$O(a)$ improved Wilson Quark Action on Anisotropic Lattice (II)

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Contents:

c.f. Matsufuru, Onogi and Umeda, hep-lat/0107001 (to appear in Phys. Rev. D)
Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215
Harada et al., hep-lat/0103026 (to appear in Phys. Rev. D)

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Introduction — Summary of Umeda's talk — (1)

We investigate $O(a)$ improved Wilson quark action on quenched anisotropic lattice, in the quark mass region from the chiral limit to around the charm quark mass.

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \;=\; & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & - \kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & - \kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

$r = 1/\xi$

(cf. Klassen (1998), Chen (2000), CP-PACS (2000): $r = 1$)

Mean-field improvement: $\kappa_\sigma = \tilde{\kappa}_\sigma / u_\sigma, \quad \kappa_\tau = \tilde{\kappa}_\tau / u_\tau$

$c_E = 1/u_\sigma^2 u_\tau, \quad c_B = 1/u_{\sigma^3}$

u_τ, u_σ : mean-field values in Landau gauge

$$\gamma_F \equiv \frac{\tilde{\kappa}_\tau}{\tilde{\kappa}_\sigma}\,, \qquad \frac{1}{\kappa} = \frac{1}{\tilde{\kappa}_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0 \gamma_F + 4))$$

This action follows the Fermilab approach.

El-Khadra et al., Phys. Rev. D 55 (1997) 3933

At the tree level, for small quark mass,

$$\frac{1}{\gamma_F} = \frac{1}{\xi} \left(1 + \frac{1}{3} m_0^2 \right)$$

— no linear term in m_0 for $m_0 \ll a_\tau^{-1}$.

Introduction — Summary of Umeda's talk — (2)

Numerical simulation

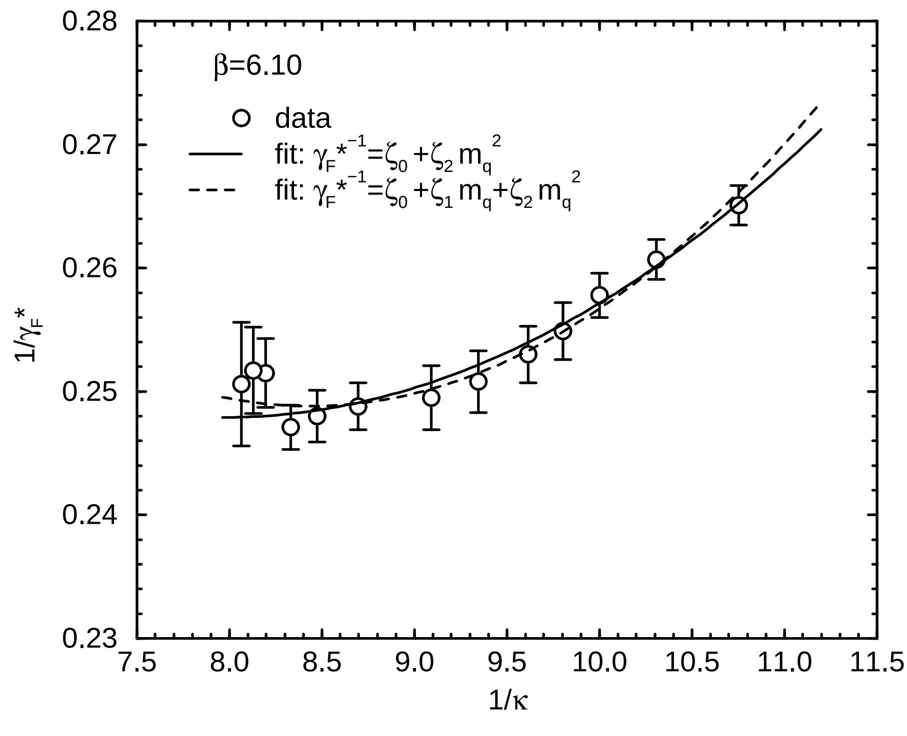
□ Gauge field: quenched, $\xi = 4$

β	γ_G	size	$a_\sigma^{-1}(r_0)$ [GeV]	$N_{conf.}$ (max)
5.75	3.072	$12^3 \times \ 96$	1.100(6)	400
5.95	3.1586	$16^3 \times 128$	1.623(9)	500
6.10	3.2108	$20^3 \times 160$	2.030(13)	600

□ Quark field:

At each β, $\kappa = 0.124 \ \sim \ 0.93$ (13 points)
(strange $\sim$ charm quark mass)
$N_{conf} = 160 \sim 600$ (lightest mass, $\beta = 6.10$)
Tadpole improvement with MF values in Landau gauge Input γ_F: 2 – 4 points $\rightarrow$ interpolation to find γ_F^* s.t. $\xi_F(\gamma_F^*) = \xi$

Result: m_q dependence is actually small in light quark region, and well fitted to linear form in m_q^2.



