V(\tau,\tau')w(\tau',\tau'')U(\omega,\tau'')^t \end{aligned}$$

$w(\tau,\tau')$: diagonal matrix

$$A(\omega) = m_0\omega^2 \prod_{i=1}^{N} \exp\{b_i u_i(\omega)\}$$

$u_i(\omega)$ $(= U(\omega,\tau_i))$: basis in singular space

b_i : parameters

▷ Figure

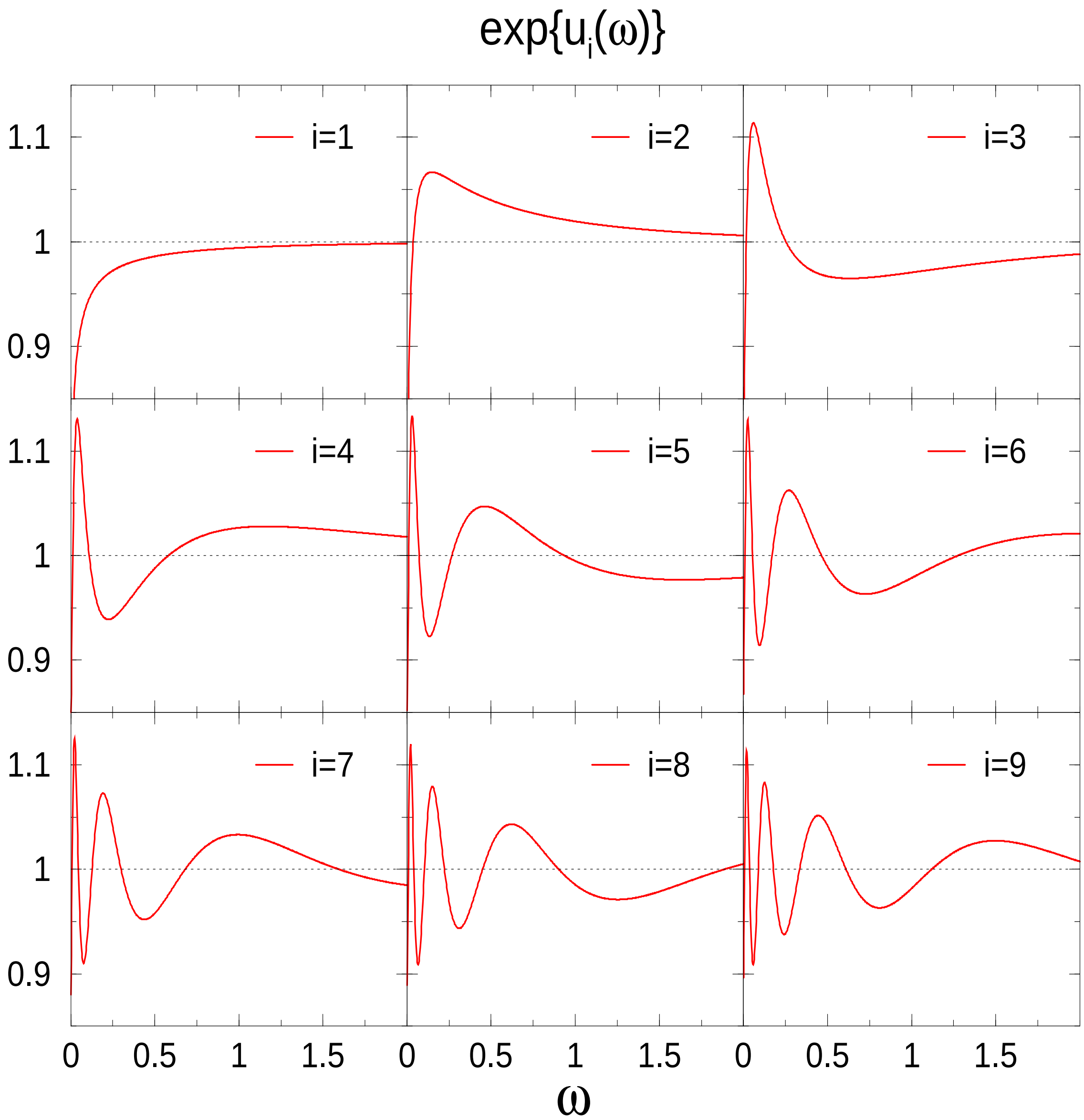
- need for large N (fine temporal resolution)
- difficult to reconstruct

narrow peak at $\omega \gg 1$

wide peak at $\omega \ll 1$

Singular Value Decomposition (2)

$$A(\omega) = m_0\omega^2 \prod_{i=1}^{N} \exp\{b_i u_i(\omega)\}$$



Approach for Finite temperature(1)

