Study of spatial meson correlations at finite temperature in quenched QCD on anisotropic lattice

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Lattice 2001

1 , Introduction

QCD phase transition at finite temperature (density)

- **Confinement $\Longleftrightarrow$ deconfinement**
- **Chiral symmetry breaking $\Longleftrightarrow$ restoration**

We study how hadronic modes change at phase transition.

- **Correlation function in Euclidean-temporal direction $\Leftrightarrow$ spectral function ($\leftrightarrow$ pole mass)**
  - **difficult to measure (Small degrees of freedom ,Temporal lattice size is 1/T, etc.)**
    cf. QCD-TARO Collab Phys. Rev. D 63 (2001) 054501
    Umeda *et al.* Int. J. Mod. Phys. A 16 (2001) 2215

- **Correlation function in spatial direction $\Leftrightarrow$ momentum spectral function ($\leftrightarrow$ screening mass)**
  - **easy to measure (often calculated in models)**
  - **relation to the temporal correlator is not clear**

Our goals:

- **Detailed analysis of correlators in spatial direction**
- **Relation between spatial and temporal correlators**

2, Studies of spatial correlators

- **Color singlet modes at $T > T_c$ — DeTar's conjecture "Confinement" feature at long range**
  DeTar, Phys. Rev. D 32 (1985) 276

- **Feature of screening mass**
  - **Strong correlation in Ps and S channels. →hadronic modes in the screening spectrum in plasma ?**
    DeTar and Kogut Phys. Rev. Lett. 59 (1987) 399

  - **Similar behavior to the free quark gas ($\rho, N\ at\ T > 1.2T_c$)**
    Born *et al.*, Phys. Rev. Lett. 67 (1991) 302

  - **No pole structure in the screening spectral function at $T > T_c$ (?)**
    S. Gupta Phys. Lett. B 288 (1992) 171

3, Our Approach

It is preferable to have large number of lattice points (degrees of freedom) for detailed analysis of spatial correlators.

$\Downarrow$

Anisotropic lattice

Small lattice spacing in z-direction
(usually in t-direction)

- **Advantage : higher resolution**
- **disadvantage : additional parameters (anisotropy, etc.) must be tuned nonperturbatively.**

Meson correlation function in z-direction

$\Downarrow$

Two kinds of analysis methods

- **Fit with ansatz**
  Shape of spectral function is assumed.
  At present we use single and double pole ansatz'.

- **Reconstruction of spectral function from the correlation function**
  $\longleftarrow$ **Maximum Entropy Method**

4, Fit with Ansatz

Hadron operator : $O(x, y, z, t)$

Correlation function

$$C(z) = \Sigma_{x,y,t} \langle O(x, y, z, t)\, O^{\dagger}(0) \rangle$$
$$\longrightarrow \quad \Sigma_i A_i \exp(-m_i z)$$

(For discrete states)

Here we assume that the spectral function is represented by a sum of poles.

Contribution from each pole :

$$y_i(A_i, m_i) = A_i \exp(-m_i z) + A_i \exp(-m_i (N_z - z))$$

- **Single pole fit :** $C(z) \longrightarrow y_0(A_0, m_0)$
- **Double pole fit :** $C(z) \longrightarrow y_0(A_0, m_0) + y_1(A_1, m_1)$

5, Spectral function

Reconstruction of spectral function by maximum entropy method

Nakahara, Asakawa and Hatsuda, Phys. Rev. D 60 (1999) 091503

$K(\omega,t)$ **: (Kernel of laplace transformation)**

$$K(\omega,t) = \exp(-\omega t) + \exp(-\omega(N_t - t))$$

$$C(t) = \int d\omega K(\omega,t) A(\omega)$$

$$C(t) \longrightarrow A(\omega)$$

lattice date spectral function

(degree of freedom $O(50) \longrightarrow O(1000)$)

— ill-posed problem

Maximum entropy method (based on Bayes' theorem)
Spectral function $A(\omega)$ is determined by maximizing

$$Q = \exp[-(\chi^2\textbf{term}) - \alpha \sum_{\omega=1}^{\omega_{max}} (A(\omega) - m(\omega) - A(\omega)\log(\frac{A(\omega)}{m(\omega)}))]$$

α **: constant (to be integrated out)**
$m(\omega)$ **: default model function**
here we set $m(\omega) = \textbf{const.} * \omega^2$
$\rho(\omega) = A(\omega)/\omega^2$ **: dimensionless combination for mesons**

Anisotropic lattice

lattice spacing in temporal direction $a_t < a_\sigma$ lattice spacing in spatial direction

$\longrightarrow$ Higher resolution in t-direction (particularly convenient at finite temperature)

At present calculation small lattice spacing in z-direction

$a_z < a_x = a_y = a_t = a$

In the following, action is described with $a_t < a_\sigma = a_x = a_y = a_z$ for simplicity.

