Hadron Spectroscopy on Anisotropic Lattice

KEK　12 Dec. 2000

中島典昭(RCNP)
松古栄夫(RCNP)
菅沼秀夫(東工大)
根本幸雄(基研)
梅田貴士(広大)

Introduction

- Hadron Physics in Lattice QCD
  Excited Hadrons, Exotic Hadrons
  - Negative Parity Baryons ($\Lambda(1405)$, N^*,)
  - H-dibaryon
  - σ (500?)
  - Glueballs

- Heavy Quark System
  - B-Physics ⟵ B-Factories (KEK, SLAC, etc.)
    Kobayashi-Maskawa Matrix Elements
    Hadronic Matrix Elements, Decay Constants,
    Quark Masses ⟵ Lattice QCD
  - Heavy Quarkonium (J/ψ production, etc.)

- Hadron Physics at Finite Temperature
  - Quark Gluon Plasma (QGP) Phase ⟵ RHIC
  - T_c 近傍のHadron PhaseでのHadronの諸性質
    (e.g. Mass shift) ⟶ Signal
    Chiral Symmetry Restoration
    (e.g. N-N^* degeneracy)

Lattice では、
Signal が見えにくい
t 方向の詳細な情報が必要

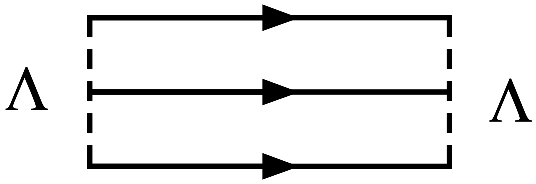
⟶ ***Anisotropic Lattice*** が有効

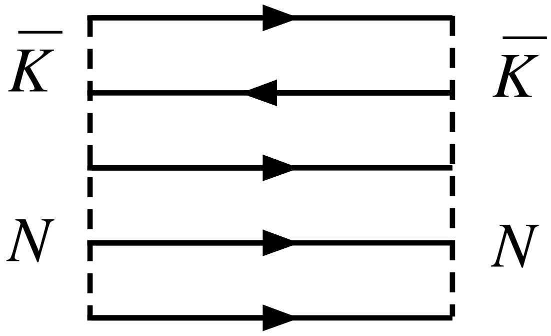
Λ(1405)

$$L_{I,2S} = S_{0,1} \quad J^P = \frac{1}{2}^-$$

(a) $SU(3)_f$ - singlet *uds* state (qqq)

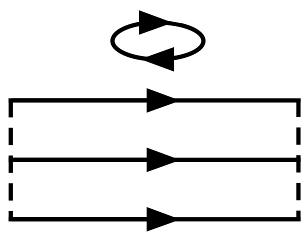
(b) $N\bar{K}$ bound state ($qqqq\bar{q}$)





Lattice QCD Study

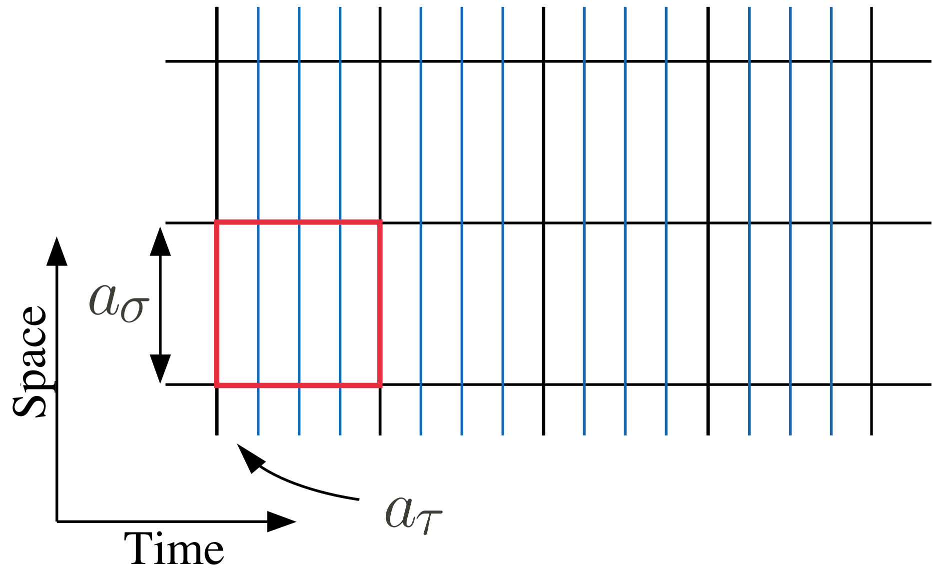
- 3 quark state or 5 quark state ?
  quenched approximation
  without



→ Full QCD

- Flavor singlet ? Λ_1(P=-1) Λ_8(P=-1)

Anisotropic Lattice



$\xi = \frac{a_\sigma}{a_\tau}$:Renormalized Anisotropy

Merits

・時間方向の詳細な情報が得られる ⟶ Fig.

Heavy Particle　Large Fluctuation

・時間方向のcutoff 大　けれど　Lattice size 小
(isotropic と比べて)

