Lattice QCD Study of Three Quark Potential

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Contents:

Introduction (1)

Static potential between quarks inside hadrons

∘ Q-$\bar{Q}$ system (∼ meson)
- Well described with short range Coulomb term
  + long range confining linear term.

$$V(r) \simeq -\frac{\alpha}{r} + \sigma r$$

String tension: $\sigma \simeq 1$ GeV/fm

∘ 3Q system (∼ baryon)
Short range: two body Coulomb potential
Long range - string picture

("Y-type" ansatz)

$$V_{3Q} = \sum_{(i,j)} V_{ij} + V_{123}$$

("Δ-type" ansatz)

$$V_{3Q} = \sum_{(i,j)} V'_{ij}$$

Introduction (2)

□ Flux tube picture prefers Y-type.

∘ l_{min} (minimum length of Y-type string) $\simeq \frac{1}{2} l_\Delta$ $(\frac{1}{2}l_\Delta \leq l_{min} \leq \frac{1}{\sqrt{3}}l_\Delta)$
$\Rightarrow$classically Y-type configuration is more stable.

∘ Regge Pole analysis
$\rightarrow$Universality of string tension: $\sigma_{Q\bar{Q}} \simeq \sigma_{3Q}$

∘ Strong coupling expansion

∘ Quark Models (Nonrelativistic/Relativistic)
e.g. *Capstick and Isgur, PRD34(1986)2809:*

$$H = \sum_i \sqrt{\vec{p}_i^2 + m^2} - \sum_{(ij)} (V_{ij}^{Coulomb} + V_{ij}^{spin}) + \sigma l_{min}$$

well describes baryon spectra.
Δ-type ansatz with $\sigma_\Delta \simeq \sigma/2$ also acceptable.

∘ Dual Ginzburg-Landau Theory (c.f. Koma's talk)
Δ-type flux tube corresponds to glueball.

□ Lattice results – support Δ-type ?

- *Sommer and Wosiek, PLB149(1984)497, NPB267(1986)531*
- *Thacker, Eichten and Sexton, in Lattice'87*
  Without smearing technique (ground satate enhancement)
  Not sufficiently latge time separation
- *Bali, hep-lat/0001312*

They claim that their data support Δ-type ansatz.
However, their data also seem to be consistent
with Y-type ansatz.

— Still not conclusive.

Our Goals

(1) Extract 3 quark potential from lattice QCD simulation.

- Determine the parameters α and σ nonperturbatively.
- discuss which of Δ and Y type is appropriate picture.

(2) Apply obtained result to phenomenological study.

- 3-body potential $\rightarrow$effective one-body potential
- Model calculation $\rightarrow$baryon spectroscopy

(3) $T > 0$ study

  – Near T_c, in confined/deconfined phase

- Universality of $\sigma_{Q\bar{Q}}$ and σ_{3Q} ?
- Baryon spectra at $T > 0$ (model calculation)
- Comparison with lattice calculation (c.f. Umeda's talk)

Lattice Simulation (1): Baryonic Wilson loop

Static potential (potential between infinitely heavy quarks)

∘ Q-$\bar{Q}$ potential ← Wilson loop

$W(r,t) \equiv$ Tr

$$\longrightarrow c \cdot \exp\left[-V_{Q\bar{Q}}(r)t\right] \qquad \text{as } t \to \infty$$

∘ $3Q$ potential ← Baryonic Wilson loop:

$W(\vec{r}_1, \vec{r}_2, \vec{r}_3, t) \equiv \frac{1}{3!}\epsilon_{abc}\epsilon_{a'b'c'}\ U_1^{aa'}U_2^{bb'}U_3^{cc'}$

$=$

$$\longrightarrow c' \cdot \exp\left[-V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3)t\right] \qquad \text{as } t \to \infty$$

In this limit, spatial path of links to connect three quark positions is unimportant.

