Charmed hadron physics in quenched anisotropic lattice QCD

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Contents:

Refs.:

J. Harada, A.S. Kronfeld, H. Matsufuru, N. Nakajima and T. Onogi,, Phys. Rev. D 64 (2001) 074501.

H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. D 64 (2001) 114503.

J. Harada, H. Matsufuru, T. Onogi and A. Sugita, in preparation.

This copy is available at

`http://www.rcnp.osaka-u.ac.jp/~matufuru/`

(24 Nov 2001 updated)

Introduction (1)

Charmed hadron physics in lattice QCD

□ Hadronic matrix elements

decay constants, form factors, etc.

Precision is required for comparison with experiments

⇒ Systematic errors are to be controlled

□ Determination of charm quark mass

Present charm quark mass: $m_c = 1.15$–1.35 GeV

— More precise determination is desired

□ Test of heavy quark expansion

$$O = O_0 + O_1 \frac{1}{m_Q} + O_2 \frac{1}{m_Q^2} + \cdots$$

Size of correction terms

Size of relativistic effect

To be compared with results around b quark mass

□ Charmed hadron spectrum

QQq, Qqq baryons

Orbitally excited states, etc.

Lattice QCD

⇒ Model independent,
nonperturbative calculation based on QCD

Introduction (2)

Problems of charm quark on lattice

Charm quark mass: $m_c \simeq 1.2$ GeV
— Not so light, not so heavy

Present available lattice cutoff: $a^{-1} = 1 \sim 3$ GeV
m_c is not sufficiently less than lattice cutoff.

Standard Wilson/clover quark action
Suffer from large $O(ma)$ errors.
Bruteforce improvement costs computational resources.

Effective theoretical approaches

- Nonrelativistic QCD (NRQCD),
  Heavy quark effective theory
  Expansion in $1/m_Q$
  Relativistic effect may be important
  No naive continuum limit
- Fermilab approach
  For quark mass not sufficiently less than a^{-1},
  temporal and spatial hopping parameters should be tuned
  rotation of quark field is to be incorporated

Yet another approach is required for systematic studies.

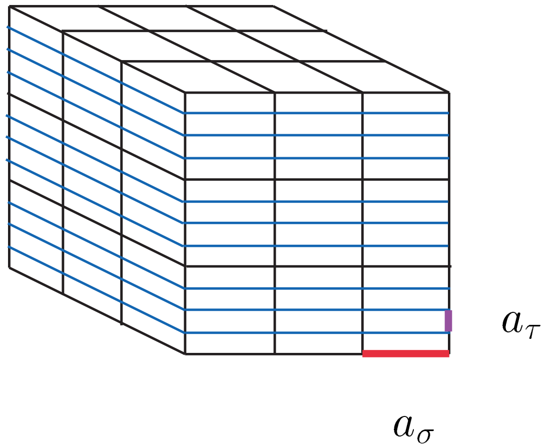
- ◦ Further improvement to remove systematic errors
  Nonperturbative renormalization, etc.
- ◦ Applications to various processes
  Simulations on relatively small resources

Our solution: Anisotropic lattice ($a_\tau < a_\sigma$)

Anisotropic lattice QCD (1)

Anisotropic lattice: $a_\tau < a_\sigma$
$\Rightarrow$ Anisotropy $\xi = a_\tau / a_\sigma$

Advantages:



○ $m_c \ll a_\tau^{-1}$ cab be achieved
while keeping total lattice
size comparably modest.
$\Rightarrow$ relativistic treatment

○ Fine temporal resolution
easy to follow change of
correlators

○ Smooth continuum limit

Instead, one needs

- Additional tuning of parameters
  ξ in general differs from bare anisotropy in actions.
  $\rightarrow$ To be determined nonperturbatively.
  Bare parameters should be tuned to retain desired ξ.
- Calculations of related quantities
  Lattice cutoffs, spectrum, renormalization constants, etc.
  (available previous works are not many)
- Tests of applicability
  Scaling, systematic errors,
  consistency with isotropic lattices

Anisotropic lattice QCD (2)

We implement an $O(a)$ improved Wilson quark action on quenched anisotropic lattice, in the quark mass region from the chiral limit to around the charm quark mass.