特に Finite Temperature

Demerit

・<u>Calibration が必要</u> ⟵

量子効果により ξ がずれる

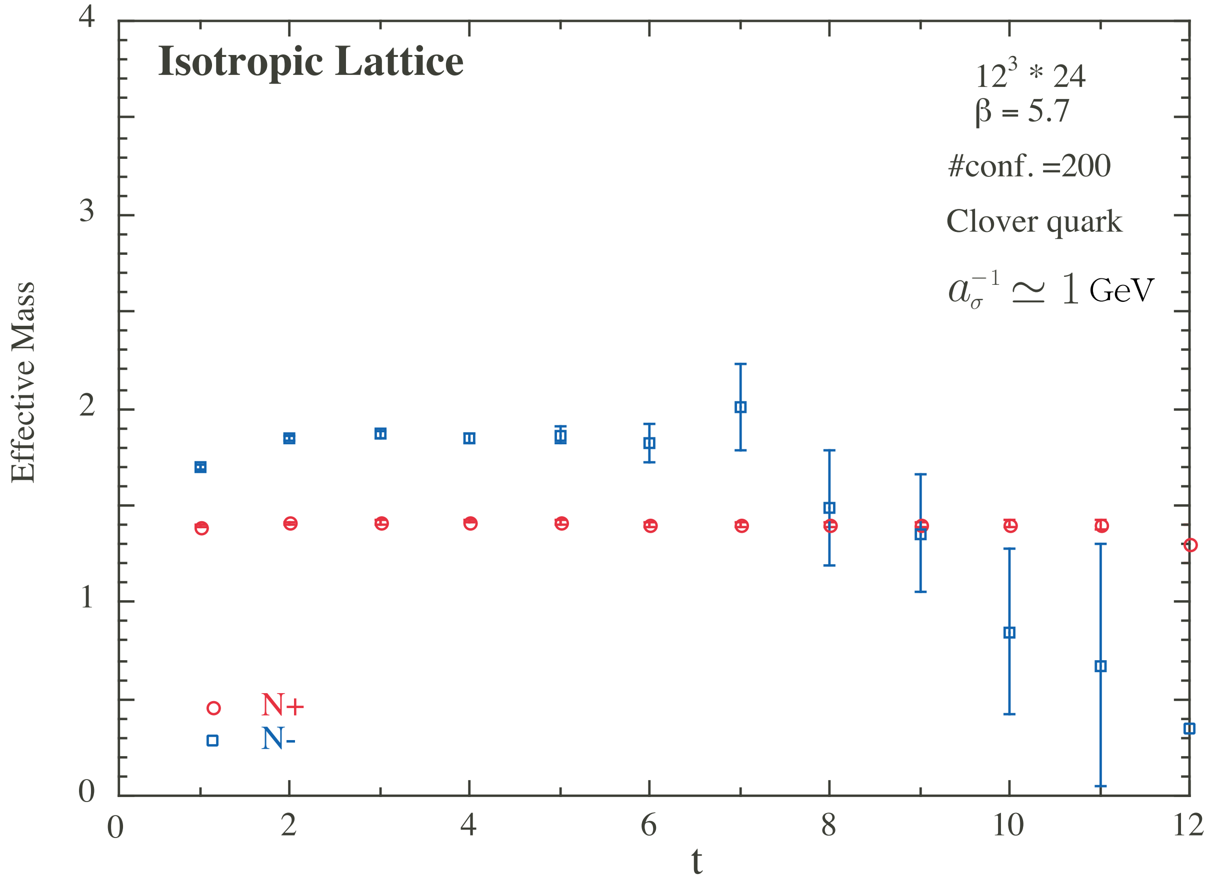
今回の話

$\mathcal{O}(a)$ - improved quark action

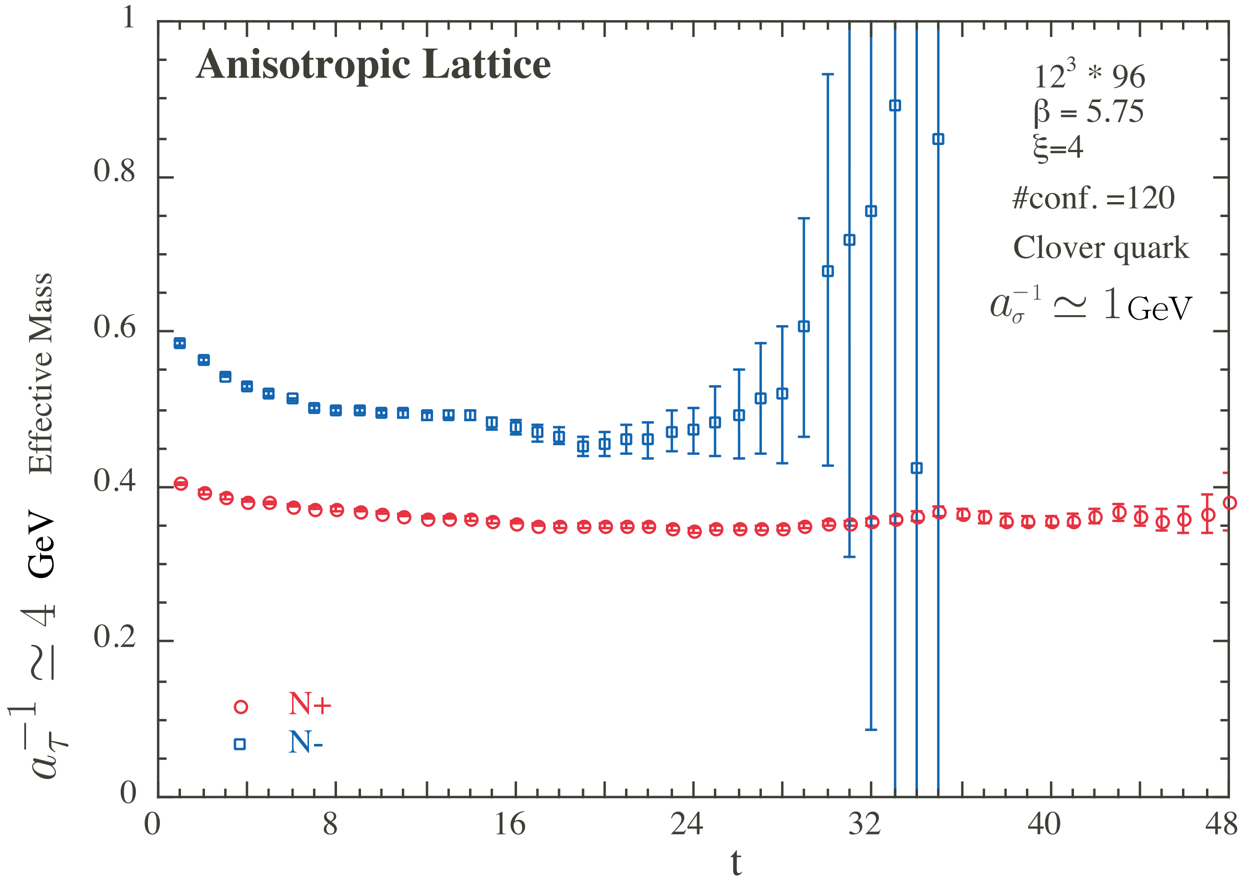
Parameter の結果 (Calibration)

Light Hadron Spectroscopy

特に、Negative Parity Baryons



Isotropic Lattice
$12^3 * 24$
$\beta = 5.7$
#conf. =200
Clover quark
$a_\sigma^{-1} \simeq 1$ GeV
Effective Mass
t
N+
N-



Anisotropic Lattice
$12^3 * 96$
$\beta = 5.75$
ξ=4
#conf. =120
Clover quark
$a_\sigma^{-1} \simeq 1$ GeV
$a_\tau^{-1} \simeq 4$ GeV Effective Mass
t
N+
N-

Quark Action

$\mathcal{O}(a)$ - improved quark action (Clover Action)

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) &= \delta_{x,y} \\ &- \kappa_\tau \left\{ (1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right\} \\ &- \kappa_\sigma \sum_i \left\{ (r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right\} \\ &- \kappa_\sigma c_E \sum_i i\sigma_{4i} F_{4i}(x) \delta_{x,y} - r\kappa_\sigma c_B \sum_{ij} \frac{1}{2} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

□ Hopping parameters

$$\kappa_\sigma = \frac{1}{2(m_0 + \gamma_F + 3r)} \quad \text{:spatial}$$

$$\kappa_\tau = \gamma_F \kappa_\sigma \quad \text{:temporal}$$

γ_F :bare anisotropy

□ Wilson parameter $r = 1/\xi$

$$\longrightarrow \quad \frac{1}{\kappa} = \frac{1}{\kappa_\sigma} - 2(\gamma_F + 3r - 4) = 2(m_0 + 4)$$

Chiral extrapolation in $\frac{1}{\kappa}$

□ Clover coefficients : $c_E = c_B = 1$

at tree level for $r = 1/\xi$

□ Mean-Field improvement

$$U_4 \to U_4/u_\tau, \quad U_i \to U_i/u_\sigma$$

$u_\tau = 0.762$

$u_\sigma = 0.987$

in Landau gauge

Note: 一般に $\gamma_F \neq \xi$

$\gamma_F = \xi$ になるような γ_F が必要である。

→ Calibration

Lattice Setup

- Gauge Action

  Anisotropic Wilson Gauge Action

  Karsch, N.P.**B205**(1982)285

  Lattice Size : $12^3 \times 96$ at Quenched Level

  Parameters ⟶ Klassen, N.P.**B533**(1998)557

  $\xi = 4$ $(\beta, \gamma_G) = (5.75, 3.072)$

  (1% level)

  Scale

  Static Quark Potential (Wilson Loop)

  $\sqrt{\sigma} = 472\ \mathrm{GeV}$ $a_\sigma^{-1} = 1.000(8)\mathrm{GeV}$

  $a_\tau^{-1} \simeq 4\mathrm{GeV}$

  Configuration

  for Calibration (400)

  for Hadron Correlator (120)

