Static Three Quark Potential in the Quenched Lattice QCD

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Contents:

Introduction (1)

Static potential between quarks inside hadrons

∘ Q-$\bar{Q}$ system (∼ meson)
- Well described with short range Coulomb term
  + long range confining linear term.

$$V(r) \simeq -\frac{A}{r} + \sigma r$$

String tension: $\sigma \simeq 1$ GeV/fm

∘ 3Q system (∼ baryon)
Short range: two body Coulomb potential
Long range - string picture

(“Y-type” ansatz)

$$V_{3Q} = \sum_{(i,j)} V_{ij} + V_{123}$$

(“Δ-type” ansatz)

$$V_{3Q} = \sum_{(i,j)} V'_{ij}$$

Introduction (2)

□ Flux tube picture prefers Y-type.

∘ L_{min} (minimum length of Y-type string) $\simeq \frac{1}{2}\, l_{\Delta}$ $\;(\frac{1}{2} l_{\Delta} \le l_{min} \le \frac{1}{\sqrt{3}} l_{\Delta})$

⇒classically Y-type configuration is more stable.

∘ Regge Pole analysis

→Universality of string tension: $\sigma_{Q\bar{Q}} \simeq \sigma_{3Q}$

∘ Strong coupling expansion

∘ Quark Models (Nonrelativistic/Relativistic)

e.g. *Capstick and Isgur, PRD34(1986)2809:*

$$H = \sum_i \sqrt{\vec{p}_i^2 + m^2} - \sum_{(ij)} (V_{ij}^{Coulomb} + V_{ij}^{spin}) + \sigma L_{min}$$

well describes baryon spectra.

Δ-type ansatz with $\sigma_{\Delta} \simeq \sigma/2$ also acceptable.

□ Lattice results

(1) *Sommer and Wosiek, PLB149(1984)497, NPB267(1986)531*

(2) *Thacker, Eichten and Sexton, in Lattice'87*

(3) *Kammesberger et al.*, in Proc. of "Few-Body Problems in Particle, Nuclear, Atomic and Molecular Physics" (1987) 529

Without smearing technique (ground satate enhancement)

Not sufficiently large time separation

(4) *Bali, hep-lat/0001312*

(1),(2) and (4) claim that their data support Δ-type ansatz.

However, their data also seem to be consistent with Y-type ansatz.

— Still not conclusive.

Our Goals

(1) Extract 3 quark potential from lattice QCD simulation.

- ∘ Determine the parameters A and σ nonperturbatively.
- ∘ discuss which of Δ and Y type is appropriate picture.

(2) Apply obtained result to phenomenological study.

- ∘ 3-body potential →effective one-body potential
- ∘ Model calculation →baryon spectroscopy

(3) $T > 0$ study

  – Near T_c, in confined/deconfined phase

- ∘ Universality of $\sigma_{Q\bar{Q}}$ and σ_{3Q} ?
- ∘ Baryon spectra at $T > 0$ (model calculation)
- ∘ Comparison with lattice calculation (c.f. Umeda's talk)

Lattice Simulation (1): Baryonic Wilson loop

Static potential (potential between infinitely heavy quarks)

∘ Q-$\bar{Q}$ potential ← Wilson loop

$W(r,t) \equiv$ Tr

$$\longrightarrow c \cdot \exp\left[-V_{Q\bar{Q}}(r)t\right] \qquad \text{as } t \to \infty$$

∘ $3Q$ potential ← Baryonic Wilson loop:

$W(\vec{r}_1, \vec{r}_2, \vec{r}_3, t) \equiv \frac{1}{3!}\epsilon_{abc}\epsilon_{a'b'c'}\ U_1^{aa'} U_2^{bb'} U_3^{cc'}$

$=$

$$\longrightarrow c' \cdot \exp\left[-V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3)t\right] \qquad \text{as } t \to \infty$$

In this limit, spatial path of links to connect three quark positions is unimportant.

Lattice Simulation (2): Simulation Parameters

□ Lattice

With standard Wilson action, in quenched approximation

∘ $12^3 \times 24$, $\beta = 5.7$ (#conf = 210)

$a^{-1} \simeq 1.0 GeV$ (from static potential)

∘ $16^3 \times 32$, $\beta = 6.0$ (#conf = 60, preliminary)