Goals of this talk

We determined $\gamma_F^*(m_q)$ in the region of $m_q = 0 \sim m_c$ at each β.

Goals of this talk are to examine:

□ systematic errors in γ_F^*

$$\gamma_F^* = \gamma_F^{*(prop)} + \delta\gamma_F^{(stat)} + \delta\gamma_F^{(O(\alpha a))} + \delta\gamma_F^{(O(a^2))}(+\delta\gamma_F^{(chiral)})$$

($\delta\gamma_F^{(stat)}$ is at 1 % level.)

□ how errors in γ_F effect on the spectrum
$\rightarrow$ light hadron spectroscopy with γ_F^* and $\gamma_F' = \gamma_F^* + \delta\gamma_F$

Calibration steps:

(a) Extraction of meson energy $E(\vec{p})$ from correlators

(b) Fit to dispersion relation

$\xi_F^{(PS)}$ and $\xi_F^{(V)}$ $\Rightarrow \gamma_F^{*(PS)}$ and $\gamma_F^{*(V)}$ respectively
average over $\gamma_F^{*(PS)}$ and $\gamma_F^{*(V)} \Rightarrow \gamma_F^*$

- ∘ Difference of $\gamma_F^{*(V)}$ and $\gamma_F^{*(PS)}$ — $O(\alpha a)$ error
- ∘ Uncertainty in assumed form of dispersion relation — $O(a^2)$ error

(c) Fit of γ_F^* and chiral extrapolation

- linear form in m_q^2
- quadratic form in m_q

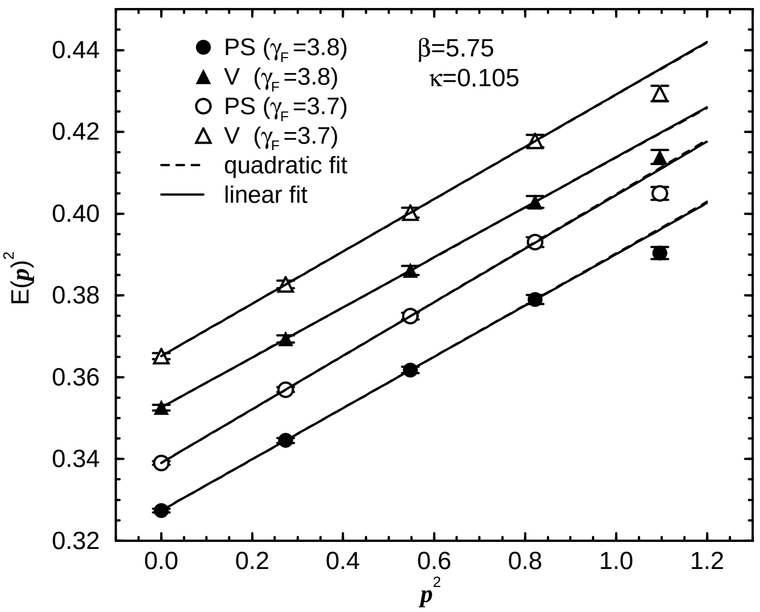
$\rightarrow$ difference: error of γ_F^* in the chiral limit

□ $O(\alpha a)$ and $O(a^2)$ errors should vanish as $a \rightarrow 0$.

□ Error in chiral limit can be improved by statistics.