  Pseudo-Heat Bath

  # thermalization = 10000

  # separation = 2000

Calibration

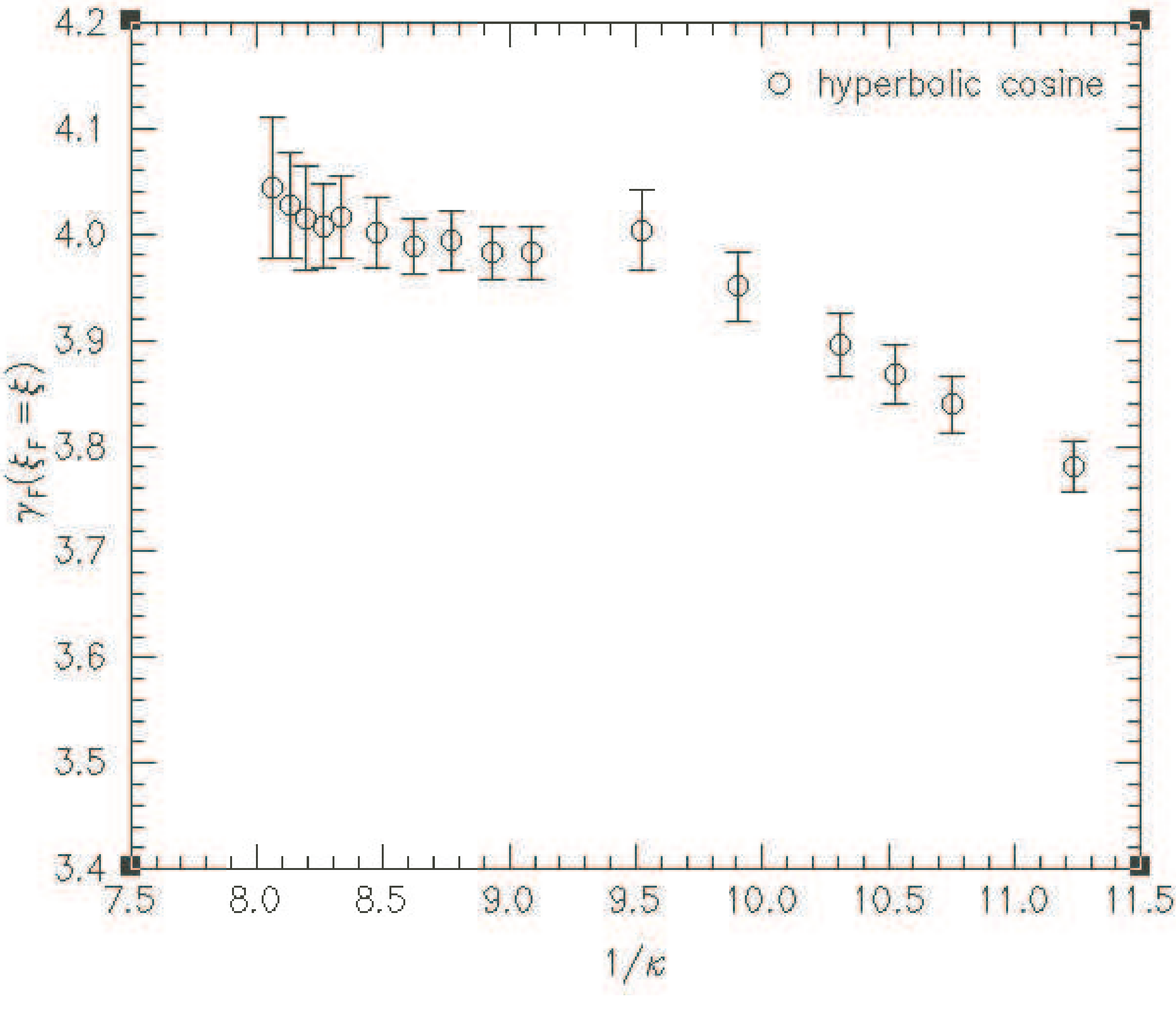
- Lattice Klein-Gordon Action for Meson Field

$$S = \sum_x \frac{1}{2\xi_F} \phi^\dagger(x) \left[-\xi_F^2 D_4^2 - \vec{D}^2 + m_0^2 \right] \phi(x)$$

- Dispersion Relation

$$\cosh E(\vec{p}) = 1 + \frac{1}{2\xi_F^2}(\vec{\hat{p}}^2 + m_0^2)$$

$$\xi_F^2 = \frac{\vec{\hat{p}}^2}{2(\cosh E(\vec{p}) - \cosh E(0))}$$



Hadron Spectrum

Two-Point function :

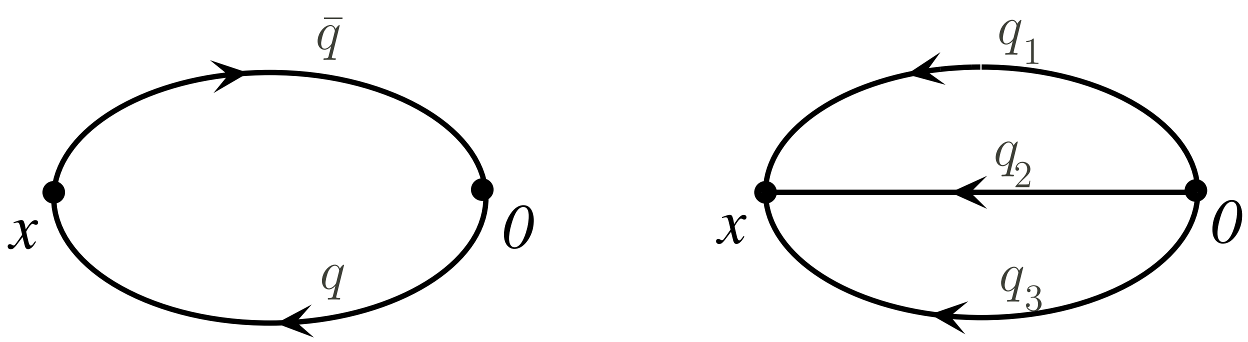
$$
\begin{aligned}
G_H(t) &= \sum_{\vec{x}} \langle 0|T\{\mathcal{O}_H(x)\overline{\mathcal{O}}_H(0)\}|0\rangle \\
&= \sum_{\vec{x}} \sum_{n} \langle 0|T\{\mathcal{O}_H(x)|n\rangle\langle n|\overline{\mathcal{O}}_H(0)\}|0\rangle \\
&\sim e^{-m_0 t} + \sum_{n\neq 0} e^{-m_n t} \\
&\to e^{-m_0 t} \qquad \text{large } t
\end{aligned}
$$

$\mathcal{O}_H$:hadron operator

$$
\begin{aligned}
\mathcal{O}_{M_{PS}} &\sim \bar{q}\gamma_5 q \\
\mathcal{O}_{M_V,k} &\sim \bar{q}\gamma_k q
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}_{B_8} &\sim \epsilon_{cde} q_c (q_d^T C\gamma_5 q_e) \\
\mathcal{O}_{B_{10},k} &\sim \epsilon_{cde} q_c (q_d^T C\gamma_k q_e) \\
\mathcal{O}_{B_1} &\sim \epsilon_{cde}\epsilon_{uds} q_c (q_d^T C\gamma_5 q_e)
\end{aligned}
$$

C :Charge Conjugation Matrix



Meson

$$G_M(t) \;=\; c_M \left[\mathrm{e}^{-tm_M} + \mathrm{e}^{-(T-t)m_M} \right]$$:Periodic Boundary