Lattice QCD

SU(3) gauge theory on Euclidean 4D lattice.
lattice spacing a_μ $\big(a_i = a_\sigma$ $(i=1,2,3)$, $a_4 = a_\tau\big)$

$$\begin{aligned}\langle O\rangle &= \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi O[U,\bar{\psi},\psi]\exp(-S_G - S_F) \\ &= \int \mathcal{D}U \det K[U] O[U]\exp(-S_G)\end{aligned}$$

$$U_\mu(x) \simeq \exp[-iga_\mu A_\mu(x)]$$

$$\Downarrow$$

$$\simeq \frac{1}{N}\sum_{i=1}^{N} O[U_i]$$

with configurations U_i generated with probability $\det K \exp(-S_G)$.
: Monte Carlo simulation

- Setting $\det K = 1$: quenched approximation
  No dynamical quark effect
- Quark propagator $S_q(x,y) = K^{-1}(x,y)$:
  obtained by solving linear equation system
- Hadron correlator:

$$C(t) = \sum_{\vec{x}}\langle H(\vec{x},t)H^\dagger(0,0)\rangle \to Z\exp(-mt)$$

$\Rightarrow$hadron mass, decay constant

Anisotropic lattice QCD (3)

Gauge field action:

$$S_G = \beta \sum_{x,i<j} \frac{1}{\gamma_G}\left[1-\frac{1}{3}\mathrm{ReTr}U_{ij}(x)\right] + \beta \sum_{x,i} \gamma_G \left[1-\frac{1}{3}\mathrm{ReTr}U_{i4}(x)\right]$$

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

F. Karsch, Nucl. Phys. B205 (1982) 285.

- $\beta = 6/g_0^2$ (g_0: bare coupling)
- γ_G: bare anisotropy
  $\gamma_G = \xi$ at the tree level.
  To be tuned for desired renormalized anisotropy ξ.

Definition of ξ: with gauge field observables
e.g., Wilson loops in space-time and space-space planes.
G. Burgers et al., Nucl. Phys. B304 (1988) 587.

In quenched calculation (no dynamical quark effect),
ξ can be determined independently of fermionic parameters.

Klassen (1998): Determination of γ_G as a function of ξ
in wide range of β and ξ with 1% accuracy
using the ratio of Wilson loops in quenched QCD.
T.R. Klassen, Nucl. Phys. B533 (1998) 557.

In this work, we make use of his result.

Cf. the same definition of ξ is adopted also in
QCD-TARO Collab., Nucl. Phys. B (Proc.Suppl.) 53 (1997) 426,
J. Engels et al., Nucl. Phys. B (Proc.Suppl.) 63 (1998) 427.

Anisotropic lattice QCD (4)

Quark action:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \; = \; & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

T. Umeda et al., Int. J. Mod. Phys. A 16 (2001) 2215

J. Harada et al., Phys. Rev. D 64 (2001) 074501

Constructed following the Fermilab approach.

El-Khadra et al., Phys. Rev. D 55 (1997) 3933

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
  (cf. another choice $r = 1$ was adopted by
  Klassen, Nucl. Phys. B (Proc. Suppl.) 73 (1998) 918,
  P. Chen, Phys. Rev. D 64 (2000) 034509,
  CP-PACS Collab., Nucl. Phys. B (Proc. Suppl.) 94 (2000) 325.)
- Tadpole improvement: $\kappa_\sigma = \tilde{\kappa}_\sigma / u_\sigma, \quad \kappa_\tau = \tilde{\kappa}_\tau / u_\tau$
  $c_E = 1/u_\sigma^2 u_\tau, \quad c_B = 1/u_{\sigma^3}$

u_σ, u_τ: mean-field values of spatial and temporal link variables.

- Parameters varied in simulations: (κ, γ_F)

$$\gamma_F \equiv \frac{\tilde{\kappa}_\tau}{\tilde{\kappa}_\sigma}, \qquad \frac{1}{\kappa} = \frac{1}{\tilde{\kappa}_\sigma} - 2(\gamma_F + 3r - 4) \;\; (= 2(m_0 \gamma_F + 4))$$

m_0: bare quark mass (in temporal lattice units)

At the tree level, with $r = 1/\xi$ and for small quark mass,

$$\frac{1}{\gamma_F} = \frac{1}{\xi} \left(1 + \frac{1}{3} m_0^2 \right).$$

— no linear term in m_0 for $m_0 \ll a_\tau^{-1}$.

Calibration (1)

Numerical simulation:

In quenched approximation (no dynamical quark effect)

Anisotropy $\xi = a_\tau / a_\sigma = 4$

□ Gauge field

Action: standard Wilson plaquette action

β	γ_G	size	$a_\sigma^{-1}(r_0)$ [GeV]	$a_\sigma^{-1}(m_{K^*})$ [GeV]	u_σ	u_σ
5.75	3.072	$12^3 \times 96$	1.100(6)	1.053(13)	0.7620(2)	0.9871
5.95	3.1586	$16^3 \times 128$	1.623(9)	1.525(27)	0.7917(1)	0.9891
6.10	3.2108	$20^3 \times 160$	2.030(13)	1.817(22)	0.8059(1)	0.9901

γ_G is chosen according to the result in

T.R. Klassen, Nucl. Phys. B 533 (1998) 557.

