# SU(3) Lattice QCD Study for Static Three-Quark Potential

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- Introduction
- Methods
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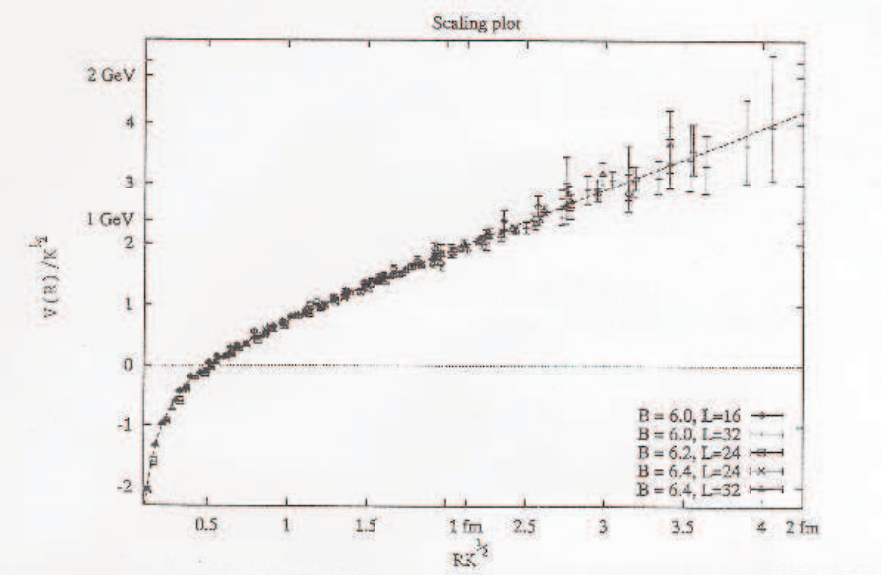
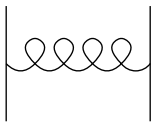
# Introduction(1)

## $Q\bar{Q}$ potential




G.S.Bali and K.Schilling Phys.Rev.D**46**(1992)2636

Coulomb term $-\dfrac{A_{Q\bar{Q}}}{R}$



$+$

linear term $\sigma_{Q\bar{Q}}R$

(string tension:$\sigma_{Q\bar{Q}} \sim 0.89\mathrm{GeV/fm}$)

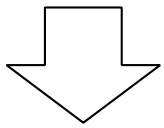
$+$

constant $C_{Q\bar{Q}}$

- Quark Confinement
- Meson Features

## Introduction(2)

3Q potential



Responsible for baryon features

- Regge trajectory
- Kogut and Susskind, PRD11(1975)395 (Strong coupling limit)
- Capstic and Isgur, PRD34(1986)2809 (Model calculation)

— early lattice works —

- Sommer and Wosiek, PLB149(1984)497, NPB267(1986)531
- Thacker, Eichten and Sexton, Lattice '87

**QCD → 3Q Potential**

**→ Baryon Properties**

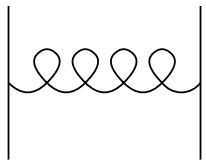
# Theoretical consideration(1)

*Theoretical Consideration*

## - short distance -

**Coulomb type due to one-gluon-exchange**

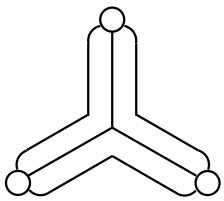
$$-\sum_{i<j}\frac{A_{3Q}}{|r_i - r_j|}$$



## - long distance -

**Three-body force proportional to the length of flux**

$\sigma_{3Q}L$
(L:length of flux)

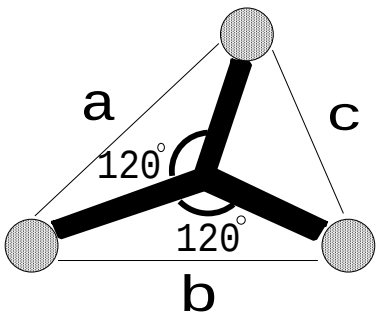


## + Constant term

# Theoretical consideration(2)

$$V_{3Q} = -\sum_{i<j} \frac{A_{3Q}}{|r_i - r_j|} + \sigma_{3Q} L_{\min} + C_{3Q}$$