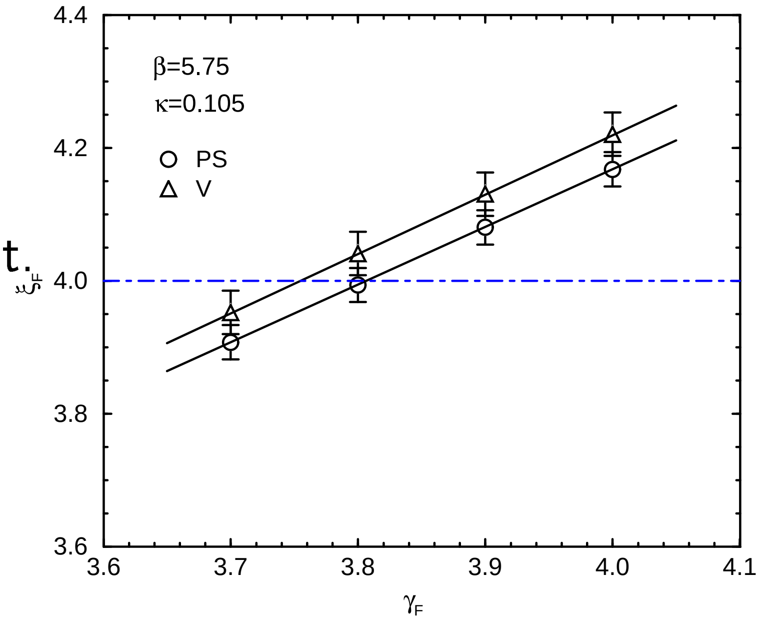
Systematic errors in calibration (1)

Energy of mesons $E(\vec{p})$:
fitted to the dispersion relation
linear/quadratic forms
$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$
$\Downarrow$
fermionic anisotropy
$\xi_F^{(PS)}$, $\xi_F^{(V)}$
for each input γ_F



Linear interpolation to $\xi_F = \xi$
$\Downarrow$
$\gamma_F^{*(PS)}$, $\gamma_F^{*(V)}$

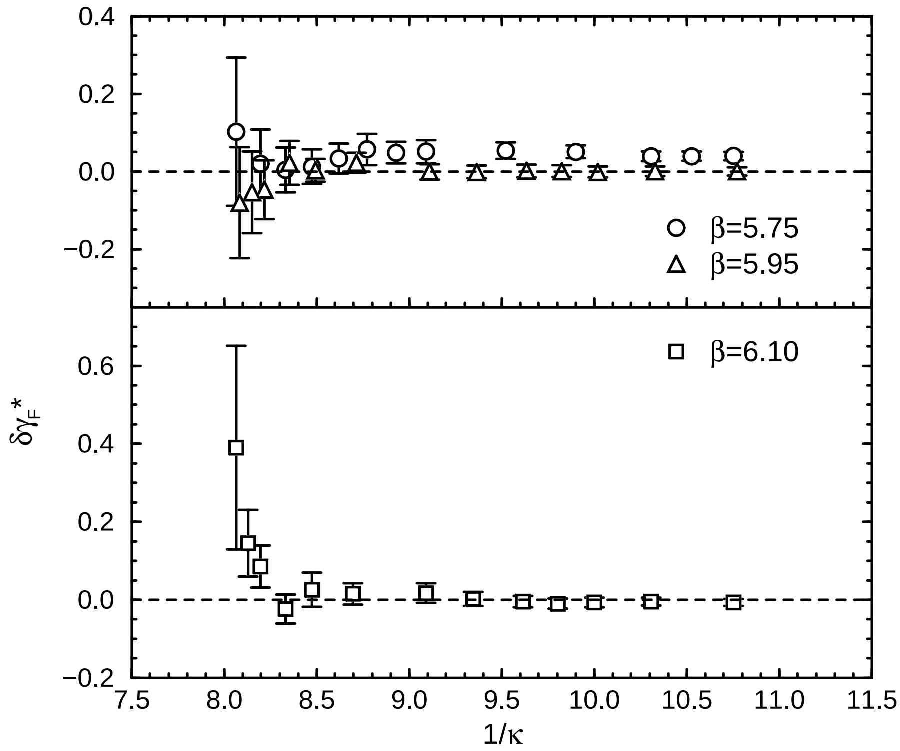
For finite a and imperfect clover coefficients,
$\gamma_F^{*(PS)}$ and $\gamma_F^{*(V)}$ are different.



Systematic errors in calibration (2)

$\delta\gamma_F^{(O(\alpha a))} = \gamma_F^{*(PS)} - \gamma_F^{*(V)}$

— represents typical size of $O(\alpha a)$ systematic error



$\delta\gamma_F^{(O(\alpha a))} \simeq$ 0.05 ($\sim$ 1 %) at $\beta = 5.75$,

and tends to disappear at higher β.

