

Hadron Spectroscopy in Anisotropic Lattice QCD

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Hadron & Nuclei, Seoul, Feb.21, 2001

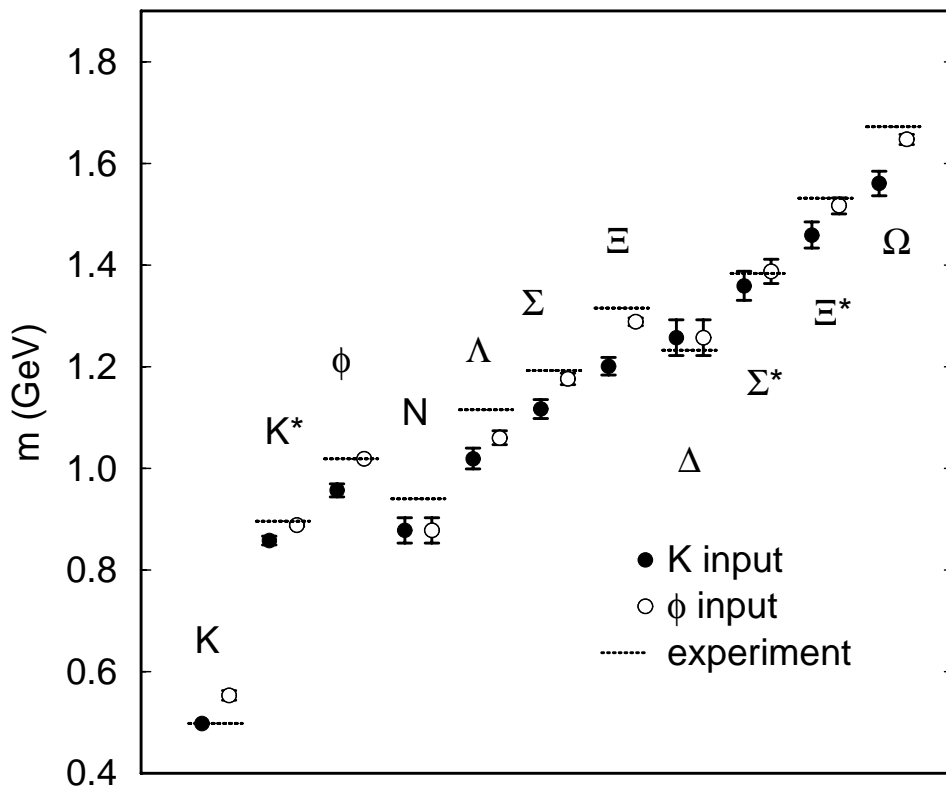
Contents

- Introduction
 - Lattice QCD
 - Anisotropic Lattice
 - Hadron Correlator
 - Numerical Results
 - Summary & Outlook
-

Introduction

■ Systematical Study of Hadron Spectrum in Lattice QCD Simulation

- mass spectrum of several kinds of hadrons by CP-PACS, UK-QCD, *etc.*



CP-PACS Collaboration, *Phys. Rev. Lett.* **84**(2000)238

○ Quenched Level Calculation

○ OUR AIM

Hadron physics based on QCD

Investigation of baryons properties

○ Mass difference

between Octet and Decuplet Baryons

- Nonrelativistic Quark Model

in One-Gluon Exchange Picture

M. Oka and K. Yazaki, Prog. Theor. Phys. 66 (1981) 556; *ibid.*572

N - Δ splitting is mainly due to spin-spin interaction

$$M_{\Delta} - M_N \propto \sum_{i < j} \frac{1}{m_i m_j}$$

m : constituent quark mass

\implies Mass difference decreases

with increasing quark masses.

- Lattice QCD

Quark mass is arbitrarily chosen

– Same behavior or not?

