Systematical Study of Octet and Decuplet Baryon Spectra in $SU(3)$ Lattice QCD

Hadron & Nuclei, Seoul, Feb.21, 2001

N. Nakajima[a], H. Matsufuru[a], Y. Nemoto[b]
and
H. Suganuma[c]

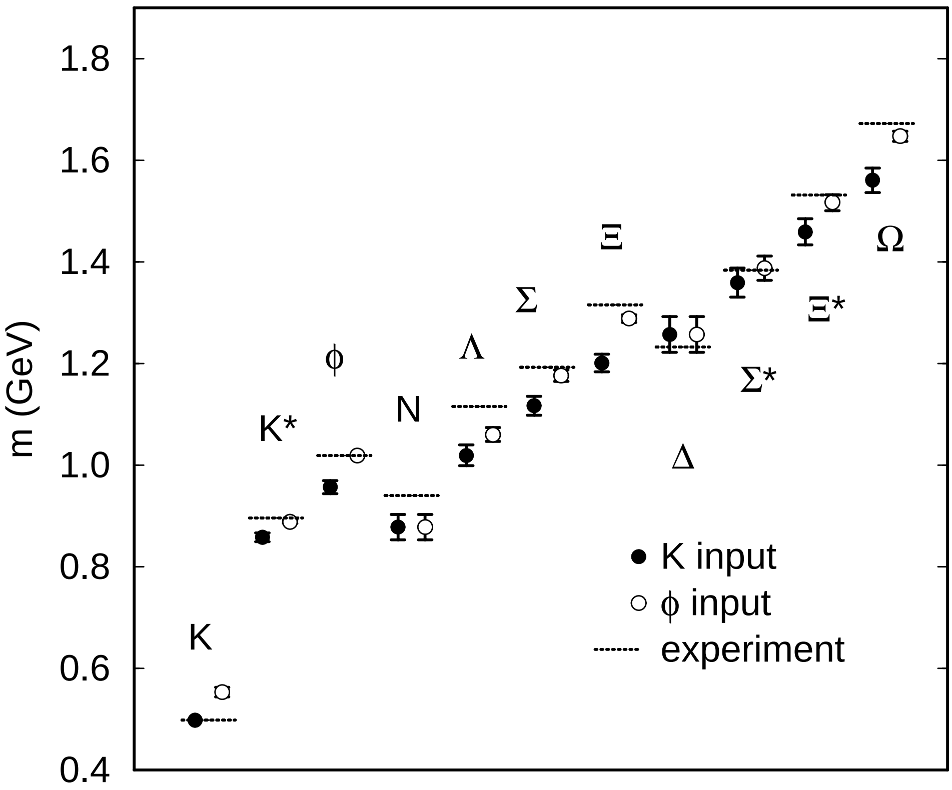
[a] *RCNP, Osaka University*
[b] *YITP, Kyoto University*
[c] *Tokyo Institute of Technology*

Contents

Introduction

■ Systematical Study of Hadron Spectrum in Lattice QCD Simulation

□ mass spectrum of several kinds of hadrons
by CP-PACS, UK-QCD, *etc.*



CP-PACS Collaboration, *Phys. Rev. Lett.* **84**(2000)238

∘ Quenched Level Calculation

○ OUR AIM

Hadron physics based on QCD

Investigation of baryons properties

○ Mass difference between Octet and Decuplet Baryons

- **Nonrelativistic Quark Model in One-Gluon Exchange Picture**

M. Oka and K. Yazaki, Prog. Theor. Phys. 66 (1981) 556; *ibid.*572

N-Δ splitting is mainly due to spin-spin interaction

$$M_\Delta - M_N \propto \sum_{i<j} \frac{1}{\mathfrak{m}_i \mathfrak{m}_j}$$

$\mathfrak{m}$: constituent quark mass

$\Longrightarrow$ Mass difference decreases with increasing quark masses.

- Lattice QCD

Quark mass is arbitrarily chosen

– Same behavior or not?