Systematic errors in calibration (3)

Uncertainty due to assumed form of dispersion relation
— represents typical $O(a^2)$ systematic error
We adopt the relativistic dispersion relation

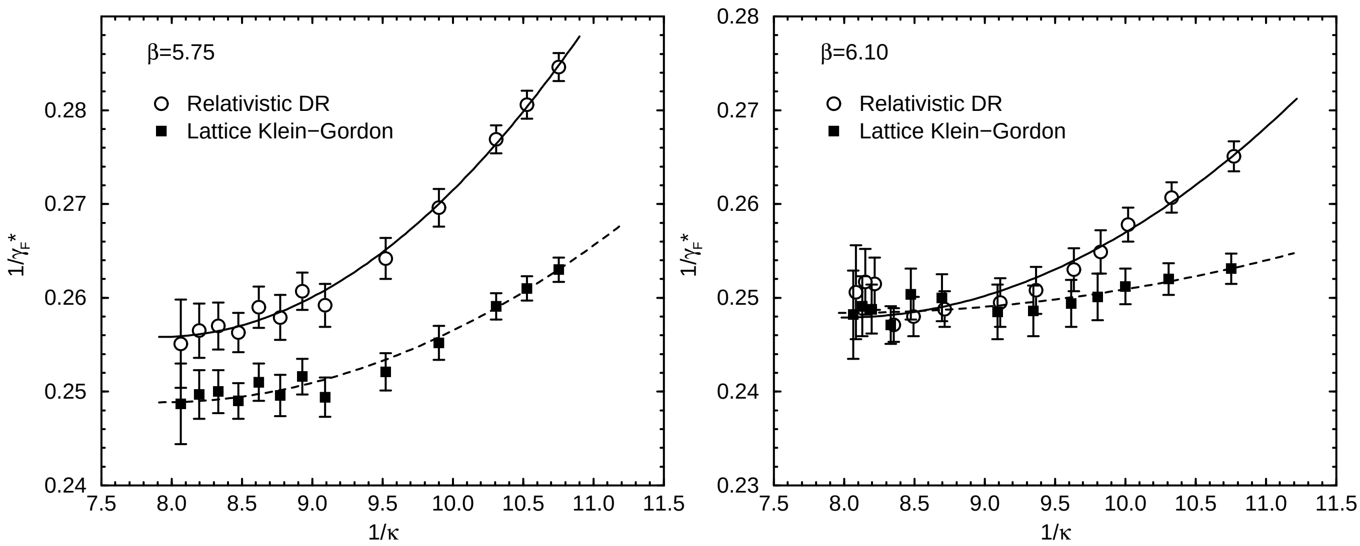
$$E(\vec{p})^2 = m^2 + \frac{1}{\xi_F^2}\vec{p}^2 + O(\vec{p}^4)$$

Comparison with lattice Klein-Gordon dispersion relation:

$$\cosh E(\vec{p}) - \cosh E(0) = 1 + \frac{1}{\xi_{KG}^2}\vec{p}^2 + O(\vec{p}^4)$$

□ For large m, relativistic dispersion relation properly incorporate quark mass dependence.
□ For small m, $\gamma^*_{F(KG)} = \gamma^*_F\,(1 + O(m^2))$

Difference: 3, 2, and 1 % at $\beta = 5.75$, 5.95 and 6.10
— tends to disappear as $a \to 0$.



Systematic errors in calibration (4)