- Minimal linking length -

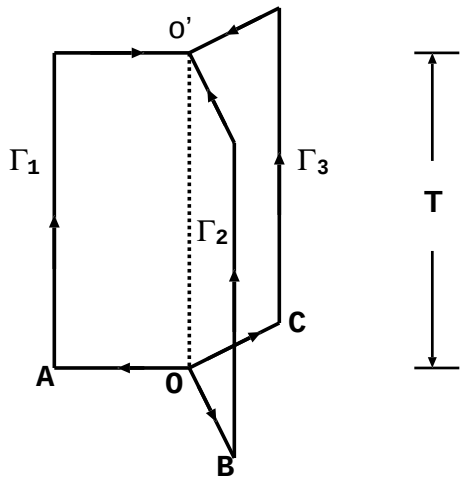



$$L_{\min} = \left( \frac{1}{2}(a^2+b^2+c^2) + \frac{\sqrt{3}}{2}\sqrt{(-a+b+c)(a-b+c)(a+b-c)(a+b+c)} \right)^{1/2}$$

Direct calculation of 3Q Potential
in lattice QCD
$\rightarrow$ Comparison with the theoretical form

# Method(1)

- 3Q Wilson loop-




$$U_{\rm i} \equiv {\rm P}\exp\left(ig\int_{\Gamma_{\rm i}} dx_\mu A_\mu(x)\right)$$

$$W_{\rm 3Q} \equiv \frac{1}{3!}\epsilon^{a_1a_2a_3}\epsilon^{b_1b_2b_3}U_1^{a_1b_1}U_2^{a_2b_2}U_3^{a_3b_3}$$

$$\Downarrow$$

$$\langle W_{\rm 3Q}\rangle = C_0 e^{-V(g.s.)T} + C_1 e^{-V(1st\ e.s.)T} + ....$$
$$\sim C_0 e^{-V(g.s.)T} \qquad ({\rm large\ T})$$

But, signal is supressed exponentially
Difficult to take large T

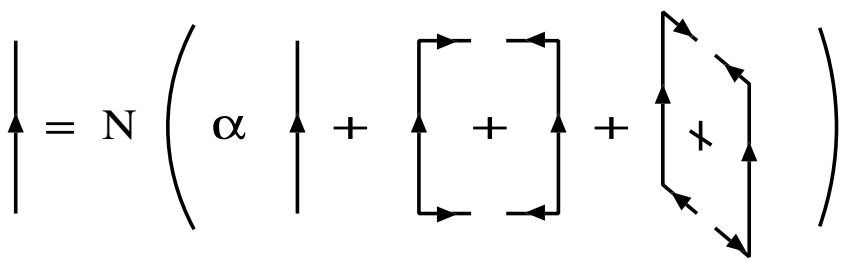
# Method(2)

## - Smearing Technique -

Enhancement of Ground-state Component

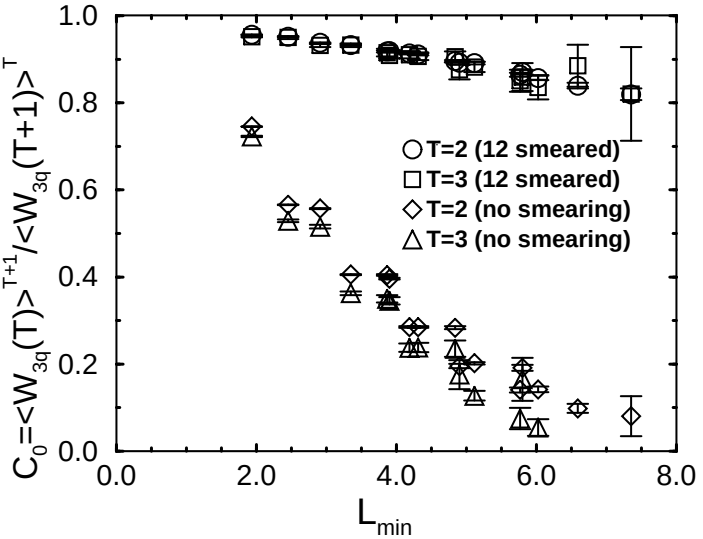
Maximize

$$\mathrm{Re\ tr}\left\{\bar{U}_i^\dagger(s)\left[\alpha U_i(s)+\sum_{j\neq i}\{U_j(s)U_i(s+\hat{j})U_j^\dagger(s+\hat{i})+U_j^\dagger(s-\hat{j})U_i(s-\hat{j})U_j(s+\hat{i}-\hat{j})\}\right]\right\}$$




$$\langle W_{3Q}\rangle = C_0 e^{-V(g.s.)T} + C_1 e^{-V(1st\ e.s.)T} + ....$$

$$C_0 \simeq 1, \quad C_1, C_2, .... \ll 1$$




We can extract ground-state potential
even with small T.

