

SU(3) Lattice QCD Study for Static Three-Quark Potential

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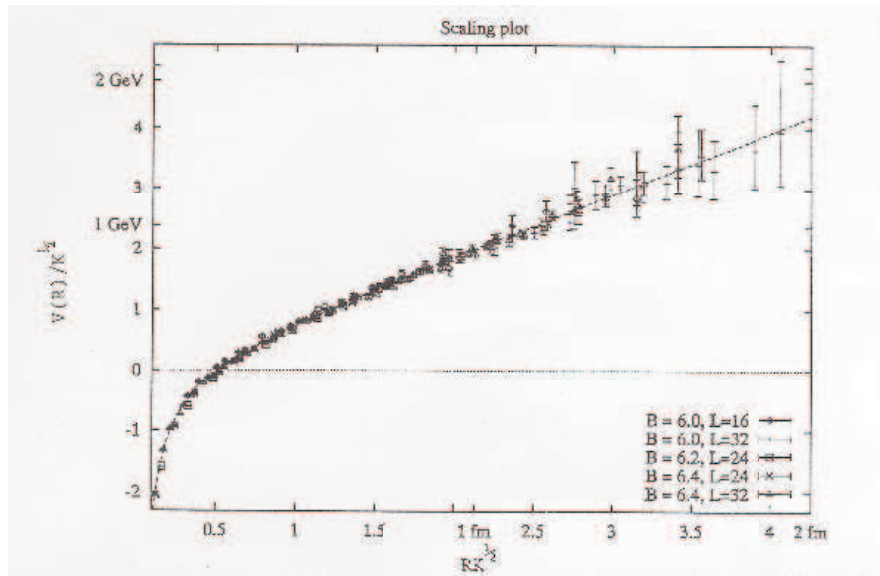
hep-lat/0006005

Phys.Rev.Lett. **86** 18-21 (2001)

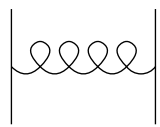
- Introduction
- Methods
- Results
- Summary


Introduction(1)

$Q\bar{Q}$ potential



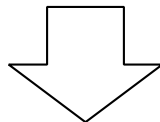
G.S.Bali and K.Schilling Phys.Rev.D46(1992)2636

Coulomb term $-\frac{A_{Q\bar{Q}}}{R}$ 

+
linear term $\sigma_{Q\bar{Q}}R$ 

(string tension: $\sigma_{Q\bar{Q}} \sim 0.89\text{GeV}/\text{fm}$)

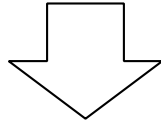
+
constant $C_{Q\bar{Q}}$



- Quark Confinement
- Meson Features

Introduction(2)

3Q potential



Responsible for baryon features

- Regge trajectory
- Kogut and Susskind, PRD11(1975)395
(Strong coupling limit)
- Capstic and Isgur, PRD34(1986)2809
(Model calculation)

— early lattice works —

- Sommer and Wosiek,
PLB149(1984)497, NPB267(1986)531
- Thacker, Eichten and Sexton, Lattice '87

QCD → 3Q Potential

→ Baryon Properties

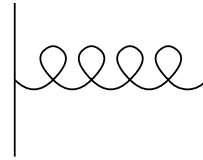
Theoretical consideration(1)

Theoretical Consideration

- short distance -

Coulomb type due to one-gluon-exchange

$$- \sum_{i < j} \frac{A_{3Q}}{|r_i - r_j|}$$

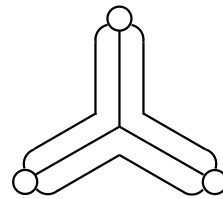


- long distance -

Three-body force proportional to the length of flux

$$\sigma_{3Q} L$$

(L:length of flux)

