J/ψ at finite temperature

- Lattice QCD result and potential model analysis

Hideo Matsufuru, *(RCNP, Osaka Univ.)*,
Osamu Miyamura *(Dept.Physics, Hiroshima Univ.)*
Takashi Umeda *(Dept.Physics, Hiroshima Univ.)*
Hideo Suganuma *(Tokyo Inst. of Technology)*,

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Contents:

Introduction

J/ψ (charmonium) states
— important signal for fomation of quark-gluon-plasma (QGP)
CERN 2000: "formation of QCD" in heavy ion experiment

c.f. NA50, Phys. Lett. B 477 (2000) 28

Theoretical understanding

□ Potential model

T. Hashimoto et al., Phys. Rev. Lett. 57 (1986) 2123

T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416

□ Lattice QCD

- Static potential

  M. Gao, Phys. Rev. D 41 (1990) 626

- Meson Correlators

  Spatial correlation of Q and $\bar{Q}$

  — *T. Umeda et al. hep-lat/0011085*

  Spectral function

  $\rightarrow$ *Hatsuda's talk*

We perform a combined study of lattice QCD and phenomenological approaches for definite understanding of charmonium properties.

Our Approaches

Lattice QCD

- ○ directly founded on QCD
- ○ to obtain ingredients of phenomenological approaches

$\rightarrow$spatial correlation between Q and $\bar{Q}$
$\rightarrow$static quark potential

Phenomenological approaches

- Potential model with $V(r)$ measured in lattice QCD

– Stationary state analysis
T-dependence of bound state energy
What is a condition of existence of bound state at $T > T_c$?

– Non-stationary state analysis (*in progress*)
Time evolution governed by Schrödinger equation
$\rightarrow$In which time scale Q and $\bar{Q}$ desolve ?

In this stage, calculations are at the quaenched level of lattice QCD simulation ($N_f = 0$).

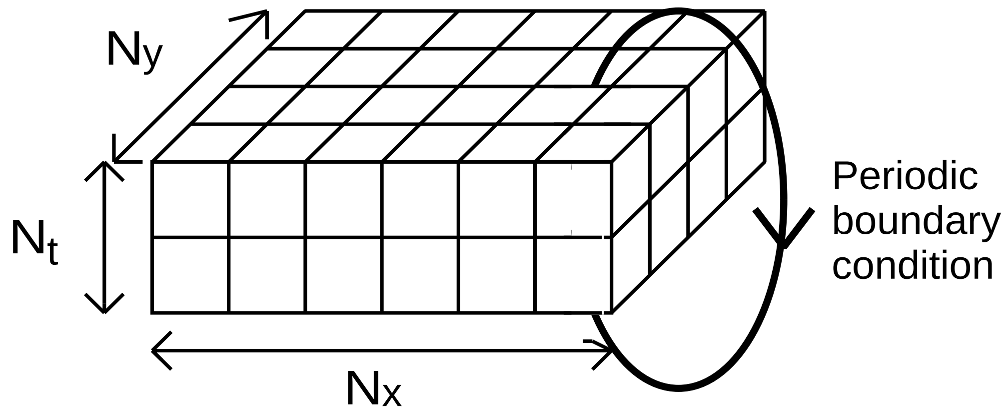
Lattice QCD (1). Introduction

Lattice QCD: nonperturvative analysis founded on QCD

Discretization of (Euclidean) space-time
with exact local gauge invariance
$\rightarrow$Path integral formalism has a well-defined meaning

$\Rightarrow$ Monte Carlo Method

At Finite Temperature lattice:



$$T = \frac{1}{N_t a} \qquad (a : \text{lattice spacing})$$

Lattice QCD (2): Setup

We employ the anisotropic lattice.

□ *Lattice*

Gauge field action: Anisotropic Symanzik (tree) action

$16^2 \times 24 \times N_t$, $\beta = 4.56$, $\gamma = 3.45$,

in quenched approximation ($N_f = 0$).