# Result(1)

QCD Standard plaquette action

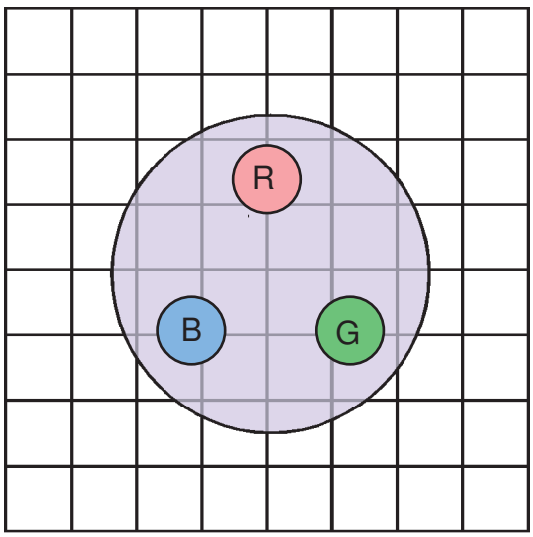
At quenched level

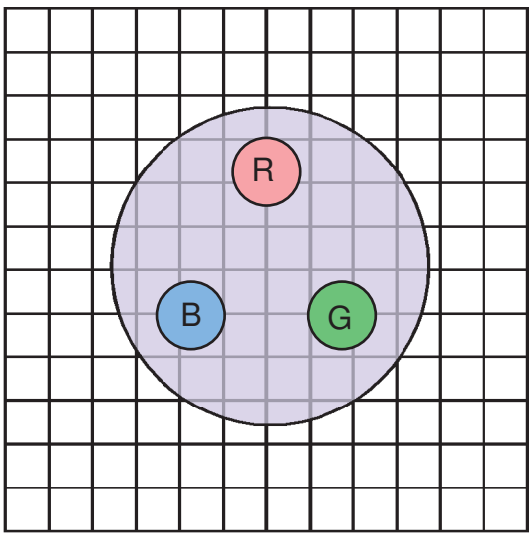
$\beta = 5.7$ $12^3 \times 24$ lattice

16 patterns of 3 quark configurations
$\alpha = 2.3$(smearing parameter), $N_{\rm smear} = 12$
210 gauge configurations