○ Structure of Negative Parity Baryon Λ(1405) in terms of quarks

- Λ(1405)

$$L_{I,2S} = S_{0,1},\ J^P = \frac{1}{2}^-$$

(a) $SU(3)_f$ - singlet uds state (qqq)

(b) $N\bar{K}$ bound state ($qqqq\bar{q}$)

- Lattice QCD

Using Quenched level calculation,

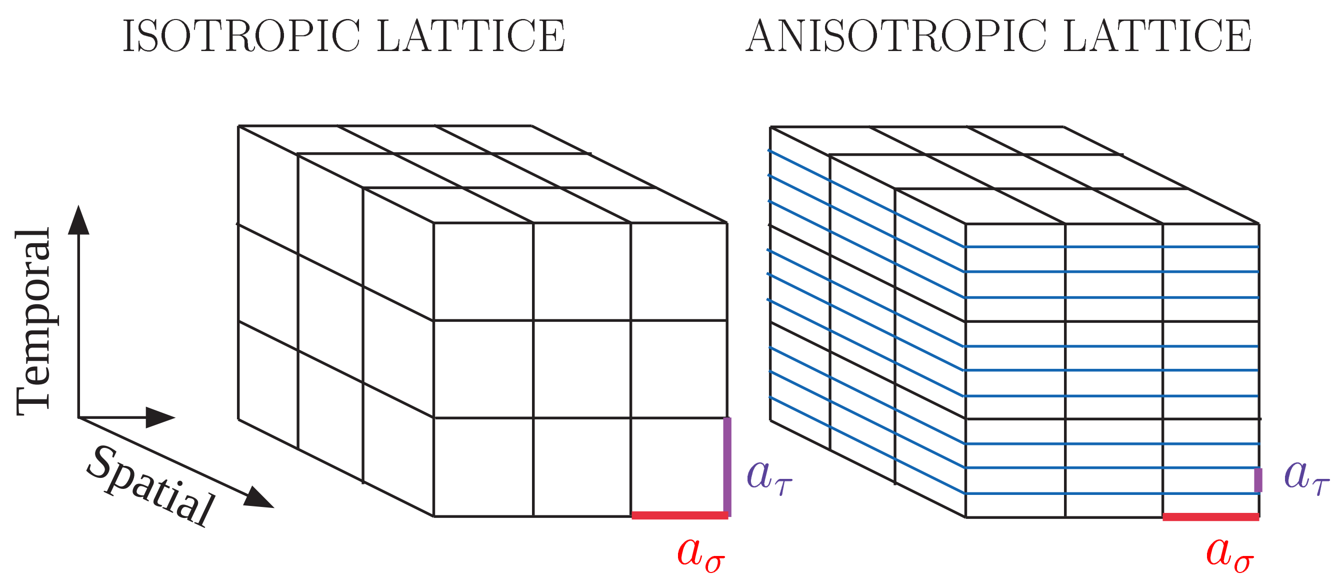
$\rightarrow$ quark-level constitution is clear

– 3 quark state or 5 quark state

– Flavor singlet

We investigate the flavor-singlet baryon spectrum with spin 1/2 and negative parity.

Anisotropic Lattice



$$\xi = \frac{a_\sigma}{a_\tau}: \text{ Anisotropy}$$

■ Anisotropic lattice

Temporal lattice spacing a_τ is finer than spatial one a_σ.

□ Advantage

- Extract detailed information on the correlators in temporal direction
  - Efficient for **excited hadrons**, *etc.* (for those correlators noise grows rapidly against signals of correlators)
- Keeping the size of lattice comparatively modest.

Lattice setup

■ Gauge Action

☐ Anisotropic Wilson Gauge Action

Karsch, N.P.B205(1982)285

☐ Lattice Size : $12^3 \times 96$ at Quenched Level

☐ Lattice Parameters : $\xi = 4,\ \beta = 5.75$

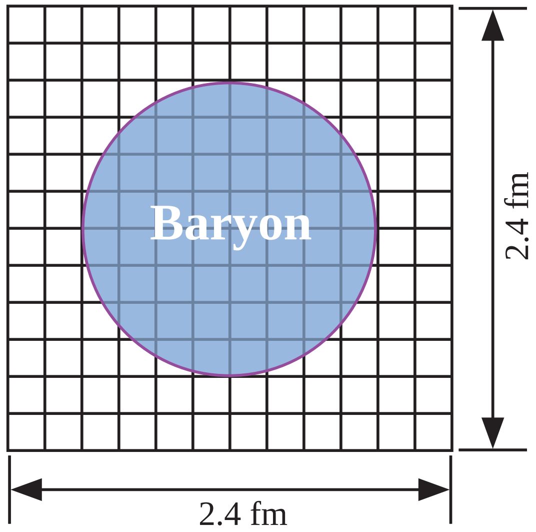
☐ Scale :