$a^{-1} \simeq 2.0 GeV$

Smearing – enhance the ground state contribution

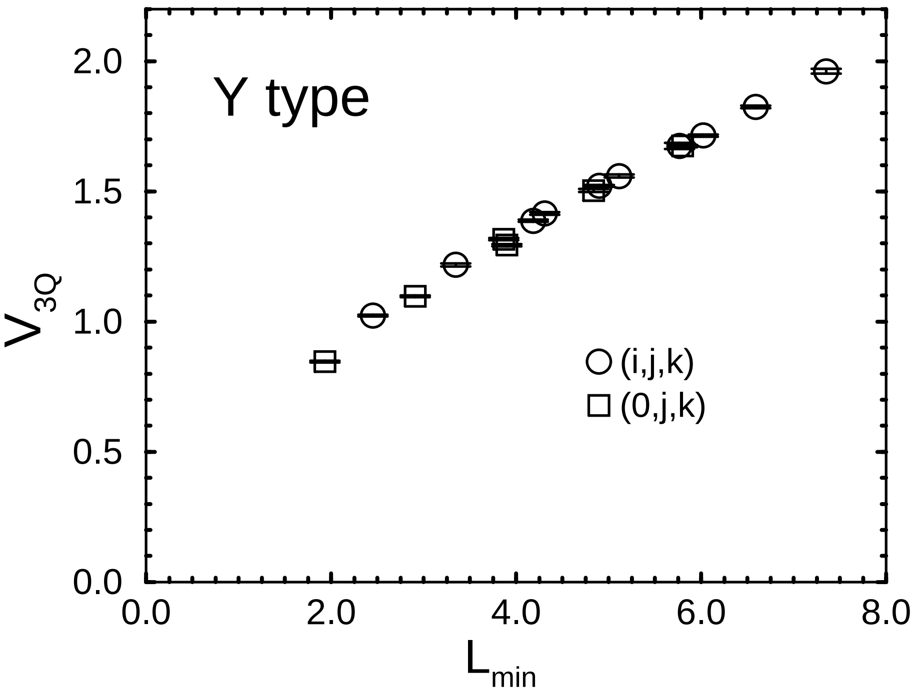
Iterate

— incorporate long range effect.

$\alpha = 2.3$, 20 sweeps for Q-$\bar{Q}$ / 12 sweeps for 3Q ($\beta = 5.7$)

Position of 3 quarks:

Lattice Simulation (3): 3Q Potential



Smearing: 12 sweeps

Fit:

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \text{const.} - A\left(\frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{31}}\right) + \sigma l_{min}$$

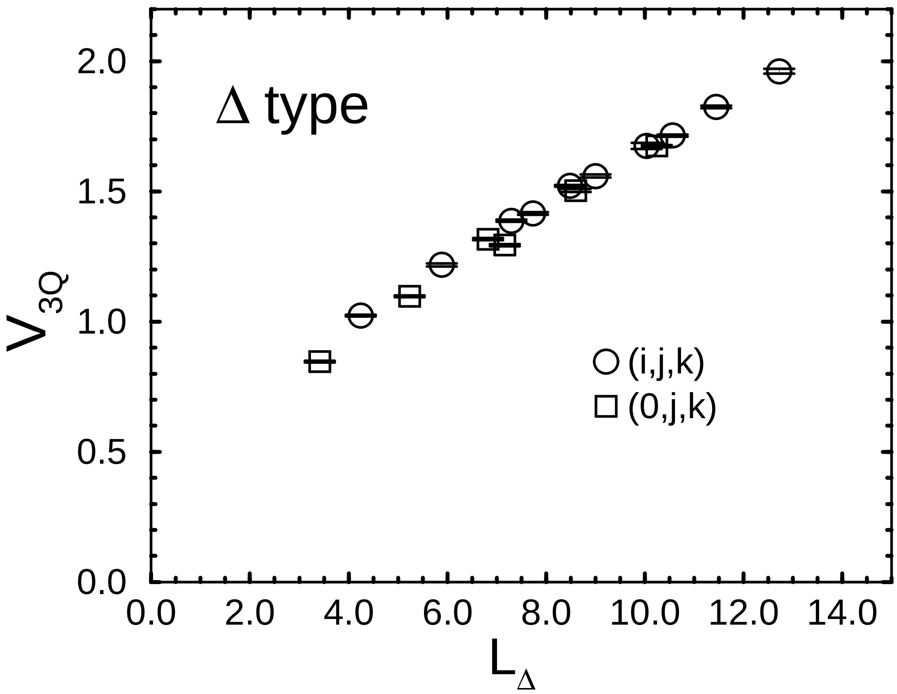
Fit range	const.	A	σ	χ^2/N_{dof}
all r	0.9140(20)	0.1316(62)	0.1528(20)	51.9/13
c.f. Q-$\bar{Q}$ $(r \geq 2)$	0.696(24)	0.395(35)	0.1561(38)	60.0/18

Lattice cutoff: $a^{-1} \simeq 1.$ GeV

Well described by Y-type form

$\sigma_{3Q} \simeq \sigma_{Q\bar{Q}}$: universality of string tension holds.