∘ Anisotropy: $\xi \equiv a_s/a_t = 3.95(2)$

∘ Cutoff: a_σ^{-1}=1.61(1) GeV ($a_\sigma \sim 0.125$ fm),

a_τ^{-1}=6.36(5) GeV

$T = 1/N_t a_t$

N_t =96 ($T \simeq 0$), 28 (0.87 T_c), 26 (0.93 T_c),

20 (1.22 T_c) (for static potential),

16 (1.52 T_c) (for correlator analysis).

Quark action:

$O(a)$ improved Wilson action onanisotropic lattice

→charm quark is treated in relativistic manner ($m_Q \ll a_\tau^{-1}$)

Quark paramters are set to be roughly m_c.

Refs:

Umeda et al., hep-lat/0011085 (charmonium at $T > 0$)

Matsufuru et al., hep-lat/0010071

and Proceedings of Confinement 2000, in press.

(static quark potential)

Lattice QCD (3): Correlator analysis-1.

Meson Correlator:

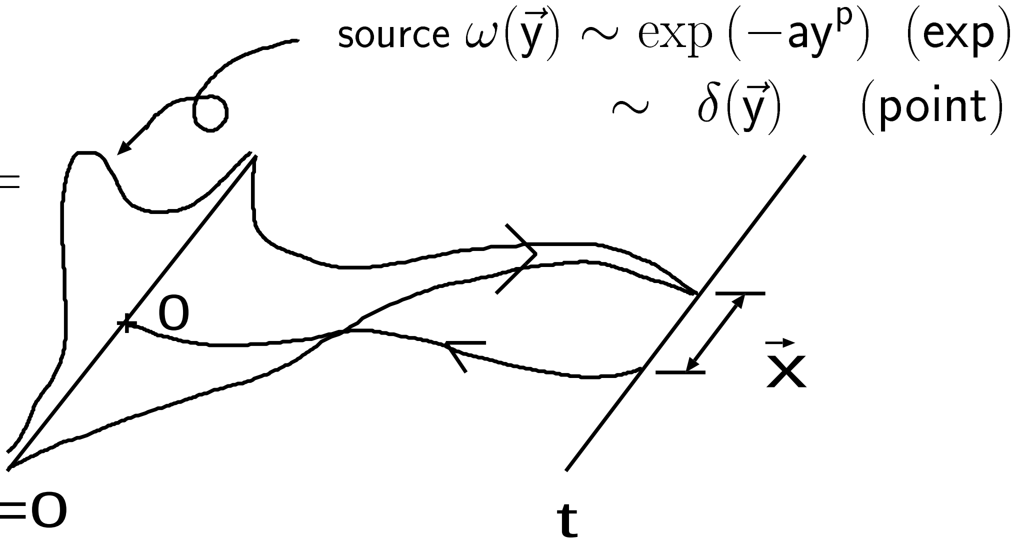
$$G_M(\vec{x},t) = \sum_{x,y_1,y_2} \omega_1(\vec{y}_1)\omega_2(\vec{y}_2)$$
$$\times\langle Tr[S(\vec{y}_1,0;\vec{z},t)\gamma_M\gamma_5 S^\dagger(\vec{y}_2,0;\vec{z}+\vec{x},t)\gamma_5\gamma_M^\dagger]\rangle$$

$S(\vec{x}_1,t_1;\vec{x}_2,t_2)$: quark propagator

$$\gamma_M \ = \ \gamma_5\ ,\ \gamma_1\ ,\ 1\ ,\ \gamma_1\gamma_5$$
$$(\ \ M \ = \ P_S\ ,\ V\ ,\ S\ ,\ A\ \ )$$

$\circ$*Gauge fixing : Coulomb gauge*

$G_M(\vec{x},t) =$



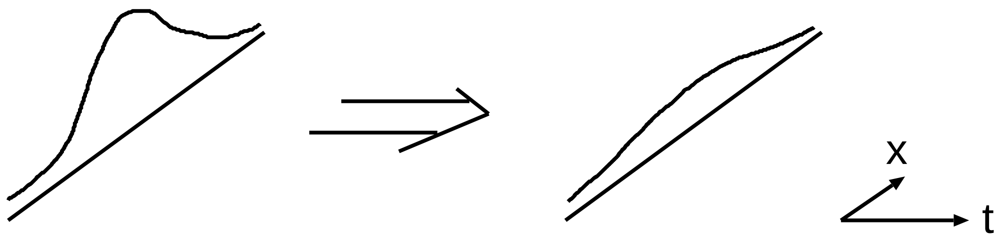
a, p are chosen to give appropriate size of meson.

