SU(3) Lattice QCD Study for Static Three-Quark Potential

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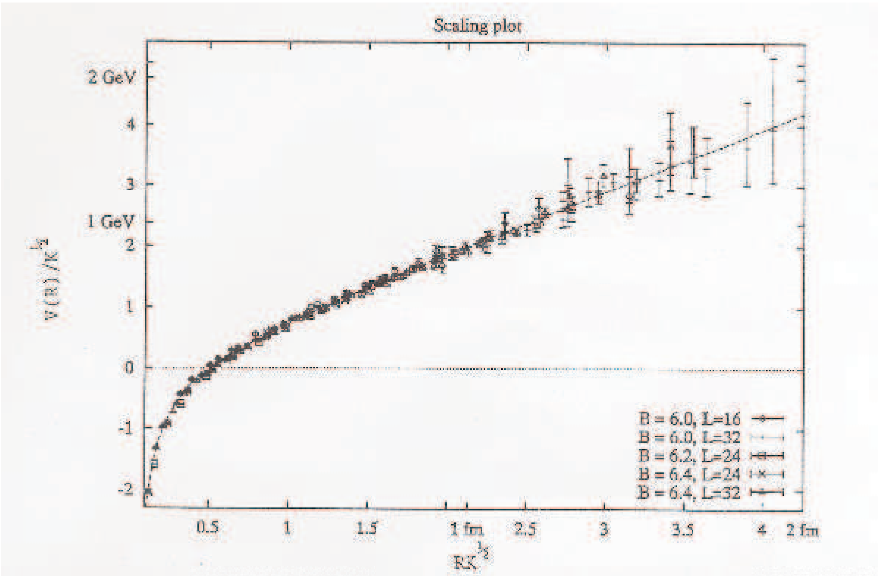
- Introduction
- Methods
- Results
- Summary

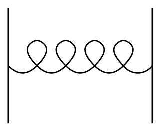
Introduction

$Q\bar{Q}$ potential



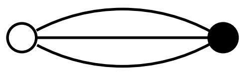
G.S.Bali and K.Schilling Phys.Rev.D**46**(1992)2636

Coulomb term $-\frac{A_{\mathrm{Q\bar{Q}}}}{R}$



$+$

linear term $\sigma_{\mathrm{Q\bar{Q}}} R$



(string tension:$\sigma_{\mathrm{Q\bar{Q}}} \sim 0.89\mathrm{GeV/fm}$)

$+$

constant $C_{\mathrm{Q\bar{Q}}}$

- Quark Confinement
- Meson Features

Introduction

3Q potential

Responsible for baryon features

- Regge trajectory
- Kogut and Susskind, PRD11(1975)395 (Strong coupling limit)
- Capstic and Isgur, PRD34(1986)2809 (Model calculation)

— early lattice works —

- Sommer and Wosiek, PLB149(1984)497, NPB267(1986)531
- Thacker, Eichten and Sexton, Lattice '87

no reliable results directly based on QCD

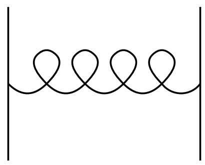
Theoretical consideration

Theoretical Consideration

- short distance -

Coulomb type due to one-gluon-exchange

$$-\sum_{i<j}\frac{A_{3Q}}{|r_i - r_j|}$$

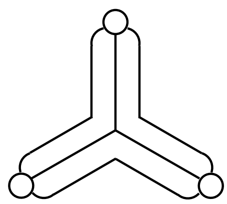


- long distance -

Three-body force proportional to the length of flux

$$\sigma_{3Q}L$$

(L:length of flux)

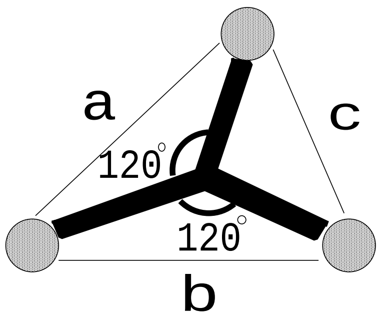


+ Constant term

Theoretical consideration

$$V_{3Q} = -\sum_{i<j} \frac{A_{3Q}}{|r_i - r_j|} + \sigma_{3Q} L_{min} + C_{3Q}$$