$a_\sigma^{-1} \simeq 1\text{GeV}$: Spatial

$a_\tau^{-1} \simeq 4\text{GeV}$: Temporal

String tension

$\sqrt{\sigma} \sim 427\text{MeV}$



■ Quark Action

☐ Anisotropic $O(a)$-improved (Clover) Quark Action

Hadron Spectrum

■ Two-Point Correlator :

$$
\begin{aligned}
G_H(t) &= \sum_{\vec{x}} \langle 0|T\{\mathcal{O}_H(x)\overline{\mathcal{O}}_H(0)\}|0\rangle \\
&= \sum_{\vec{x}} \sum_{n} \langle 0|T\{\mathcal{O}_H(x)|n\rangle\langle n|\overline{\mathcal{O}}_H(0)\}|0\rangle \\
&\sim e^{-m_0 t} + \sum_{n\neq 0} e^{-m_n t} \\
&\to e^{-m_0 t} \qquad \text{at large } t
\end{aligned}
$$

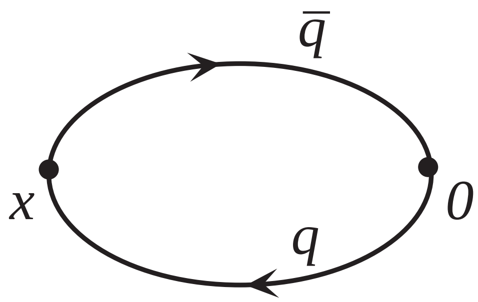
m_0 : Ground State Mass

$\mathcal{O}_H$: Hadron Operator

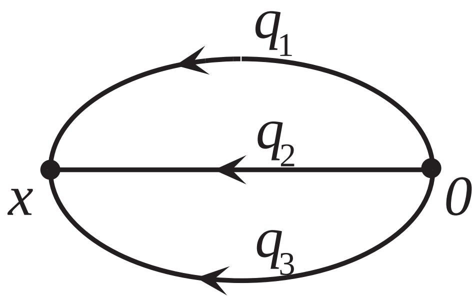
■ Hadron Operators

MESON	Pseudoscalar	$\bar{q}_c\gamma_5 q_c$
	Vector	$\bar{q}_c\gamma_k q_c$
BARYON	Octet	$\epsilon_{abc}q_a(q_b^T C\gamma_5 q_c)$
	Decuplet	$\epsilon_{abc}q_a(q_b^T C\gamma_k q_c)$
	Singlet	$\epsilon_{abc}\epsilon_{uds}q_a(q_b^T C\gamma_5 q_c)$

C : Charge Conjugation Matrix



MESON



BARYON

■ Correlators on the Lattice

□ Meson

$$G_M(t) \;=\; c_M \left[\mathrm{e}^{-tm_M} + \mathrm{e}^{-(T-t)m_M} \right]$$

:Periodic Boundary Condition

$\Longrightarrow$ m_M : meson mass

□ Baryon

$$\begin{aligned} G_B(t) \;&=\; (1+\gamma_4) \left[c_{B^+}\mathrm{e}^{-tm_{B^+}} + c_{B^-} b \mathrm{e}^{-(T-t)m_{B^-}} \right] \\ &+\; (1-\gamma_4) \left[c_{B^+} b \mathrm{e}^{-(T-t)m_{B^+}} + c_{B^-}\mathrm{e}^{-tm_{B^-}} \right] \end{aligned}$$

$b = +1$: Periodic Boundary Condition
$b = -1$: Antiperiodic Boundary Condition

Combination of correlators
with either boundary condition.

$\Longrightarrow$
m_{B^+} : Positive parity baryon mass
m_{B^-} : Negative parity baryon mass

Numerical Results of Hadron Spectrum

■ Mesons

☒ **Pseudoscalar**
$\pi(u\bar{d},\bar{u}d,\ldots)$, $K(u\bar{s},\bar{u}s,\ldots)$,...

☒ **Vector**
$\rho(u\bar{u},d\bar{d})$, $K^*(u\bar{s},\ldots)$,$\phi(s\bar{s})$

■ Baryons

☐ **Positive Parity Baryon**

☒ **Decuplet**
$\Delta(uuu,\ldots)$,$\Sigma^*(uus,\ldots)$, $\Xi^*(uss,\ldots)$,$\Omega(sss)$

☒ **Octet**
$N(uud,udd)$,$\Sigma(uus,\ldots)$, $\Xi(uss,\ldots)$,$\Lambda(uds)$

☒ **Singlet**

☐ **Negative Parity Baryon**

☒ **Decuplet**

☒ **Octet**

☒ **Singlet** $\Lambda(uds) \to \Lambda(1405)$?