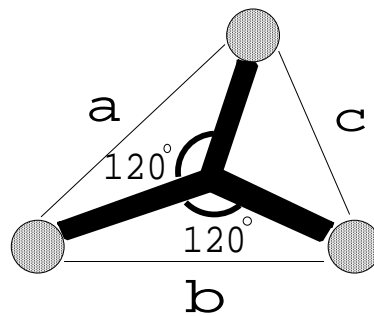


+ Constant term

Theoretical consideration(2)

$$V_{3Q} = - \sum_{i < j} \frac{A_{3Q}}{|r_i - r_j|} + \sigma_{3Q} L_{\min} + C_{3Q}$$

- Minimal linking length -



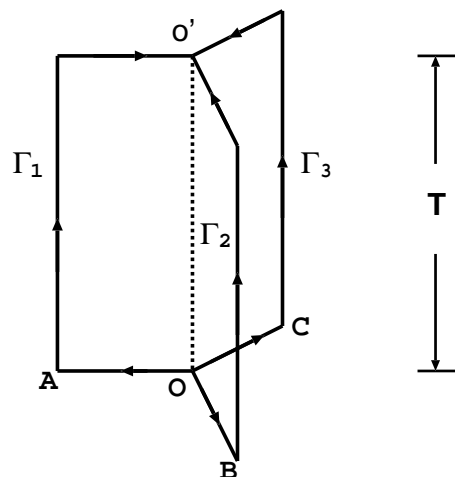
$$L_{\min} = \left(\frac{1}{2}(a^2 + b^2 + c^2) + \frac{\sqrt{3}}{2} \sqrt{(-a + b + c)(a - b + c)(a + b - c)(a + b + c)} \right)^{1/2}$$

Direct calculation of 3Q Potential
in lattice QCD

→ Comparison with the theoretical form

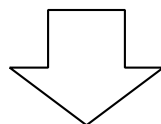
Method(1)

- 3Q Wilson loop-



$$U_i \equiv \text{P exp} \left(ig \int_{\Gamma_i} dx_\mu A_\mu(x) \right)$$

$$W_{3Q} \equiv \frac{1}{3!} \epsilon^{a_1 a_2 a_3} \epsilon^{b_1 b_2 b_3} U_1^{a_1 b_1} U_2^{a_2 b_2} U_3^{a_3 b_3}$$



$$\begin{aligned} \langle W_{3Q} \rangle &= C_0 e^{-V(g.s.)T} + C_1 e^{-V(1st\ e.s.)T} + \dots \\ &\sim C_0 e^{-V(g.s.)T} \quad (\text{large } T) \end{aligned}$$

But, signal is suppressed exponentially
Difficult to take large T

Method(2)

- Smearing Technique -

Enhancement of Ground-state Component

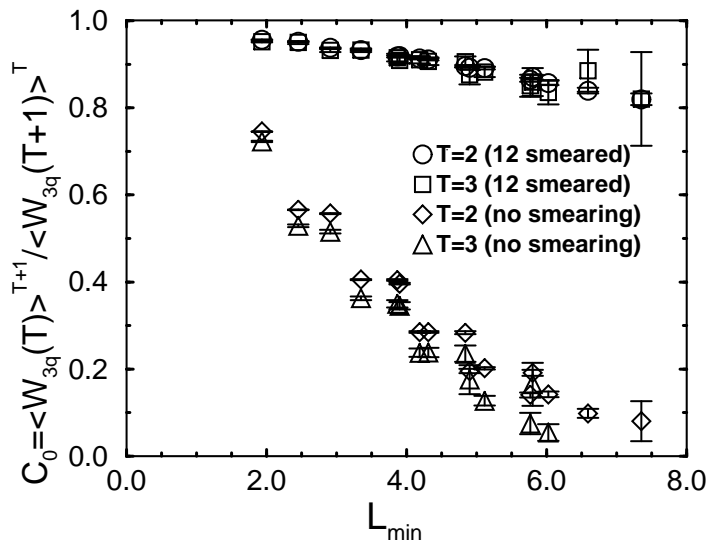
Maximize

$$\text{Re tr} \left\{ \bar{U}_i^\dagger(s) \left[\alpha U_i(s) + \sum_{j \neq i} \{ U_j(s) U_i(s + \hat{j}) U_j^\dagger(s + \hat{i}) + U_j^\dagger(s - \hat{j}) U_i(s - \hat{j}) U_j(s + \hat{i} - \hat{j}) \} \right] \right\}$$

