Charmonium physics at finite temperature on the lattice

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Contents

Introduction

Finite temperature hadron properties:

What happens on hadrons
or what new effects above T_c ?

Quark-Gluon plasma formation
$\Downarrow$

- J/ψ suppression above T_c
  Matsui and Satz (1986)
- Mass shift near T_c
  Hashimoto et al (1986)

We study the Charmonium
(heavy quarkonium) at $T \geq 0$

$\Longrightarrow$ Lattice QCD is useful

What is the Lattice QCD ?

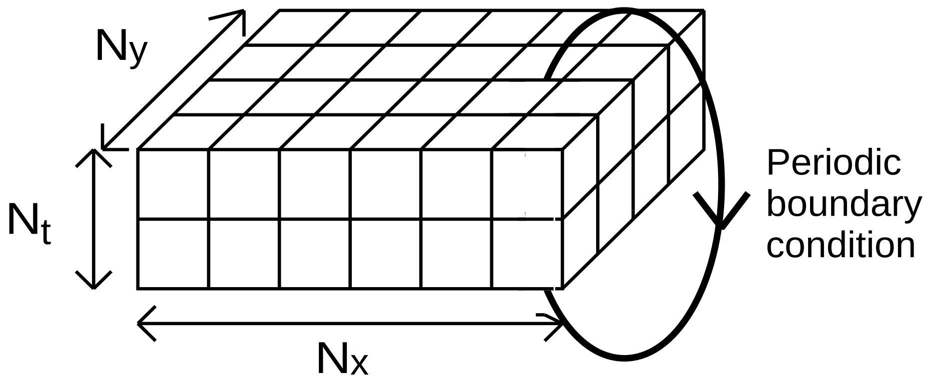
Discretization of space-time (Euclidean space)
with exact local gauge invariance

$$\Downarrow$$

Path integral formalism has
a well-defined meaning

$\Rightarrow$ Monte Carlo Method

Finite Temperature lattice QCD



$$T = \frac{1}{N_t a} \qquad (\ a \ : \text{lattice spacing}\ )$$

How efficient is the Lattice QCD ?

Merits

- Calculation by the first principle !

$\Longrightarrow$ Available for non-perturbative calculation

  - phase transition
  - hadron mass
  - etc.

Demerits

- Finite volume
  - It is difficult to treat hadron gas .
- Without dynamical quarks

Our results is without dynamical quark.

We are planning to the full QCD

at KEK supercomputer on this March.

$\Longrightarrow$ phase transition is first order.

What can the Lattice QCD predicts ?

Quark propagetors with interaction ($T \geq 0$)

$\Downarrow$

Hadron correlator

$\Longrightarrow$ hadron masses

- Pole mass (temporal mass)
- Screening mass (spatial mass)

mass shift

$\Longrightarrow$ wave function

hadronic bound state

$\Longrightarrow$ spectral function

mass shift & width

Problem

- Small d.o.f. in temporal direction
  $\Longrightarrow$ Anisotropic lattice $a_t < a_s$

- $O(a)$ discretization error
  $\Longrightarrow$ Clover quark action

- Heavy quark mass $>$ lattice cutoff
  $\Longrightarrow$ Fermilab action

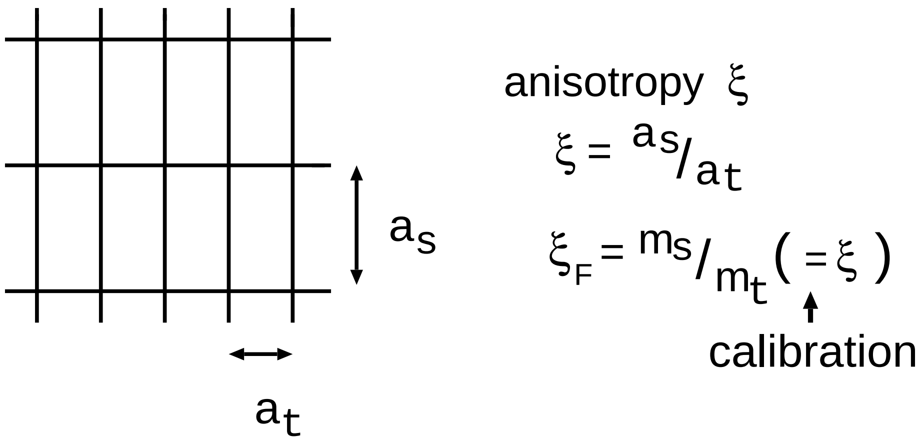
- Short physical scale in temporal direction
  $\Longrightarrow$ Smearing for hadronic operator
  $\Longrightarrow$ Variational analysis

Anisotropic lattice

Need detailed information
in temporal(temperature) direction
→ Anisotropic Lattice

Karsch (1982)

Burgers, Karsch, Nakamura and Stamatescu (1988)