Lattice Simulation (2): Simulation Parameters

□ Lattice

Action: standard Wilson action

$12^3 \times 24$, $\beta = 5.7$ in quenched approximation

$a^{-1} = 1.0 GeV$ (from static potential)

#conf = 210

Smearing – enhance the ground state contribution

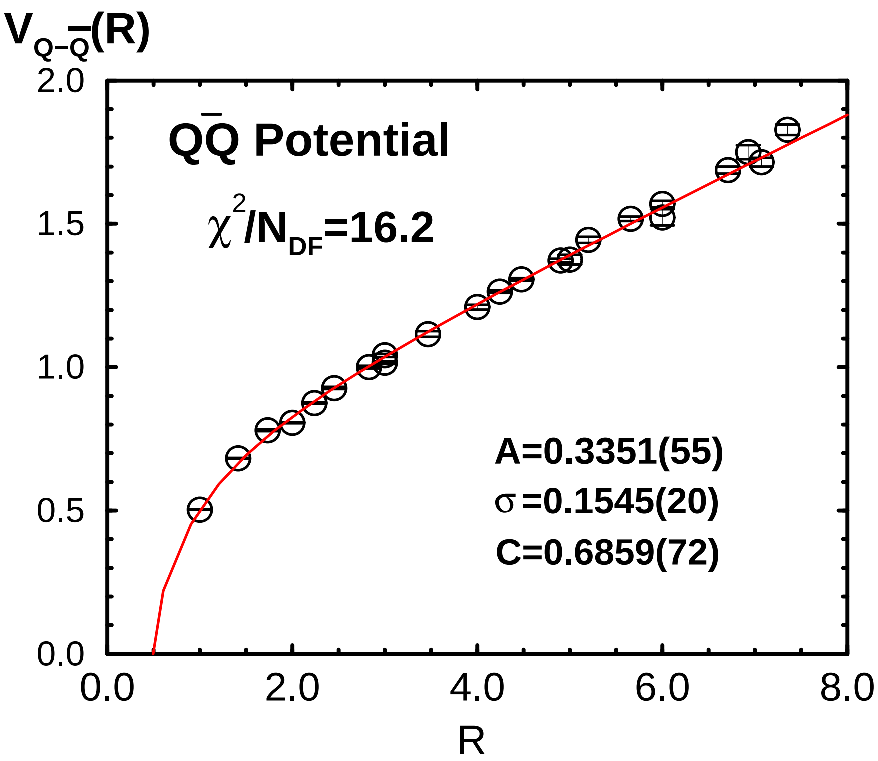
Iterate

— incorporate long range effect.

$\alpha = 2.3$, 20 sweeps for Q-$\bar{Q}$ / 12 sweeps for $3Q$

Position of 3 quarks:

Lattice Simulation (3): Q-$\bar{Q}$ potential



Smearing: 20 sweeps

Fit:

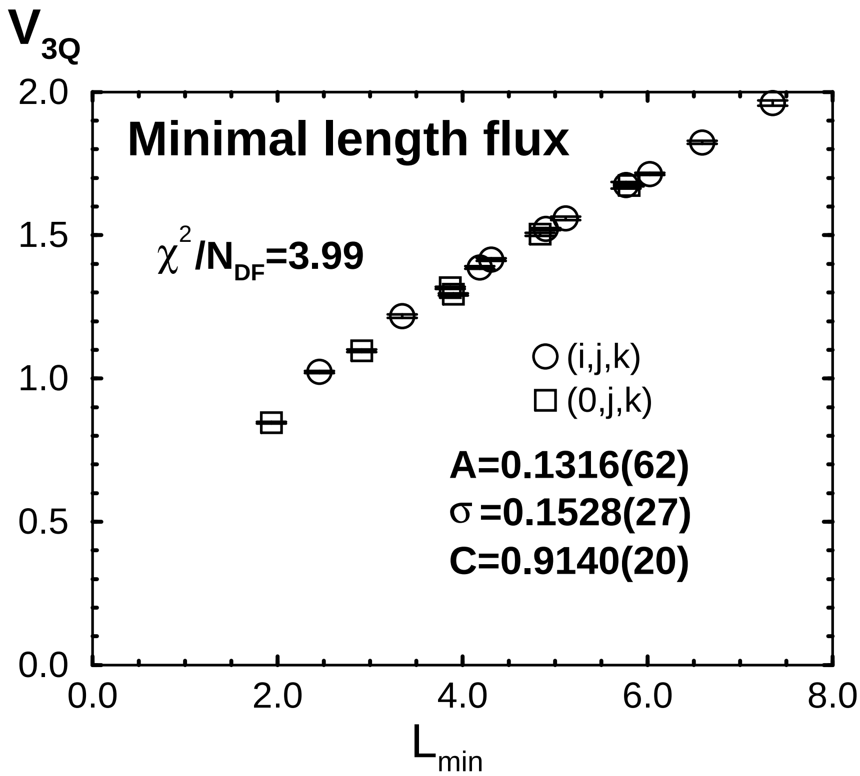
$$V(r) = \text{const.} - \frac{\alpha}{r} + \sigma r$$