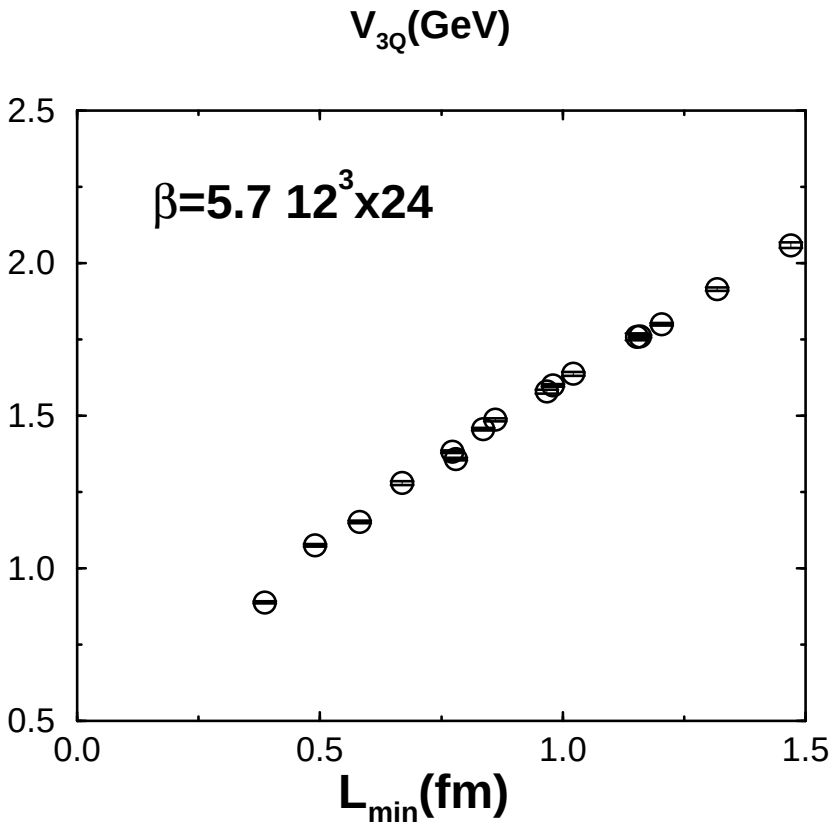
$\beta = 5.8$ $16^3 \times 32$ lattice

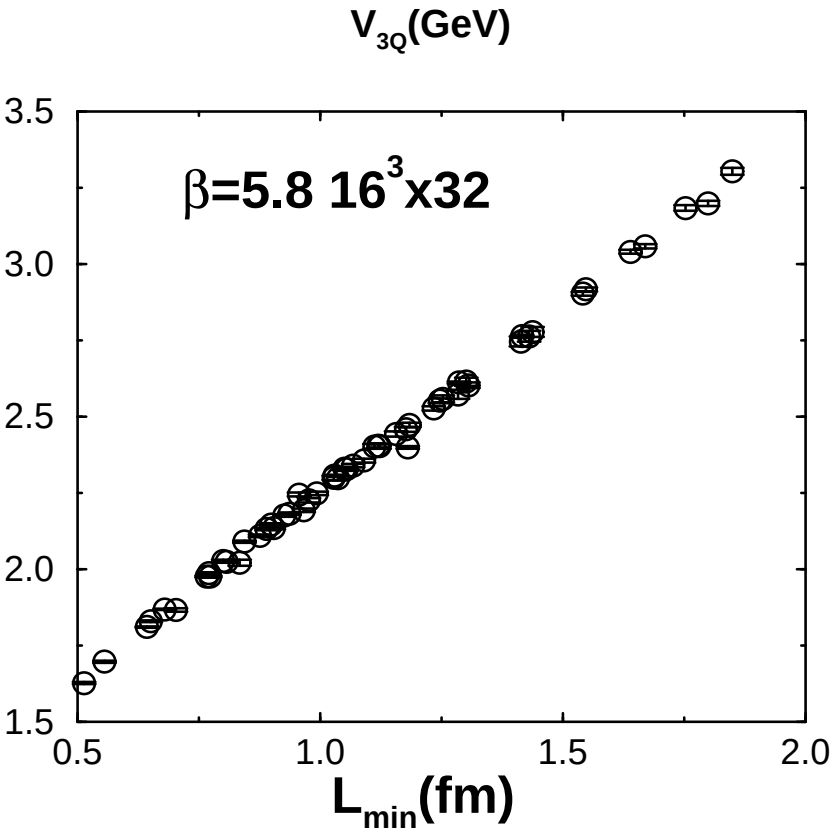
66 patterns of 3 quark configurations
$\alpha = 2.3$(smearing parameter), $N_{\rm smear} = 25$
200 gauge configurations



# Result(2)


$V_{3Q}$(GeV)
β=5.7 $12^3$x24
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
$L_{min}$(fm)


$V_{3Q}$(GeV)
β=5.8 $16^3$x32
3.5
3.0
2.5
2.0
1.5
0.5
1.0
1.5
2.0
$L_{min}$(fm)

# Result(3)

$\beta = 5.7$ (lattice unit $a \simeq 0.19$ fm)

| | $\sigma$(Gev/fm) | $A$ | $C$ |
|---|---|---|---|
| 3Q | 0.844(15) | 0.1331(66) | 0.9182(213) |
| $Q\bar{Q}$ | 0.902(26) | 0.2793(116) | 0.6203(161) |

$\beta = 5.8$ (lattice unit $a \simeq 0.14$ fm)

| | $\sigma$(Gev/fm) | $A$ | $C$ |
|---|---|---|---|
| 3Q | 1.019( 7) | 0.1304(17) | 0.9326(53) |
| $Q\bar{Q}$ | 1.109(27) | 0.2580(159) | 0.6081(182) |

- universality of string tension $\sigma_{Q\bar{Q}} \simeq \sigma_{3Q}$
- consistency with P-QCD $A_{Q\bar{Q}} \simeq 2A_{3Q}$

## Summary

We have carried out the detailed analysis of 3Q potential using the SU(3) lattice QCD at quenched level. Here, smearing was essential for this analysis. $V_{3\mathrm{Q}}$ is well described with the following form

$$V_{3\mathrm{Q}} = -\sum_{i<j} \frac{A_{3\mathrm{Q}}}{|r_i - r_j|} + \sigma_{3\mathrm{Q}} L_{\min} + C_{3\mathrm{Q}}$$

$L_{\min}$:minimal linking length for 3 quarks

Here, we have observed 2 remarkable features.