Lattice QCD (4): Correlator analysis-2.

□ *t-dependence of the wave function*

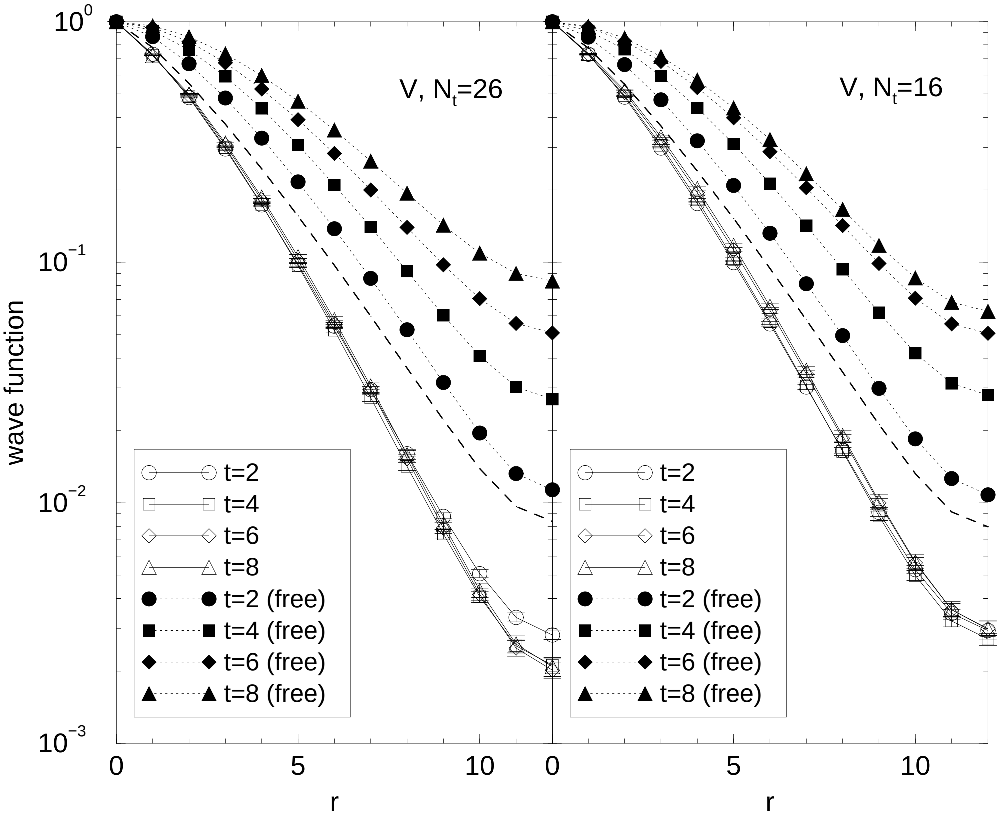
$$w_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t) O^\dagger(0)\rangle$$

If there is no bound state (like free quark case), wave function becomes broader as t (Euclidean time).



In this case, $\phi_\Gamma(r,t) = w_\Gamma(r,t)/w_\Gamma(r=0,t)$
(normalized at spatial origin)

increase as t.