$$\uparrow = N \left(\alpha \uparrow + \begin{array}{c} \text{---} \text{---} \text{---} \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{---} \text{---} \end{array} \right)$$

$$\langle W_{3Q} \rangle = C_0 e^{-V(g.s.)T} + C_1 e^{-V(1st\ e.s.)T} + \dots$$

$$C_0 \simeq 1, \quad C_1, C_2, \dots \ll 1$$



We can extract ground-state potential even with small T.

Result(1)

QCD Standard plaquette action

At quenched level

$\beta = 5.7$ $12^3 \times 24$ lattice

16 patterns of 3 quark configurations

$\alpha = 2.3$ (smearing parameter), $N_{\text{smear}} = 12$

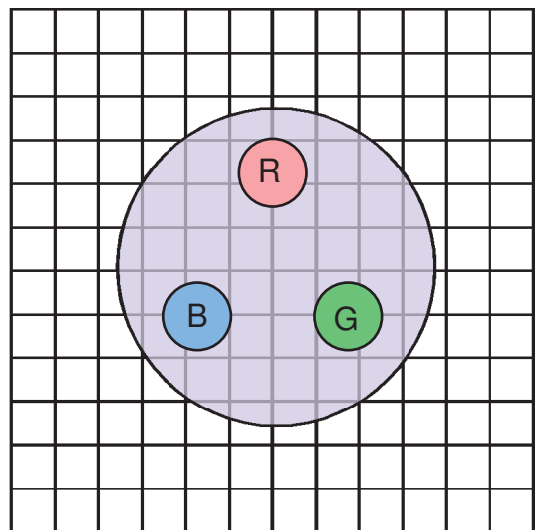
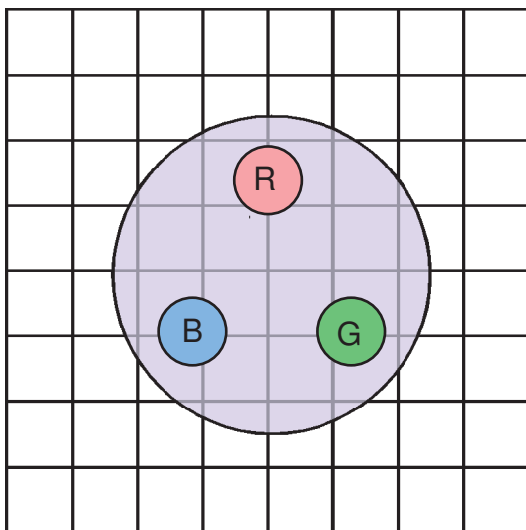
210 gauge configurations