Fit range	const.	α	σ	χ^2/N_{dof}
all r	0.6859(71)	0.3351(55)	0.1545(20)	341./ 21
$r \geq 2$	0.696(24)	0.395(35)	0.1561(38)	60.0/18 ~ 3

Lattice cutoff: $a^{-1} \simeq 1.$ GeV

∘ Large χ^2 signals large $O(a^2)$ discretization effect
(Rotationally symmetry is broken)

Lattice Simulation (4): 3Q Potential



Smearing: 12 sweeps

Fit:

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \text{const.} - \frac{\alpha}{2}\left(\frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{31}}\right) + \sigma l_{min}$$

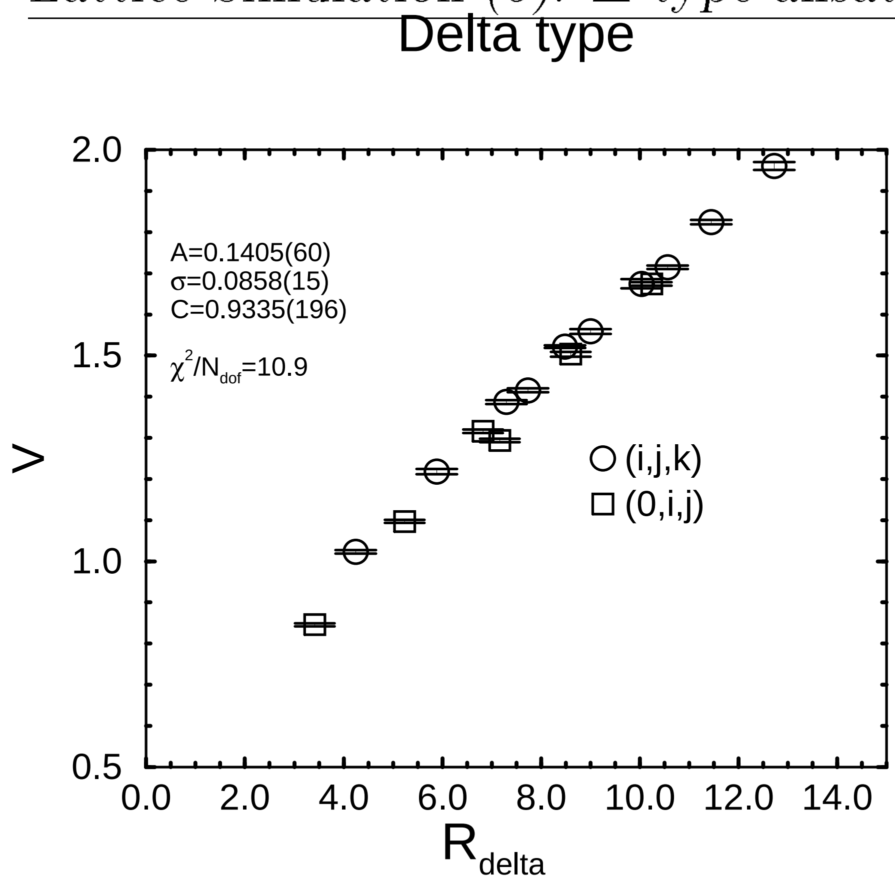
Fit range	const.	α	σ	χ^2/N_{dof}
all r	0.9140(20)	0.263(12)	0.1528(20)	51.9/13
c.f. Q-$\bar{Q}$ $(r \geq 2)$	0.696(24)	0.395(35)	0.1561(38)	60.0/18