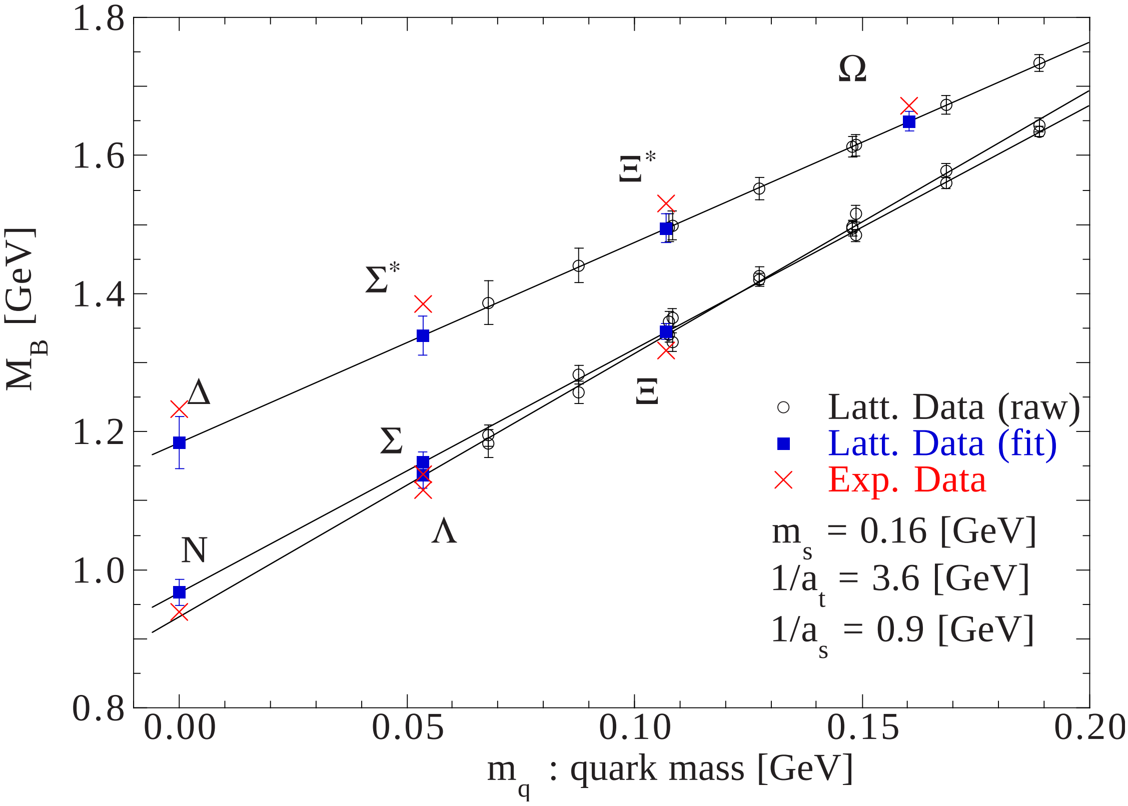
◎ Result 1

- Octet and decuplet positive parity baryons
- Mass difference

◎ Result 2

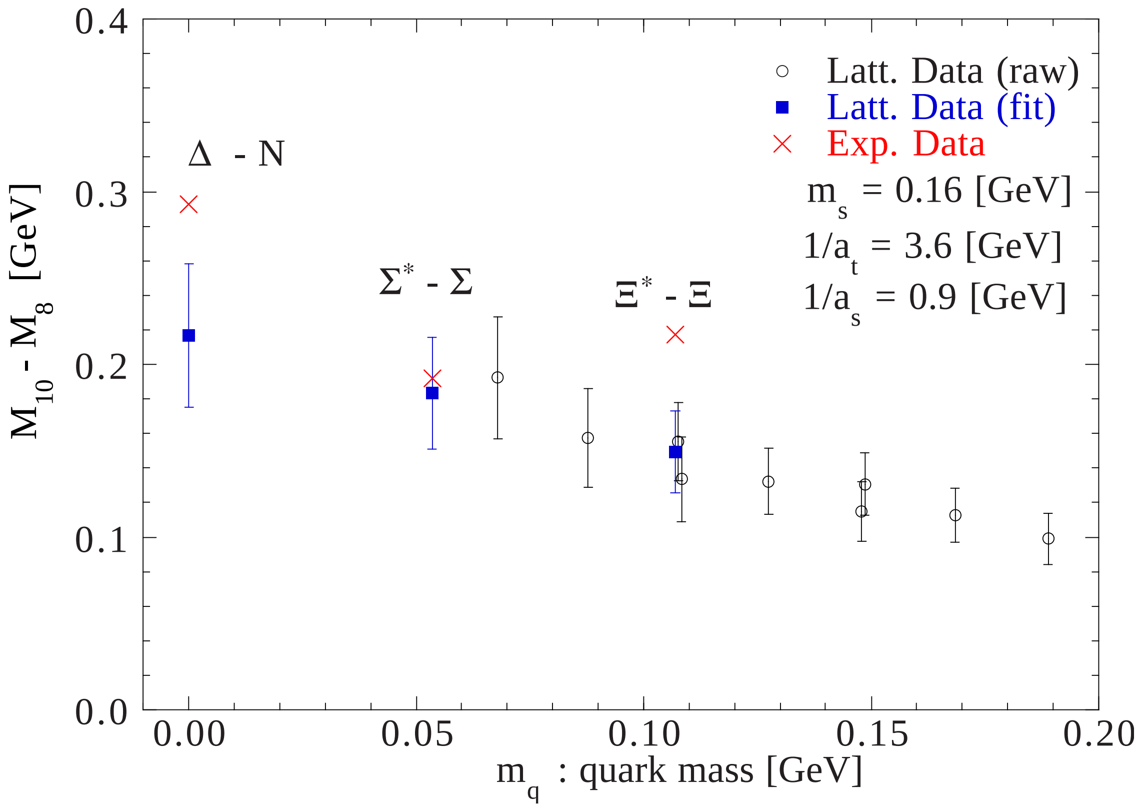
- Flavor-Singlet Negative Parity Baryon

Octet and Decuplet Baryon spectra



- ◦ u,d-quark masses are zero in chiral limit