○ Structure of Negative Parity Baryon $\Lambda(1405)$

in terms of quarks

- $\Lambda(1405)$

$$L_{I,2S} = S_{0,1}, J^P = \frac{1}{2}^-$$

(a) $SU(3)_f$ - singlet uds state (qqq)

(b) $N\bar{K}$ bound state ($qqqq\bar{q}$)

- Lattice QCD

Using Quenched level calculation,

\rightarrow quark-level constitution is clear

– 3 quark state or 5 quark state

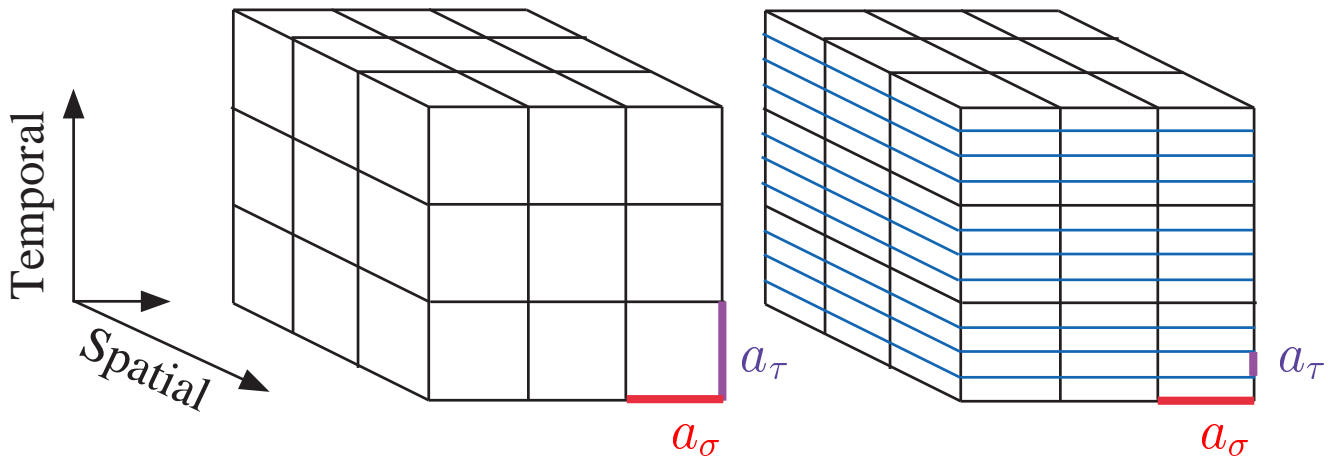
– Flavor singlet

We investigate the flavor-singlet baryon spectrum with spin 1/2 and negative parity.

Anisotropic Lattice

ISOTROPIC LATTICE

ANISOTROPIC LATTICE



$$\xi = \frac{a_\sigma}{a_\tau}: \text{Anisotropy}$$

■ Anisotropic lattice

Temporal lattice spacing a_τ is finer than spatial one a_σ .

□ Advantage

- Extract detailed information

on the correlators in temporal direction

– Efficient for **excited hadrons**, *etc.*

(for those correlators noise grows rapidly against signals of correlators)

- Keeping the size of lattice comparatively modest.

Lattice setup

■ Gauge Action

- Anisotropic Wilson Gauge Action

Karsch, N.P.B205(1982)285

- Lattice Size : $16^3 \times 128$ at Quenched Level

- Lattice Parameters : $\xi = 4, \beta = 5.95$

- Scale :

$$a_\sigma^{-1} \simeq 1.46\text{GeV} : \text{Spatial}$$

$$a_\tau^{-1} \simeq 5.84\text{GeV} : \text{Temporal}$$

String tension

$$\sqrt{\sigma} \sim 427\text{MeV}$$

■ Quark Action

- Anisotropic $O(a)$ -improved (Clover) Quark Action

Hadron Spectrum

■ Two-Point Correlator :

$$\begin{aligned}
 G_H(t) &= \sum_{\vec{x}} \langle 0 | T \{ \mathcal{O}_H(x) \bar{\mathcal{O}}_H(0) \} | 0 \rangle \\
 &= \sum_{\vec{x}} \sum_n \langle 0 | T \{ \mathcal{O}_H(x) | n \rangle \langle n | \bar{\mathcal{O}}_H(0) \} | 0 \rangle \\
 &\sim e^{-m_0 t} + \sum_{n \neq 0} e^{-m_n t} \\
 &\rightarrow e^{-m_0 t} \quad \text{at large } t
 \end{aligned}$$