Lattice cutoff: $a^{-1} \simeq 1.$ GeV

Well described by Y-type form

$\sigma_{3Q} \simeq \sigma_{Q\bar{Q}}$: universality of string tension holds.

Lattice Simulation (5): Δ type ansatz



Smearing: 12 sweeps

Fit:

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \mathrm{const.} - \frac{\alpha}{2}\left(\frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{31}}\right) + \sigma(r_{12} + r_{23} + r_{31})$$

Fit range	const.	α	σ	χ^2/N_{dof}
all r	0.934(20)	0.281(12)	0.0858(15)	142./13
c.f. Q-$\bar{Q}$ $(r \geq 2)$	0.696(24)	0.395(35)	0.1561(38)	60.0/18

Lattice cutoff: $a^{-1} \simeq 1.$ GeV

Fit seems not so bad.

$\sigma_\Delta \simeq \frac{1}{2}\sigma_{Q\bar{Q}}$ — No clear explanation.

(color factor like as Coulomb term ?)

∘ χ^2 is much larger than Y-type fit.

Lattice Simulation (6): Discussion

Which of Y- and Δ-type is appropriate description ?

□ Fit:

$$\chi^2\ (Y\text{-type})/N_{dof} = 51.9/13 = 3.99$$
$$\chi^2\ (\Delta\text{-type})/N_{dof} = 142./13 = 10.9$$

wihch are compared with

$$\chi^2\ (Q\text{-}\bar{Q};\, r>2)/N_{dof} = 60.0/18 = 3.33$$

$\rightarrow$signals typical size of finite a effect

Y-type is preferable, but Δ-type is not strongly excluded.

□ String tension:

$$\sigma(\Delta) \simeq \frac{1}{2}\sigma(Y) \simeq \frac{1}{2}\sigma_{Q\bar{Q}}$$

– In the fit, this is because geometrically $l_{min} \simeq \frac{1}{2} l_\Delta$ roughly holds in most cases. $(\frac{1}{2} l_\Delta \leq l_{min} \leq \frac{1}{\sqrt{3}} l_\Delta)$

- Coulomb term: Factor $\frac{1}{2}$ comes from the color factor in perturbation (one gluon exchange).
- String: nonperturbative picture
  - – No clear physical explanation of origin of $\frac{1}{2}$.

Our calculation:

Y-type ansatz is better for numerical result and physical reason, although Δ-type ansatz is not completely excluded yet.

Lattice Simulation (7): Outlook

To distinguish the $Y-$ and Δ-type ansatz clearly,
following analyses will work:

- ◦ 4 quark system ($QQ\bar{Q}\bar{Q}$)
  - – Difference of Y-type and Δ-type ansatz are more clear.
- ◦ Scaling analysis by varying β ($\leftrightarrow a$), or using improved action.
  - – Define the quantity which signals the deviation from the fitting form and compare it at various a.

For physical applications,

- ◦ Precise results / extrapolation to the continuum limit
- ◦ Excited states of string
  - $\longleftarrow$Variational analysis
- ◦ With dynamical quarks
  - – String breaking effect (c.f. Pennanen's talk)
- • At finite temperature $\rightarrow$ next part.