Lattice Simulation (4): Δ type ansatz



Smearing: 12 sweeps

Fit:

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \text{const.} - A\left(\frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{31}}\right) + \sigma(r_{12} + r_{23} + r_{31})$$

Fit range	const.	A	σ	χ^2/N_{dof}
all r	0.934(20)	0.1405(60)	0.0858(15)	142./13
c.f. Q-$\bar{Q}$ $(r \geq 2)$	0.696(24)	0.395(35)	0.1561(38)	60.0/18

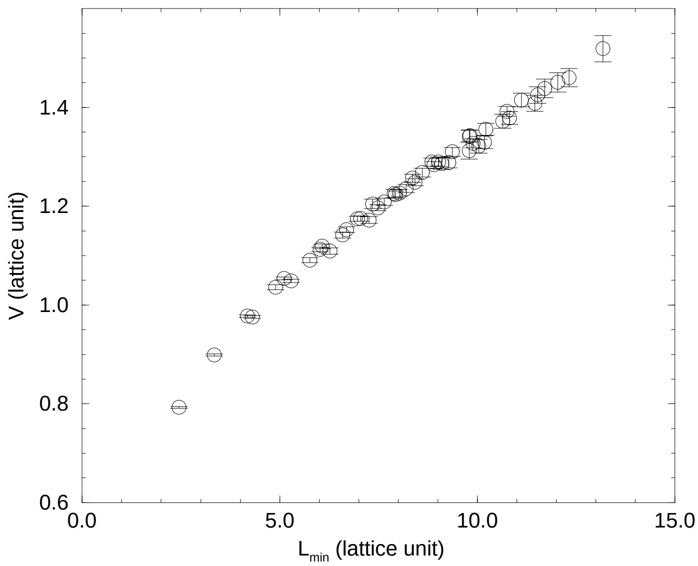
Lattice cutoff: $a^{-1} \simeq 1.$ GeV

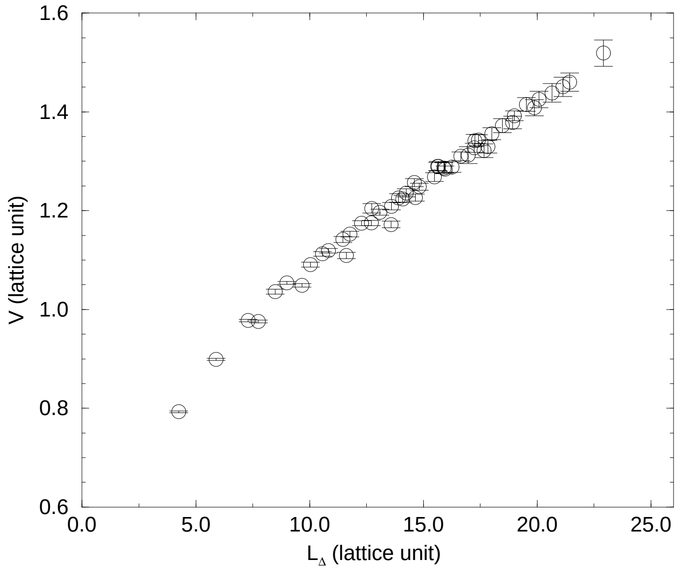
Fit seems not so bad.

$\sigma_\Delta \simeq \frac{1}{2}\sigma_{Q\bar{Q}}$ — No clear explanation.
(color factor like as Coulomb term ?)

∘ χ^2 is much larger than Y-type fit.

Lattice Simulation (5): Preliminary result at $\beta = 6.0$





Smearing: 42 sweeps $(3Q)$ / 36 sweeps $(Q\bar{Q})$

Fit	const.	A	σ	χ^2/N_{dof}
Y	0.933(12)	0.1203(46)	0.0475(14)	54.1/46
Δ	0.932(12)	0.1195(45)	0.02685(75)	25.9/46
Q-$\bar{Q}$ (on-axis)	0.6247(63)	0.2674(50)	0.0532(16)	0.51/3

Lattice cutoff: $a^{-1} \simeq 2.$ GeV

Lattice Simulation (6): Summary and Outlook

We calculated the three quark potential with the baryonic Wilson loop, making use of smearing technique.

- Which of Y- and Δ-type is appropriate description ?
□ Fit result at $\beta = 5.7$:

χ^2 (Y-type)$/N_{dof} = 51.9/13 = 3.99$

χ^2 (Δ-type)$/N_{dof} = 142./13 = 10.9$

χ^2 (Q-$\bar{Q}$; $r > 2$)$/N_{dof} = 60.0/18 = 3.33$ (→guide of finite a effect)

Y-type is preferable, but Δ-type is not strongly excluded.