$\boxed{\text{Gauge field}}$ — Anisotropic plaquette action

Karsch, Nucl. Phys. B 205 (1982) 285

e.g. determination of ξ by matching of Wilson loops $W_{\sigma\sigma}, W_{\sigma t}$

Bungers *et al.* Nucl. Phys. B 304 (1988) 587

$\boxed{\text{Quark field}}$ — $O(a)$ improved Wilson quark action

Here we use type of Umeda *et al.* Nucl. Phys. B 304 (1988) 587

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) = \delta_{x,y} - \kappa_t[(1-\gamma_4)U_4(x)\delta_{x+\hat{4},y} + (1+\gamma_4)U_4^\dagger(x-\hat{4})\delta_{x-\hat{4},y}] \\ -\kappa_\sigma \sum_i [(\frac{1}{\xi} - \gamma_i)U_i(x)\delta_{x+\hat{i},y} + (\frac{1}{\xi} + \gamma_i)U_i^\dagger(x-\hat{i})\delta_{x-\hat{i},y}] \\ -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x)\delta_{x,y} - \frac{1}{\xi}\kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

$\gamma_F = \kappa_t/\kappa_\sigma$: bare quark anisotropy

calibration : We determine γ_F so that anisotropy ξ_F defined by fermionic observable satisfy $\xi_F = \xi$.

We use meson dispersion relation (Lattice Klein-Gordon type).

Simulation parameters

$\boxed{\text{Gauge field}}$ — Standard plaquette action

Lattice size :$12^2 \times 96 \times N_t$

z-direction : lattice spacing $a_z < a_x = a_y = a_t = a$

Quenched approximation (No dynamical quark effect)

$(\beta = 5.75, \gamma_G = 3.072) \longrightarrow \xi = 4.00(4)$ (anisotropy)

Klassen, Nucl. Phys. B 533 (1998) 557

N_t	T/T_c	$conf.$
12	0	150
5	0.8	150
3	1.3	150
2	2.0	150

Cutoff:

$a^{-1} = 1.000(8) GeV$

$a_z^{-1} = 4.00(5) GeV$

(set by string tension)

$\boxed{\text{Quark field}}$ — $O(a)$ improved Wilson quark action

quark mass region : $m_q \simeq (0.5 \sim 1.5) m_s$

Meson dispersion relation is used for calibration

(From Lattice Klein-Gordon eq.)

ik	κ	$m_{PS}[GeV]$	$m_V[GeV]$
1	0.1200	1.000(24)	1.168(41)
2	0.1220	0.814(25)	1.040(54)
3	0.1240	0.602(27)	0.896(93)

cf. Umeda *et al.*, Int. J. Mod. Phys. A 16 (2001) 2215

Matsufuru, Onogi, Umeda, hep-lat/0107001

At present, only the result at ik=1 and PS channel is shown.

Result : $T \simeq 0$ (Nt=12) pole fit

Fit range : Lmin$\sim$Lmax : Lmax=Nz/2 (=48)

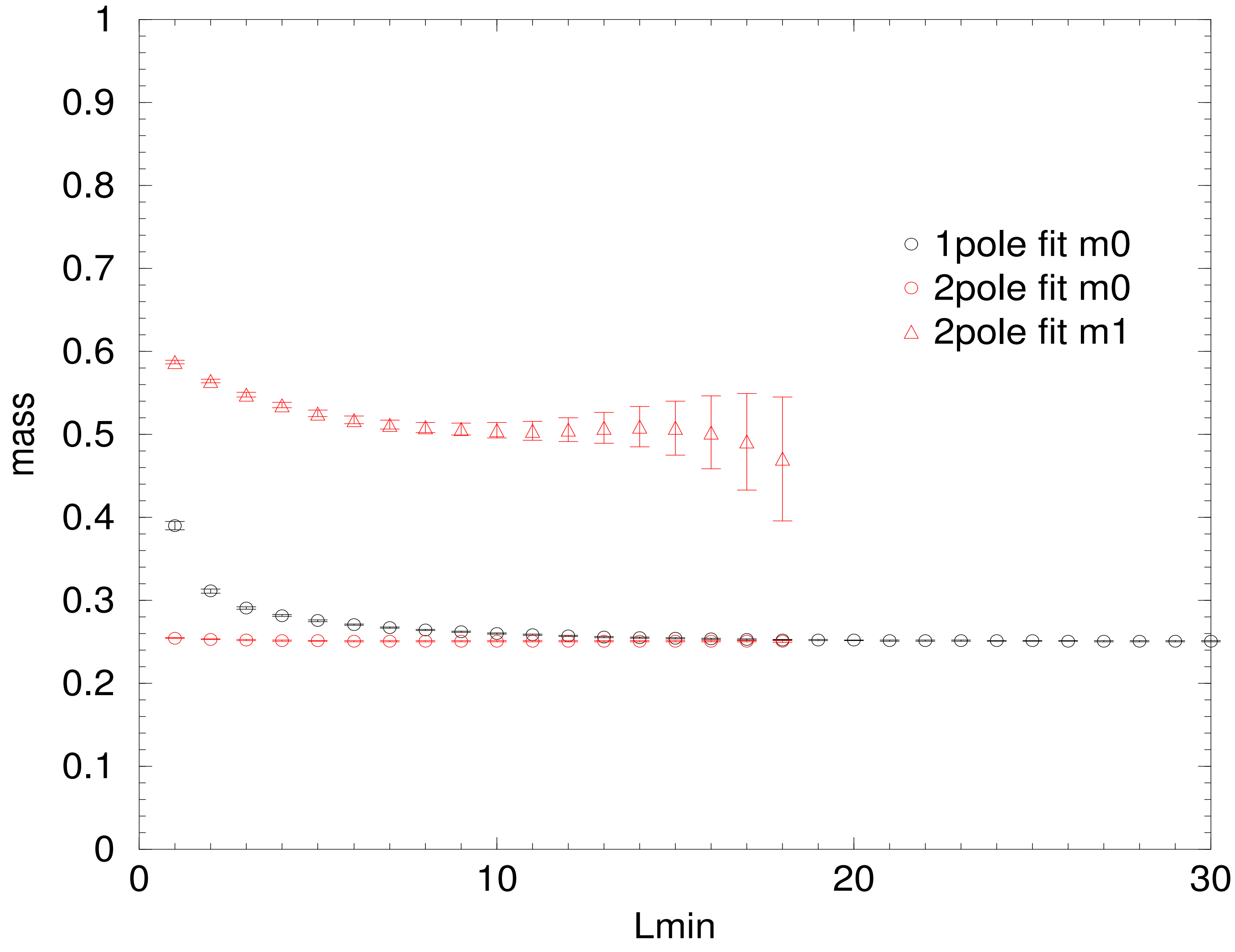
Single pole fit is applicable in Lmin$\geq$ 20