Lattice scale a_σ^{-1} is set by:

- Hadronic radius r_0

  Defined with $r_0^2 F(r_0) = 1.65 \quad \left(r_0^{-1} = 495 \text{ MeV}\right)$

  R. Sommer, Nucl. Phys. B 411 (1994) 839.

- K^* meson mass

  From the result of light hadron spectroscopy

□ Quark field

Action: $O(a)$ improved Wilson (clover) action

Tadpole improvement with MF values in Landau gauge

In calibration:

At each β, $\kappa = 0.124 \sim 0.93$ (13 points)

quark mass range: strange $\sim$ charm quark mass

$N_{conf} = 160 \sim 600$ (lightest mass, $\beta = 6.10$)

Simulations are performed on

NEC SX-5 at RCNP, Osaka Univ. and Hitachi SR8000 at KEK.

Calibration (2)

Calibration at each quark mass (κ):

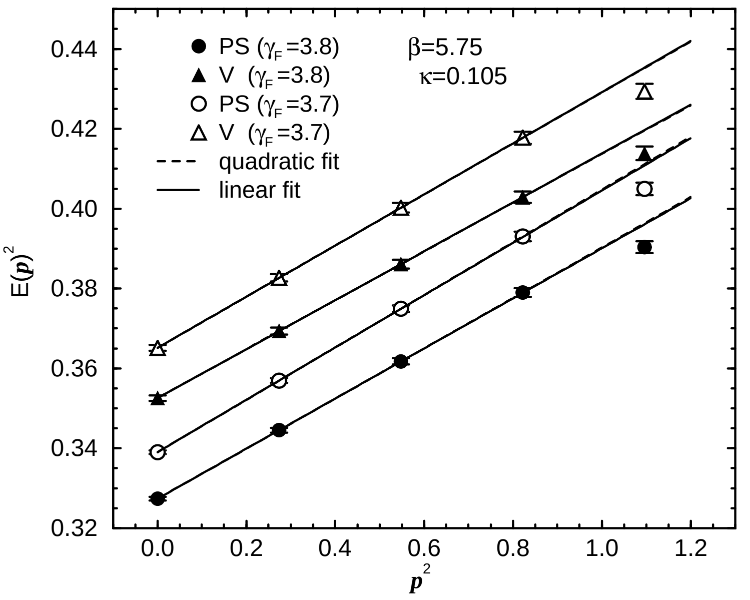
For each input γ_F,
Energy of mesons $E(\vec{p})$ is fitted to dispersion relation

$$E(\vec{p})^2 = m^2 + \frac{\vec{p}^2}{\xi_F^2} + O(\vec{p}^4)$$

(linear/quadratic forms)

$\Downarrow$

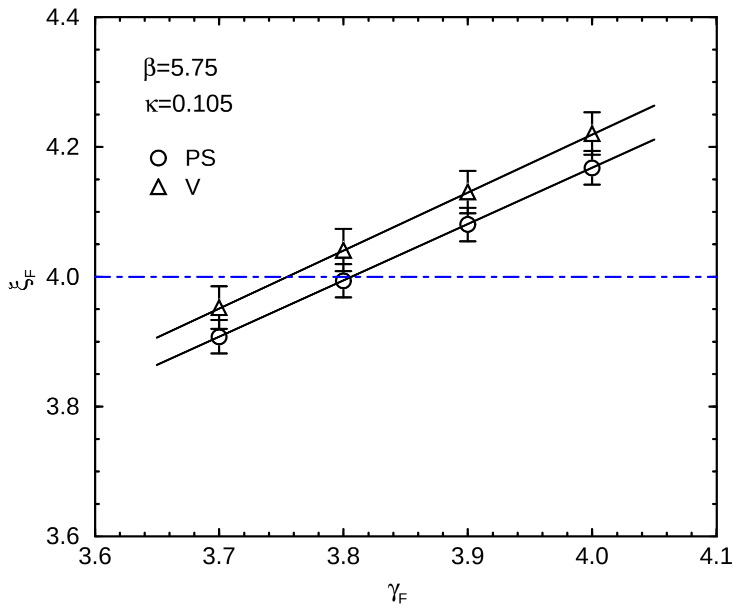
fermionic anisotropy
$\xi_F^{(PS)}$, $\xi_F^{(V)}$
for each γ_F



Linear interpolation
to $\xi_F(\gamma_F) = \xi$
$\Rightarrow \gamma_F^{*(PS)}$, $\gamma_F^{*(V)}$

(For finite a and imperfect clover coefficients, $\gamma_F^{*(PS)}$ and $\gamma_F^{*(V)}$ are in general different.)