- ◦ Scale and s-quark masses are determined by fitting the intercept to experimental data.

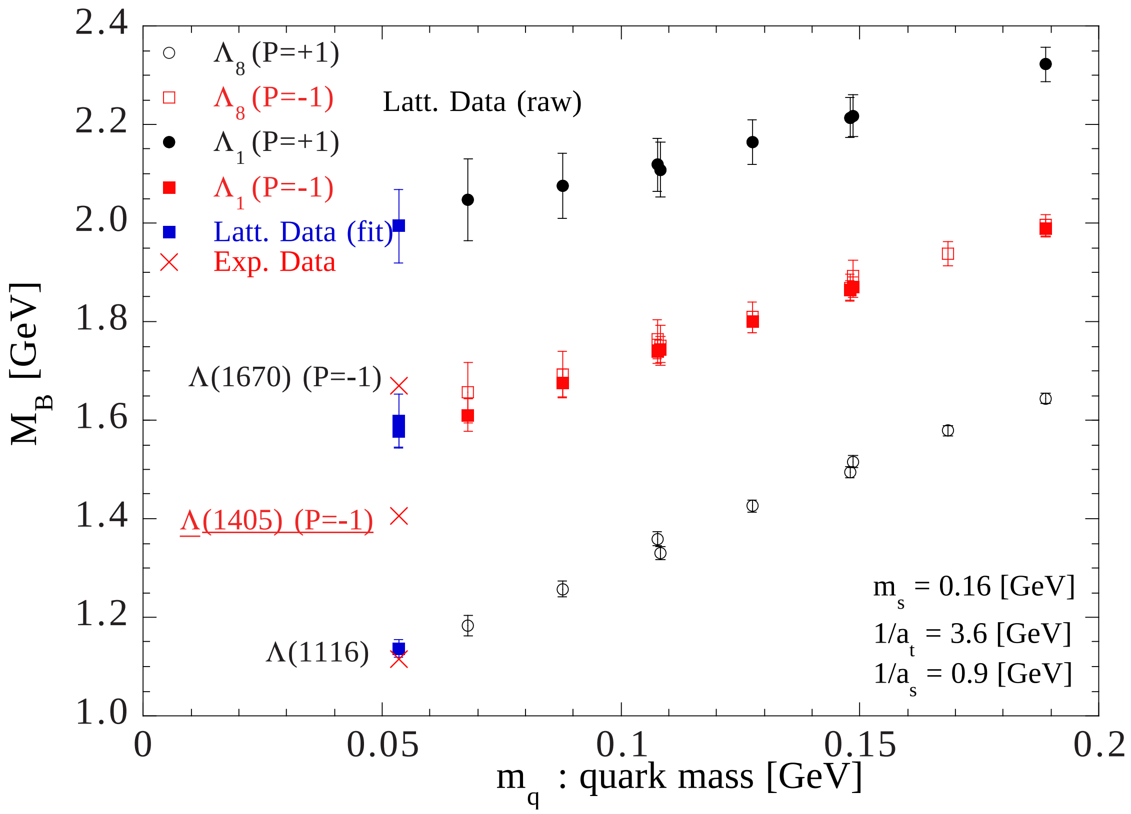
Mass Difference between Octet and Decuplet Baryons



∘ Octet-decuplet mass difference decreases as quark mass increases.