- Minimal linking length -

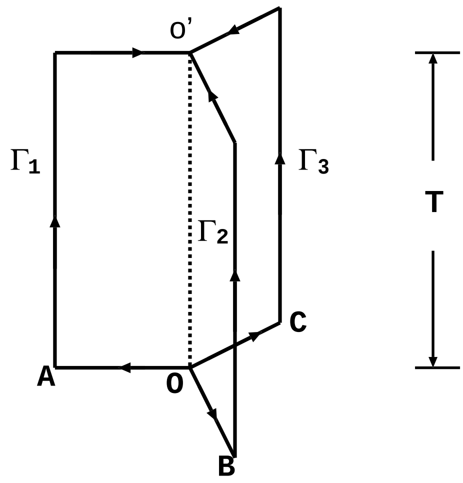


$$L_{min} = \left(\frac{1}{2}(a^2+b^2+c^2) + \frac{\sqrt{3}}{2}\sqrt{(-a+b+c)(a-b+c)(a+b-c)(a+b+c)} \right)^{1/2}$$

Direct calculation of 3Q Potential
in lattice QCD
$\rightarrow$ Comparison with the theoretical form

Method

- 3Q Wilson loop-



$$U_{\rm i} \equiv {\rm P}\exp\left(ig \int_{\Gamma_{\rm i}} dx_\mu A_\mu(x)\right)$$

$$W_{3\rm Q} \equiv \frac{1}{3!}\epsilon^{a_1 a_2 a_3}\epsilon^{b_1 b_2 b_3} U_1^{a_1 b_1} U_2^{a_2 b_2} U_3^{a_3 b_3}$$

$$\langle W_{3\rm Q}\rangle = C_0 e^{-V(g.s.)T} + C_1 e^{-V(1st\ e.s.)T} + \ldots.$$

$$\sim C_0 e^{-V(g.s.)T} \qquad \text{(large T)}$$

But, signal is supressed exponentially
Difficult to take large T

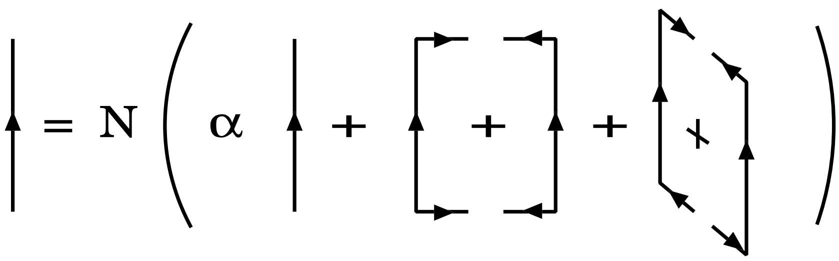
Method

- Smearing Technique -

Enhancement of Ground-state Component

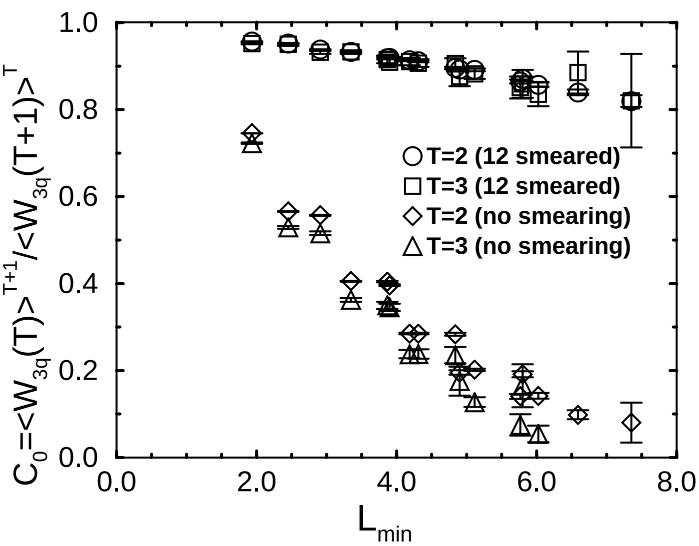
Maximize

$$\mathrm{Re\ tr}\left\{\bar{U}_i^\dagger(s)\left[\alpha U_i(s)+\sum_{j\neq i}\{U_j(s)U_i(s+\hat{j})U_j^\dagger(s+\hat{i})+U_j^\dagger(s-\hat{j})U_i(s-\hat{j})U_j(s+\hat{i}-\hat{j})\}\right]\right\}$$