Average $\gamma_F^{*(PS)}$ and $\gamma_F^{*(V)}$
$\Rightarrow \gamma_F^*$ for each κ



Calibration (3)

$\gamma_F^*(\kappa)$ is fitted to a linear form in m_q^2:

$$\frac{1}{\gamma_F^*} = \zeta_0 + \zeta_2\, m_q^2, \qquad m_q = \frac{1}{2\xi}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right),$$

where m_q is naive quark mass.
(κ_c is determined by linear extrapolation of m_{PS}^2 in $1/\kappa$ to $m_{PS}^2 = 0$ using largest two κ.)

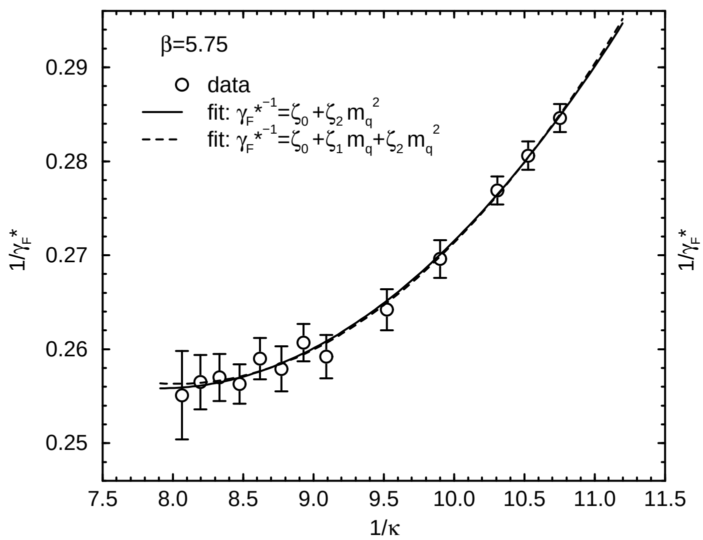
β	ζ_0	ζ_2	χ^2/N_{df}	$\gamma_F^*(m_q=0)$	κ_c
5.75	0.2558(9)	0.230(12)	1.83/11	3.909(14)	0.12640(5)
5.95	0.2490(8)	0.189(15)	3.52/11	4.016(13)	0.12592(6)
6.10	0.2479(9)	0.143(14)	4.44/11	4.034(14)	0.12558(4)

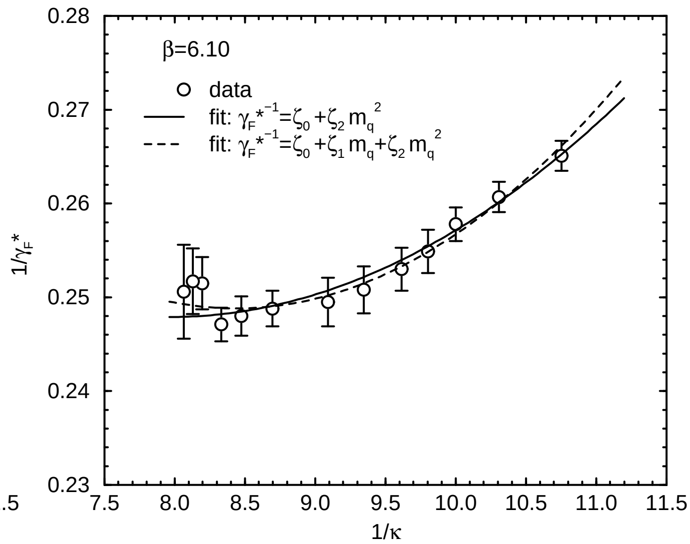
(Result of uncorrelated fit.)

m_q dependence is actually small in light quark region, and well fitted to linear form in m_q^2.
Result of linear fit in $m_q^2 \Rightarrow$ result of calibration

Fit γ_F^* to a quadratic form in m_q:

- ○ Coefficient of linear term is almost consistent with zero.
- ○ Difference in ζ_0 from linear fit is at 1% level.





Calibration (4)

Systematic errors in γ_F^*

$$\gamma_F^* = \gamma_F^{*(prop)} + \delta\gamma_F^{(stat)} + \delta\gamma_F^{(O(\alpha a))} + \delta\gamma_F^{(O(a^2))} (+\delta\gamma_F^{(chiral)})$$