□ String tension: $2\sigma(\Delta) \simeq \sigma(Y) \simeq \sigma_{Q\bar{Q}}$
– In the fit, this is because geometrically $l_{min} \simeq \frac{1}{2} l_\Delta$ roughly holds in most cases. $(\frac{1}{2} l_\Delta \leq l_{min} \leq \frac{1}{\sqrt{3}} l_\Delta)$

String: nonperturbative picture
— Physical reason of $\frac{1}{2}$ is unclear.

Our conclusion: Y-type ansatz is better for numerical result, although Δ-type ansatz is not completely excluded yet.
⇒Simulation at larger β ($\beta = 6.0$ is in progress)

Outlook

- Precise results / extrapolation to the continuum limit
- 4 quark system $(QQ\bar{Q}\bar{Q})$
  – Difference of Y-type and Δ-type ansatz are more clear.
- Excited states of string ←Variational analysis
- With dynamical quarks – String breaking effect

At Finite Temperature (1): Polyakov Loop Correlators

Finite temperature lattice QCD

We use anisotropic lattice:

$a_\sigma > a_\tau$ ($\xi \equiv a_\sigma/a_\tau$: anisotropy)

$N_\sigma a_\sigma \gg N_\tau a_\tau = 1/T$

Polyakov loop

$$P(\vec{x}) = \mathrm{Tr} \prod_{t=0}^{N_\tau - 1} U_4(\vec{x})$$

– $\langle P \rangle$ is the order parameter of deconfining transition

Static quark potentials

∘ Q-$\bar{Q}$ potential:

$$P_2(\vec{r}) = \langle P(0) P^\dagger(\vec{r}) \rangle \simeq c \cdot \exp(-V_{Q\bar{Q}}(\vec{r}) N_\tau)$$

∘ $3Q$ potential:

$$\begin{aligned} P_3(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \langle P(\vec{r}_1) P(\vec{r}_2) P(\vec{r}_3) \rangle \\ &\simeq c' \cdot \exp(-V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3) N_\tau) \end{aligned}$$

At Finite Temperature (2): Simulation

□ *Lattice*

Gauge field action: Anisotropic Symanzik (tree) action

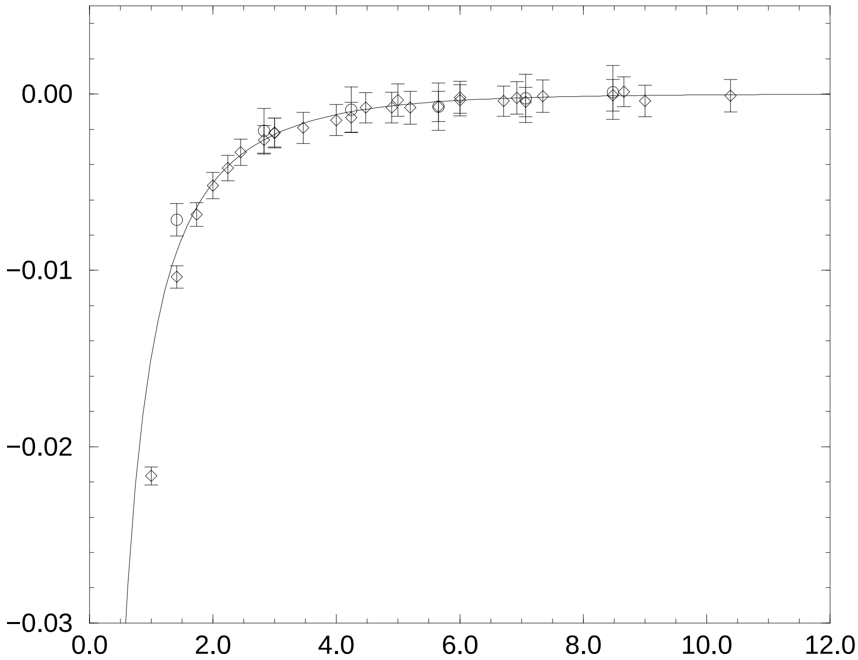
$16^2 \times 24 \times N_t$, $\beta = 4.56$, $\gamma = 3.45$, quenched

- Cutoff: a_σ^{-1}=1.61(1) GeV ($a_\sigma \sim 0.125$ fm),
  a_τ^{-1}=6.36(5) GeV
- Anisotropy: $\xi \equiv a_s/a_t = 3.95(2)$
  (from the ratio of Wilson loops)
  [Engels, Karsch and Scheideler (1997), Klassen (1998)]