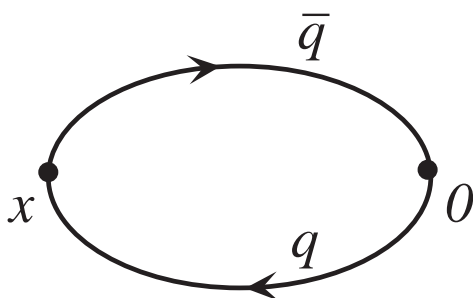
m_0 : Ground State Mass

\mathcal{O}_H : Hadron Operator

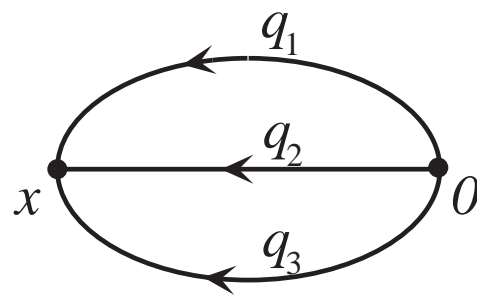
■ Hadron Operators

MESON	Pseudoscalar	$\bar{q}_c \gamma_5 q_c$
	Vector	$\bar{q}_c \gamma_k q_c$
BARYON	Octet	$\epsilon_{abc} q_a (q_b^T C \gamma_5 q_c)$
	Decuplet	$\epsilon_{abc} q_a (q_b^T C \gamma_k q_c)$
	Singlet	$\epsilon_{abc} \epsilon_{uds} q_a (q_b^T C \gamma_5 q_c)$

C : Charge Conjugation Matrix



MESON



BARYON

■ Correlators on the Lattice

□ Meson

$$G_M(t) = c_M \left[e^{-tm_M} + e^{-(T-t)m_M} \right]$$

:Periodic Boundary Condition

\implies m_M : meson mass

□ Baryon

$$G_B(t) = (1 + \gamma_4) \left[c_{B^+} e^{-tm_{B^+}} + c_{B^-} b e^{-(T-t)m_{B^-}} \right] \\ + (1 - \gamma_4) \left[c_{B^+} b e^{-(T-t)m_{B^+}} + c_{B^-} e^{-tm_{B^-}} \right]$$

$b = +1$: Periodic Boundary Condition

$b = -1$: Antiperiodic Boundary Condition

Combination of correlators

with either boundary condition.

\implies m_{B^+} : Positive parity baryon mass
 m_{B^-} : Negative parity baryon mass