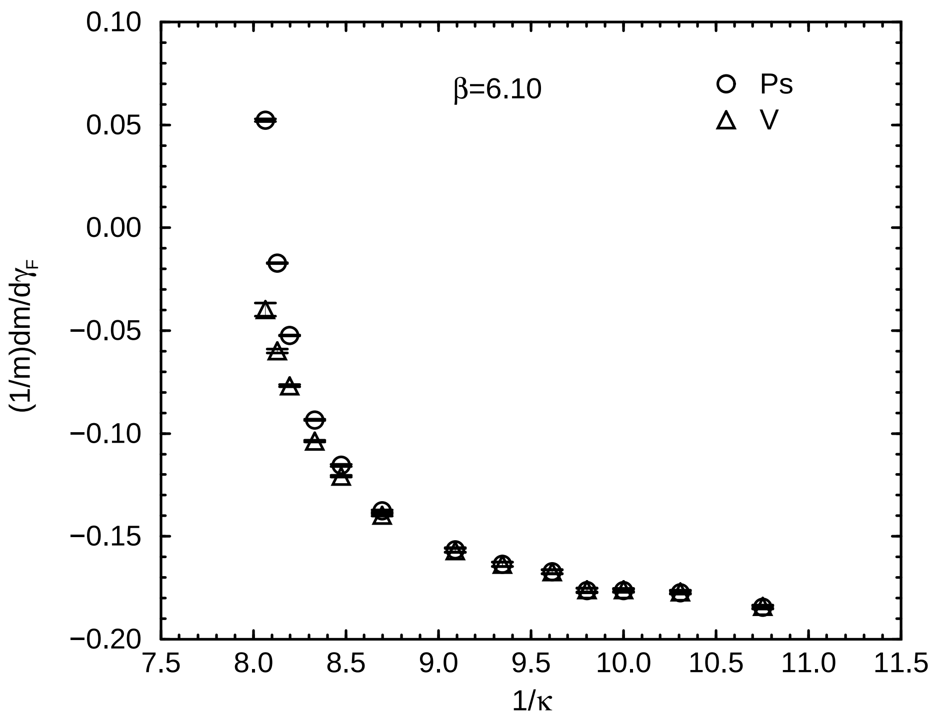
$\delta\gamma_F^{(O(\alpha a))} \leftarrow$ difference of $\gamma_F^{*(PS)}$ and $\gamma_F^{*(V)}$
$\sim$ 1 % at $\beta = 5.75$, and almost vanish at higher β.

$\delta\gamma_F^{(O(a^2))} \leftarrow$ relativistic and lattice K-G dispersion relation
$\sim$ 3, 2 and 1% at β =5.75, 5.95 and 6.10, respectively.

$\delta\gamma_F^{(chiral)} \leftarrow$ two types of fit: linear in m_q^2 and quadratic in m_q
At 1% level at each β

All systematic errors are under control in the continuum limit.

Effect on the meson masses:
— 2 % error in γ_F causes errors in masses less than 1 %.



Light Hadron Spectroscopy (1)

Light hadron spectrum is obtained with standard procedures.

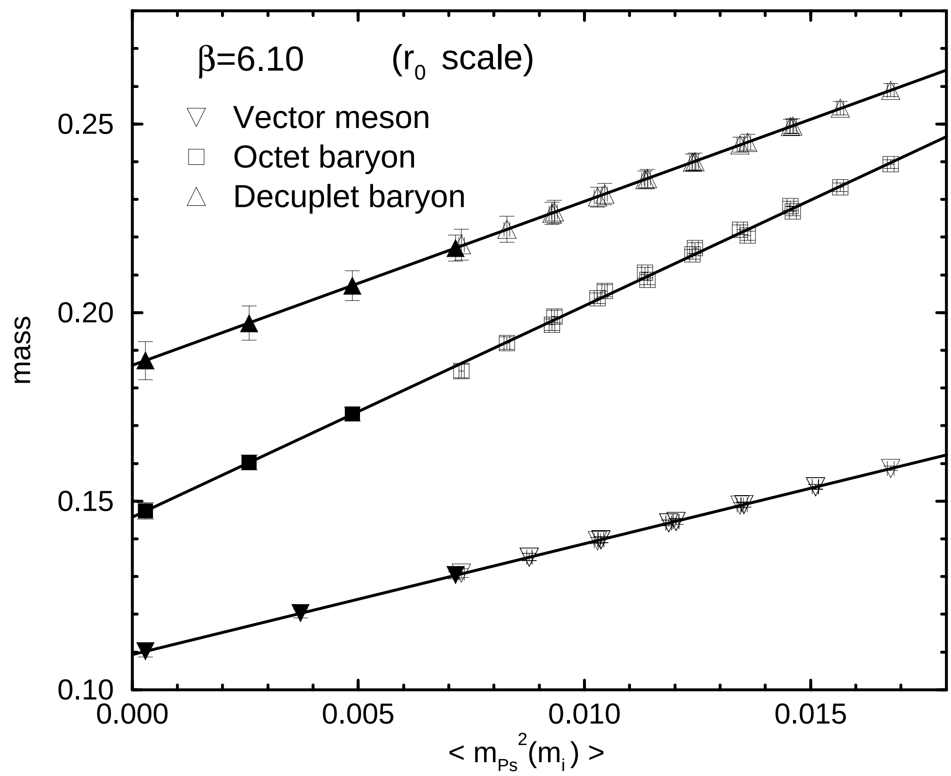
Gauge parameters: are same as in calibration.
Quark: 4 κ's in the strange quark mass region,
with γ_F^* in the chiral limit.