Systematic error in the chiral extrapolation

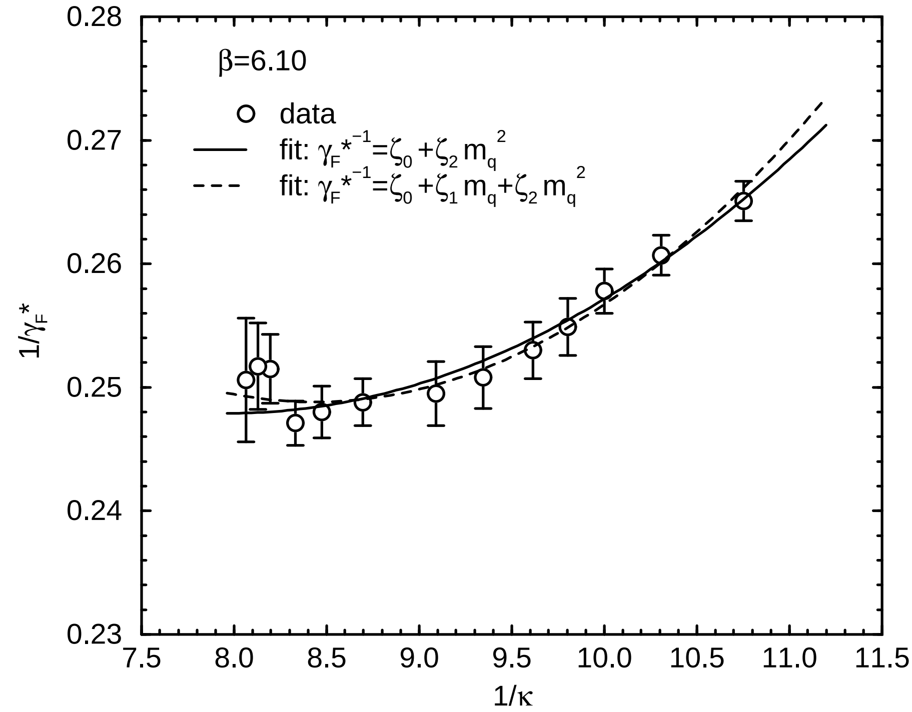
We fit γ_F^* to functions of m_q

$$\frac{1}{\gamma_F^*} = \zeta_0 + \zeta_1\, m_q + \zeta_2\, m_q^2, \qquad m_q = \frac{1}{2\xi}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

- linear form in m_q^2
- quadratic form in m_q

Difference of ζ_0 with two fits: estimate of $\delta\gamma_F^{(chiral)}$
$\rightarrow$ $\delta\gamma_F^{(chiral)}$ is at 1% level.

β	fit-type	ζ_0	ζ_1	ζ_2	γ_F^* ($m_q = 0$)
5.75	linear	0.2558(9)	—	0.230(12)	3.909(14)
	quad.	0.2564(23)	−0.007(28)	0.247(68)	3.901(34)
5.95	linear	0.2490(8)	—	0.189(15)	4.016(13)
	quad.	0.2465(18)	0.036(23)	0.095(61)	4.057(30)
6.10	linear	0.2479(9)	—	0.143(14)	4.034(14)
	quad.	0.2493(18)	−0.022(24)	0.200(63)	4.011(28)



Systematic errors in calibration (5)

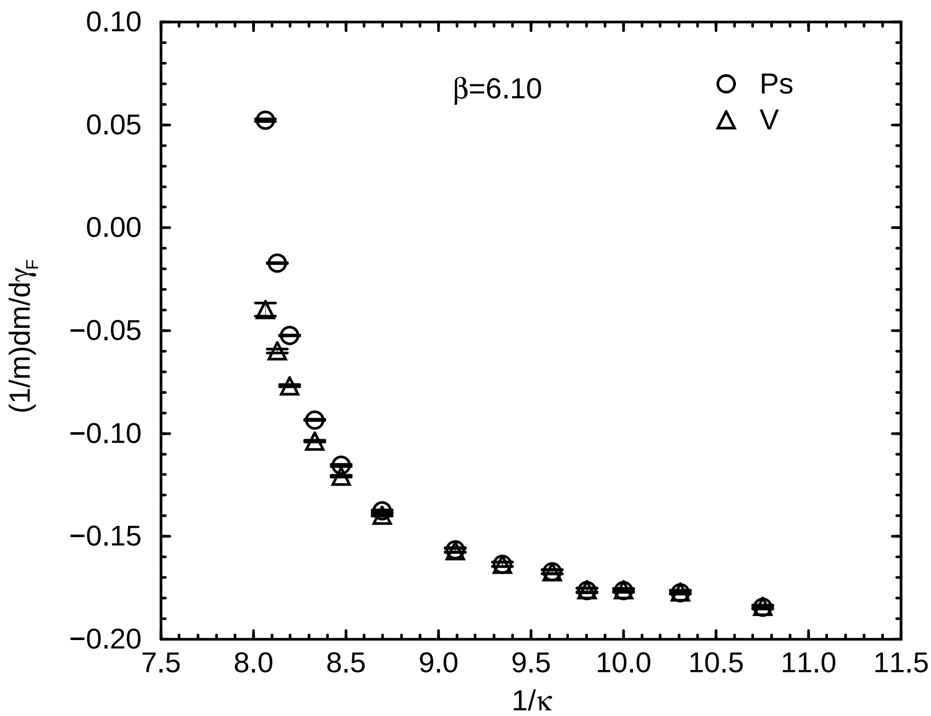
Summary of systematic errors in γ_F^*

$$\gamma_F^* = \gamma_F^{*(prop)} + \delta\gamma_F^{(stat)} + \delta\gamma_F^{(O(\alpha a))} + \delta\gamma_F^{(O(a^2))}(+\delta\gamma_F^{(chiral)})$$