- Large data points for reconstruction of $A(\omega)$

  $\Longrightarrow$ Anisotropic lattice

- Restriction from SVD
  - High statistics
  - Shift of zero point $\omega \rightarrow \omega + \mathrm{dm}$

$$e^{\mathrm{dm}\cdot\tau} C(\tau) = \int d\omega \; K(\tau,\omega) A'(\omega)$$

$$\Longrightarrow A(\omega) = A'(\omega - \mathrm{dm})$$

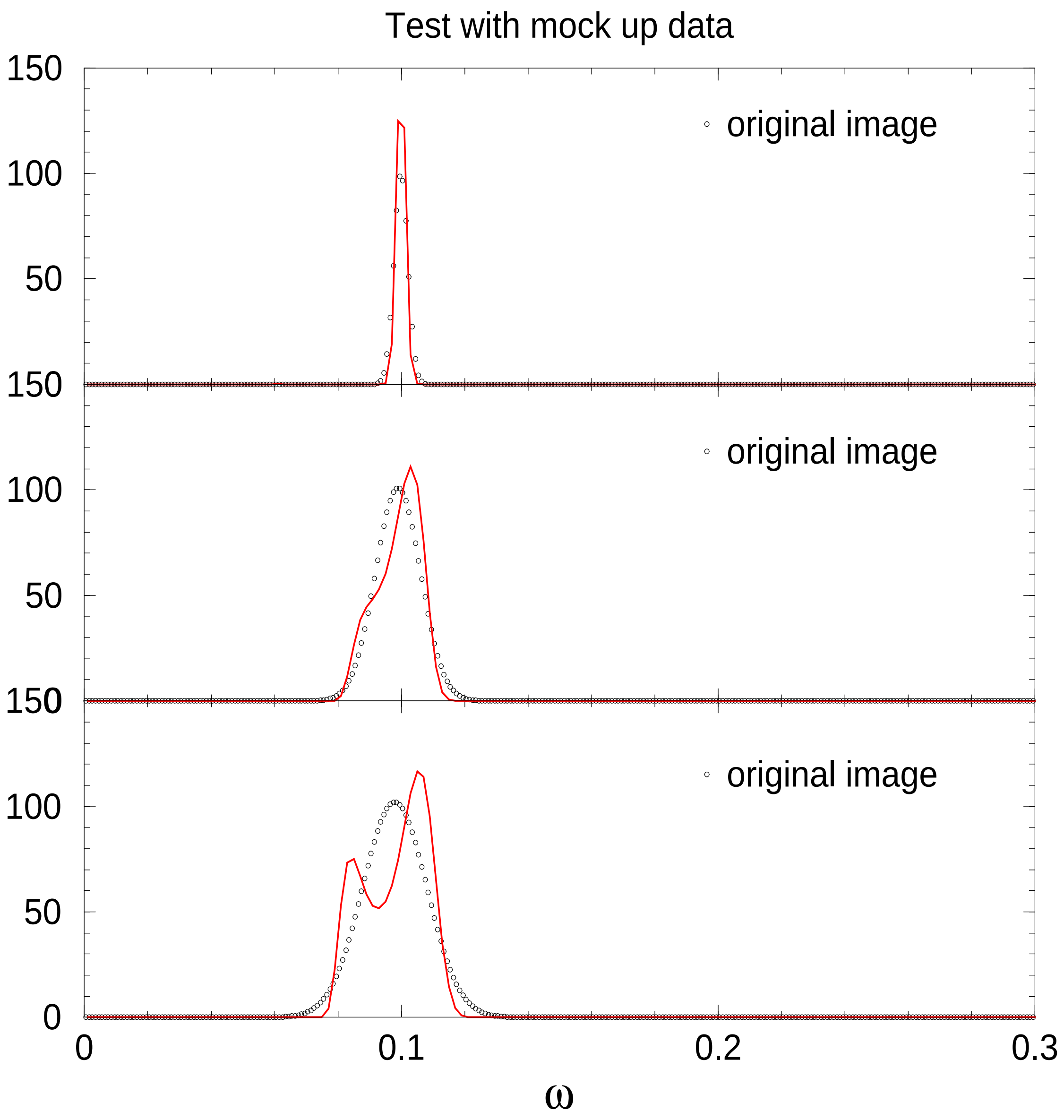
(real*8 code works on heavy quarks)