At Finite Temperature (1): Polyakov Loop Correlators

Finite temperature lattice QCD

We use anisotropic lattice:

$a_\sigma > a_\tau$ ($\xi \equiv a_\sigma / a_\tau$: anisotropy)

$N_\sigma a_\sigma \gg N_\tau a_\tau = 1/T$

Polyakov loop

$$P(\vec{x}) = \mathrm{Tr} \prod_{t=0}^{N_\tau - 1} U_4(\vec{x})$$

– $\langle P \rangle$ is the order parameter of deconfining transition

Static quark potentials

∘ Q-$\bar{Q}$ potential:

$$P_2(\vec{r}) = \langle P(0) P^\dagger(\vec{r}) \rangle \simeq c \cdot \exp(-V_{Q\bar{Q}}(\vec{r}) N_\tau)$$

∘ $3Q$ potential:

$$\begin{aligned} P_3(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \langle P(\vec{r}_1) P(\vec{r}_2) P(\vec{r}_3) \rangle \\ &\simeq c' \cdot \exp(-V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3) N_\tau) \end{aligned}$$

At Finite Temperature (2): Simulation

□ *Lattice*

Gauge field action: Anisotropic Symanzik (tree) action

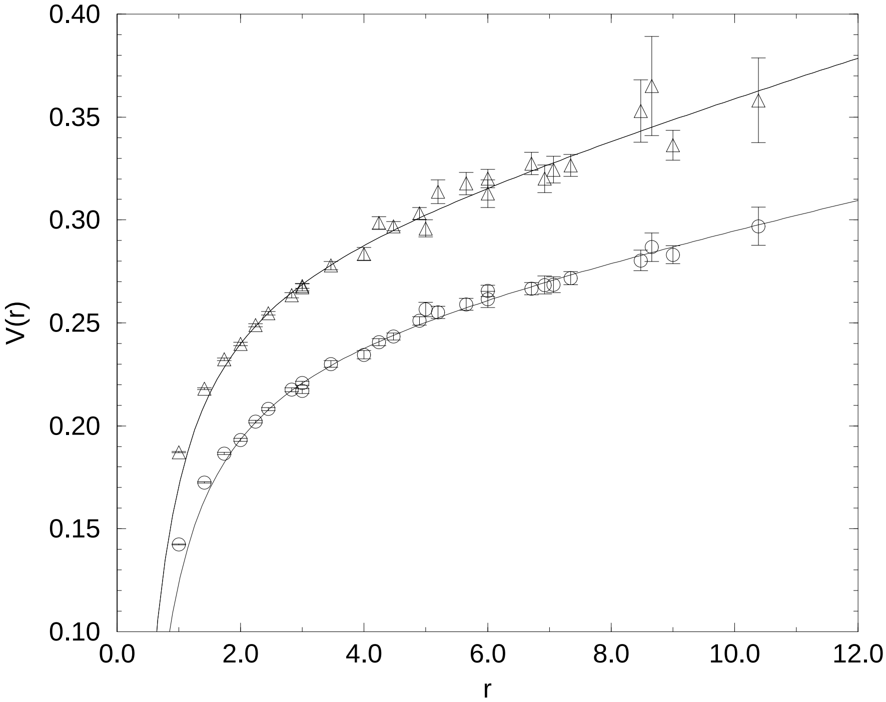
$16^2 \times 24 \times N_t$, $\beta = 4.56$, $\gamma = 3.45$, quenched

- Cutoff: a_σ^{-1}=1.61(1) GeV ($a_\sigma \sim 0.125$ fm),
  a_τ^{-1}=6.36(5) GeV
- Anisotropy: $\xi \equiv a_s/a_t = 3.95(2)$
  (from the ratio of Wilson loops)
  [Engels, Karsch and Scheideler (1997), Klassen (1998)]

$N_t = 96$ ($T \simeq 0$), 28, 26 ($T < T_c$), 20 ($T > T_c$) : $T = 1/N_t a_t$

N_t	T	#conf	observable	$\sqrt{\sigma_{Q\bar{Q}}}$ *[MeV]*
96	$\simeq 0$	120	Wilson loop	420
28	$0.87T_c$	440	Polyakov loop	$\sim$ 300
26	$0.93T_c$	450	Polyakov loop	$\sim$ 260
20	$1.22T_c$	60	Polyakov loop	-