β	κ	γ_F	$N_{conf.}$
5.75	0.1210, 0.1220, 0.1230, 0.1240	3.909	200
5.95	0.1230, 0.1235, 0.1240, 0.1245	4.016	100
6.10	0.1230, 0.1235, 0.1240, 0.1245	4.034	100

Lattice scale:

- ☐ Hadronic radius r_0
- ☐ K^* meson mass

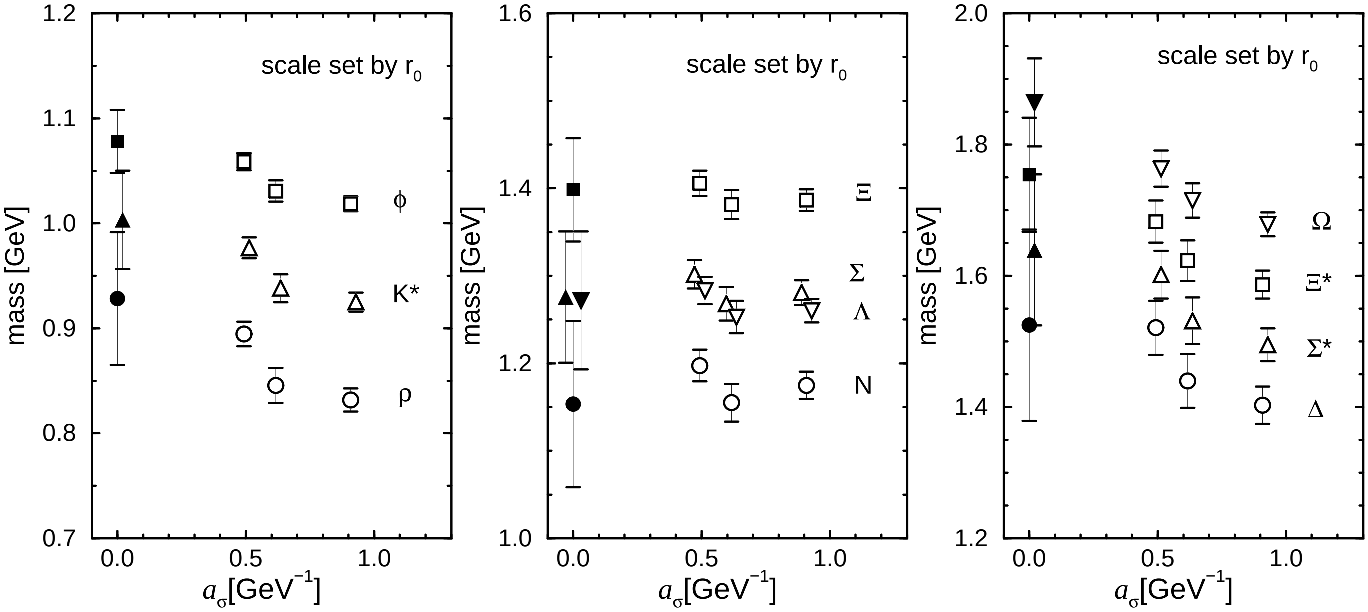
Quark masses m_n (u and d) and m_s are set by m_π and m_K.
Chiral extrapolation: in terms of m_{PS}^2.



Light Hadron Spectroscopy (2)

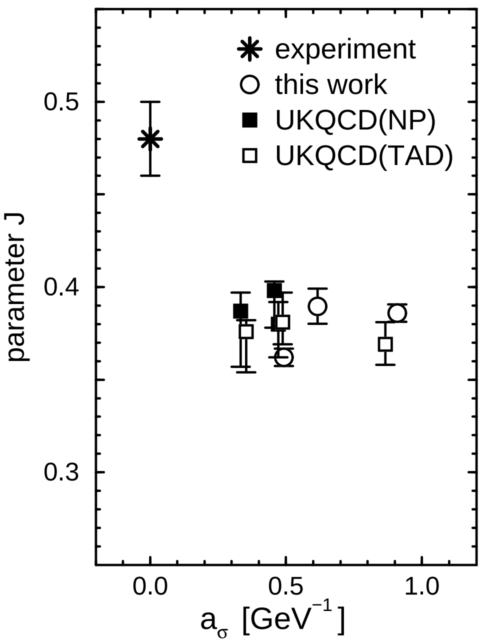
Spectrum for physical quark masses (scale set by r_0)
— Compared with the result on isotropic lattice (in $a \to 0$)

UKQCD Collab., Phys.Rev. D 62(2000) 054506.