— ground state is dominant

Two pole fit is applicable in Lmin$\geq$ 10

But fit is unstable in Lmin$\geq$ 20

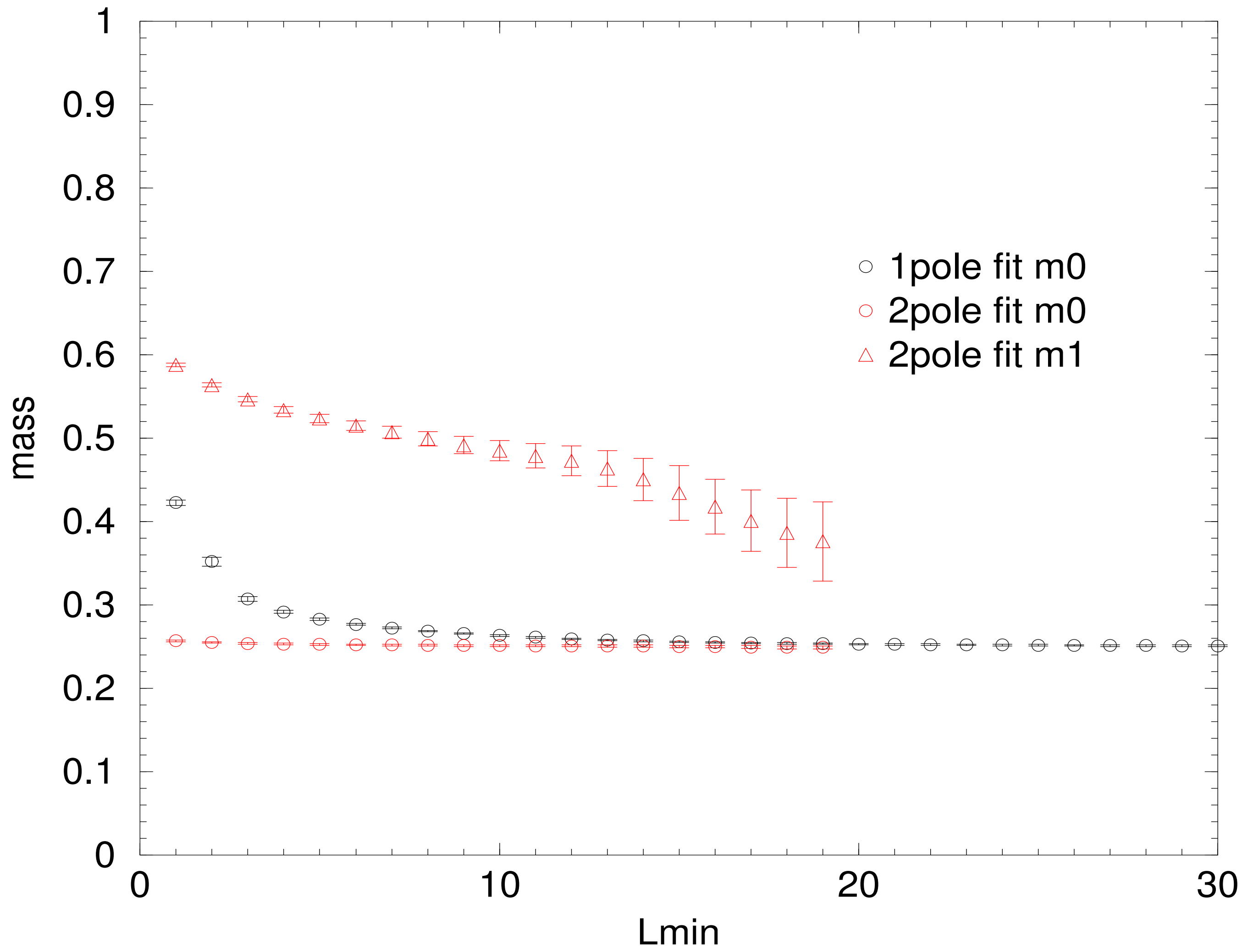
(because contribution of excited state is too small)