$\rightarrow$ consistent with the one-gluon-exchange

Λ baryons spectrum



We have calculated

- both particles (positive & negative parity); $P = \pm 1$
- both singlet and octet ; Λ_1,Λ_8

- Mass of $\Lambda_1(-1)$ is bigger than $\Lambda(1405)$
- Degeneracy of $\Lambda_1(-1)$ and $\Lambda_8(-1)$

Summary & Outlook

■ Lattice QCD

Anisotropic Lattice

$O(a)$-improved quark action

Lattice size : $12^3 \times 96$, $\beta = 5.75$, $\xi = 4$

■ Light Hadron Spectrum

□ Decuplet,Octet and Singlet Baryons

- □ Positive Parity Baryons
  N, Σ, Ξ, Λ, Δ, Σ^*, Ξ^*, Ω
- □ Singlet & Octet Λ Baryons

☒ Mass Difference of Octet and Decuplet Baryon

- Behavior is consistent with OGE explanation.

$\Longrightarrow$ determination of model parameters (future)

☒ Excited Hadron (Negative Parity Baryon), $\Lambda(1405)$

- Mass of $\Lambda_1(\frac{1}{2}^-)$ is bigger than $\Lambda(1405)$.
- Degeneracy of $\Lambda_1(\frac{1}{2}^-)$ and $\Lambda_8(\frac{1}{2}^-)$

$\Longrightarrow$ comparing with a 5 quark state (future)

($\rightarrow$ full QCD calculation)

$\Longrightarrow$ need calculations on <u>finer lattices</u>

Calibration

☐ Lattice Klein-Gordon Action for Meson Field

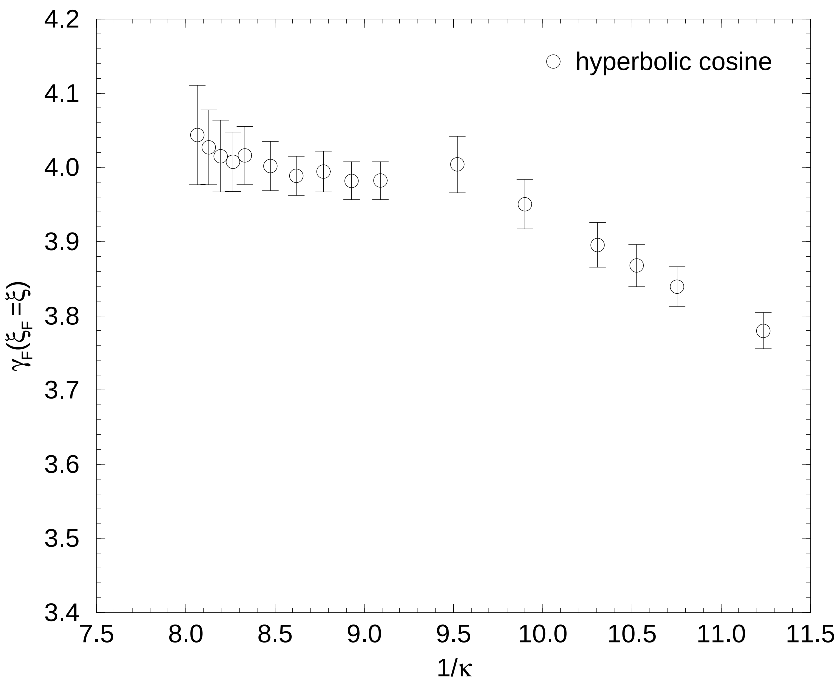
$$S = \sum_x \frac{1}{2\xi_F} \phi^\dagger(x) \left[-\xi_F^2 D_4^2 - \vec{D}^2 + m_0^2 \right] \phi(x),$$

where m_0 is in the unit of a_σ.