$\beta = 5.8$ $16^3 \times 32$ lattice

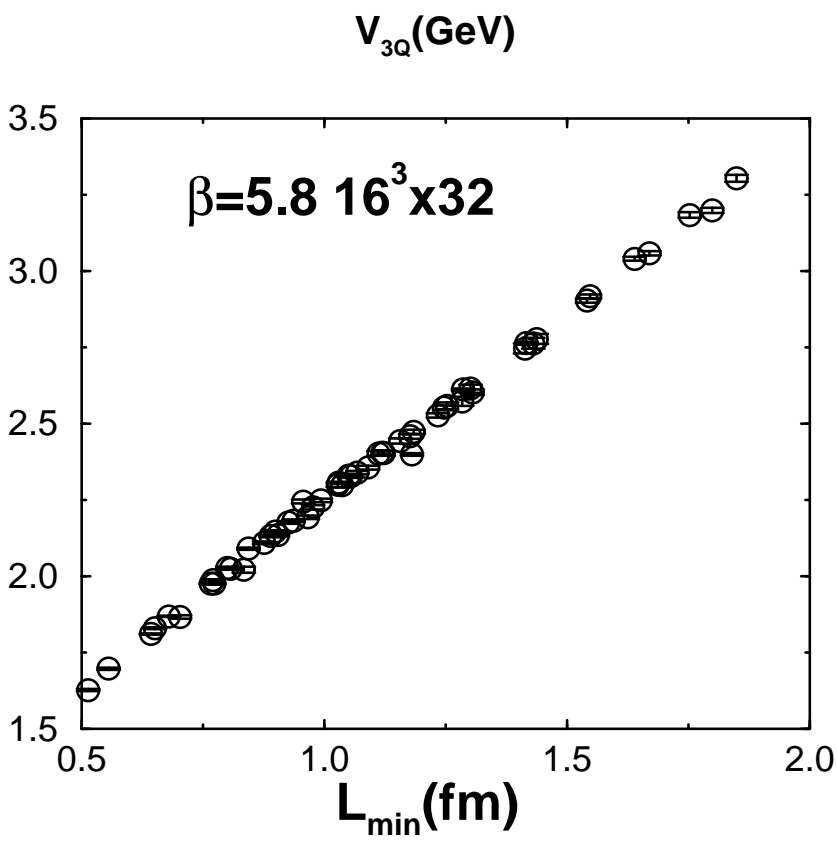
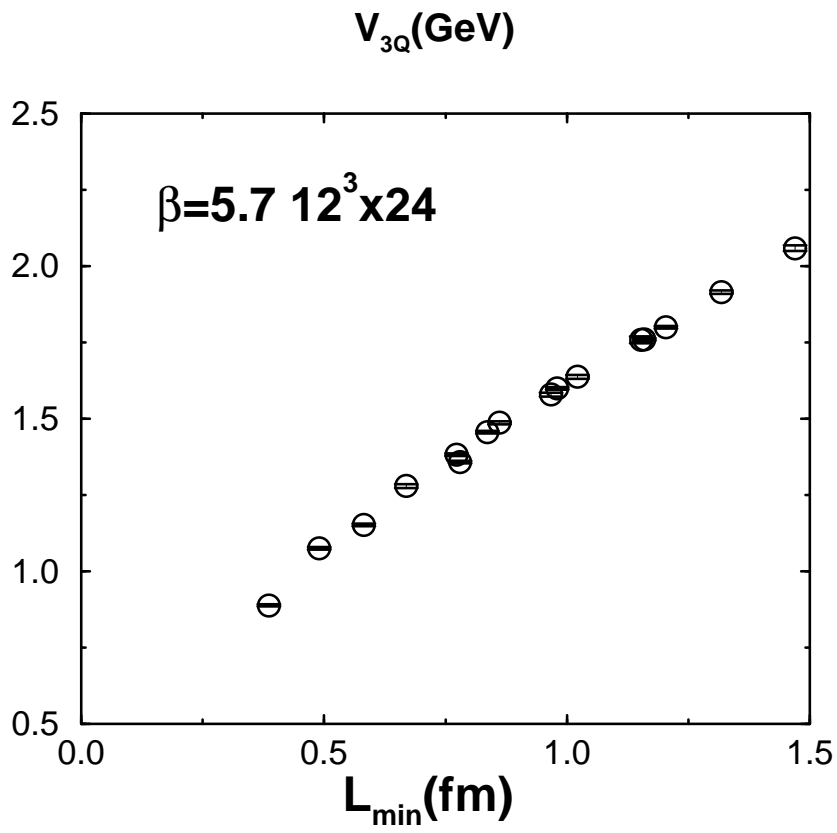
66 patterns of 3 quark configurations