Numerical Results of Hadron Spectrum

■ Mesons

⊗ Pseudoscalar

$$\pi(u\bar{d}, \bar{u}d, \dots), K(u\bar{s}, \bar{u}s, \dots), \dots$$

⊗ Vector

$$\rho(u\bar{u}, d\bar{d}), K^*(u\bar{s}, \dots), \phi(s\bar{s})$$

■ Baryons

□ Positive Parity Baryon

⊗ Decuplet

$$\Delta(uuu, \dots), \Sigma^*(uus, \dots), \Xi^*(uss, \dots), \Omega(sss)$$

⊗ Octet

$$N(uud, udd), \Sigma(uus, \dots), \Xi(uss, \dots), \Lambda(uds)$$

⊗ Singlet

□ Negative Parity Baryon

⊗ Decuplet

⊗ Octet

⊗ Singlet $\Lambda(uds) \rightarrow \Lambda(1405)?$

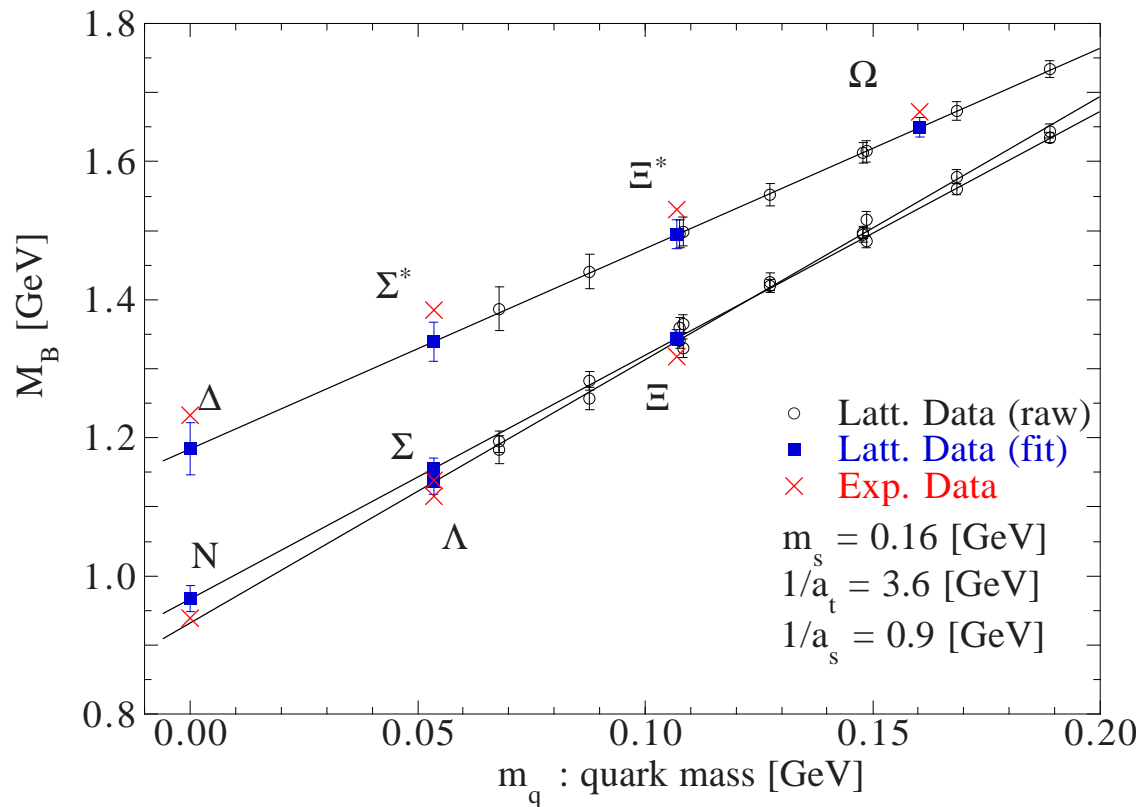
⊙ Result 1

- Octet and decuplet positive parity baryons
- Mass difference

⊙ Result 2

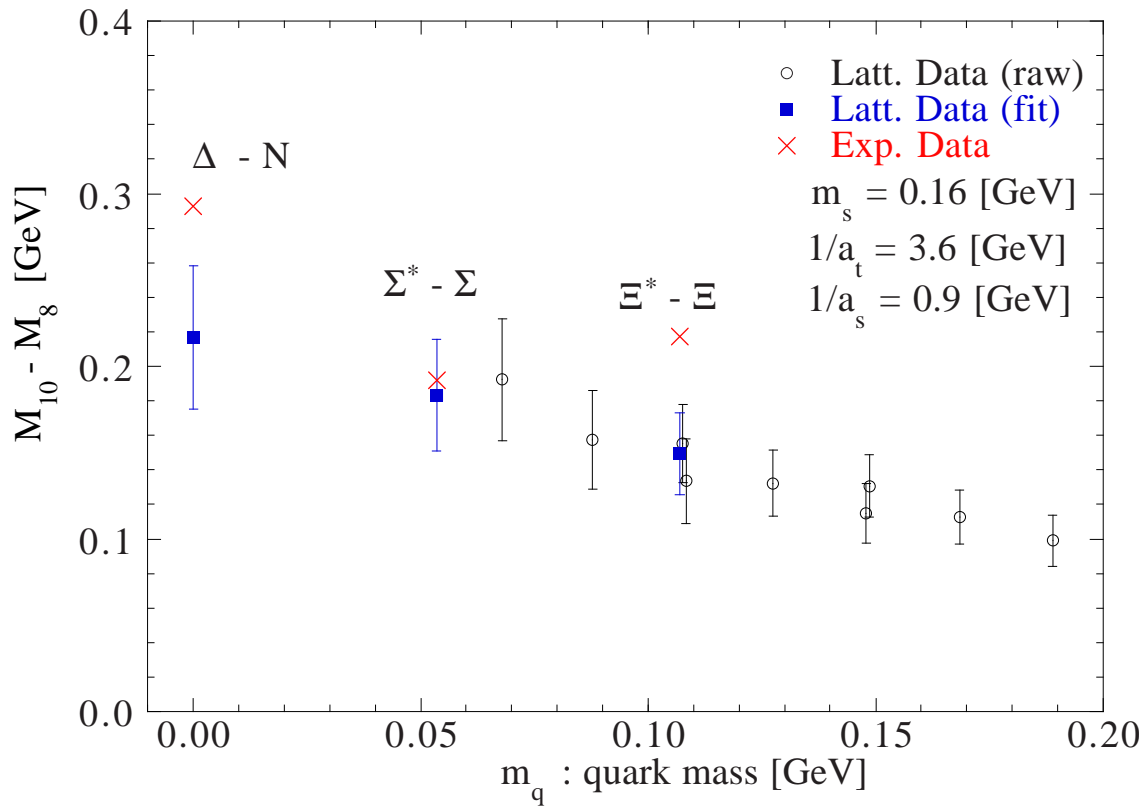
- Flavor-Singlet Negative Parity Baryon

Octet and Decuplet Baryon spectra



- u, d -quark masses are zero in chiral limit
- Scale and s -quark masses are determined by fitting the intercept to experimental data.

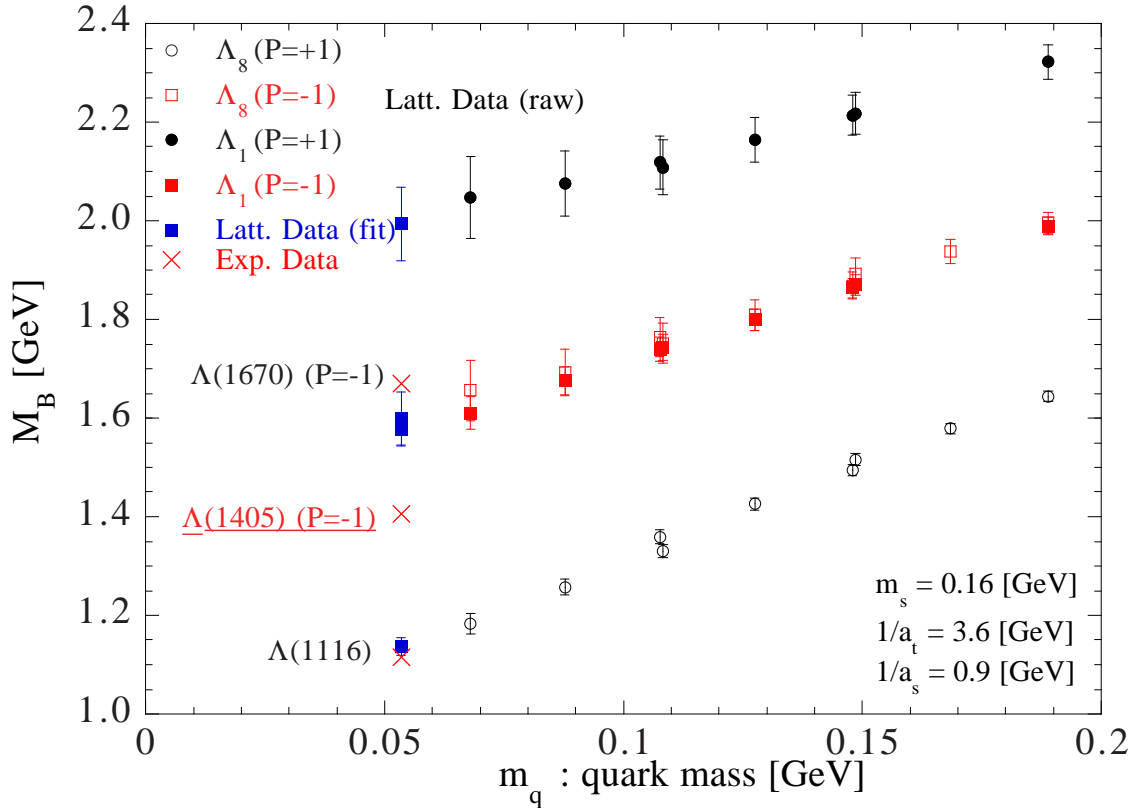
Mass Difference between Octet and Decuplet Baryons



○ Octet-decuplet mass difference decreases as quark mass increases.