Baryon

$$\begin{aligned} G_B(t) \;&=\; (1+\gamma_4) \left[c_{B^+}\mathrm{e}^{-tm_{B^+}} + c_{B^-} b \mathrm{e}^{-(T-t)m_{B^-}} \right] \\ &+\; (1-\gamma_4) \left[c_{B^+} b \mathrm{e}^{-(T-t)m_{B^+}} + c_{B^-}\mathrm{e}^{-tm_{B^-}} \right] \end{aligned}$$

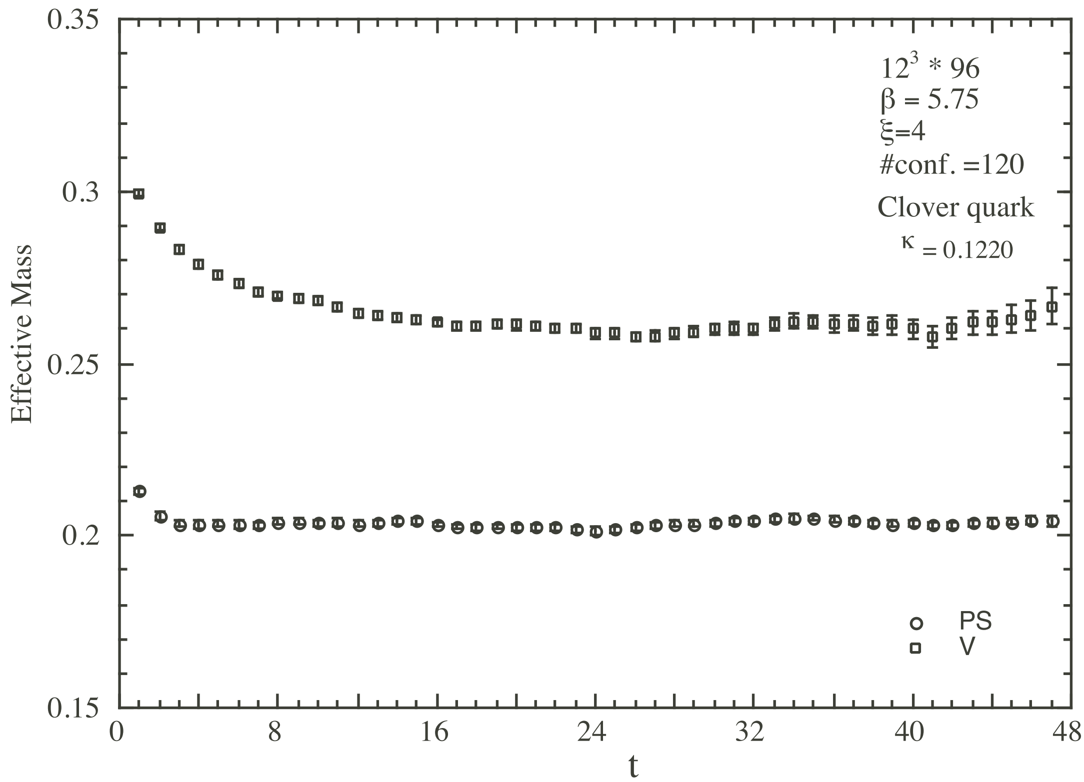
$b = +1$:Periodic Boundary

$b = -1$:Antiperiodic Boundary

□Effective Mass

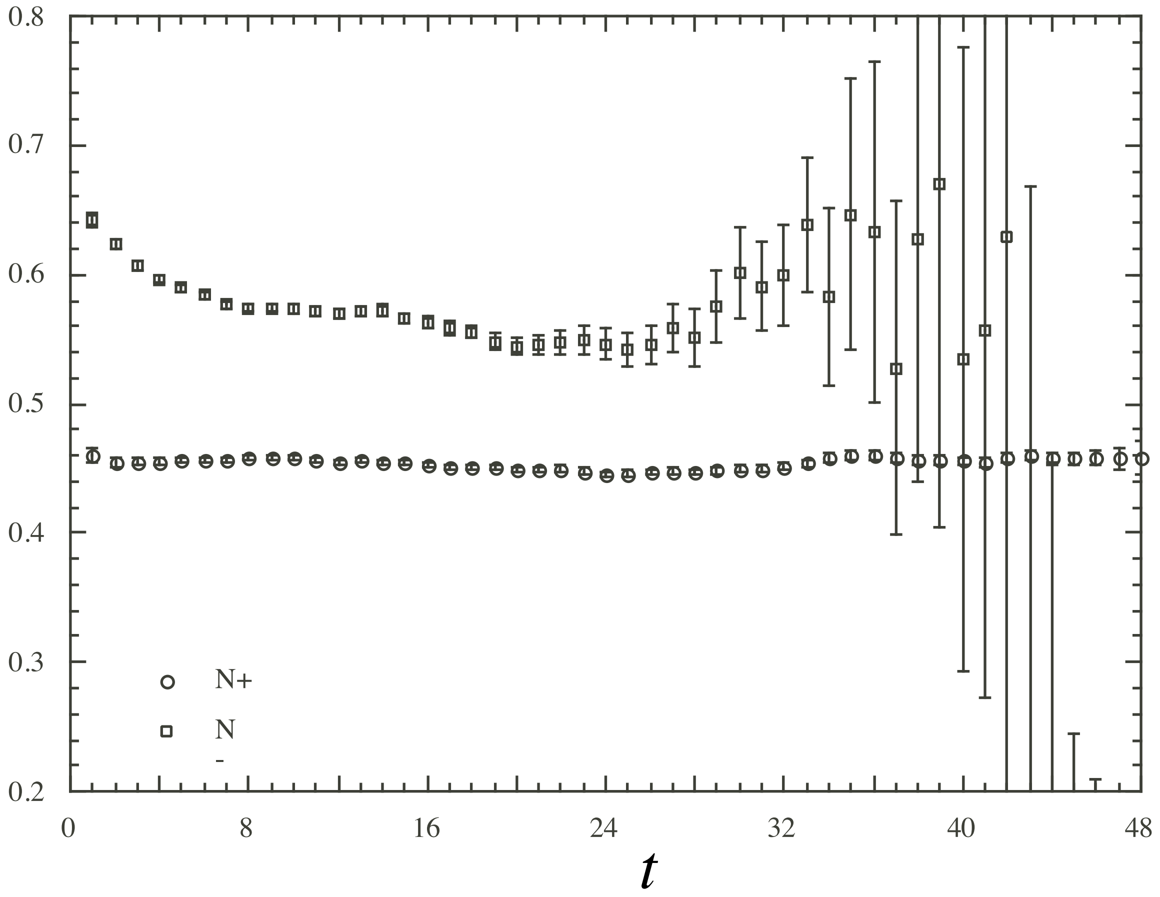
$$m_{eff}(t) = \ln \frac{G_H(t)}{G_H(t+1)}$$