Merit

- Many degree of freedom in temporal direction for small computing power

Demerit

- Complicate analysis from a new parameter "anisotropy ξ"

Heavy quark action

Fermilab (Clover) action on Anisotropic lattice

$$
\begin{aligned}
S_F(\kappa_s, \gamma_F) &= \sum_x \bar{q}(y) K[U](x,y) q(x) \\
K[U](x,y) &= 1 - \kappa_s \sum_i \left[\left(\frac{1}{\xi} - \gamma_i \right) T_{+i} + \left(\frac{1}{\xi} + \gamma_i \right) T_{-i} \right] \\
&- \kappa_t \left[(1 - \gamma_4) \, T_{+4} + (1 + \gamma_4) T_{-4} \right] \\
&- \kappa_s \frac{1}{2\xi} c_B \vec{\Sigma} \cdot \vec{B} + \kappa_t \frac{1}{2\xi} c_E \vec{\alpha} \cdot \vec{E}
\end{aligned}
$$

$$
\begin{aligned}
\kappa_t &= \gamma_F \kappa_s \\
c_B, c_E &: \text{ Clover coefficients} \\
& \quad (\text{ Mean-field improved }) \\
T_{+\mu}(x,y) &= U_\mu(x) \delta_{x+\hat{\mu},y} \\
T_{-\mu}(x,y) &= U_\mu(x - \hat{\mu}) \delta_{x-\hat{\mu},y}
\end{aligned}
$$

Clover action:Sheikholeslami, Wohlert (1985)

Fermilab action : El-Khadra, Kronfeld, Mackenzie (1997)

c.f. Klassen hep-lat/9809174

Simulation parameters

Lattice:

$12^3 \times N_t$,$\beta = 5.68$, $\gamma = 4.0$, quenched

$N_t = 72$ $(T \simeq 0)$, 20 $(T < T_c)$, 16, 12 $(T > T_c)$: $T = 1/N_t a_t$

- #conf. = 60
- Anisotropy: $\xi \equiv a_s/a_t = 5.3(1)$
  from the ratio of Wilson loops

  Engels, Karsch and Scheideler (1997), Klassen (1998)
- Cutoff: $a_s^{-1} = 0.85(3)$ GeV,$a_t^{-1} = 4.5(2)$ GeV
  from heavy quark potential

Quark: Anisotropic Fermilab (Clover) action

- Hopping parameter and bare anisotropy:

κ	γ_F	m_{PS}	m_V [GeV]
0.0985	3.580	3.51(16)	3.56(16)
0.1032	3.670	3.04(13)	3.10(14)
0.1075	3.750	2.63(12)	2.70(12)
0.1159	3.867	1.91(09)	2.00(08)