At Finite Temperature (3): Q-$\bar{Q}$ Potential



Fit:

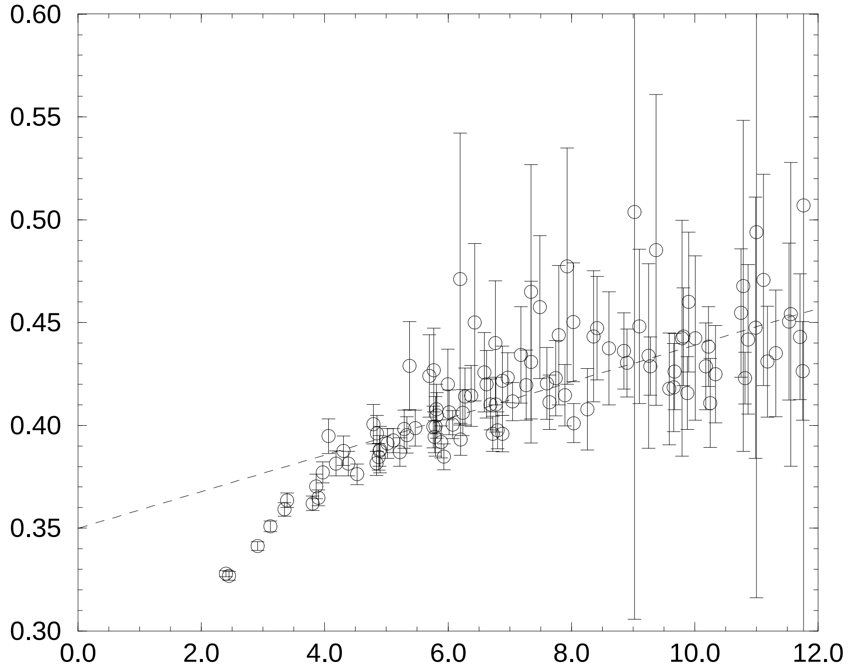
$$V(r) = \text{const.} - \frac{\alpha}{r} + \sigma r$$

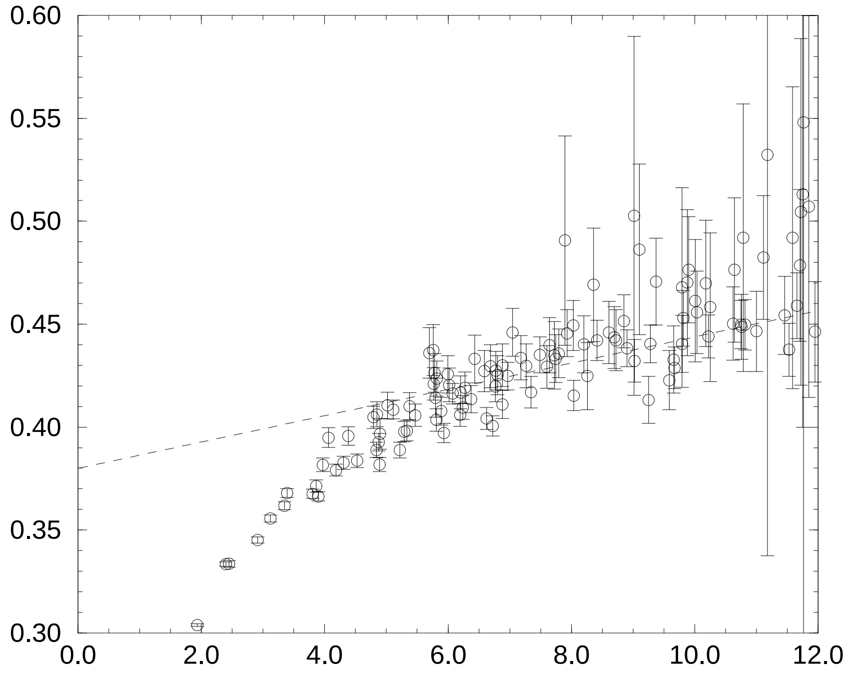
N_τ	T	Fit range	const.	α	σ	χ^2/N_{dof}
96	$r \geq 2.8$	~ 0	0.0587(49)	0.0587(49)	0.01781(29)	78.12/18
28	$r \geq 2$	$0.87T_c$	0.2817(70)	0.119 (11)	0.0089 (11)	25.2/21
26	$r \geq 2$	$0.93T_c$	0.2934(41)	0.1253(63)	0.00638(69)	23.5/21