$\alpha = 2.3$ (smearing parameter), $N_{\text{smear}} = 25$

200 gauge configurations



Result(2)



Result(3)

$\beta = 5.7$ (lattice unit $a \simeq 0.19$ fm)

	$\sigma(\text{Gev}/\text{fm})$	A	C
3Q	0.844(15)	0.1331(66)	0.9182(213)
Q \bar{Q}	0.902(26)	0.2793(116)	0.6203(161)

$\beta = 5.8$ (lattice unit $a \simeq 0.14$ fm)

	$\sigma(\text{Gev}/\text{fm})$	A	C
3Q	1.019(7)	0.1304(17)	0.9326(53)
Q \bar{Q}	1.109(27)	0.2580(159)	0.6081(182)

- universality of string tension $\sigma_{Q\bar{Q}} \simeq \sigma_{3Q}$
- consistency with P-QCD $A_{Q\bar{Q}} \simeq 2A_{3Q}$

Summary

We have carried out the detailed analysis of 3Q potential using the SU(3) lattice QCD at quenched level. Here, smearing was essential for this analysis. V_{3Q} is well described with the following form

$$V_{3Q} = - \sum_{i < j} \frac{A_{3Q}}{|r_i - r_j|} + \sigma_{3Q} L_{\min} + C_{3Q}$$

L_{\min} : minimal linking length for 3 quarks

Here, we have observed 2 remarkable features.

- universality of string tension $\sigma_{Q\bar{Q}} \sim \sigma_{3Q}$
- consistency with P-QCD $A_{Q\bar{Q}} \sim 2A_{3Q}$

Potential Table

$\beta=5.7$ ($a\sim 0.2$ fm)

$12^3 \times 24$ lattice

16 patterns of 3 quark configurations

	σ	A	C
3Q	0.1524(28)	0.1331(66)	0.9182(213)
QQ \bar{Q}	0.1629(47)	0.2793(116)	0.6203(161)

(i, j, k)	V_{3Q}^{latt}	V_{3Q}^{fit}	$V_{3Q}^{\text{latt}} - V_{3Q}^{\text{fit}}$
(1, 1, 1)	1.0231	0.0038	0.0141
(1, 1, 2)	1.2179	0.0061	0.0035
(1, 1, 3)	1.4154	0.0049	0.0197
(1, 2, 2)	1.3870	0.0046	-0.0016
(1, 2, 3)	1.5588	0.0060	0.0008
(1, 3, 3)	1.7141	0.0043	-0.0053
(2, 2, 2)	1.5216	0.0033	-0.0013
(2, 2, 3)	1.6745	0.0115	-0.0010
(2, 3, 3)	1.8242	0.0054	0.0074
(3, 3, 3)	1.9607	0.0092	0.0171
(0, 1, 1)	0.8459	0.0036	-0.0069
(0, 1, 2)	1.0970	0.0041	-0.0052
(0, 1, 3)	1.2935	0.0039	0.0010
(0, 2, 2)	1.3164	0.0041	-0.0098
(0, 2, 3)	1.5032	0.0058	-0.0036
(0, 3, 3)	1.6741	0.0040	-0.0066

Potential Table

$\beta=5.8$ ($a\sim 0.14$ fm)

$16^3 \times 32$ lattice

66 patterns of 3 quark configurations

	σ	A	C
3Q	0.0999(7)	0.1304(17)	0.9326(53)
Q \bar{Q}	0.1087(26)	0.2580(159)	0.6081(182)