- γ_F determined by calibration
- Periodic b. c. for spatial direction

Calibration

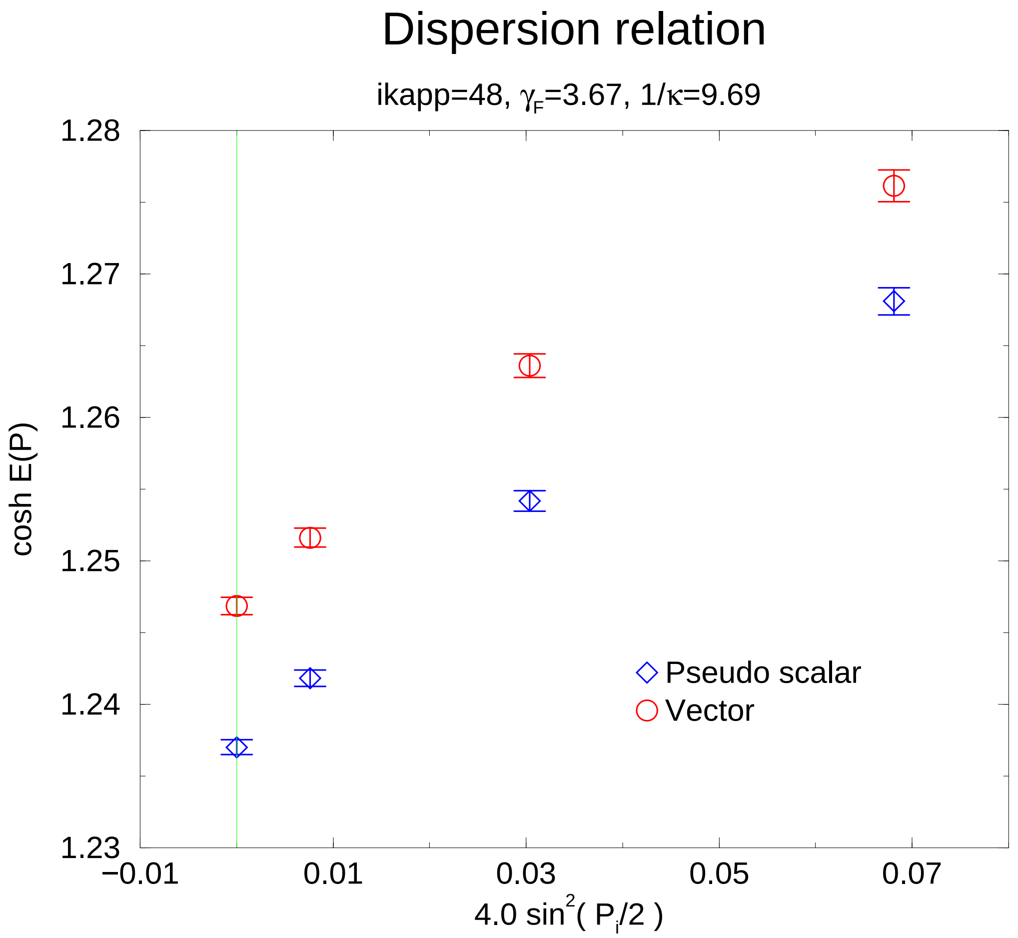
∘ Anisotropy of heavy quark field : ξ_F

$$\xi_F = \xi (= 5.3(1))$$

·Dispersion relation of free meson

$$\cosh\left(E(\vec{p})\right) = 1 + \frac{1}{2\xi^2}(\hat{\vec{p}}^2 + m^2)$$

$$\hat{p}_i = 2 \sin p_i / 2$$

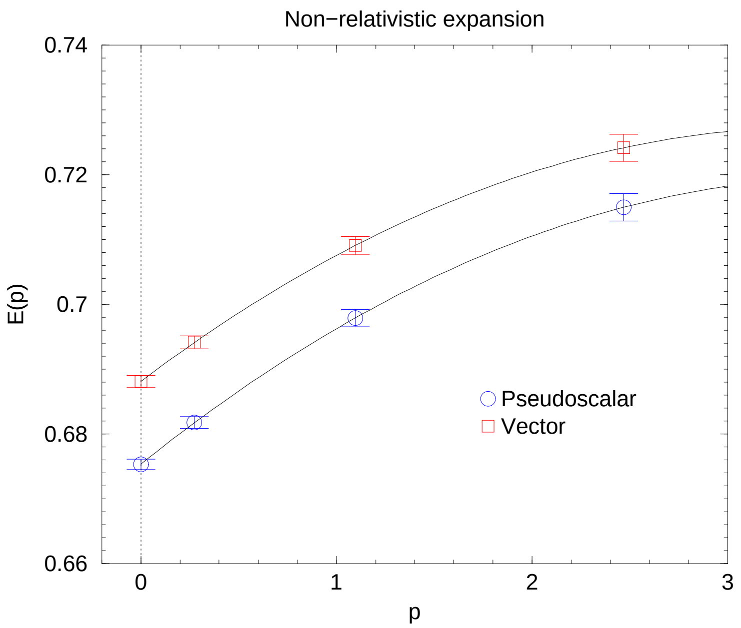


Dispersion check

$\xi_F = 5.3$ from free meson propagator

I) Non-relativistic expansion

$$E = M + \frac{p^2}{2M\xi_F^2} - \frac{p^4}{8M^3\xi_F^4} + \cdots$$



$\xi_F \sim 5.5$

II) Mass ratio ($\star$ case of light quark)

$$\xi_F = \frac{m_s}{m_t}$$

κ	γ_F	m_{PS}	m_V [GeV]
0.1288	4.077	0.836(3)	1.015(5)

$\xi_F = 5.52(12)$ from dispersion relation

$\xi_F = 5.437(44)$ from mass ratio

Meson Correlator

$$G_M(\vec{x},t) = \sum_{x,y_1,y_2} \omega_1(\vec{y}_1)\omega_2(\vec{y}_2)$$
$$\times \langle Tr[S(\vec{y}_1,0;\vec{z},t)\gamma_M\gamma_5 S^\dagger(\vec{y}_2,0;\vec{z}+\vec{x},t)\gamma_5\gamma_M^\dagger]\rangle$$

$S(\vec{x}_1,t_1;\vec{x}_2,t_2)$: quark propagator

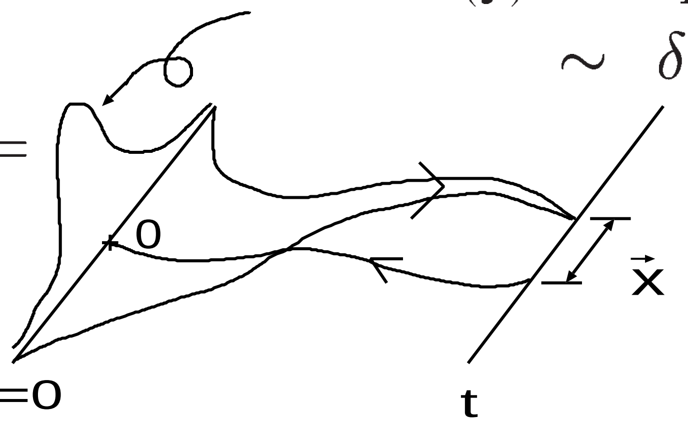
$$\gamma_M = \gamma_5\ ,\ \gamma_1\ ,\ 1\ ,\ \gamma_1\gamma_5$$
$$(\ M = P_S\ ,\ V\ ,\ S\ ,\ A\ )$$