- universality of string tension $\sigma_{\mathrm{Q}\bar{\mathrm{Q}}} \sim \sigma_{3\mathrm{Q}}$
- consistency with P-QCD $A_{\mathrm{Q}\bar{\mathrm{Q}}} \sim 2A_{3\mathrm{Q}}$

# Potential Table

$\beta$=5.7 (a$\sim$0.2 fm)
$12^3 \times 24$ lattice
16 patterns of 3 quark configurations

| | $\sigma$ | $A$ | $C$ |
|---|---|---|---|
| 3Q | 0.1524(28) | 0.1331(66) | 0.9182(213) |
| Q$\bar{\text{Q}}$ | 0.1629(47) | 0.2793(116) | 0.6203(161) |

| $(i,j,k)$ | $V_{3Q}^{\text{latt}}$ | $V_{3Q}^{\text{fit}}$ | $V_{3Q}^{\text{latt}} - V_{3Q}^{\text{fit}}$ |
|---|---|---|---|
| $(1,1,1)$ | 1.0231 | 0.0038 | 0.0141 |
| $(1,1,2)$ | 1.2179 | 0.0061 | 0.0035 |
| $(1,1,3)$ | 1.4154 | 0.0049 | 0.0197 |
| $(1,2,2)$ | 1.3870 | 0.0046 | $-0.0016$ |
| $(1,2,3)$ | 1.5588 | 0.0060 | 0.0008 |
| $(1,3,3)$ | 1.7141 | 0.0043 | $-0.0053$ |
| $(2,2,2)$ | 1.5216 | 0.0033 | $-0.0013$ |
| $(2,2,3)$ | 1.6745 | 0.0115 | $-0.0010$ |
| $(2,3,3)$ | 1.8242 | 0.0054 | 0.0074 |
| $(3,3,3)$ | 1.9607 | 0.0092 | 0.0171 |
| $(0,1,1)$ | 0.8459 | 0.0036 | $-0.0069$ |
| $(0,1,2)$ | 1.0970 | 0.0041 | $-0.0052$ |
| $(0,1,3)$ | 1.2935 | 0.0039 | 0.0010 |
| $(0,2,2)$ | 1.3164 | 0.0041 | $-0.0098$ |
| $(0,2,3)$ | 1.5032 | 0.0058 | $-0.0036$ |
| $(0,3,3)$ | 1.6741 | 0.0040 | $-0.0066$ |

# Potential Table

$\beta$=5.8 (a$\sim$0.14 fm)

$16^3 \times 32$ lattice

66 patterns of 3 quark configurations

| | $\sigma$ | $A$ | $C$ |
|---|---|---|---|
| 3Q | 0.0999(7) | 0.1304(17) | 0.9326(53) |
| Q$\bar{\text{Q}}$ | 0.1087(26) | 0.2580(159) | 0.6081(182) |