(i, j, k)	V_{3Q}^{latt}	V_{3Q}^{fit}	$V_{3Q}^{\text{latt}} - V_{3Q}^{\text{fit}}$
(1, 1, 1)	0.9144	0.9008	0.0136
(1, 1, 2)	1.0657	1.0582	0.0075
(1, 1, 3)	1.1962	1.1883	0.0079
(1, 1, 4)	1.3049	1.3054	-0.0005
(1, 1, 5)	1.4134	1.4162	-0.0029
(1, 1, 6)	1.5339	1.5236	0.0103
(1, 2, 2)	1.1866	1.1877	-0.0011
(1, 2, 3)	1.3062	1.3076	-0.0014
(1, 2, 4)	1.4153	1.4203	-0.0050
(1, 2, 5)	1.5259	1.5287	-0.0029
(1, 2, 6)	1.6360	1.6347	0.0013
(1, 3, 3)	1.4178	1.4209	-0.0031
(1, 3, 4)	1.5216	1.5296	-0.0080
(1, 3, 5)	1.6276	1.6357	-0.0081
(1, 3, 6)	1.7296	1.7401	-0.0105
(1, 4, 4)	1.6291	1.6356	-0.0064
(1, 4, 5)	1.7190	1.7396	-0.0206
(1, 4, 6)	1.8203	1.8425	-0.0222
(2, 2, 2)	1.2793	1.2838	-0.0046
(2, 2, 3)	1.3815	1.3903	-0.0089
(2, 2, 4)	1.4904	1.4969	-0.0064
(2, 3, 3)	1.4751	1.4880	-0.0129
(2, 3, 5)	1.6814	1.6912	-0.0098
(2, 3, 6)	1.7847	1.7931	-0.0084
(2, 4, 4)	1.6800	1.6869	-0.0069
(2, 4, 5)	1.7669	1.7862	-0.0193
(3, 3, 3)	1.5562	1.5746	-0.0185
(3, 3, 4)	1.6475	1.6692	-0.0217
(3, 4, 6)	1.9425	1.9496	-0.0071
(3, 5, 5)	1.9322	1.9446	-0.0124

(i, j, k)	V_{3Q}^{latt}	V_{3Q}^{fit}	$V_{3Q}^{\text{latt}} - V_{3Q}^{\text{fit}}$
(4, 5, 6)	2.1263	2.1094	0.0169
(4, 6, 6)	2.2264	2.1979	0.0285
(5, 6, 6)	2.3105	2.2733	0.0372
(0, 1, 1)	0.7701	0.7727	-0.0027
(0, 1, 2)	0.9659	0.9694	-0.0035
(0, 1, 3)	1.1107	1.1071	0.0037
(0, 1, 4)	1.2323	1.2268	0.0055
(0, 1, 5)	1.3339	1.3388	-0.0049
(0, 1, 6)	1.4566	1.4469	0.0097
(0, 1, 7)	1.5730	1.5527	0.0203
(0, 1, 8)	1.6779	1.6570	0.0209
(0, 2, 2)	1.1375	1.1422	-0.0047
(0, 2, 3)	1.2660	1.2711	-0.0050
(0, 2, 4)	1.3815	1.3871	-0.0056
(0, 2, 5)	1.4928	1.4972	-0.0043
(0, 2, 6)	1.6069	1.6041	0.0028
(0, 2, 8)	1.8285	1.8128	0.0157
(0, 3, 3)	1.3893	1.3940	-0.0047
(0, 3, 4)	1.5007	1.5066	-0.0059
(0, 3, 5)	1.6135	1.6145	-0.0010
(0, 4, 4)	1.6079	1.6165	-0.0086
(0, 4, 5)	1.7085	1.7226	-0.0141
(0, 4, 6)	1.8263	1.8267	-0.0004
(0, 4, 7)	1.9328	1.9297	0.0031
(0, 4, 8)	2.0399	2.0319	0.0080
(0, 5, 5)	1.7994	1.8271	-0.0277
(0, 5, 6)	1.9205	1.9300	-0.0096
(0, 6, 6)	2.0302	2.0319	-0.0018
(0, 6, 7)	2.1390	2.1330	0.0060
(0, 6, 8)	2.2371	2.2337	0.0035