◇Gauge fixing : Coulomb gauge

source $\omega(\vec{\mathrm{y}}) \sim \exp(-\mathrm{ay}^{\mathrm{p}})$ (exp)

$\sim \delta(\vec{\mathrm{y}})$ (point)

$G_M(\vec{x},t) =$

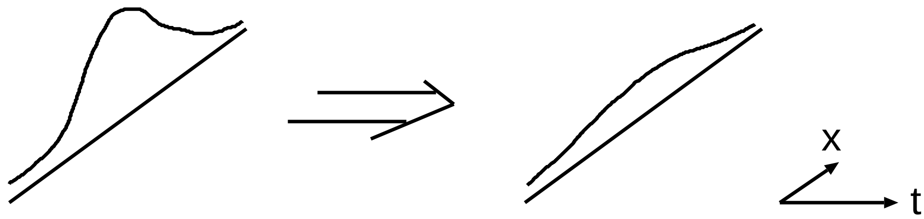


a, p are determined from $\vec{x}$-dependence
in point-point P_S correlator at $T \simeq 0$

t-dependence of the wave function

$$w_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t) O^\dagger(0)\rangle$$

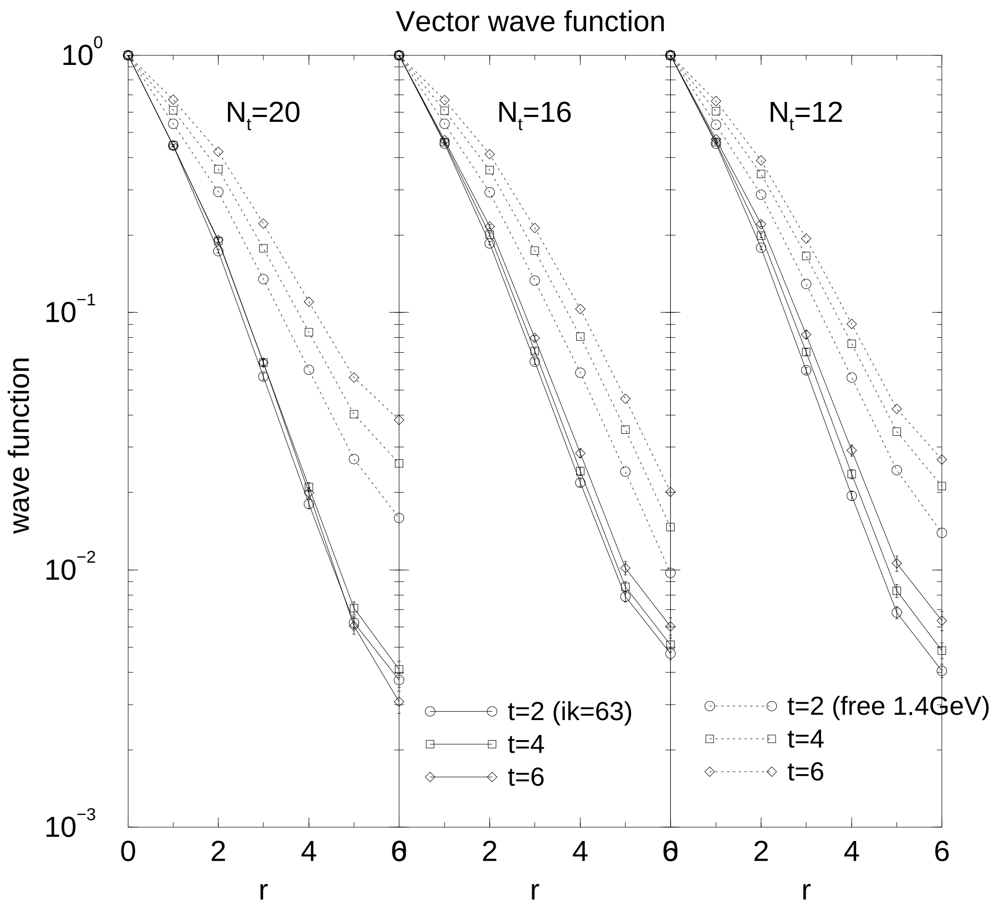
If there is no bound state (like free quark case), wave function becomes broader as t.