Parameter J

$$J \equiv m_V \left. \frac{dm_V}{dm_{PS}^2} \right|_{\frac{m_V}{m_{PS}}=\frac{m_{K^*}}{m_K}}$$

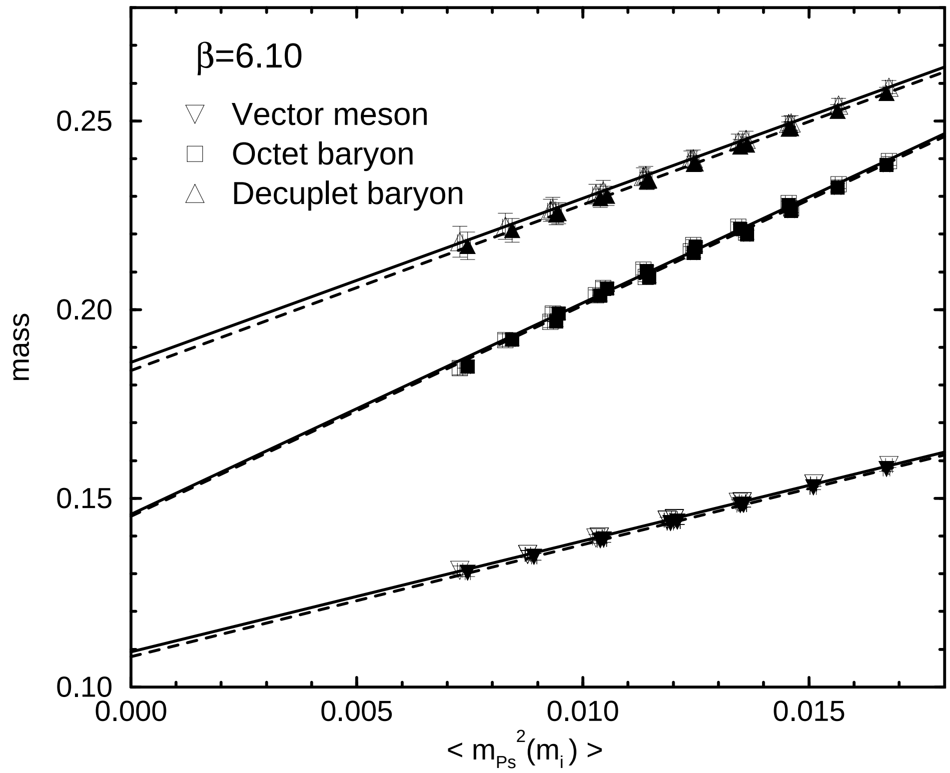


Light hadron spectrum is consistent with the results on isotropic lattices.

Light Hadron Spectroscopy (3)

Effect of systematic errors on the spectrum:
the same simulation at

$$\gamma_F' = \gamma_F^* + \delta\gamma_F \qquad \delta\gamma_F = 0.1 \quad \text{(2.5\% shift)}$$



The change of spectrum for physical quark masses is less than 1%.

Heavy-light meson decay constant (1)

Simulation:

On the same lattices as calibration at $\beta = 5.95$ and 6.10.

□ Quark parameters

(κ, γ_F) and meson masses with degenerate quarks:

β		κ	γ_F	m_{PS}	m_V
5.95	heavy	0.1020	3.858	0.6016(28)	0.6113(28)
		0.1060	3.913		
		0.1100	3.954	0.4297(19)	0.4427(19)
		0.1130	3.977		$\simeq 0.385$
	light	0.1235	4.016	0.13190(43)	0.1754(11)
		0.1240	4.016	0.11702(47)	0.1654(14)
		0.1245	4.016	0.10063(53)	0.1555(19)
6.10	heavy	0.1060	3.957		$\simeq 0.498$
		0.1090	3.981		$\simeq 0.436$
		0.1120	4.000		$\simeq 0.377$
		0.1150	4.014	0.3067(12)	0.3176(13)
	light	0.1235	4.034	0.11595(31)	0.1489(7)
		0.1240	4.034	0.10142(33)	0.1395(8)
		0.1245	4.034	0.08535(36)	0.1310(12)

(Meson masses are in temporal lattice units.)

- γ_F for light quarks are γ_F^* in the massless limit.
- γ_F for heavy quarks are from the linear fit of γ_F^* in m_q^2.

Present results are at preliminary level.

- ◦ Matching: tadpole improved tree level.
- ◦ Light quark mass: in the chiral limit.
- ◦ Only PS meson decay constants.
- ◦ Systematic uncertainties are not examined.

Cf. Matching coefficients with one-loop perturbation:
J. Harada et al., Phys. Rev. D 64 (2001) 074501.

Heavy-light meson decay constant (2)

Heavy-light meson dispersion relation:
Same analysis as in the calibration
$\Rightarrow \xi_F$ for heavy light meson.