- Check of $\mathrm{nt_{max}}$ dependence at $T = 0$

  τ-range : $1 \sim \mathrm{nt_{max}}$

Approach for Finite temperature(2)

- Estimation for width of Spectral function



Simulation parameters

Gauge parameters

Anisotropic plaquette action $(a_\sigma/a_\tau = 4)$

Scale is determined from Sommer scale r_0

β	γ_G	size	a_σ^{-1} (GeV)
6.10	3.2108	$20^3 \times N_t$	2.030(13)
	N_t	T/T_c	conf.#
	160	~ 0	500
	32	~ 0.9	500
	26	~ 1.1	500

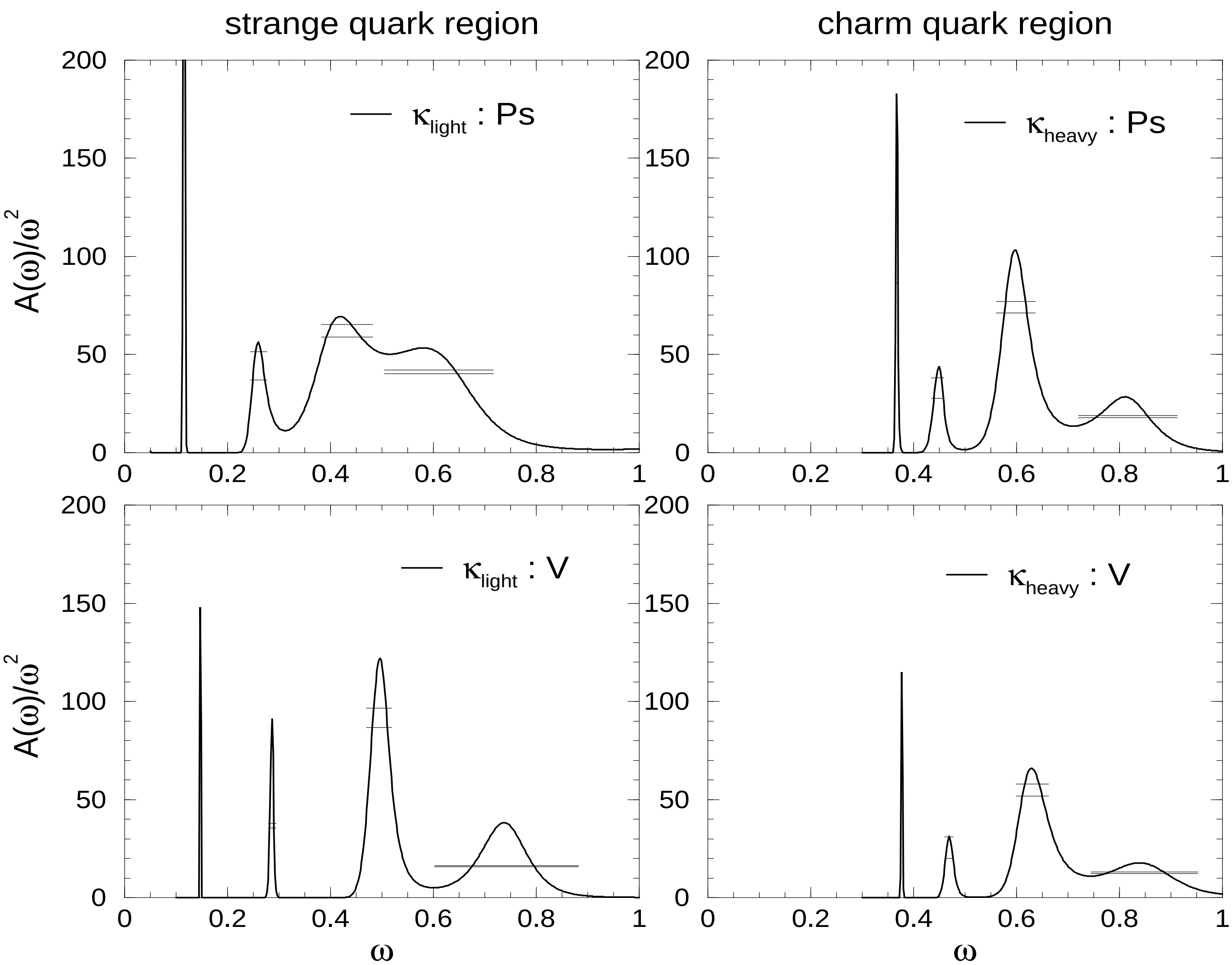
Quark parameters

$O(a)$ improved Wilson quark action

Clover coefficients : Tadpole improved tree level

	κ	γ_F	quark mass
κ_{light}	0.1235	4.034	$\sim$ strange quark
κ_{heavy}	0.1120	4.000	$\sim$ charm quark