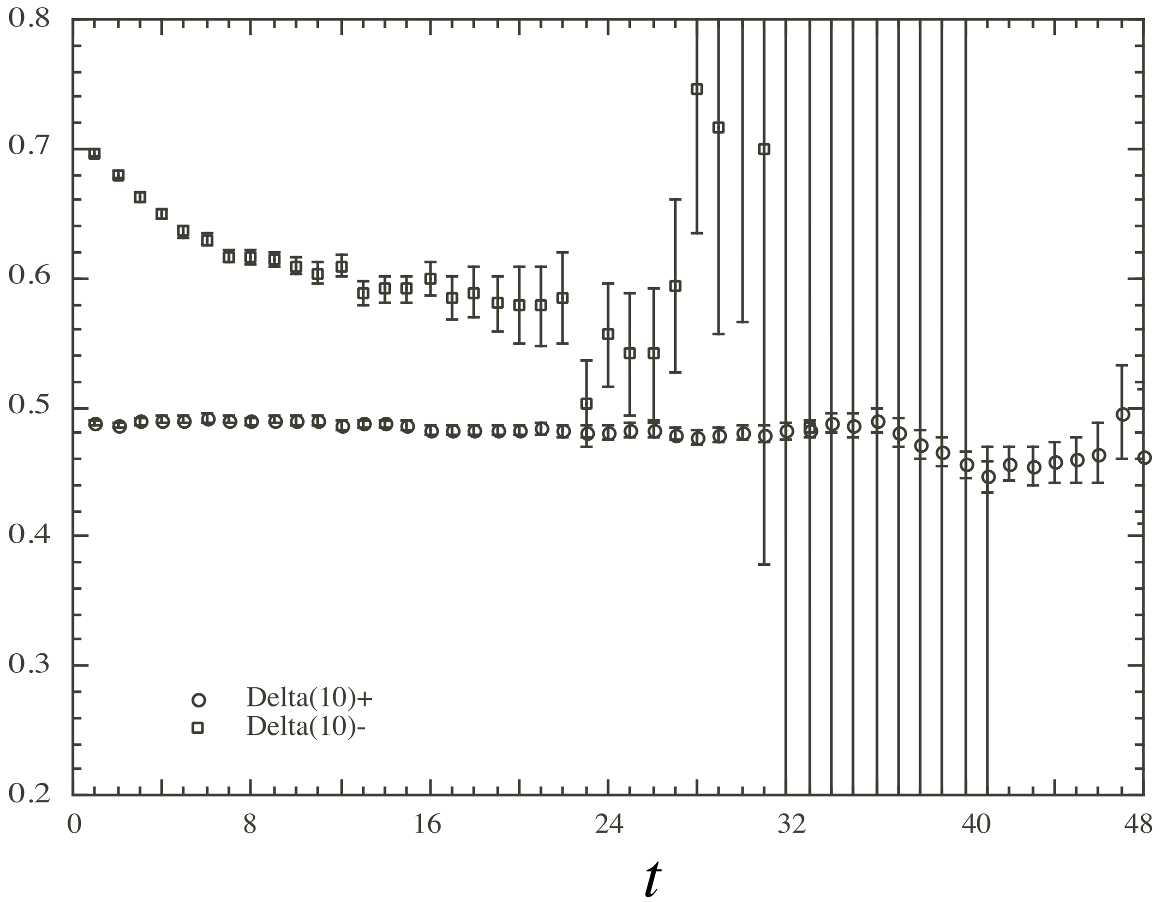
$\rightarrow$ constant (= m_0) at large t



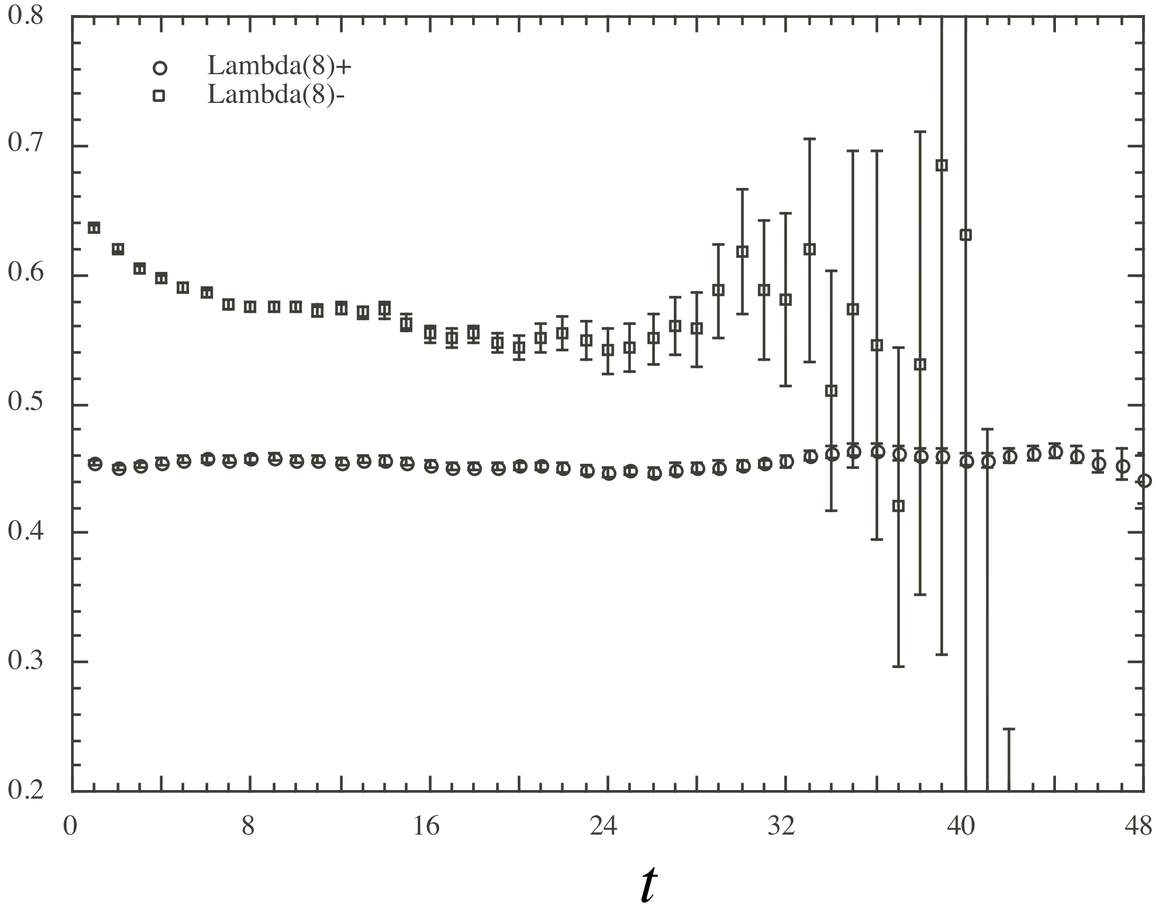
■ Smearing Gaussian with $\sigma \sim 2a_\sigma$