Lattice cutoffs: $a_\sigma^{-1} \simeq 1.6$ GeV, $a_\tau^{-1} \simeq 6.4$ GeV.

String tension slowly decreases toward T_c.

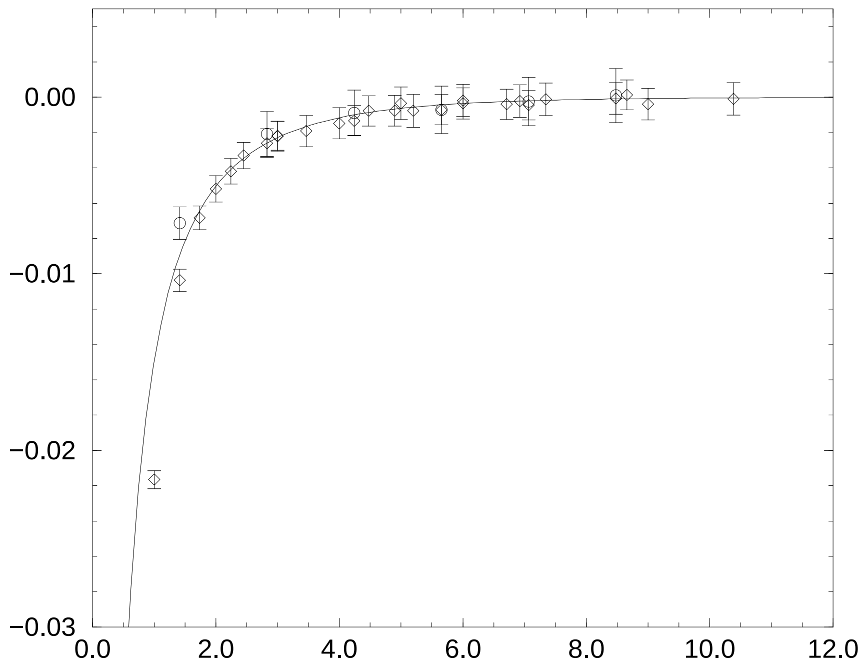
At Finite Temperature (4): 3Q Potential





- ◦ $3Q$ potential from Polyakov loop correlator suffer from large statistical fluctuations.
- ◦ Slope at large distance is consistent with Q-$\bar{Q}$ case.

At Finite Temperature (5): Above T_c



∘ Q-$\bar{Q}$ system — Debye screening

$$V_{Q\bar{Q}} = V_{singlet} + V_{octet}$$
$$\rightarrow c \cdot \exp(-mr)/r \quad : \text{Yukawa potential}$$

[Gao, PRD41(1990)626]

$m \sim$ (electric) screening mass $\sim$ 0.5-1 GeV

∘ $3Q$ system:

— Sum of two-quark (Q-Q) potential ?

Well fitted to the form

$$V_{3Q} = \sum c \frac{\exp(-mr_{ij})}{r_{ij}}.$$

$V_{3Q}/3$ vs r_{ij} for equilateral triangle configuration

$\rightarrow$Almost same behavior as Q-$\bar{Q}$ system.

At Finite Temperature (6): Summary

□ $T > T_c$

∘ Debye screening.

∘ V_{3Q} →sum of two-body potential.

□ $T < T_c$

Three quark potentail from Polyakov loop correlator suffer from large statistical fluctuations.

Need more study !