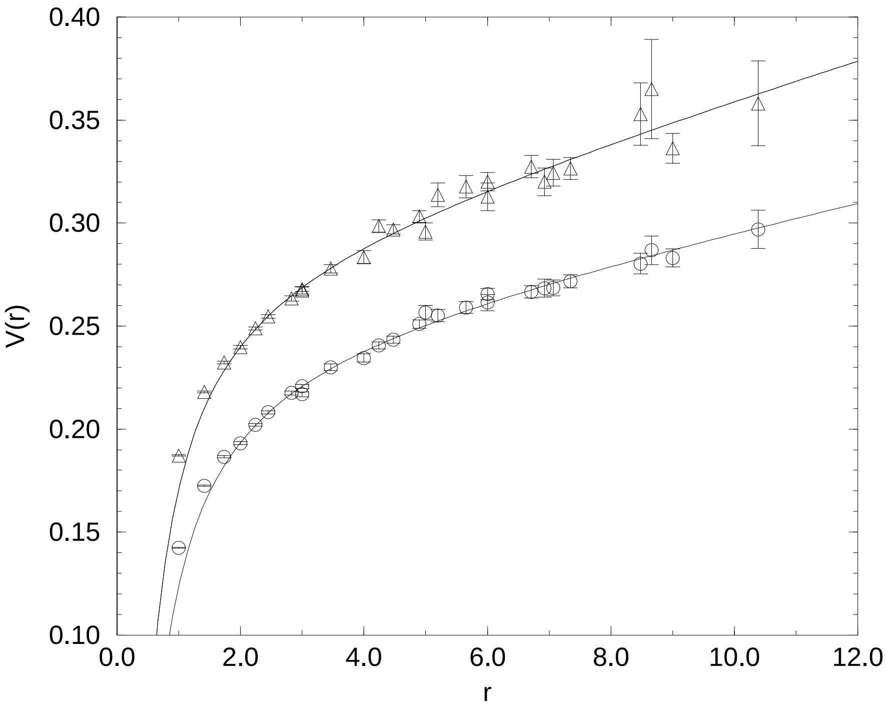
Lattice QCD (5): Static potential

Polyakov loop (order parameter of deconfining transition)

$$P(\vec{x}) = \mathrm{Tr} \prod_{t=0}^{N_\tau - 1} U_4(\vec{x}, t)$$

Static quark potential

$$P_2(\vec{r}) = \langle P(0) P^\dagger(\vec{r}) \rangle \simeq c \cdot \exp(-V_{Q\bar{Q}}(\vec{r}) N_\tau)$$



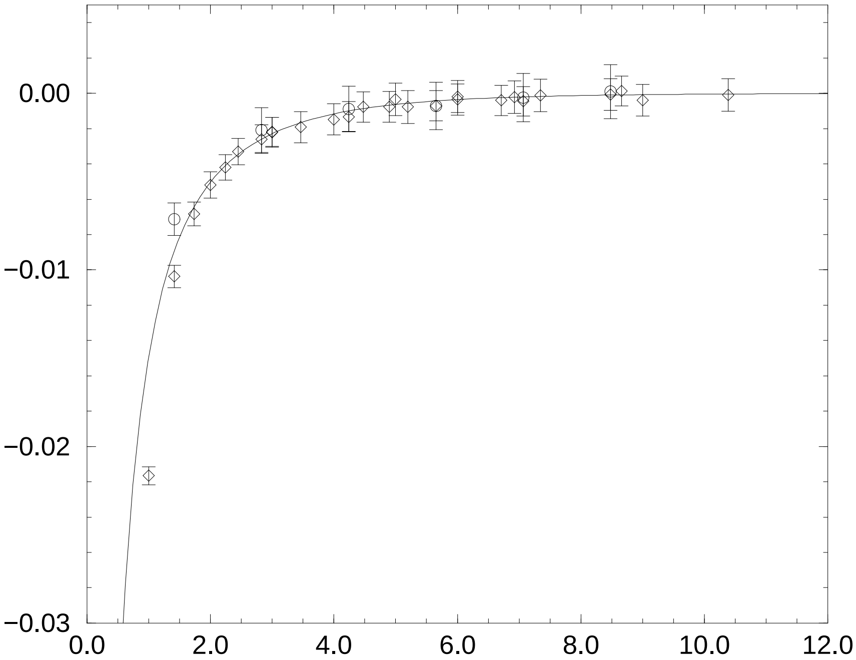
$\rightarrow$ Fit: $V(r) = \mathrm{const.} - A/r + \sigma r$

N_τ	T	Fit range	const.	A	σ	χ^2/N_{dof}
96	~ 0	$r \geq 2.8$	0.0587(49)	0.0587(49)	0.01781(29)	78.12/18
28	$0.87 T_c$	$r \geq 2$	0.2817(70)	0.119 (11)	0.0089 (11)	25.2/21
26	$0.93 T_c$	$r \geq 2$	0.2934(41)	0.1253(63)	0.00638(69)	23.5/21

Lattice cutoffs: $a_\sigma^{-1} \simeq 1.6$ GeV, $a_\tau^{-1} \simeq 6.4$ GeV.