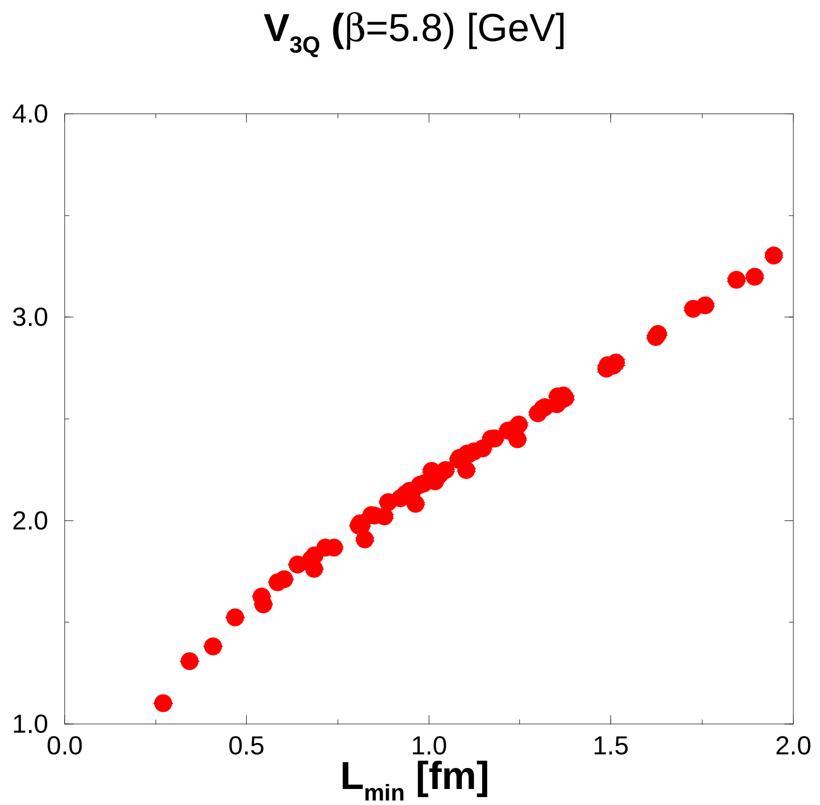
$$\langle W_{3Q}\rangle = C_0 e^{-V(g.s.)T} + C_1 e^{-V(1st\ \ e.s.)T} + ...$$

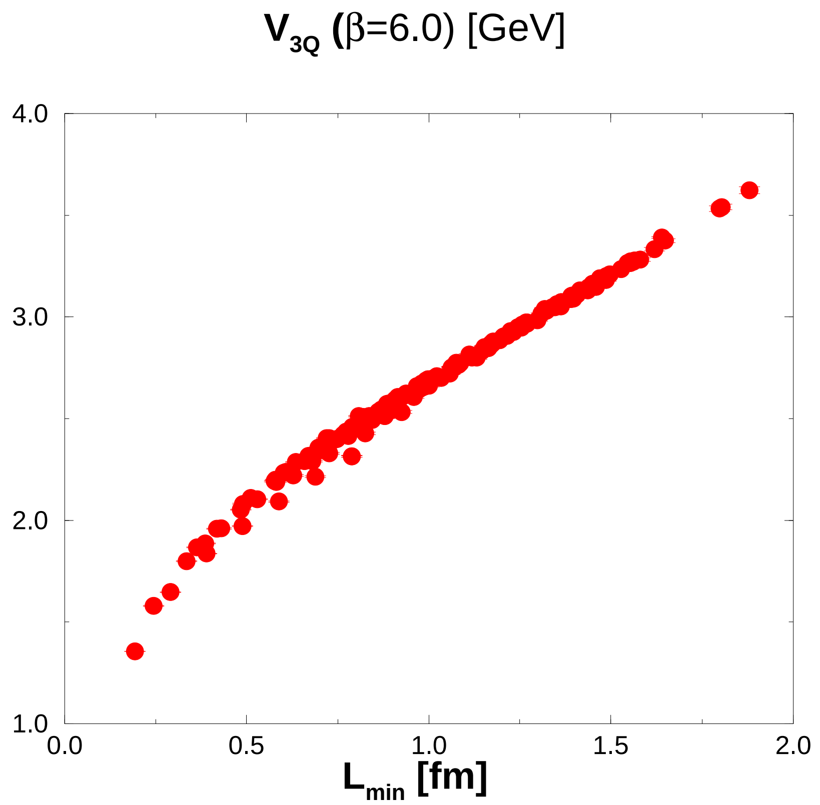
$$C_0 \simeq 1, \quad C_1, C_2, ... \ll 1$$