$N_t = 96$ ($T \simeq 0$), 28, 26 ($T < T_c$), 20 ($T > T_c$) : $T = 1/N_t a_t$

N_t	T	#conf	observable	$\sqrt{\sigma_{Q\bar{Q}}}$ [MeV]
96	$\simeq 0$	120	Wilson loop	420
28	$0.87T_c$	440	Polyakov loop	$\sim$ 300
26	$0.93T_c$	450	Polyakov loop	$\sim$ 260
20	$1.22T_c$	60	Polyakov loop	-

At Finite Temperature (3): Above T_c



- Q-$\bar{Q}$ system — Debye screening

  $V_{Q\bar{Q}} = V_{singlet} + V_{octet}$

  $\rightarrow c \cdot \exp(-mr)/r$: Yukawa potential

  [Gao, PRD41(1990)626]

  $m \sim$ screening mass $\sim$ 0.5-1 GeV

- $3Q$ system:

  — Sum of two-quark (Q-Q) potential ?

Well fitted to the form

$$V_{3Q} = \sum c \frac{\exp(-mr_{ij})}{r_{ij}}.$$

$V_{3Q}/3$ vs r_{ij} for equilateral triangle configuration

$\rightarrow$Almost same behavior as Q-$\bar{Q}$ system.

At Finite Temperature (4): Summary

□ At $T > T_c$

We calculated the static three quark potential above T_c
with equilateral constellations.

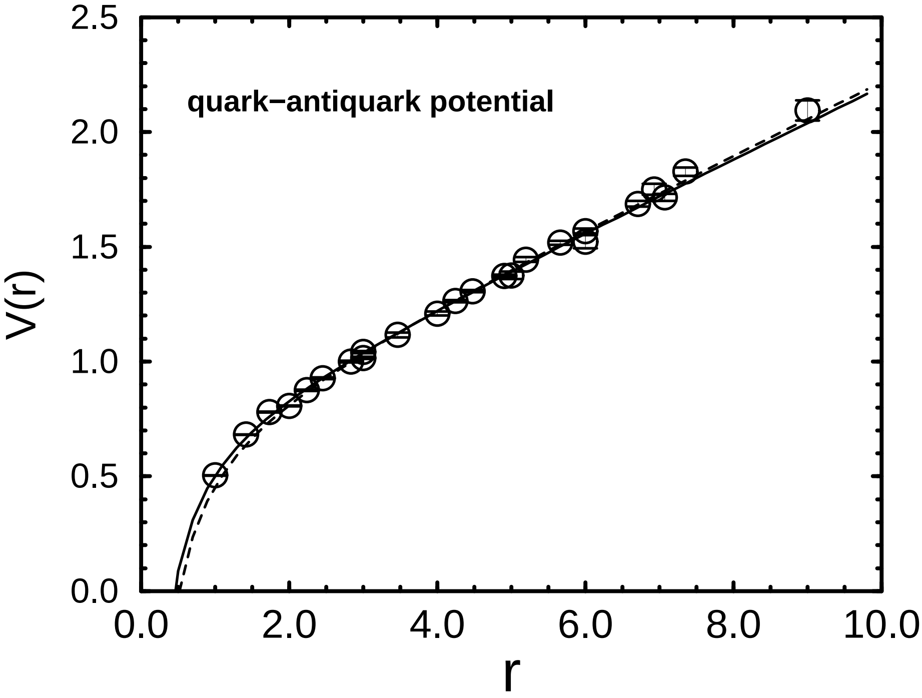
∘ Debye screening observed.
∘ $V_{3Q}/3$ shows almost same behavior as $Q\bar{Q}$ potential.

□ At $T < T_c$

$3Q$ potential from Polyakov loop correlator suffer from large statistical fluctuations.

— More study is needed.

$Appendix\ (1){:}\ Q\text{-}\bar{Q}\ potential\ (T=0)$



Smearing: 20 sweeps

Fit:

$$V(r) = \text{const.} - \frac{A}{r} + \sigma r$$

Fit range	const.	A	σ	χ^2/N_{dof}
on-axis	0.629 (16)	0.2793(12)	0.1629(47)	1.77/ 3
all r	0.6859(71)	0.3351(55)	0.1545(20)	341./ 21
$r \geq 2$	0.696 (24)	0.395(35)	0.1561(38)	60.0/18 ~ 3

Lattice cutoff: $a^{-1} \simeq 1.$ GeV

◦ Large χ^2 signals large $O(a^2)$ discretization effect
(Rotationally symmetry is broken)