String tension slowly decreases toward T_c.

Lattice QCD (6): Static potential-2



○ Q-$\bar{Q}$ system — Debye screening

$$V_{Q\bar{Q}} = V_{singlet} + V_{octet}$$

$$\rightarrow\ -A\,\exp(-\mu r)/r \quad : \text{ Yukawa potential}$$

[Gao, PRD41(1990)626]

$\mu \sim$ screening mass $\sim$ 0.5-1 GeV

Fit (lattice unit):

range	const	A	μ	χ^2 / N_{dof}
all r	0.20755(88)	0.0469(15)	0.803(40)	5.29 / 24
$r \geq 2$	0.20794(87)	0.0217(33)	0.385(73)	1.03 / 21

Potential Model (1)

Schrödinger equation for stationary state (spin averaged):

$$\left(-\frac{\Delta}{2m_R} + V(x)\right)\psi(x) = E\psi(x)$$

(m_R: reduced mass)

For S-state,

$$\left(-\frac{1}{2m_R}\frac{d^2}{dr^2} + V(r)\right)u(r) = Eu(r)$$

$u(r) = r\psi_r(r)$, $\psi_r(r)$: radial wave function

Solve this equation numerically with $V(x)$ measured in lattice simulation.

Potential Model (2): Stationary state ($T < T_c$)

Potential $V(r)$ from lattice data (fitted paramters):

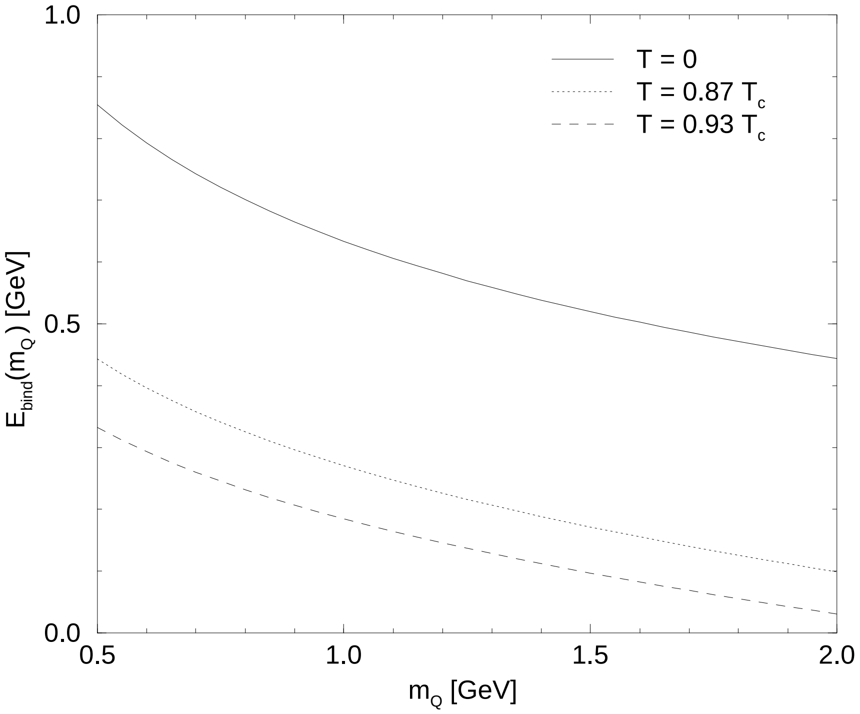
$$V(r) = -\frac{A}{r} + \sigma r$$

charm quark mass $m_c \simeq 1.3$ GeV.