$\delta\gamma_F^{(O(\alpha a))}$ ← difference of $\gamma_F^{*(PS)}$ and $\gamma_F^{*(V)}$
 $\sim$ 1 % at $\beta = 5.75$, and almost vanish at higher β.
$\delta\gamma_F^{(O(a^2))}$ ← relativistic and lattice K-G dispersion relation
 $\sim$ 3, 2 and 1% at β =5.75, 5.95 and 6.10, respectively.
$\delta\gamma_F^{(chiral)}$ ← two types of fit: linear in m_q^2 and quadratic in m_q
 At 1% level at each β

All systematic errors are under control in the continuum limit.

Effect on the meson masses:
 — 2 % error in γ_F causes errors in masses less than 1 %.



Light Hadron Spectroscopy (1)

Light hadron spectrum is obtained with standard procedures.

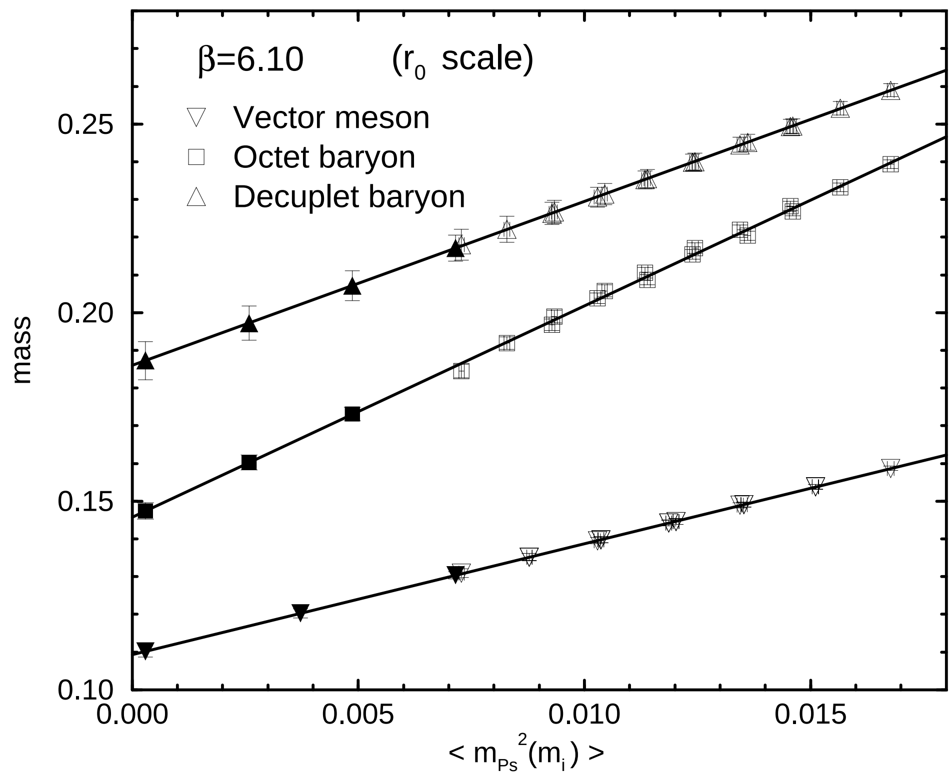
Gauge parameters: are same as in calibration.
Quark: 4 κ's in the strange quark mass region,
with γ_F^* in the chiral limit.