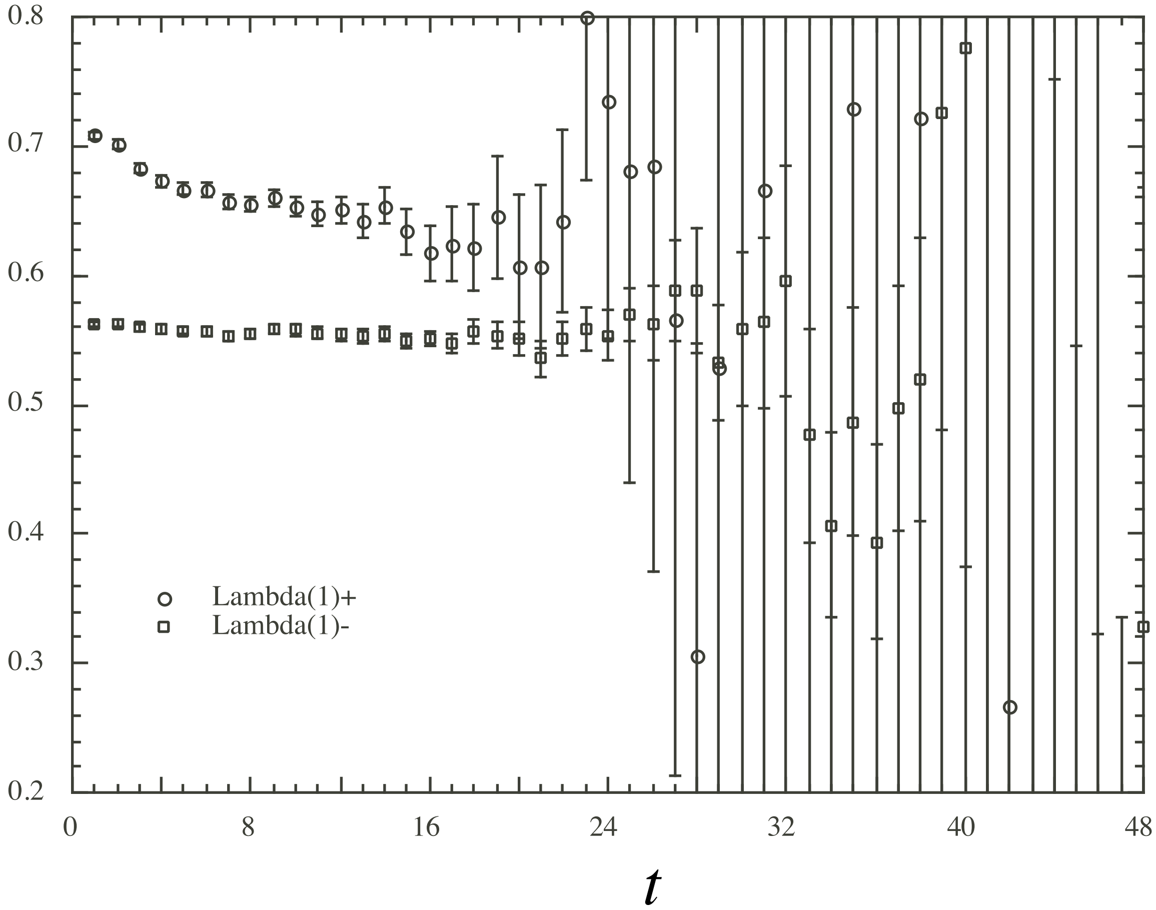
Effective Mass





Effective Mass





Chiral extrapolation

- Quark Mass

$$m_q = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

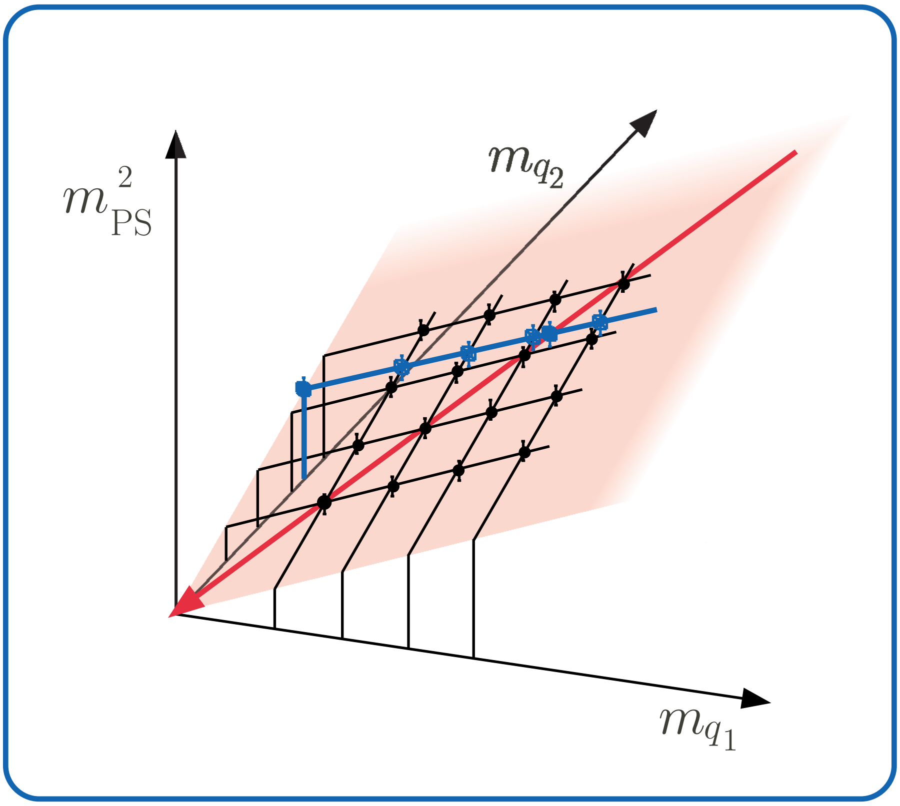
- Hadron Mass

$$m_{\rm PS}^2 \propto m_{q_1} + m_{q_2} \rightarrow \frac{1}{\kappa_c}$$

(CHPT)

$$m_{\rm V} = a_{\rm V} + b_{\rm V}\,(m_{q_1} + m_{q_2})$$

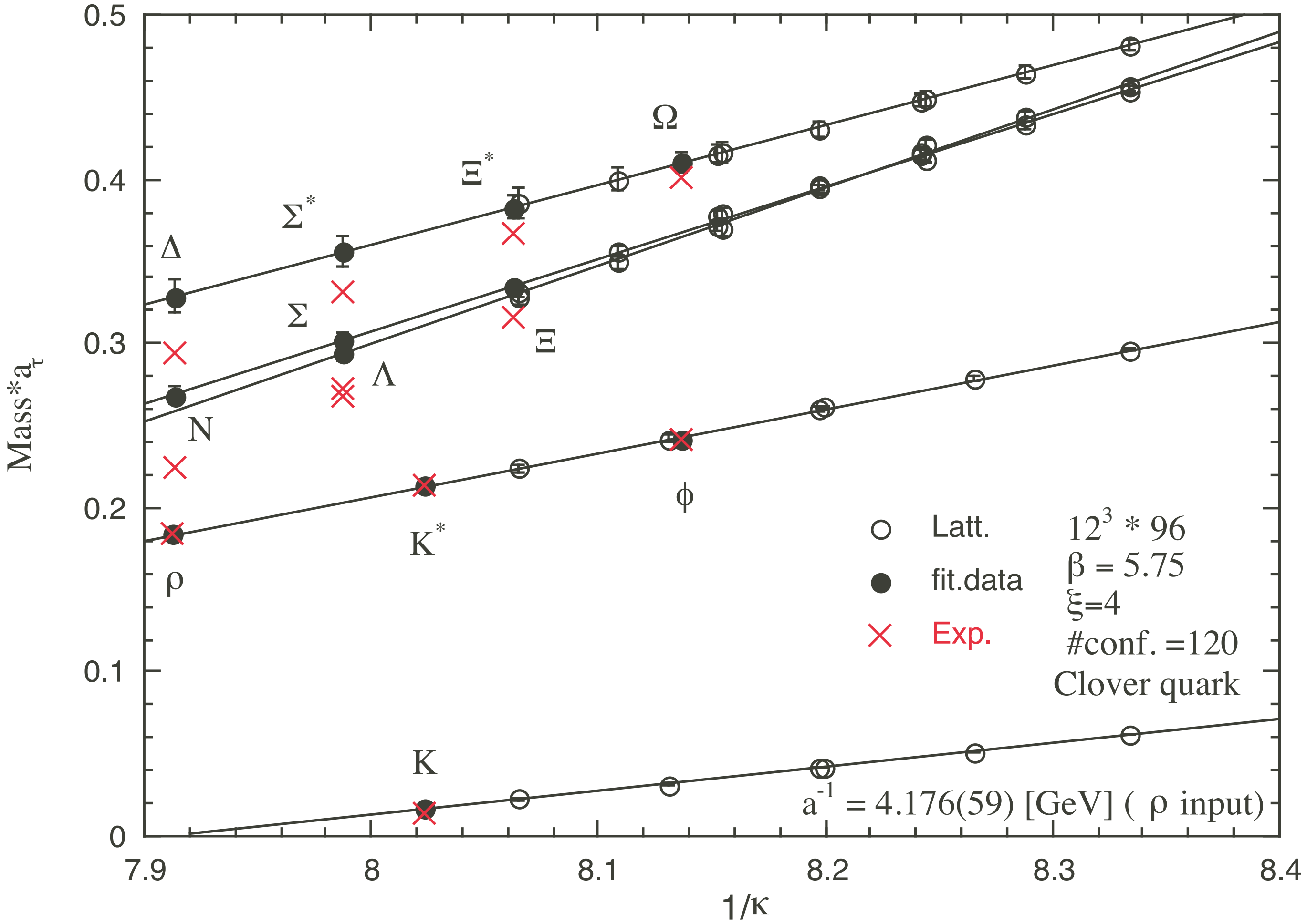
$$m_{\rm B} = a_{\rm B} + b_{\rm B}\,(m_{q1} + m_{q2} + m_{q3})$$