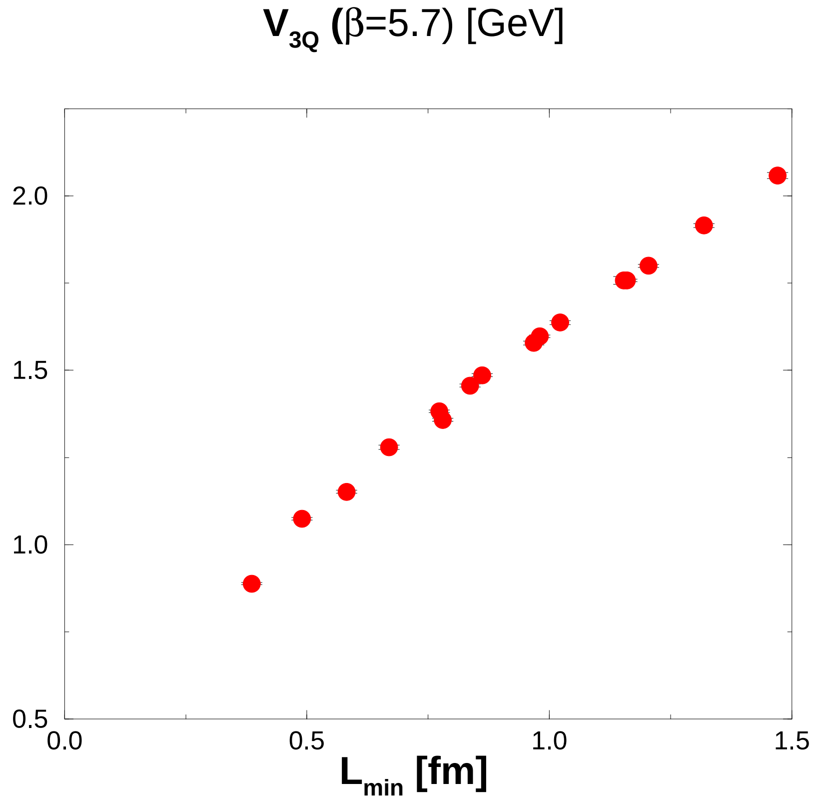
We can extract ground-state potential even with small T.

Result

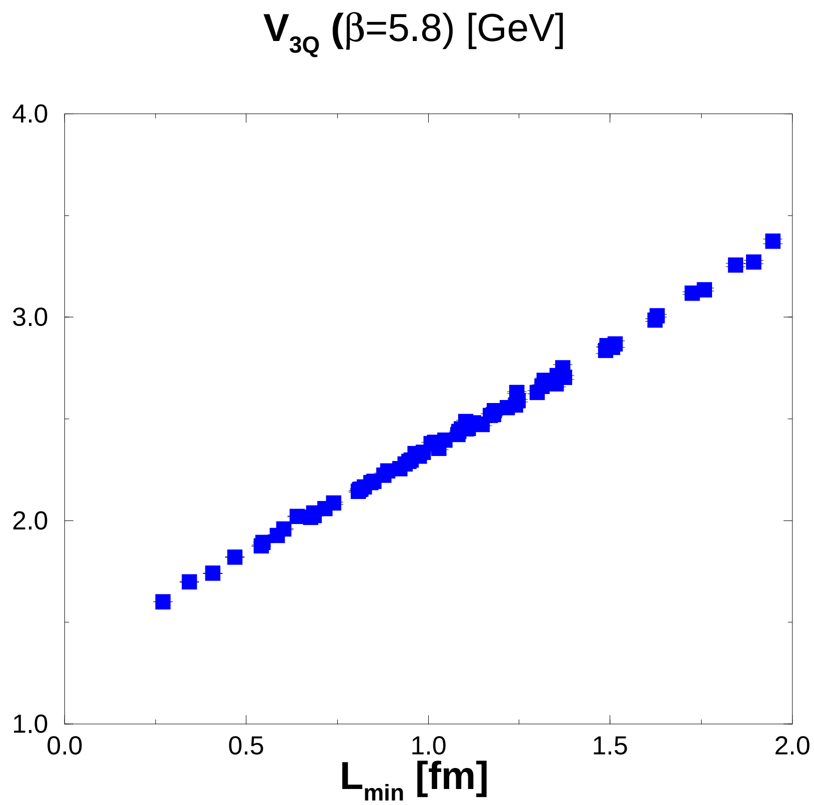

V3Q (β=5.8) [GeV]
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
Lmin [fm]