Result at $\beta = 5.95$ (with $N_{conf} = 200$):

κ_h	κ_l	ξ_{PS}	ξ_V
0.1020	0.1235	3.912(56)	3.912(92)
0.1020	0.1245	3.914(79)	3.89(13)
0.1130	0.1235	3.949(57)	3.953(93)
0.1130	0.1245	3.946(73)	3.90(12)

— Almost consistent with $\xi = 4$.
Small difference from ξ is partially due to the systematic effect.
$\Rightarrow$ Effect on the spectrum and decay constants is to be examined.

Heavy-light meson decay constant (3)

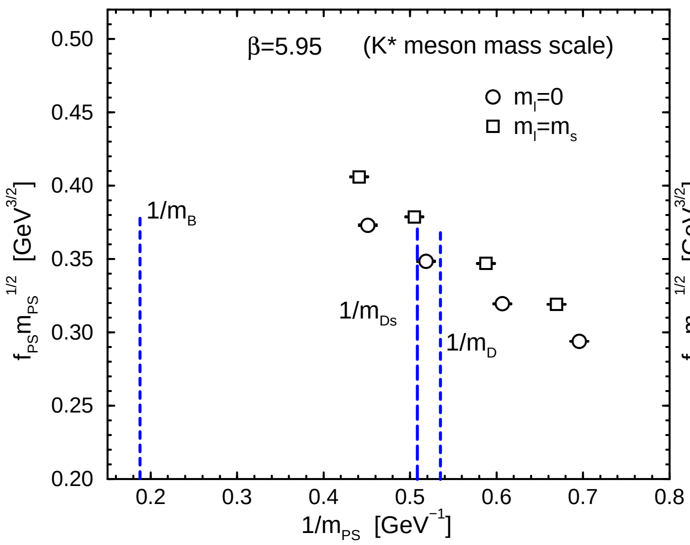
Result for PS decay constant:

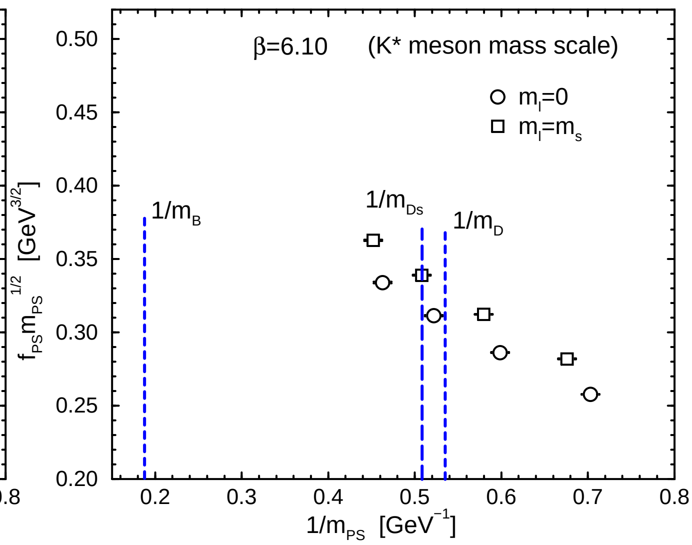
- Lattice scale is set by K^* meson mass.
- Tadpole improved tree level.
- (u, d) quark mass is approximated with massless limit.
- s quark mass is set by K meson mass.

decay const.	$\beta = 5.95$	$\beta = 6.10$
f_π	160.9(2) MeV	139.4(2) MeV
f_K	186.7(2) MeV	165.2(2) MeV
f_D	251.7(3) MeV	225.5(3) MeV
f_{D_s}	269.8(2) MeV	242.2(2) MeV
f_D/f_π	1.564(3)	1.618(3)
f_{D_s}/f_K	1.445(1)	1.466(2)
f_{D_s}/f_D	1.0721(3)	1.0740(4)

Scaling of $f_{PS}\sqrt{m_{PS}}$ ($\Leftarrow$ heavy quark symmetry):

Consistent behavior with other lattice works.





Heavy-light meson decay constant (4)

Comparison with other works:

Conclusion and Outlook

(1). We implemented the $O(a)$ improved Wilson quark action on the quenched anisotropic lattice at three β in the quark mass region from the chiral limit to the charm quark mass region.

□ Systematic errors are totally a few percent, and decrease toward the continuum limit.

□ Light hadron spectrum at physical quark mass:
consistent with results on isotropic lattice.
changes 1 % for the 2 % change of bare anisotropy.

— The systematic uncertainties due to the anisotropy are under control.

(2). Preliminary result of heavy-light meson decay constant is encouraging for further applications.

□ Other decay constants, form factors, excited state spectrum.

□ Further improvement to removing systematic uncertainties such as nonperturbative renormalization

□ Simulations with dynamical quark effect
— Need more efficient procedures for calibration