β	size	κ	γ_F	a_σ^{-1} [GeV]	$N_{conf.}$
5.75	$12^3 \times 96$	0.1210, 0.1220, 0.1230, 0.1240	3.909	1.100(6)	200
5.95	$16^3 \times 128$	0.1230, 0.1235, 0.1240, 0.1245	4.016	1.623(9)	100
6.10	$20^3 \times 160$	0.1230, 0.1235, 0.1240, 0.1245	4.034	2.030(13)	100

Scale:

- ☐ Hadronic radius r_0
- ☐ K^* meson mass

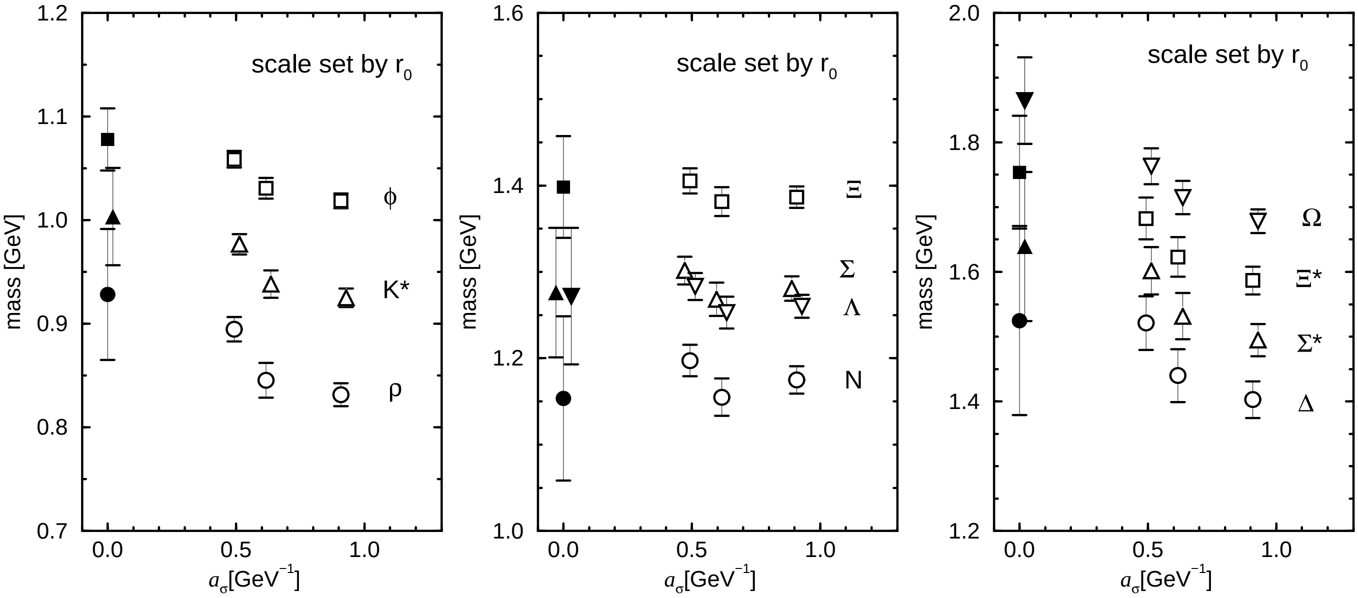
Quark masses m_n (u and d) and m_s are set by m_π and m_K.
Chiral extrapolation: in terms of m_{PS}^2.



Light Hadron Spectroscopy (2)

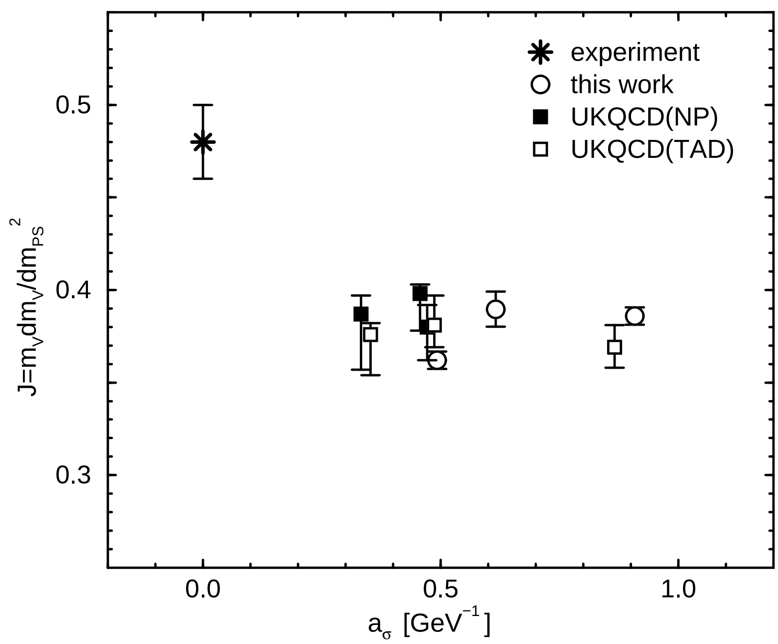
Spectrum for physical quark masses (scale set by r_0)
— Compared with the result on isotropic lattice (in $a \to 0$)

UKQCD Collab., Phys.Rev. D 62(2000) 054506.