V3Q (β=6.0) [GeV]
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
Lmin [fm]

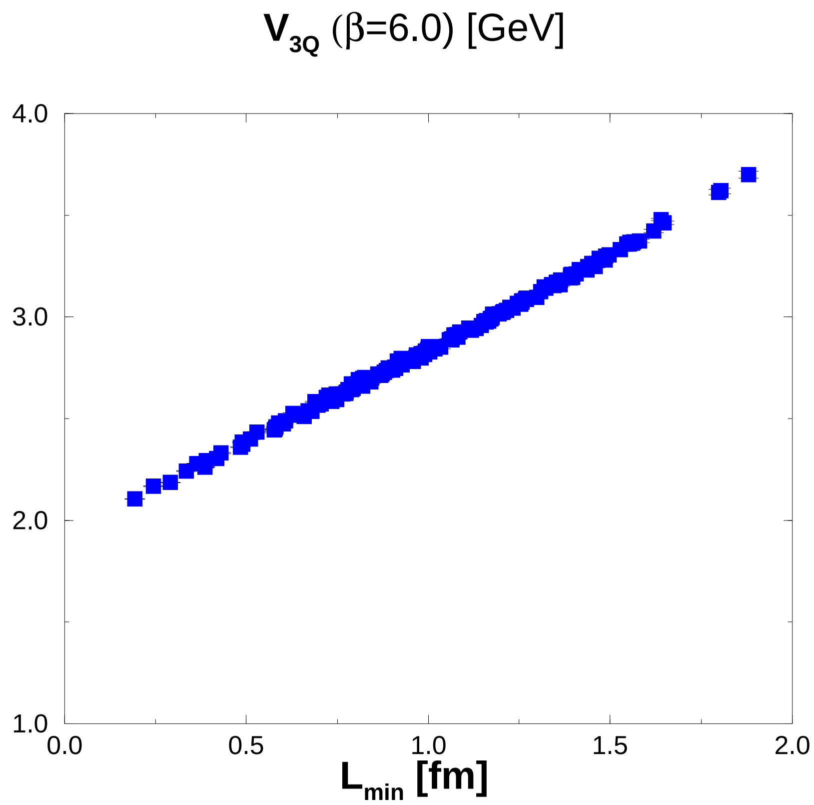
Result



Result

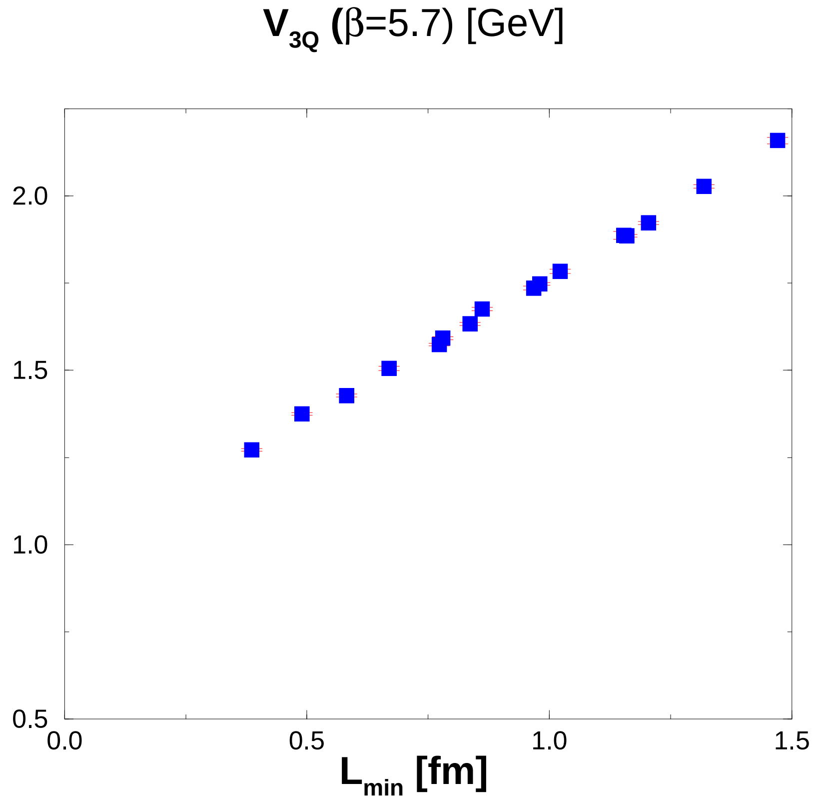


V_{3Q} (β=5.8) [GeV]
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
L_{min} [fm]