$$m_u = m_d = 0$$

$$m_s \longleftarrow \frac{m_\phi}{m_\rho} \text{ or } m_K$$

Results I



$$\frac{1}{\kappa_c} = 7.913(1)$$

$$a_\tau^{-1} = \frac{m_{\rho,Exp.}}{m_{\rho,Latt.}} = 4.176(59) \text{ GeV}$$

s-quark ⟵ $\frac{m_\phi}{m_\rho}$

$$\frac{1}{\kappa_s} = 8.137(7)$$

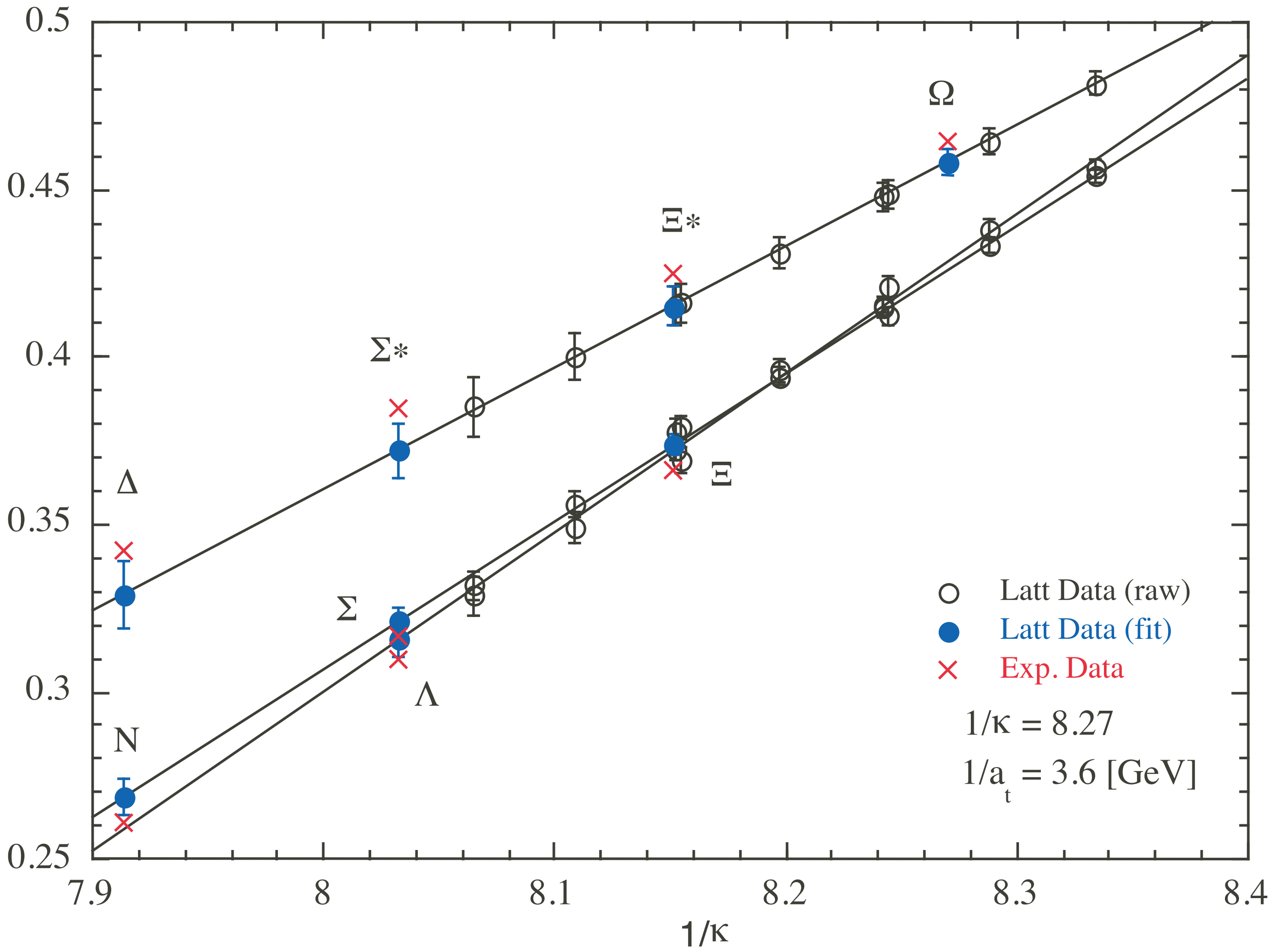
$$\frac{1}{\kappa} = \frac{1}{2}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right]$$

Meson

$$\frac{1}{\kappa} = \frac{1}{3}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2} + \frac{1}{\kappa_3}\right]$$