→ consistent with the one-gluon-exchange

Λ baryons spectrum



We have calculated

- \circ both particles (positive & negative parity); $P = \pm 1$
- \circ both singlet and octet ; Λ_1, Λ_8

- \circ Mass of $\Lambda_1(-1)$ is bigger than $\Lambda(1405)$
- \circ Degeneracy of $\Lambda_1(-1)$ and $\Lambda_8(-1)$

Summary & Outlook

■ Lattice QCD

Anisotropic Lattice

$O(a)$ -improved quark action

Lattice size : $12^3 \times 96$, $\beta = 5.75$, $\xi = 4$

■ Light Hadron Spectrum

□ Decuplet, Octet and Singlet Baryons

□ Positive Parity Baryons

N , Σ , Ξ , Λ , Δ , Σ^* , Ξ^* , Ω

□ Singlet & Octet Λ Baryons

⊗ Mass Difference of Octet and Decuplet Baryon

- Behavior is consistent with OGE explanation.

⇒ determination of model parameters (future)

⊗ Excited Hadron (Negative Parity Baryon), $\Lambda(1405)$

- Mass of $\Lambda_1(\frac{1}{2}^-)$ is bigger than $\Lambda(1405)$.

- Degeneracy of $\Lambda_1(\frac{1}{2}^-)$ and $\Lambda_8(\frac{1}{2}^-)$

⇒ comparing with a 5 quark state (future)

(→ full QCD calculation)

⇒ need calculations on finer lattices

Calibration

□ Lattice Klein-Gordon Action for Meson Field

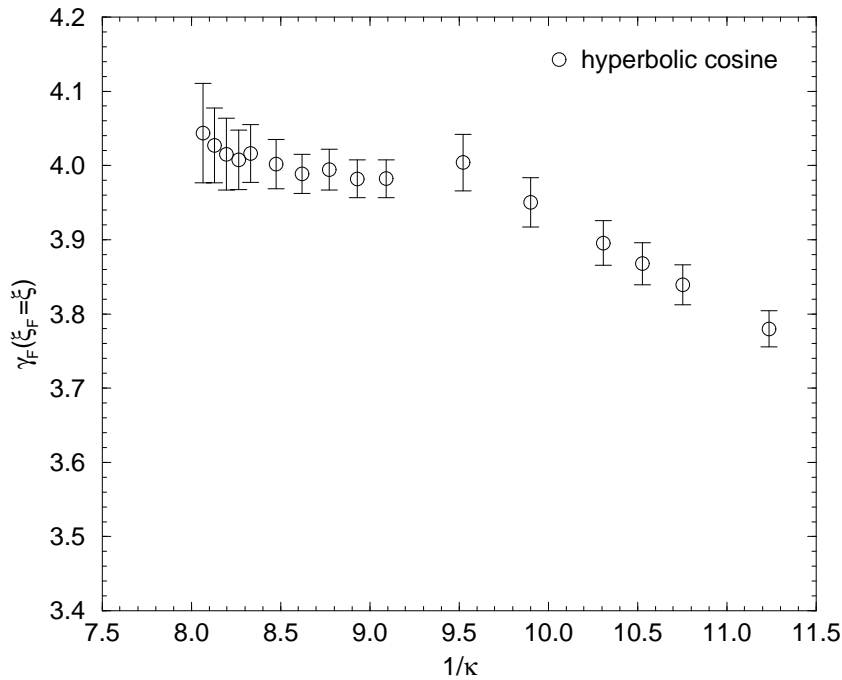
$$S = \sum_x \frac{1}{2\xi_F} \phi^\dagger(x) \left[-\xi_F^2 D_4^2 - \vec{D}^2 + m_0^2 \right] \phi(x),$$

where m_0 is in the unit of a_σ .

□ Dispersion Relation

$$\cosh E(\vec{p}) = 1 + \frac{1}{2\xi_F^2} (\hat{p}^2 + m_0^2).$$

$$\xi_F^2 = \frac{\hat{p}^2}{2(\cosh E(\vec{p}) - \cosh E(0))}$$



Lattice setup

Gauge Action

- Anisotropic Wilson Gauge Action

Karsch, N.P.B205(1982)285

- Lattice Size : $12^3 \times 96$ at Quenched Level

- Parameters

Klassen, N.P.B533(1998)557

for $\xi = 4$, $(\beta, \gamma_G) = (5.75, 3.072)$

(1% level)

- Scale Static Quark Potential (Wilson Loop)
 $\sqrt{\sigma} = 427 \text{ GeV}$

$$a_\sigma^{-1} = 1.000(8) \text{ GeV}$$

$$a_\tau^{-1} \simeq 4 \text{ GeV}$$

- Configuration

for Calibration (400)

for Hadron Correlator (120)