In this case, $\phi_\Gamma(r,t) = w_\Gamma(r,t)/w_\Gamma(r=0,t)$

(normalized at spatial origin)

increase as t.

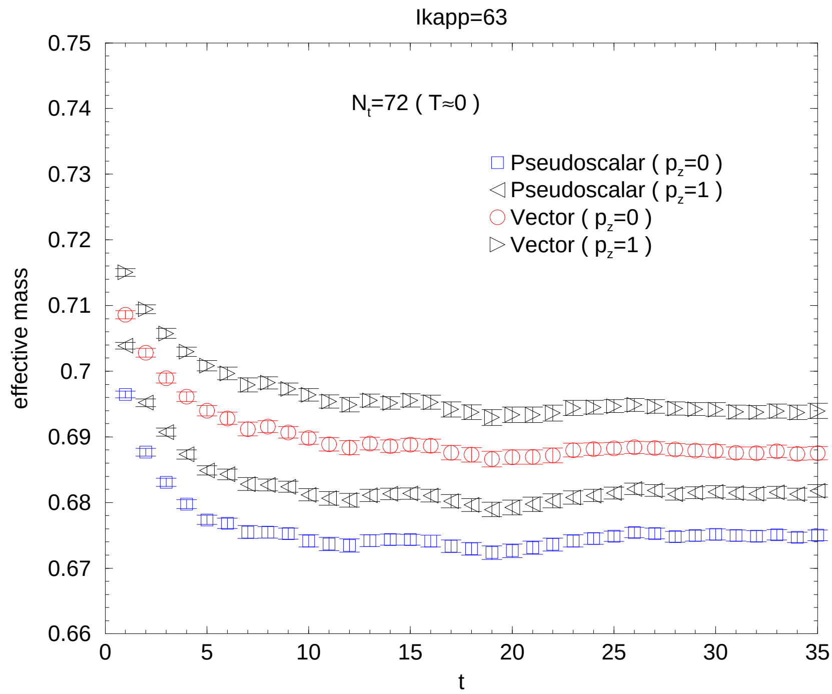


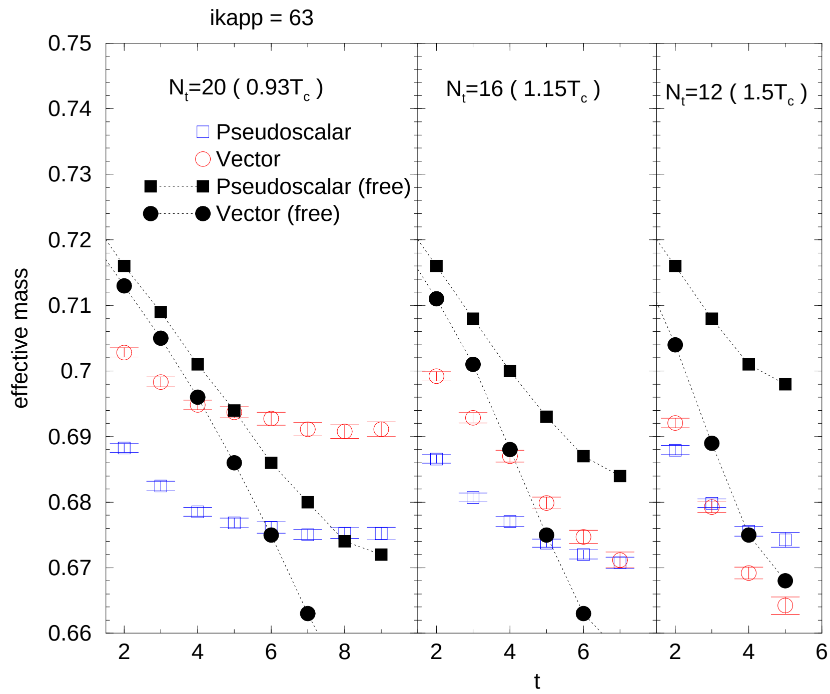
Effective mass

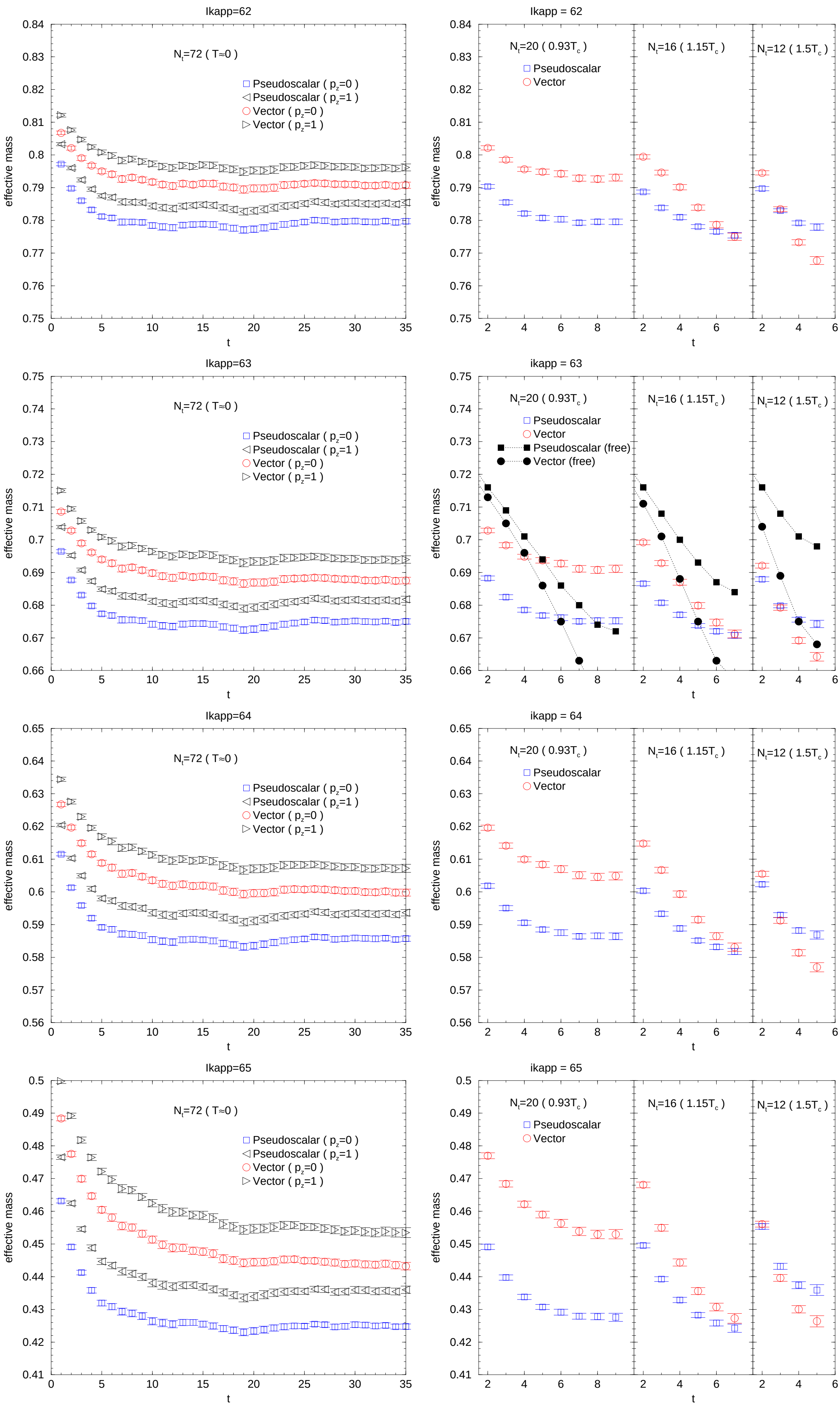
Effective mass : m_{eff}

$$\frac{G_M(\vec{x}=0,t)}{G_M(\vec{x}=0,t+1)} = \frac{\cosh\left[m_{\text{eff}}(N_t/2-t)\right]}{\cosh\left[m_{\text{eff}}(N_t/2-t-1)\right]}$$

◦ effective mass at $T > 0$






Ikapp=62
N_t=72 (T≈0)
Pseudoscalar (p_z=0)
Pseudoscalar (p_z=1)
Vector (p_z=0)
Vector (p_z=1)
effective mass
t
Ikapp = 62
N_t=20 (0.93T_c)
N_t=16 (1.15T_c)
N_t=12 (1.5T_c)
Pseudoscalar
Vector
Ikapp=63
ikapp = 63
Pseudoscalar (free)
Vector (free)
Ikapp=64
ikapp = 64
Ikapp=65
ikapp = 65

Variational analysis

$$\mathcal{O}_1^\dagger|0\rangle = z_1|1'\rangle, \quad \mathcal{O}_2^\dagger|0\rangle = z_2|2'\rangle$$

$|1'\rangle, |2'\rangle$ are linear combinations of $|1\rangle, |2\rangle$
Here $|1\rangle, |2\rangle$ are eigen states of Hamiltonian.