V_{3Q} (β=6.0) [GeV]
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
L_{min} [fm]

Result



Result

$\beta = 5.7$ (lattice unit $a \simeq 0.19$ fm)

	σ(Gev/fm)	A	C
3Q	0.825(15)	0.1331(66)	0.9182(213)
$Q\bar{Q}$	0.882(25)	0.2793(116)	0.6203(161)

$\beta = 5.8$ (lattice unit $a \simeq 0.14$ fm)

	σ(Gev/fm)	A	C
3Q	0.811(6)	0.1304(17)	0.9326(53)
$Q\bar{Q}$	0.882(21)	0.2580(159)	0.6081(182)

$\beta = 6.0$ (lattice unit $a \simeq 0.1$ fm)

	σ(Gev/fm)	A	C
3Q	0.804(7)	0.1363(11)	0.9590(35)
$Q\bar{Q}$	0.882(12)	0.2768(24)	0.6374(30)

- universality of string tension $\sigma_{Q\bar{Q}} \simeq \sigma_{3Q}$
- consistency with P-QCD $A_{Q\bar{Q}} \simeq 2A_{3Q}$

Summary

We have carried out the detailed analysis of 3Q potential using the SU(3) lattice QCD at quenched level. Here, smearing was essential for this analysis. $V_{3\mathrm{Q}}$ is well described with the following form

$$V_{3\mathrm{Q}} = -\sum_{i<j} \frac{A_{3\mathrm{Q}}}{|r_i - r_j|} + \sigma_{3\mathrm{Q}} L_{\mathrm{min}} + C_{3\mathrm{Q}}$$

L_{min}:minimal linking length for 3 quarks

Here, we have observed 2 remarkable features.

- universality of string tension $\sigma_{\mathrm{Q}\bar{\mathrm{Q}}} \sim \sigma_{3\mathrm{Q}}$
- consistency with P-QCD $A_{\mathrm{Q}\bar{\mathrm{Q}}} \sim 2A_{3\mathrm{Q}}$

Potential Table

β=5.7 (a$\sim$0.2 fm)
$12^3 \times 24$ lattice
16 patterns of 3 quark configurations

	σ	A	C
3Q	0.1524(28)	0.1331(66)	0.9182(213)
Q$\bar{\text{Q}}$	0.1629(47)	0.2793(116)	0.6203(161)

(i,j,k)	$V_{3Q}^{\rm latt}$	$V_{3Q}^{\rm fit}$	$V_{3Q}^{\rm latt} - V_{3Q}^{\rm fit}$
$(1,1,1)$	1.0231	0.0038	0.0141
$(1,1,2)$	1.2179	0.0061	0.0035
$(1,1,3)$	1.4154	0.0049	0.0197
$(1,2,2)$	1.3870	0.0046	−0.0016
$(1,2,3)$	1.5588	0.0060	0.0008
$(1,3,3)$	1.7141	0.0043	−0.0053
$(2,2,2)$	1.5216	0.0033	−0.0013
$(2,2,3)$	1.6745	0.0115	−0.0010
$(2,3,3)$	1.8242	0.0054	0.0074
$(3,3,3)$	1.9607	0.0092	0.0171
$(0,1,1)$	0.8459	0.0036	−0.0069
$(0,1,2)$	1.0970	0.0041	−0.0052
$(0,1,3)$	1.2935	0.0039	0.0010
$(0,2,2)$	1.3164	0.0041	−0.0098
$(0,2,3)$	1.5032	0.0058	−0.0036
$(0,3,3)$	1.6741	0.0040	−0.0066