Result : $T = 0.8T_c$ (Nt=5) pole fit

Fit range : Lmin$\sim$Lmax : Lmax=Nz/2 (=48)
single pole fit : result is almost same as at $T \simeq 0$
Two pole fit : parameters corresponding to the 1st excited state is not constants in whole Lmin.
— To be clarified

- 3 pole fit
- Other form of spectral function



Result : $T = 1.3T_c$ (Nt=3) pole fit

Fit range : Lmin$\sim$Lmax : Lmax=Nz/2 (48)

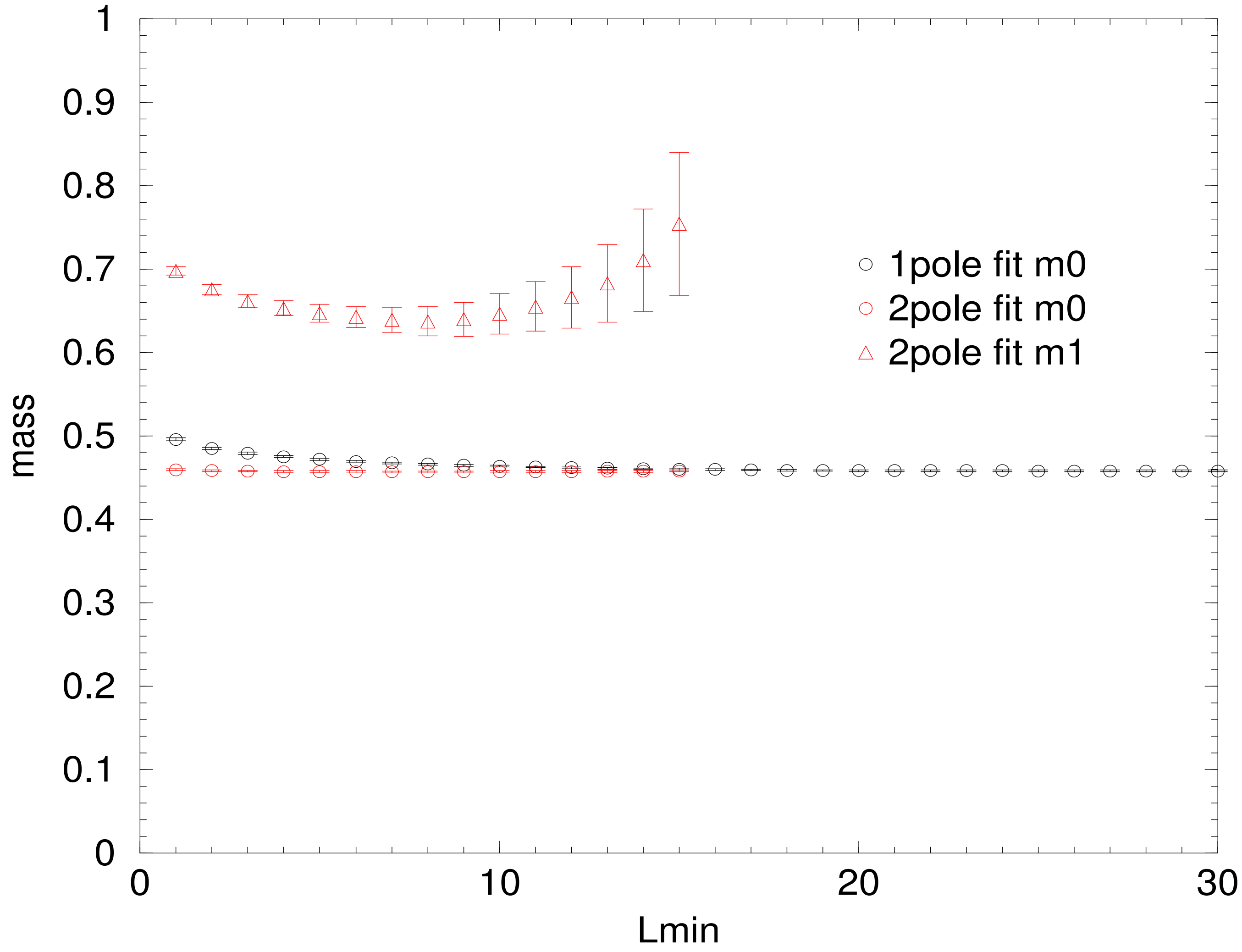
Single pole fit is applicable in Lmin$\geq$ 15.

$m_{gr} \cong 0.45 a_z^{-1} = 1.8$ GeV $\quad$ (m_{gr} : ground state mass)