☐ Dispersion Relation

$$\cosh E(\vec{p}) = 1 + \frac{1}{2\xi_F^2}(\hat{\vec{p}}^2 + m_0^2).$$

$$\xi_F^2 = \frac{\hat{\vec{p}}^2}{2(\cosh E(\vec{p}) - \cosh E(0))}$$



Lattice setup

Gauge Action

☐ Anisotropic Wilson Gauge Action

Karsch, N.P.B205(1982)285

☐ Lattice Size : $12^3 \times 96$ at Quenched Level

☐ Parameters

Klassen, N.P.B533(1998)557

for $\xi = 4$, $(\beta, \gamma_G) = (5.75, 3.072)$

(1% level)

☐ Scale Static Quark Potential (Wilson Loop)

$\sqrt{\sigma} = 427$ GeV

$a_\sigma^{-1} = 1.000(8)\text{GeV}$

$a_\tau^{-1} \simeq 4\text{GeV}$

☐ Configuration

for Calibration (400)

for Hadron Correlator (120)

Pseudo-Heat Bath

$\#$ thermalization $= 10000$

$\#$ separation $= 2000$

Quark Action

$O(a)$-improved Quark Action (Clover Quark Action)

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y),$$

$$\begin{aligned} K(x,y) &= \delta_{x,y} - \kappa_\tau \left\{ (1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right\} \\ & -\kappa_\sigma \sum_i \left\{ (r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right\} \\ & -\kappa_\sigma c_E \sum i\sigma_{4i} F_{4i}(x) \delta_{x,y} - r\kappa_\sigma c_B \sum_{ij} \frac{1}{2} \sigma_{ij} F_{ij}(x) \delta_{x,y}. \end{aligned}$$

☐ Hopping Parameters

$$\kappa_\sigma = \frac{1}{2(m_0 + \gamma_F + 3r)} \ : \text{ spatial} \qquad \gamma_F\text{: bare anisotropy}$$

$$\kappa_\tau = \gamma_F \kappa_\sigma \ : \text{ temporal}$$

☐ Wilson Parameter $\quad r = \dfrac{1}{\xi}$

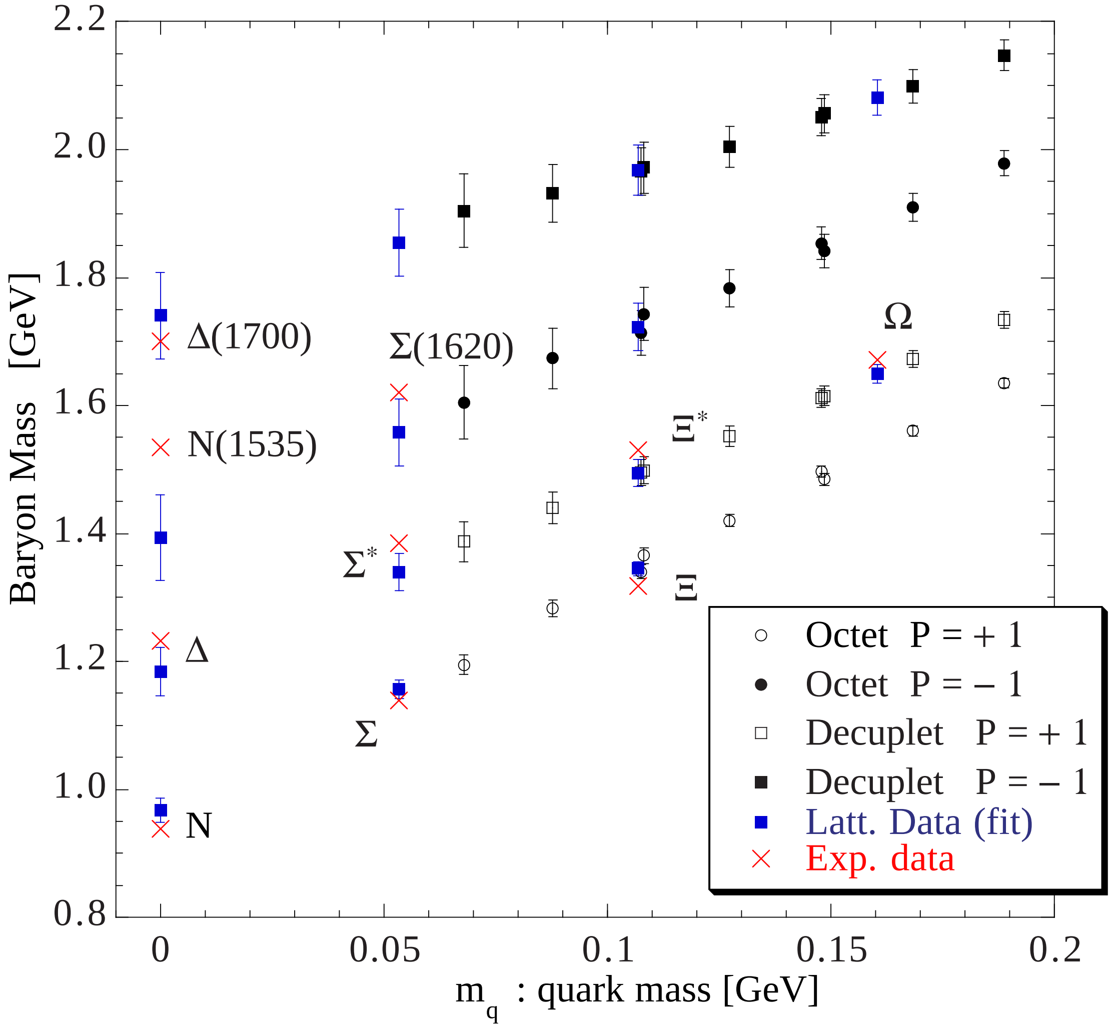
$$\rightarrow \frac{1}{\kappa} = \frac{1}{\kappa_\sigma} - 2(\gamma_F + 3r - 4) = 2(m_0 + 4)$$