Parameter J

$$J \equiv m_V \left. \frac{dm_V}{dm_{PS}^2} \right|_{\frac{m_V}{m_{PS}} = \frac{m_{K^*}}{m_K}}$$

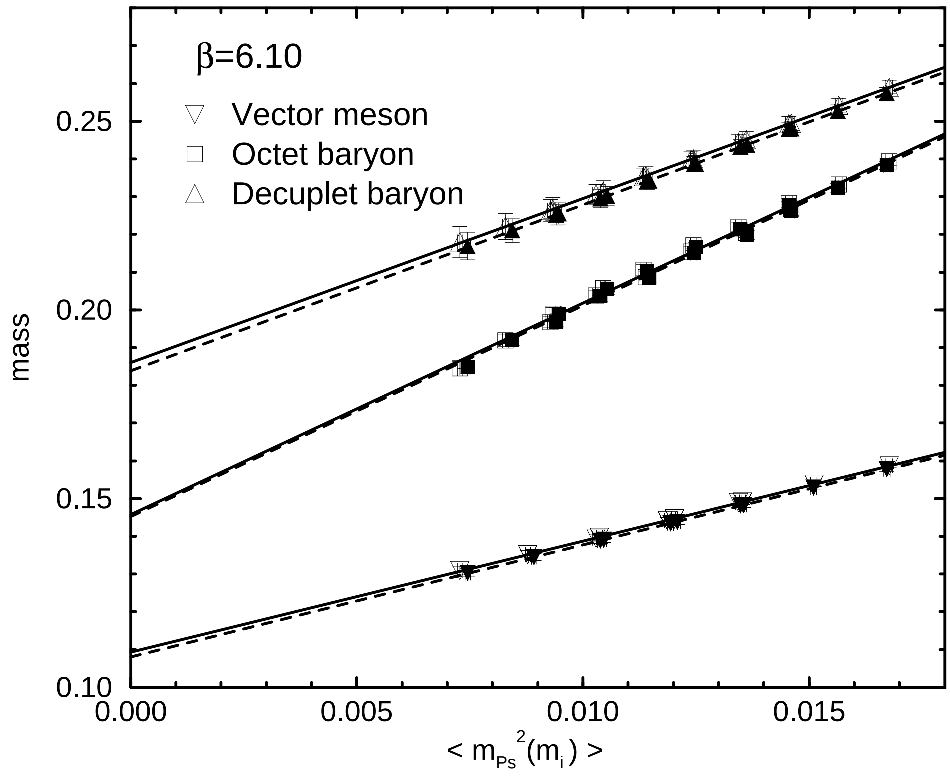


Light hadron spectrum is consistent with the results on isotropic lattices.

Light Hadron Spectroscopy (3)

Effect of systematic errors on the spectrum:
the same simulation at

$$\gamma_F' = \gamma_F^* + \delta\gamma_F \qquad \delta\gamma_F = 0.1 \quad \text{(2.5\% shift)}$$



The change of spectrum for physical quark masses is less than 1%.

Conclusion and Outlook

Conclusion

We implemented the $O(a)$ improved Wilson quark action on the quenched anisotropic lattice at three β in the quark mass region from the chiral limit to the charm quark mass region.

☐ Systematic errors are totally a few percent, and decrease toward the continuum limit.

☐ Light hadron spectrum at physical quark mass:
consistent with results on isotropic lattice.
changes 1 % for the 2 % change of bare anisotropy.

— The systematic uncertainties due to the anisotropy are under control.

Outlook

☐ Applications

Finite temperature QCD (cf. Nomura's poster)
Heavy-light systems (decay const., excited states, etc.)
Detailed studies of hadron structure

☐ Further implementation

With dynamical quark
Nonperturbative renormalization