cf. $2\pi T$(free two quarks)$= 2\pi a_z^{-1}/3 = 2.1$ GeV

In the case of two pole fit, 1st excited state mass dose not become constant

— To be clarified



Result : $T = 2T_c$ (Nt=2) pole fit

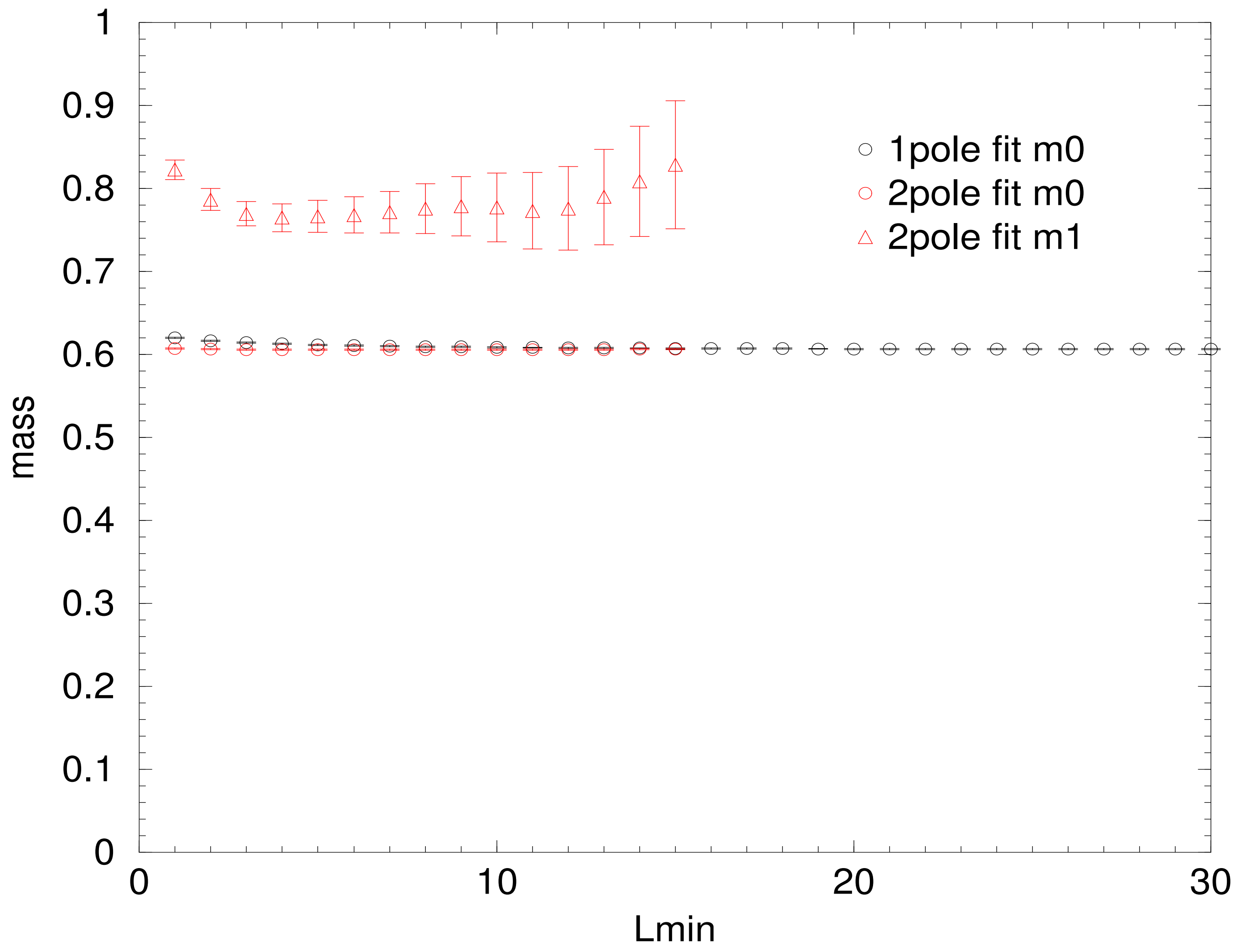
Fit range : Lmin$\sim$Lmax : Lmax=Nz/2 (48)

Single pole fit is applicable in Lmin$\geq$ 15.