Chiral extrapolation in $\dfrac{1}{\kappa}$

☐ Clover Coefficients

$c_E = c_B = 1$, at tree level for $r = \dfrac{1}{\xi}$

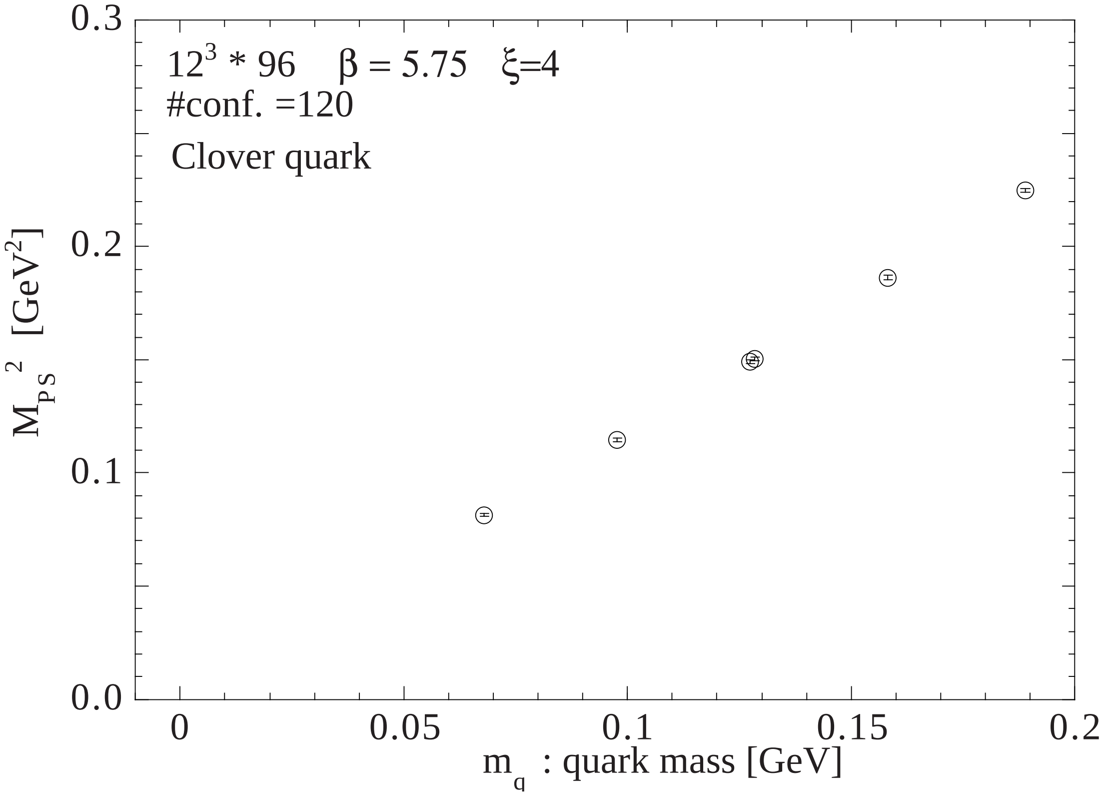
Octet & Decuplet baryon spectra



We have calculated

- both particles (positive & negative parity); $P = \pm 1$
- both singlet and octet

Pseudoscalar meson squard vs. quark mass



$$m_\pi^2 \sim m_q \text{ (CHPT)}$$