Fit of lattice data (physical unit):

T	A	σ [GeV]
~ 0	0.24	0.18
$0.87T_c$	0.48	0.091
$0.93T_c$	0.50	0.066

Binding energy:



Precise determination of A is significant to obtain T-dependence of charmonium mass.
(*present result is at preliminary level.*)

Potential Model (3): Stationary state ($T < T_c$)

Screened potential

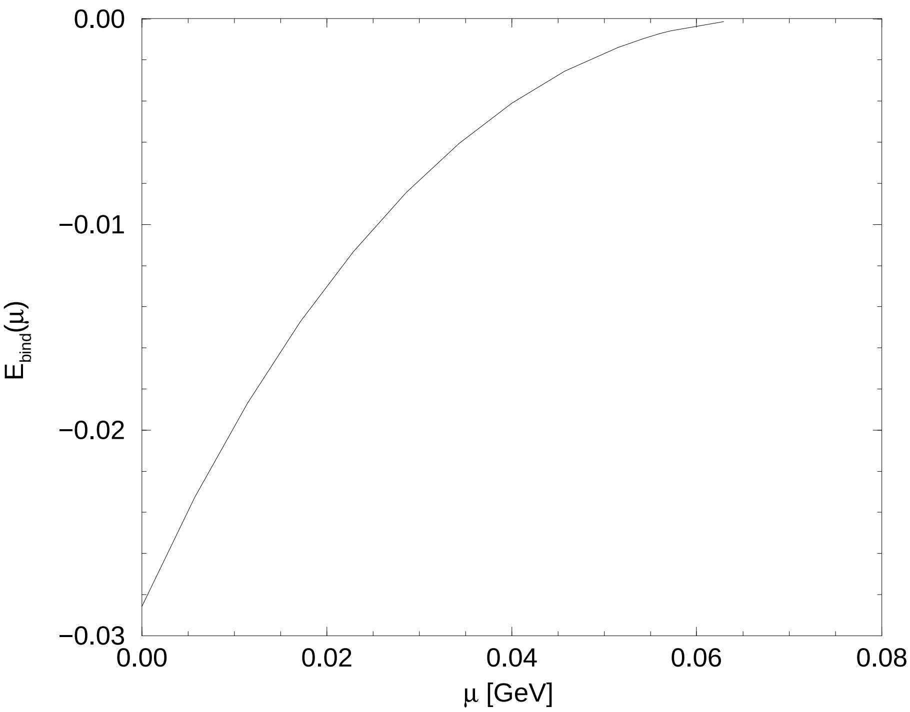
$$V(r) = -A\,\frac{\exp(-\mu r)}{r}$$

At $T \simeq 1.22 T_c$ (lattice data):

fit range	A	μ [GeV]
all r	0.088	0.628
$r \geq 2$	0.188	1.288

(paramters strongly depends on the fit range)

For $A = 0.088$ and $m_Q = 1.3$ GeV,
μ-dependence of binding energy:



Similar result for $A = 0.188$ and $m_Q = 1.3$ GeV

— c and $\bar{c}$ are not bound at this temperature.

c.f Karsch, Mehr and Satz, Z. Phys. C 37 (1988) 617

Potential Model (4): Toward non-stationary state

Time-dependent Schrödinger equation:

$$i\frac{\partial}{\partial t}\psi(x) = \left(-\frac{\Delta}{2m_R} + V(x)\right)\psi(x)$$

— In which time scale, Q and $\bar{Q}$ are well separated ?

To avoid singularity of Coulomb(-like) potential at $x \sim 0$, modified cylindrical coordinate is appropriate.

$$x = \xi^{2/3}\cos\phi, \quad y = \xi^{2/3}\sin\phi, \quad z = z$$

Kono et al., J. Comput. Phys. 130 (1997) 148

— *Work in progress.*

Conclusion and Outlook

We investigated charmonium state from two points of view.

Lattice Results:

∘ Even above T_c ($T \simeq 1.5T_c$), wave function shows qualitatively different behavior from the free quarks.
$\rightarrow Q$-$\bar{Q}$ strongly correlate even at $T \simeq 1.5T_c$ in the time scale of $1/T$.

∘ Static quark potential was measured.

Potential model:
Stationary state problem was studied with the potential obtained in lattice simulation.
At $T \simeq 1.22T_c$, no bound state is formed for c and $\bar{c}$.

$\Downarrow$

Puzzling results
More studies are needed.
∘ Non-stationary state problem:
evolution of wave function along Schrödinger equation

Outlooks
∘ Precise determination of potential is significant.
∘ Lattice simulation with $N_f > 0$
∘ Charmonium spectroscopy in the potential model at $T > 0$