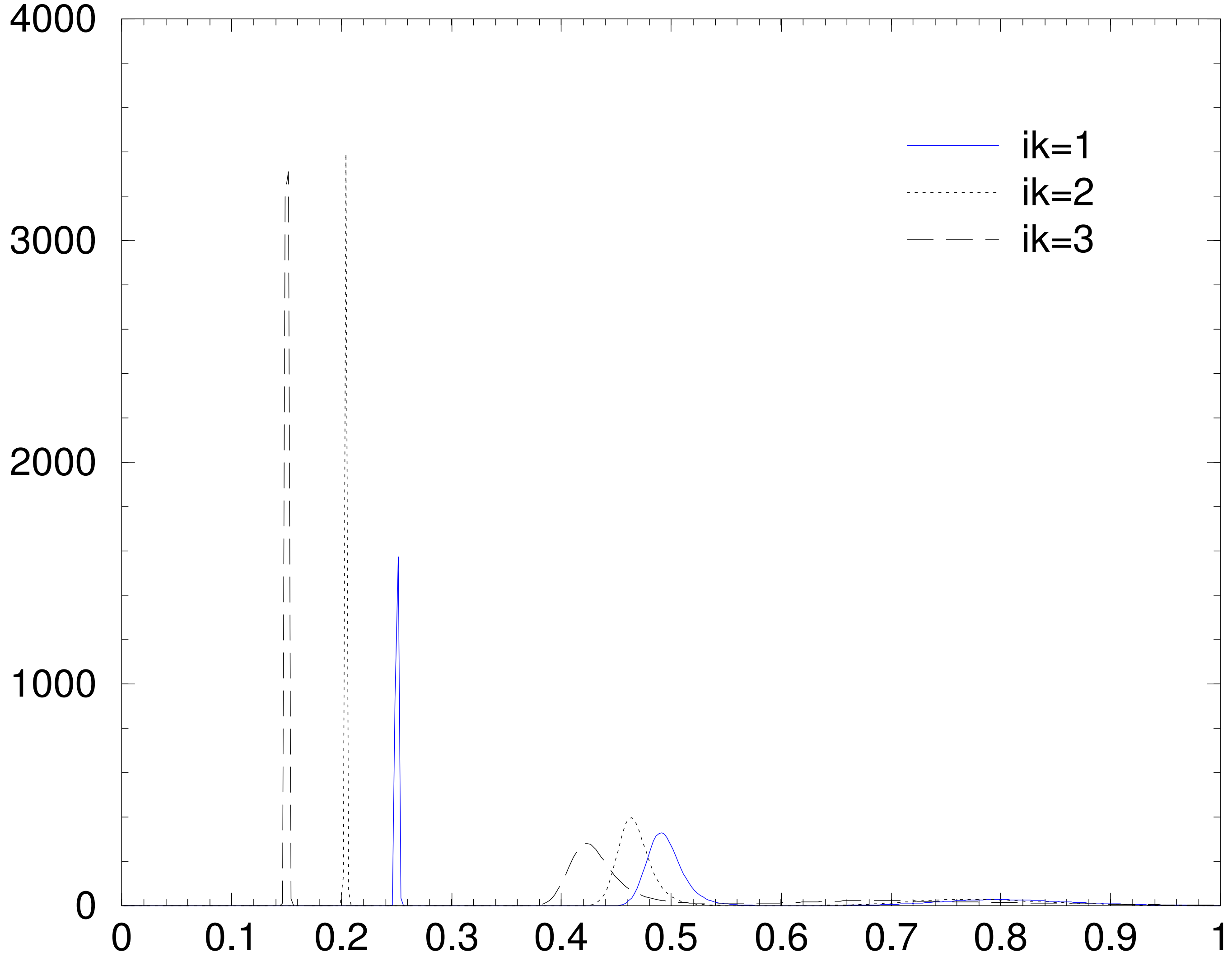
$m_{gr} \cong 0.6 a_z^{-1} = 2.4$ GeV $\quad$ (m_{gr} : ground state mass)