In the numerical calculation,

$$\begin{aligned} C_{ij}(t) &= \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle \\ &= \begin{pmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{pmatrix} \end{aligned}$$

$$\Downarrow$$

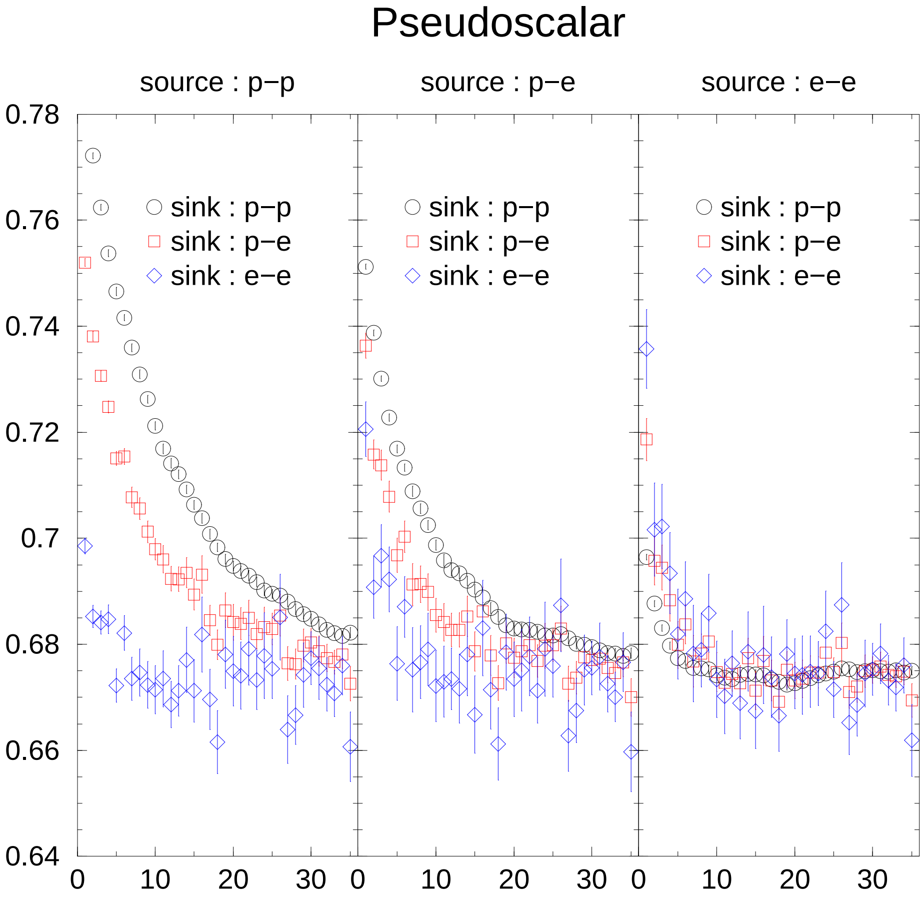
Diagonalize

$|i'\rangle$ basis $\Longrightarrow$ $|i\rangle$ basis

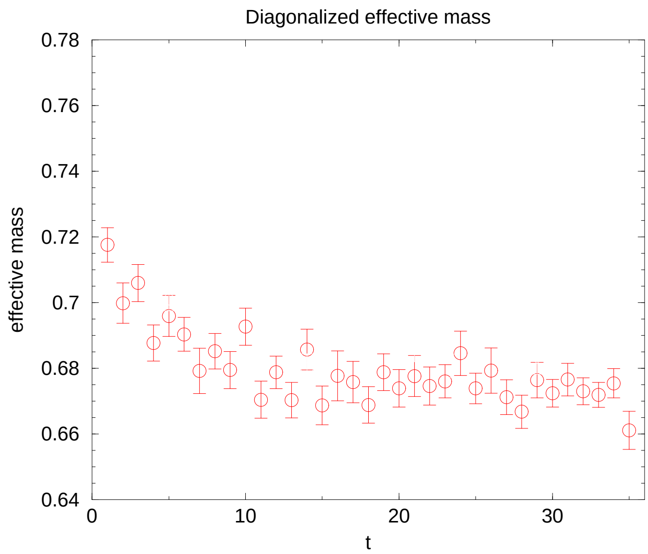
In this talk, We used

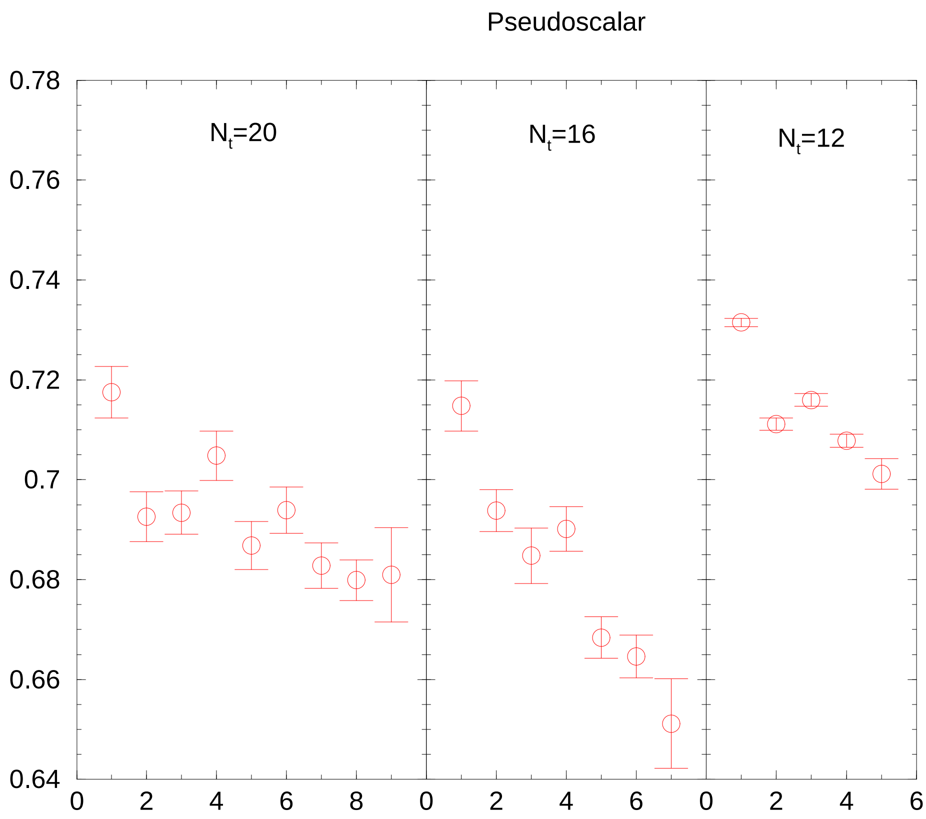
- $\mathcal{O}_1$: point-point
- $\mathcal{O}_2$: point-exp
- $\mathcal{O}_3$: exp-exp

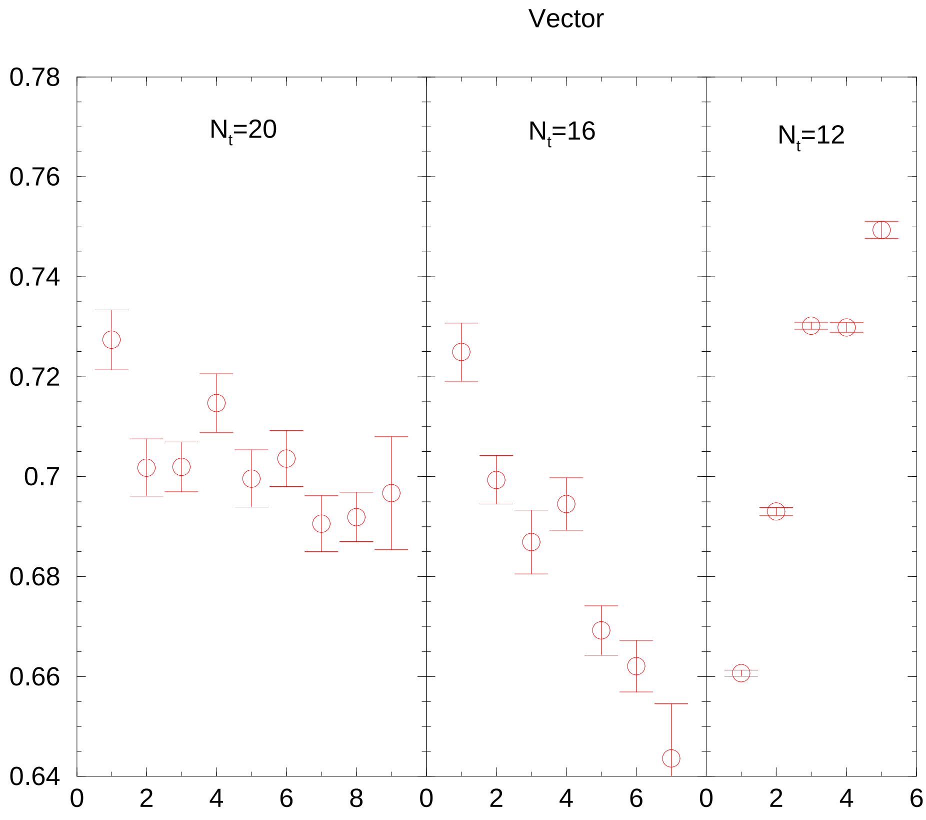
Results of variational analysis (Preliminary!)



⇓ Diagonalization




Pseudoscalar
N_t=20
N_t=16
N_t=12
0.78
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0 2 4 6 8 0 2 4 6 0 2 4 6


Vector
N_t=20
N_t=16
N_t=12
0.78
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0 2 4 6 8 0 2 4 6 0 2 4 6

Summary

- We use the anisotropic Fermilab action
  for heavy quark system.
- About charmonium
  - wave function
  - hadron mass
    - effective mass
    - variational analysis

Outlook

- Improvement of hadronic operator
- Fine lattice
- Other channels : baryons etc.
- With dynamical quarks