| $(i,j,k)$ | $V_{3Q}^{latt}$ | $V_{3Q}^{fit}$ | $V_{3Q}^{latt} - V_{3Q}^{fit}$ |
|---|---|---|---|
| $(1,1,1)$ | 0.9144 | 0.9008 | 0.0136 |
| $(1,1,2)$ | 1.0657 | 1.0582 | 0.0075 |
| $(1,1,3)$ | 1.1962 | 1.1883 | 0.0079 |
| $(1,1,4)$ | 1.3049 | 1.3054 | −0.0005 |
| $(1,1,5)$ | 1.4134 | 1.4162 | −0.0029 |
| $(1,1,6)$ | 1.5339 | 1.5236 | 0.0103 |
| $(1,2,2)$ | 1.1866 | 1.1877 | −0.0011 |
| $(1,2,3)$ | 1.3062 | 1.3076 | −0.0014 |
| $(1,2,4)$ | 1.4153 | 1.4203 | −0.0050 |
| $(1,2,5)$ | 1.5259 | 1.5287 | −0.0029 |
| $(1,2,6)$ | 1.6360 | 1.6347 | 0.0013 |
| $(1,3,3)$ | 1.4178 | 1.4209 | −0.0031 |
| $(1,3,4)$ | 1.5216 | 1.5296 | −0.0080 |
| $(1,3,5)$ | 1.6276 | 1.6357 | −0.0081 |
| $(1,3,6)$ | 1.7296 | 1.7401 | −0.0105 |
| $(1,4,4)$ | 1.6291 | 1.6356 | −0.0064 |
| $(1,4,5)$ | 1.7190 | 1.7396 | −0.0206 |
| $(1,4,6)$ | 1.8203 | 1.8425 | −0.0222 |
| $(2,2,2)$ | 1.2793 | 1.2838 | −0.0046 |
| $(2,2,3)$ | 1.3815 | 1.3903 | −0.0089 |
| $(2,2,4)$ | 1.4904 | 1.4969 | −0.0064 |
| $(2,3,3)$ | 1.4751 | 1.4880 | −0.0129 |
| $(2,3,5)$ | 1.6814 | 1.6912 | −0.0098 |
| $(2,3,6)$ | 1.7847 | 1.7931 | −0.0084 |
| $(2,4,4)$ | 1.6800 | 1.6869 | −0.0069 |
| $(2,4,5)$ | 1.7669 | 1.7862 | −0.0193 |
| $(3,3,3)$ | 1.5562 | 1.5746 | −0.0185 |
| $(3,3,4)$ | 1.6475 | 1.6692 | −0.0217 |
| $(3,4,6)$ | 1.9425 | 1.9496 | −0.0071 |
| $(3,5,5)$ | 1.9322 | 1.9446 | −0.0124 |

| $(i,j,k)$ | $V_{3Q}^{latt}$ | $V_{3Q}^{fit}$ | $V_{3Q}^{latt} - V_{3Q}^{fit}$ |
|---|---|---|---|
| $(4,5,6)$ | 2.1263 | 2.1094 | 0.0169 |
| $(4,6,6)$ | 2.2264 | 2.1979 | 0.0285 |
| $(5,6,6)$ | 2.3105 | 2.2733 | 0.0372 |
| $(0,1,1)$ | 0.7701 | 0.7727 | −0.0027 |
| $(0,1,2)$ | 0.9659 | 0.9694 | −0.0035 |
| $(0,1,3)$ | 1.1107 | 1.1071 | 0.0037 |
| $(0,1,4)$ | 1.2323 | 1.2268 | 0.0055 |
| $(0,1,5)$ | 1.3339 | 1.3388 | −0.0049 |
| $(0,1,6)$ | 1.4566 | 1.4469 | 0.0097 |
| $(0,1,7)$ | 1.5730 | 1.5527 | 0.0203 |
| $(0,1,8)$ | 1.6779 | 1.6570 | 0.0209 |
| $(0,2,2)$ | 1.1375 | 1.1422 | −0.0047 |
| $(0,2,3)$ | 1.2660 | 1.2711 | −0.0050 |
| $(0,2,4)$ | 1.3815 | 1.3871 | −0.0056 |
| $(0,2,5)$ | 1.4928 | 1.4972 | −0.0043 |
| $(0,2,6)$ | 1.6069 | 1.6041 | 0.0028 |
| $(0,2,8)$ | 1.8285 | 1.8128 | 0.0157 |
| $(0,3,3)$ | 1.3893 | 1.3940 | −0.0047 |
| $(0,3,4)$ | 1.5007 | 1.5066 | −0.0059 |
| $(0,3,5)$ | 1.6135 | 1.6145 | −0.0010 |
| $(0,4,4)$ | 1.6079 | 1.6165 | −0.0086 |
| $(0,4,5)$ | 1.7085 | 1.7226 | −0.0141 |
| $(0,4,6)$ | 1.8263 | 1.8267 | −0.0004 |
| $(0,4,7)$ | 1.9328 | 1.9297 | 0.0031 |
| $(0,4,8)$ | 2.0399 | 2.0319 | 0.0080 |
| $(0,5,5)$ | 1.7994 | 1.8271 | −0.0277 |
| $(0,5,6)$ | 1.9205 | 1.9300 | −0.0096 |
| $(0,6,6)$ | 2.0302 | 2.0319 | −0.0018 |
| $(0,6,7)$ | 2.1390 | 2.1330 | 0.0060 |
| $(0,6,8)$ | 2.2371 | 2.2337 | 0.0035 |