cf. $2\pi T \cong 3.1$ GeV

Two pole fit is roughly stable

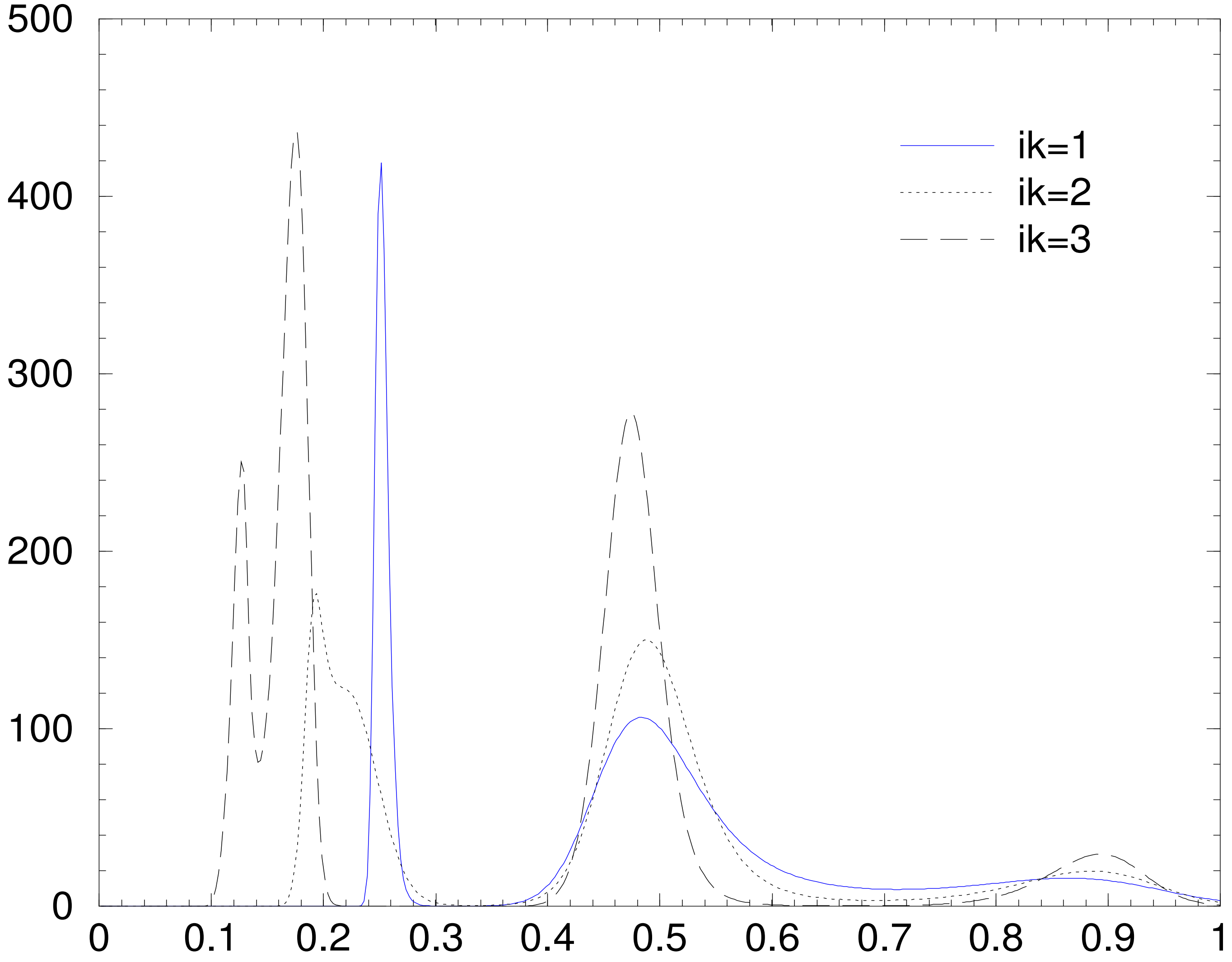


Result : $T \simeq 0T_c$ (Nt=12) spectral function



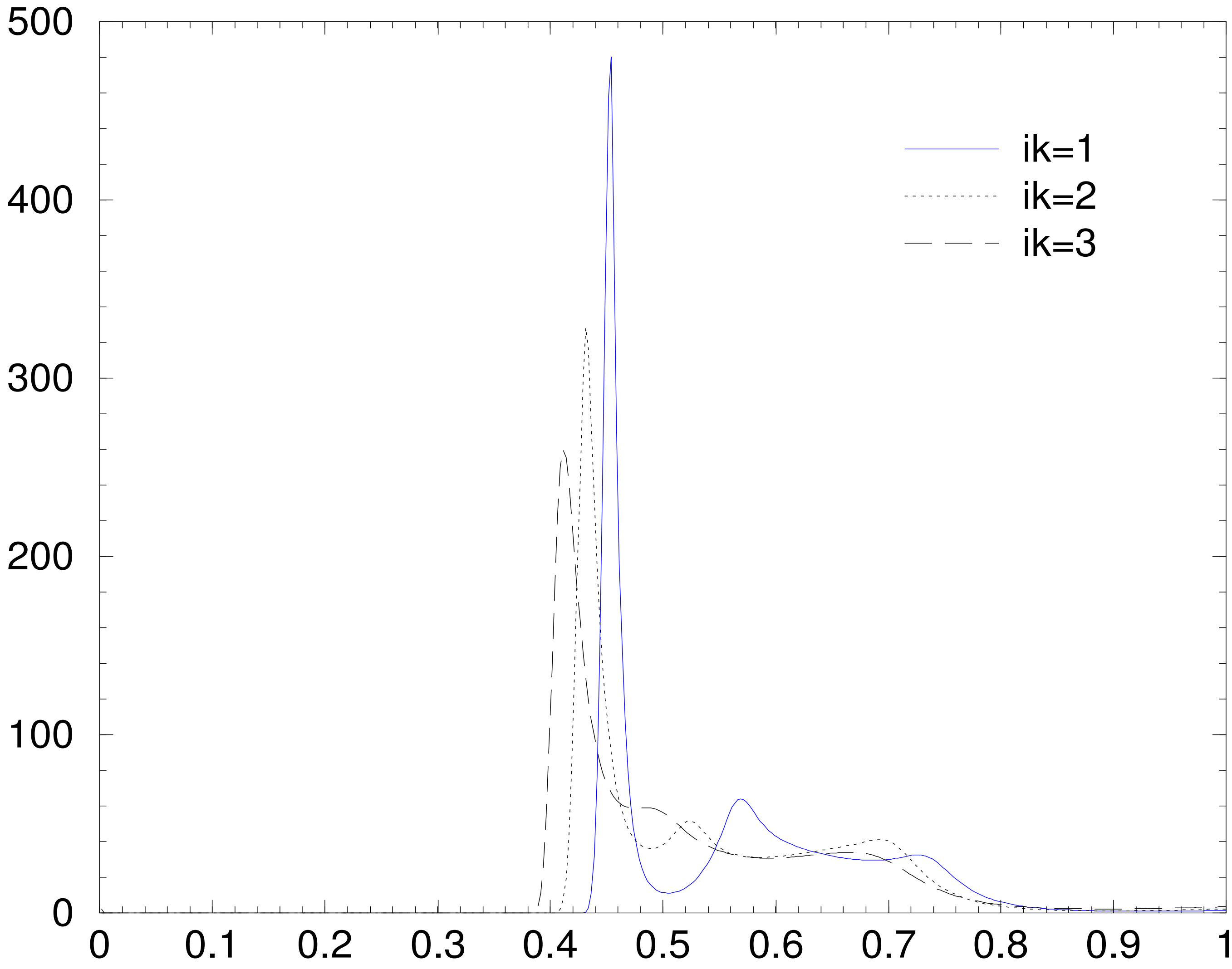
Lmin=1, Lmax=30, $m_0 = 1.0$
Peak location is consistent with results of pole fits.
(In present analysis, peak width dose not display physical width.)