Pseudo-Heat Bath

thermalization = 10000

separation = 2000

Quark Action

$O(a)$ -improved Quark Action (Clover Quark Action)

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y),$$

$$K(x,y) = \delta_{x,y} - \kappa_\tau \left\{ (1 - \gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1 + \gamma_4) U_4^\dagger(x - \hat{4}) \delta_{x-\hat{4},y} \right\} \\ - \kappa_\sigma \sum_i \left\{ (r - \gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r + \gamma_i) U_i^\dagger(x - \hat{i}) \delta_{x-\hat{i},y} \right\} \\ - \kappa_\sigma c_E \sum_i i \sigma_{4i} F_{4i}(x) \delta_{x,y} - r \kappa_\sigma c_B \sum_{ij} \frac{1}{2} \sigma_{ij} F_{ij}(x) \delta_{x,y}.$$

□ Hopping Parameters

$$\kappa_\sigma = \frac{1}{2(m_0 + \gamma_F + 3r)} : \text{spatial} \quad \gamma_F: \text{bare anisotropy}$$

$$\kappa_\tau = \gamma_F \kappa_\sigma : \text{temporal}$$

□ Wilson Parameter $r = \frac{1}{\xi}$

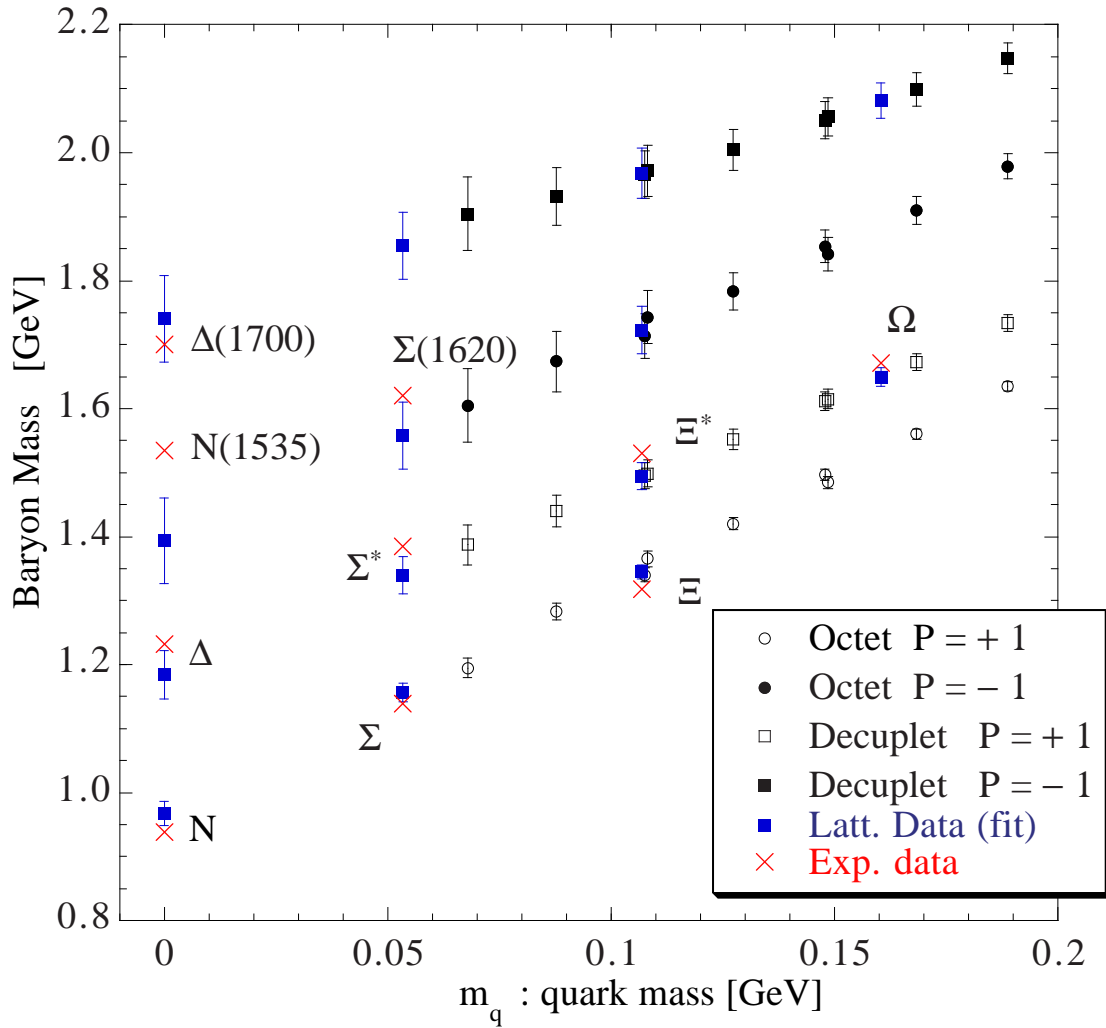
$$\rightarrow \frac{1}{\kappa} = \frac{1}{\kappa_\sigma} - 2(\gamma_F + 3r - 4) = 2(m_0 + 4)$$