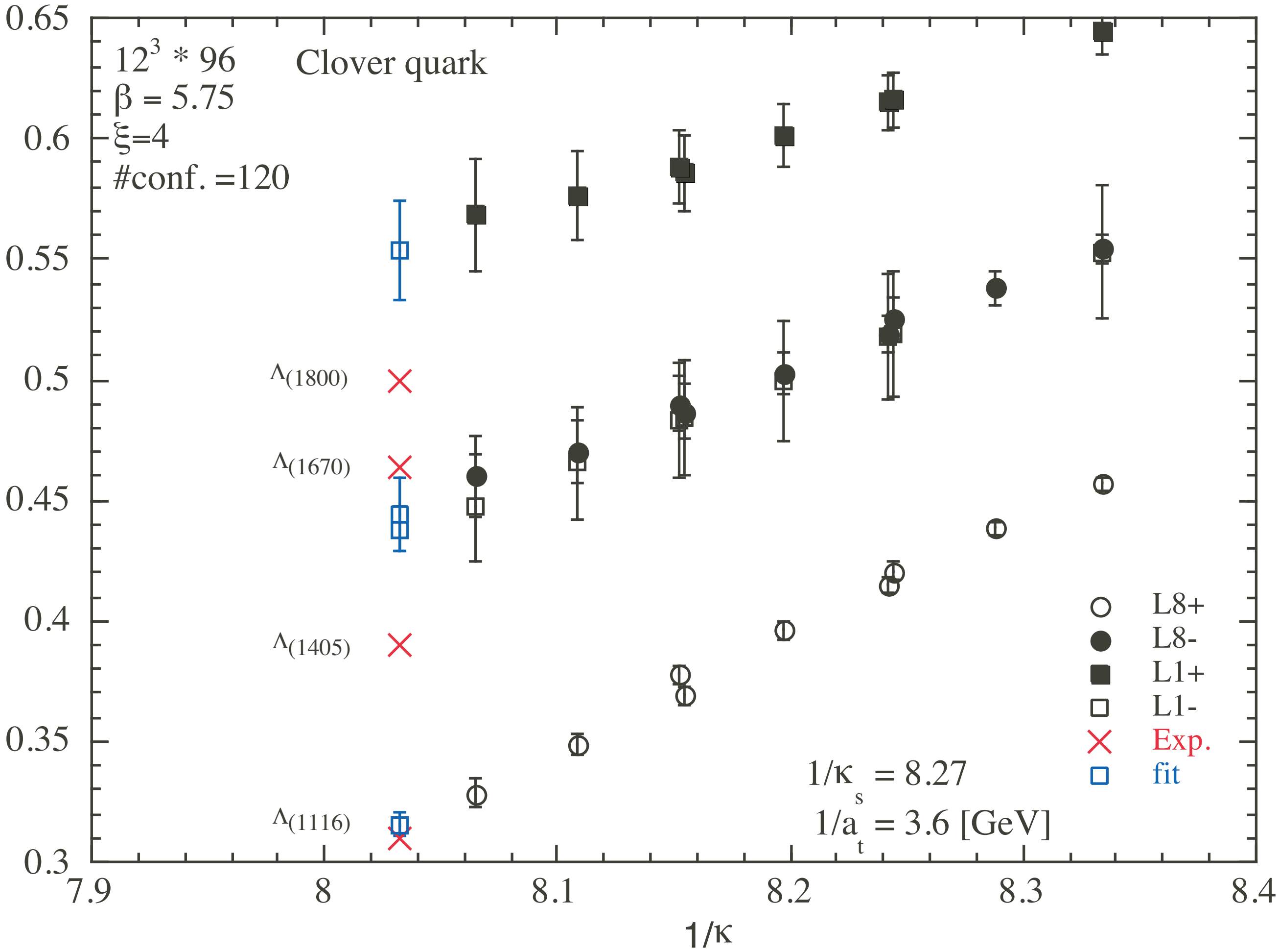
Baryon

Results II



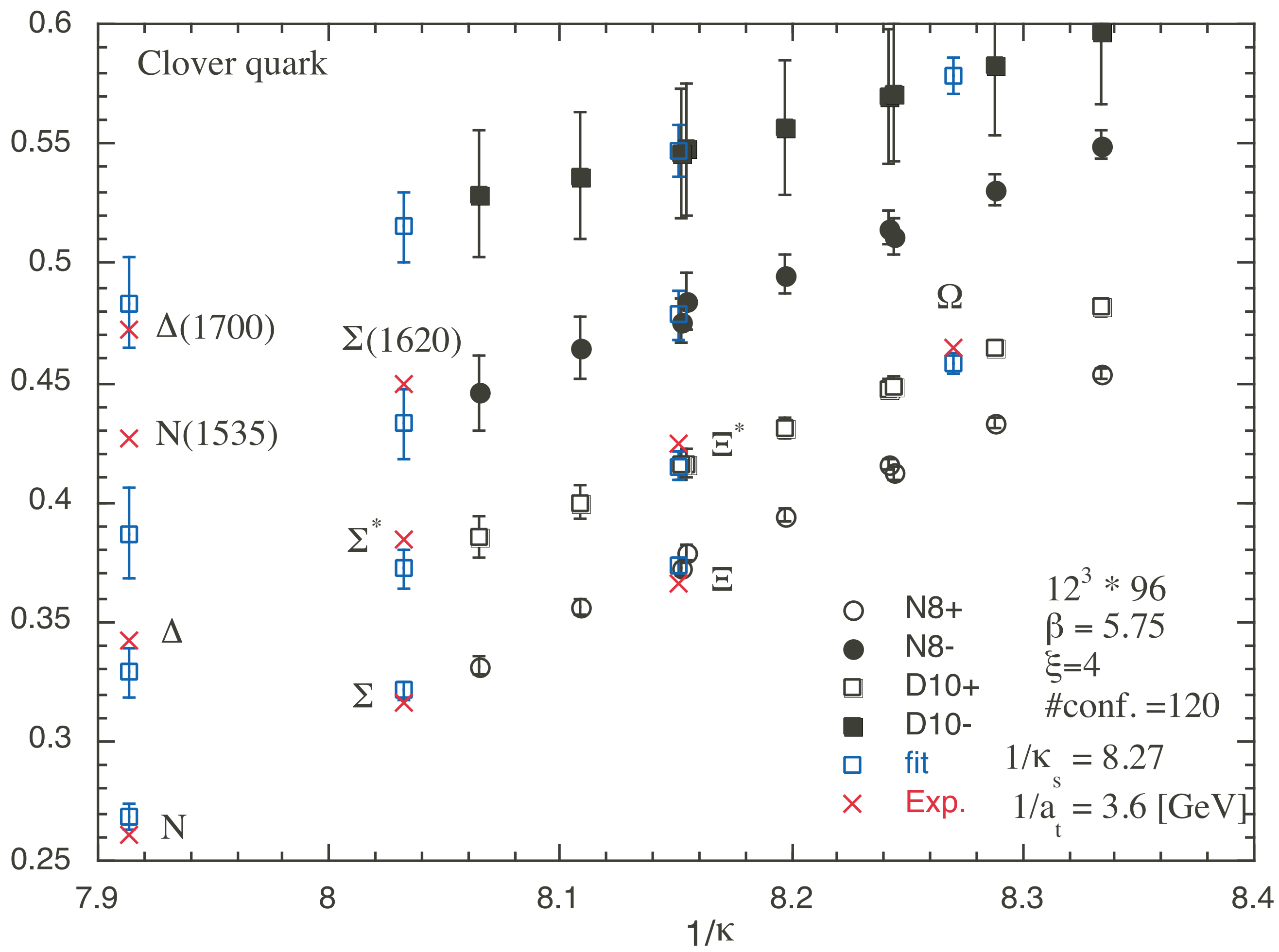
0.5
0.45
0.4
0.35
0.3
0.25
7.9
8
8.1
8.2
8.3
8.4
1/κ
Ω
Ξ*
Σ*
Δ
Ξ
Σ
Λ
N
Latt Data (raw)
Latt Data (fit)
Exp. Data
1/κ = 8.27
1/a_t = 3.6 [GeV]

Results III



0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
7.9
8
8.1
8.2
8.3
8.4
1/κ
12^3 * 96
β = 5.75
ξ=4
#conf. =120
Clover quark
Λ(1800)
Λ(1670)
Λ(1405)
Λ(1116)
L8+
L8-
L1+
L1-
Exp.
fit
1/κs = 8.27
1/at = 3.6 [GeV]

Results IV



Clover quark
Δ(1700)
N(1535)
Δ
N
Σ(1620)
Σ*
Σ
Ξ*
Ξ
Ω
N8+
N8-
D10+
D10-
fit
Exp.
12^3 * 96
β = 5.75
ξ=4
#conf. =120
1/κs = 8.27
1/at = 3.6 [GeV]
1/κ
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
7.9
8
8.1
8.2
8.3
8.4

Summary & Outlook

- Anisotropic Lattice
  - $\mathcal{O}(a)$ - improved quark action

  ——▷ ξ , β dependence

- Light Hadron Spectrum
  - Excited Hadron (Negative Parity Baryon)

  ——▷ • high β

  ——▷ • another baryon operator

  ——▷ • 5 quark system

  - - - -▷ • full QCD

- □ Heavy Quark System ◀—— in progress
- □ Finite Temperature ◀—— in progress