Result : $T = 0.8T_c$ (Nt=5) spectral function



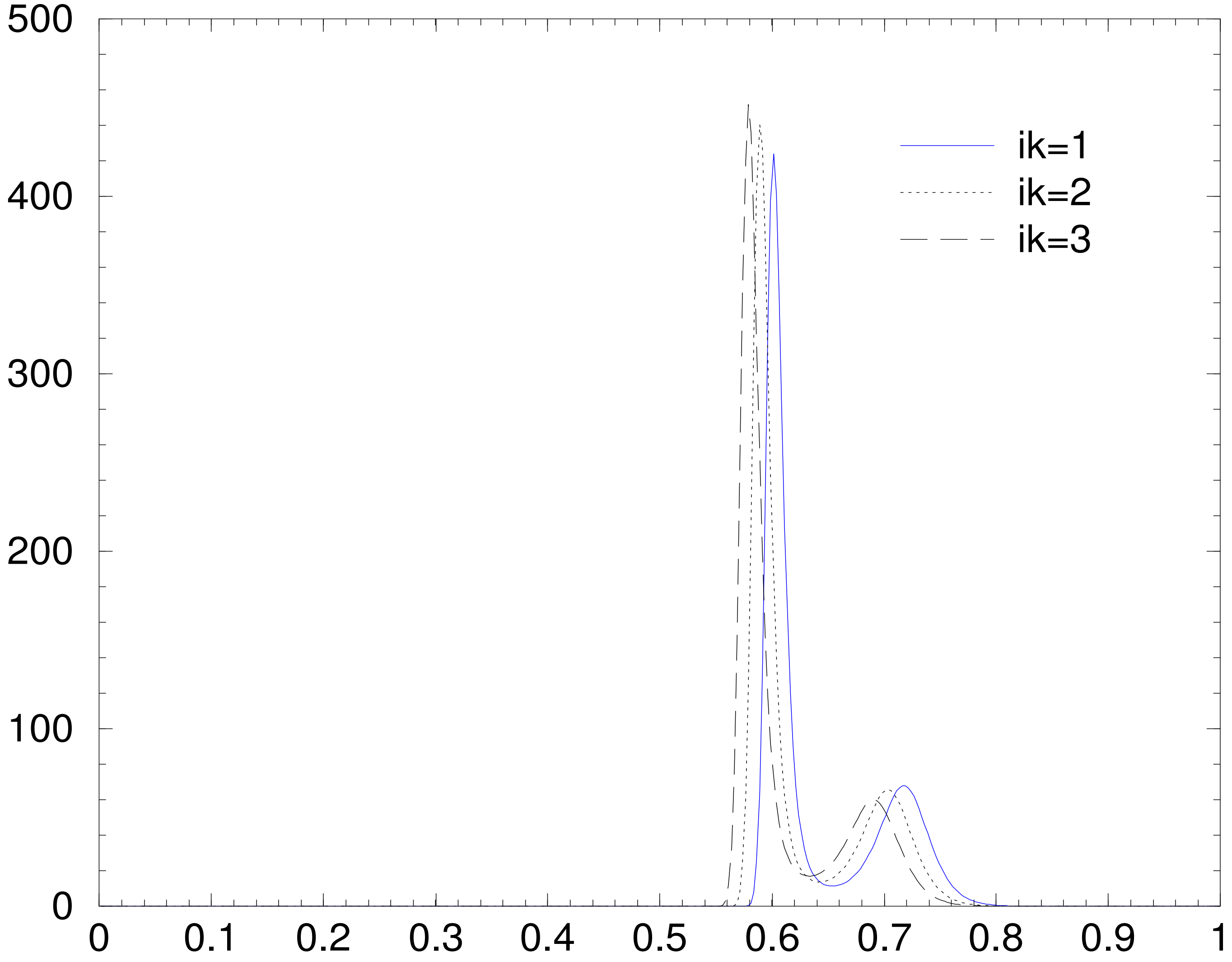
Lmin=1, Lmax=30, $m_0 = 1.0$
Peak location of ground and 1st. excited state is almost same as at $T \simeq 0$.
(But 1st. excited pole couldn't be isolated.)
In the case of ik=2,3 (lighter quark mass),
shape of spectral function is unstable.
$\longrightarrow$ It need more statistics.

Result : $T = 1.3T_c$ (Nt=3) spectral function



Lmin=1, Lmax=30, $m_0 = 1.0$
This is a peak structure which correspond to ground state (?)
They are consistent with result of pole fit.
Structure is not clear in $\omega \geq 0.5$
$\longrightarrow$ Need more statistics

Result : $T = 2T_c$ (Nt=2) spectral function



Lmin=1, Lmax=30, $m_0 = 1.0$
These are peak structures which respond to ground state and 1st. excited state (?)
Peak locations are consistent with results of pole fit.

Summary and Outlook

We analyzed spatial correlators on quenched anisotropic lattice at finite temperature.

Present analysis procedures :

- **single/double pole fits**
- **Reconstruction of spectral function (by MEM)**

We found qualitatively consistent results
$\Longrightarrow$ It seems that our analysis wark at finite temperature.

Detailed analysis with higher statistics (other types of fit ansatz , etc.)
$\Longrightarrow$ Quantitative analysis would be possible.

Outlook :

- **Calculations with high statistics and with larger lattices (higher $\beta's$).**
- **Baryonic modes**
- **Effects of dynamical quarks**
- **Relation between spatial and temporal correlators**
- **Analysis of spatial Wilson loop**