Chiral extrapolation in $\frac{1}{\kappa}$

□ Clover Coefficients

$$c_E = c_B = 1, \text{ at tree level for } r = \frac{1}{\xi}$$

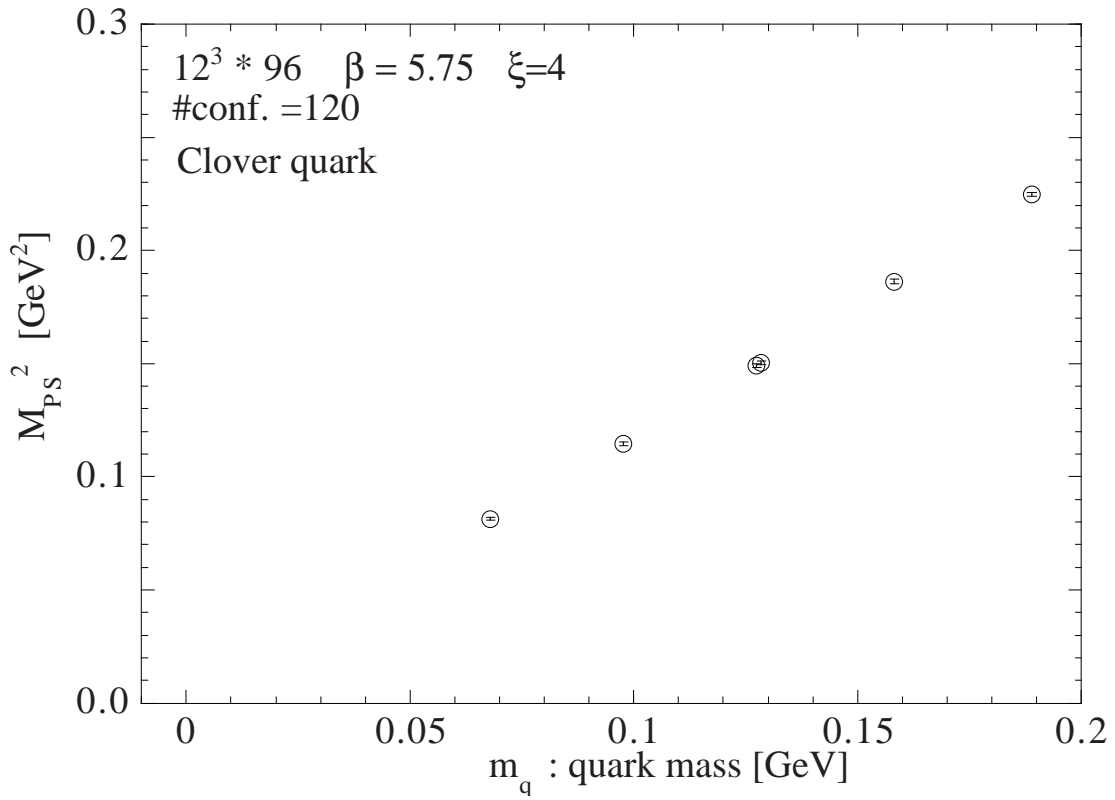
Octet & Decuplet baryon spectra



We have calculated

- both particles (positive & negative parity); $P = \pm 1$
- both singlet and octet

Pseudoscalar meson squared vs. quark mass



$$m_\pi^2 \sim m_q \text{ (CHPT)}$$