Potential Table

β=5.8 (a∼0.14 fm)
$16^3 \times 32$ lattice
66 patterns of 3 quark configurations

	σ	A	C
3Q	0.0999(7)	0.1304(17)	0.9326(53)
$Q\bar{Q}$	0.1087(26)	0.2580(159)	0.6081(182)

(i,j,k)	V_{3Q}^{latt}	V_{3Q}^{fit}	$V_{3Q}^{latt} - V_{3Q}^{fit}$
$(1,1,1)$	0.9144	0.9008	0.0136
$(1,1,2)$	1.0657	1.0582	0.0075
$(1,1,3)$	1.1962	1.1883	0.0079
$(1,1,4)$	1.3049	1.3054	−0.0005
$(1,1,5)$	1.4134	1.4162	−0.0029
$(1,1,6)$	1.5339	1.5236	0.0103
$(1,2,2)$	1.1866	1.1877	−0.0011
$(1,2,3)$	1.3062	1.3076	−0.0014
$(1,2,4)$	1.4153	1.4203	−0.0050
$(1,2,5)$	1.5259	1.5287	−0.0029
$(1,2,6)$	1.6360	1.6347	0.0013
$(1,3,3)$	1.4178	1.4209	−0.0031
$(1,3,4)$	1.5216	1.5296	−0.0080
$(1,3,5)$	1.6276	1.6357	−0.0081
$(1,3,6)$	1.7296	1.7401	−0.0105
$(1,4,4)$	1.6291	1.6356	−0.0064
$(1,4,5)$	1.7190	1.7396	−0.0206
$(1,4,6)$	1.8203	1.8425	−0.0222
$(2,2,2)$	1.2793	1.2838	−0.0046
$(2,2,3)$	1.3815	1.3903	−0.0089
$(2,2,4)$	1.4904	1.4969	−0.0064
$(2,3,3)$	1.4751	1.4880	−0.0129
$(2,3,5)$	1.6814	1.6912	−0.0098
$(2,3,6)$	1.7847	1.7931	−0.0084
$(2,4,4)$	1.6800	1.6869	−0.0069
$(2,4,5)$	1.7669	1.7862	−0.0193
$(3,3,3)$	1.5562	1.5746	−0.0185
$(3,3,4)$	1.6475	1.6692	−0.0217
$(3,4,6)$	1.9425	1.9496	−0.0071
$(3,5,5)$	1.9322	1.9446	−0.0124

(i,j,k)	V_{3Q}^{latt}	V_{3Q}^{fit}	$V_{3Q}^{latt} - V_{3Q}^{fit}$
$(4,5,6)$	2.1263	2.1094	0.0169
$(4,6,6)$	2.2264	2.1979	0.0285
$(5,6,6)$	2.3105	2.2733	0.0372
$(0,1,1)$	0.7701	0.7727	−0.0027
$(0,1,2)$	0.9659	0.9694	−0.0035
$(0,1,3)$	1.1107	1.1071	0.0037
$(0,1,4)$	1.2323	1.2268	0.0055
$(0,1,5)$	1.3339	1.3388	−0.0049
$(0,1,6)$	1.4566	1.4469	0.0097
$(0,1,7)$	1.5730	1.5527	0.0203
$(0,1,8)$	1.6779	1.6570	0.0209
$(0,2,2)$	1.1375	1.1422	−0.0047
$(0,2,3)$	1.2660	1.2711	−0.0050
$(0,2,4)$	1.3815	1.3871	−0.0056
$(0,2,5)$	1.4928	1.4972	−0.0043
$(0,2,6)$	1.6069	1.6041	0.0028
$(0,2,8)$	1.8285	1.8128	0.0157
$(0,3,3)$	1.3893	1.3940	−0.0047
$(0,3,4)$	1.5007	1.5066	−0.0059
$(0,3,5)$	1.6135	1.6145	−0.0010
$(0,4,4)$	1.6079	1.6165	−0.0086
$(0,4,5)$	1.7085	1.7226	−0.0141
$(0,4,6)$	1.8263	1.8267	−0.0004
$(0,4,7)$	1.9328	1.9297	0.0031
$(0,4,8)$	2.0399	2.0319	0.0080
$(0,5,5)$	1.7994	1.8271	−0.0277
$(0,5,6)$	1.9205	1.9300	−0.0096
$(0,6,6)$	2.0302	2.0319	−0.0018
$(0,6,7)$	2.1390	2.1330	0.0060
$(0,6,8)$	2.2371	2.2337	0.0035