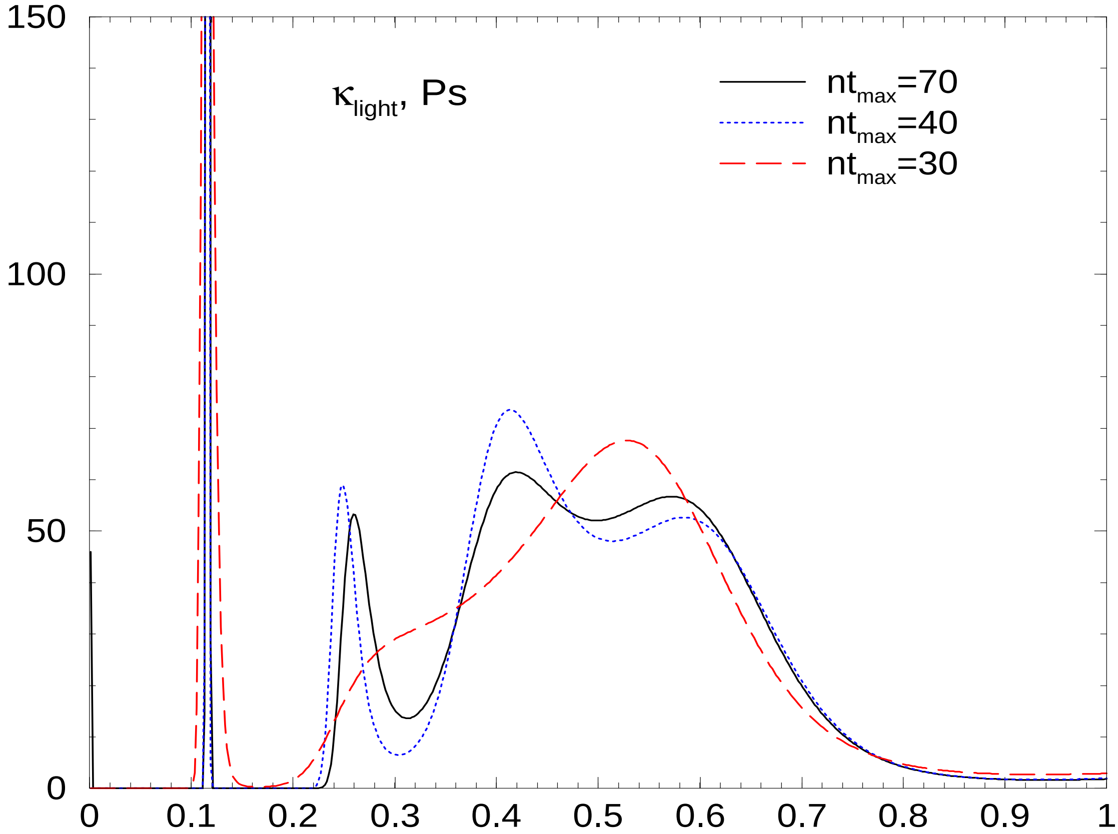
Spectral function at T = 0



These are consistent with

pole fit or previous works

nt_{max} dependence $(T = 0)$

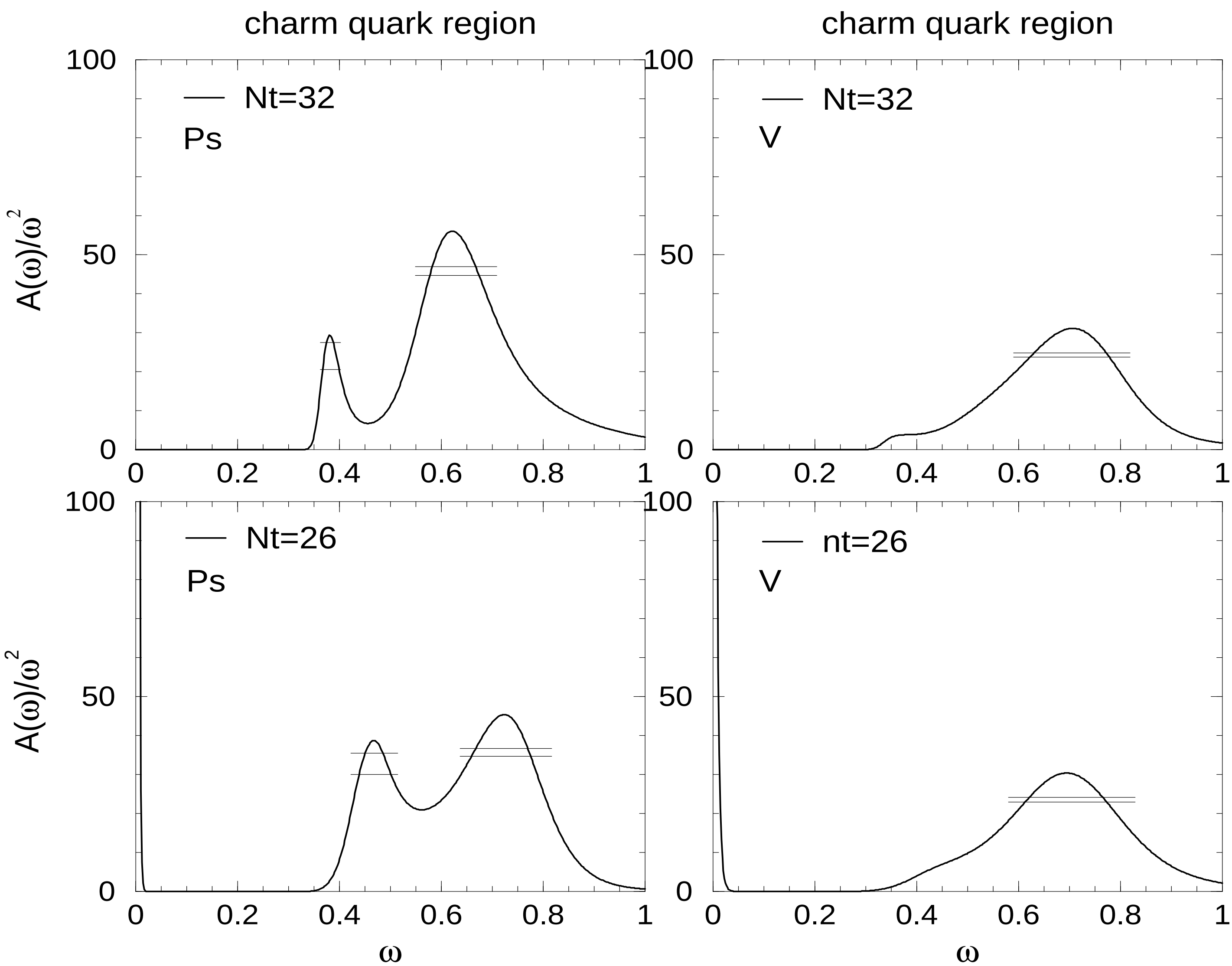


▷ MEM work $(nt_{max} \cdot a_T) \gtrsim 1\text{fm}$

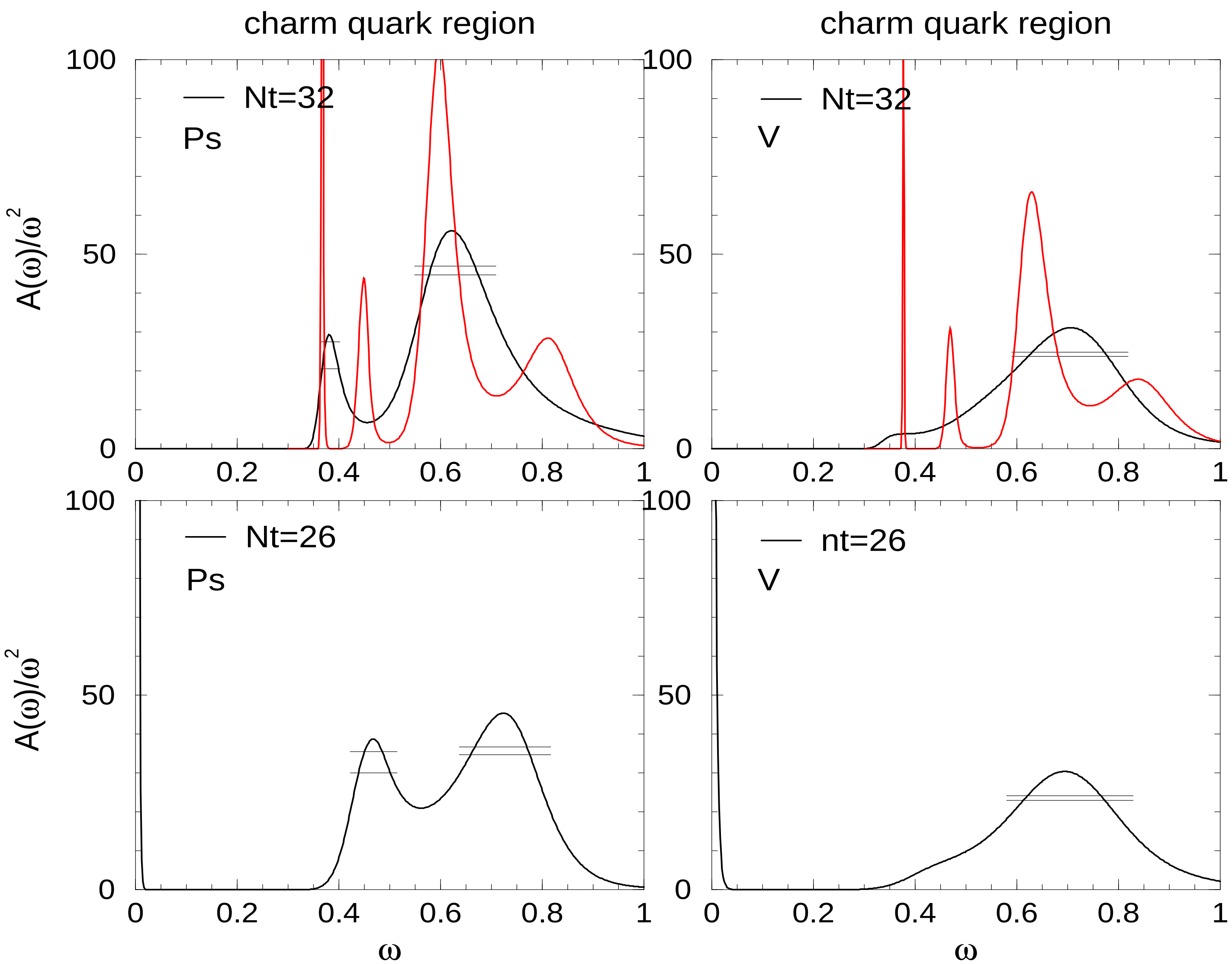
It is difficult to reconstruct

from correlator with multi states

Spectral function at T > 0



Spectral function at T > 0



Not correctly reconstructed at T > 0

Discussion

It is difficult to reconstruct
a Spectral Function at $T > 0$ with MEM

- ▷ temporal resolution is enough
  $a_{\mathcal{T}} \simeq 8\mathrm{GeV} \Longrightarrow 1/T_c \sim 28 a_{\mathcal{T}} \quad (N \leq 14)$
- ▷ larger statics is needed
  present stat.$=$ 500conf. $\Longrightarrow$??
- ▷ MEM works $(nt_{max} \cdot a_{\mathcal{T}}) \gtrsim 1\mathrm{fm}$
  $\Longrightarrow$ using smeared operators (?)
- ▷ Improvement of default model $m(\omega)$