A Study of Meson Correlators at Finite Temperature

Abstract : We study 4-point mesonic correlators in (euclidean) time and space directions at finite temperature using anisotropic lattices. We investigate meson masses and wave functions, both in the heavy quark and in the light quark mass region (Wilson fermionic action). We also use a clover action as effective action for heavy quarks.

The QCD-TARO Collaboration

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Introduction

Finite temperature hadron properties:

What happens on hadrons or what new effect above T_c ?

Change of masses and width near T_c

Example of effective theory:

NJL model analysis

c.f. Hatsuda and Kunihiro, Phys. Rep. 247 (1994) 221

◦ Chiral restoration
◦ parity partner degeneracy
◦ Above T_c : Soft modes

Our Approach

Temporal correlator ;

$\Longrightarrow$ Pole mass (mass of temporal direction)

$\Updownarrow$

Screening mass (mass of spatial direction)

$\Longrightarrow$ Wave function

Is there bound state at $T > T_c$?

How can we show that ?

$\Longrightarrow$ Spectral function

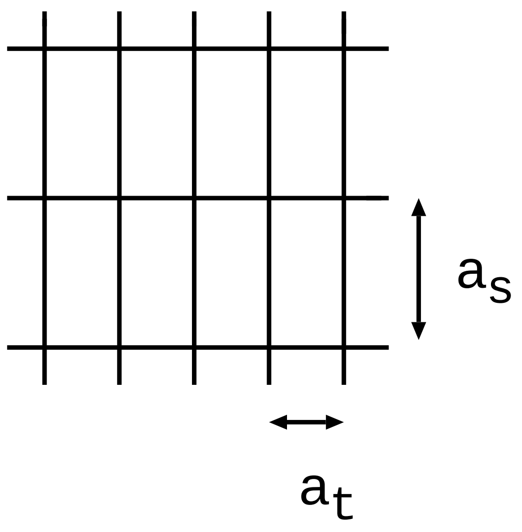
Need detailed information

in temporal(temperature) direction

$\rightarrow$ Anisotropic Lattice

Karsch ('82)

Burgers, Karsch, Nakamura and Stamatescu ('88)



anisotropy ξ

$\xi = a_s / a_t$

$\xi_F = m_s / m_t \, (= \xi)$

calibration

Strategy

Difficulty;

Mass is extracted at $t \gg 1$.

However, at $T > 0$, temporal extent is short.

$\Longrightarrow$ Choice of hadronic operator is significant.

∘ Investigate following questions.

- Define the "hadronic operator" as one which has sufficiently large overlap with corresponding states. Then, what happens on this operator at $T > 0$?

- Is there bound state at $T > T_c$?
  How can we show that ?

- Develop reliable procedure to extract the pole masses with short extent in t-direction.

Meson Correlator

$$G_M(\vec{x},t) = \sum_{x,y_1,y_2} \omega_1(\vec{y}_1)\omega_2(\vec{y}_2)$$
$$\times \langle Tr[S(\vec{y}_1,0;\vec{z},t)\gamma_M\gamma_5 S^\dagger(\vec{y}_2,0;\vec{z}+\vec{x},t)\gamma_5\gamma_M^\dagger]\rangle$$

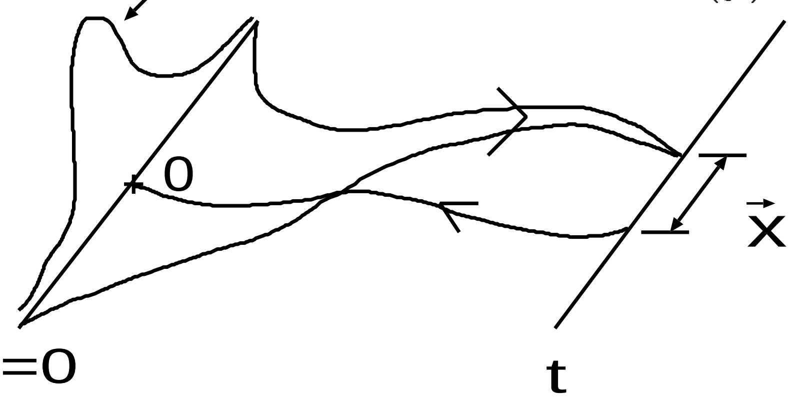
$S(\vec{x}_1,t_1;\vec{x}_2,t_2)$: quark propagator

$$\gamma_M = \gamma_5\ ,\ \gamma_1\ ,\ 1\ ,\ \gamma_1\gamma_5$$
$$(\ M = P_S\ ,\ V\ ,\ S\ ,\ A\ )$$

◇Gauge fixing : Coulomb gauge

source $\omega(\vec{y}) \sim \exp(-ay^p)$ (exp)
$\sim \delta(\vec{y})$ (point)

$G_M(\vec{x},t) =$



a, p are determined from $\vec{x}$-dependence
in point-point P_S correlator at $T \simeq 0$

(I) Meson with light quark

Simulation parameters

Lattice:
$12^3 \times N_t$,$\beta = 5.68$, $\gamma = 4.0$, quenched
$N_t = 72$ $(T \simeq 0)$, 20 $(T < T_c)$, 16, 12 $(T > T_c)$: $T = 1/N_t a_t$

- #conf. = 60
- Anisotropy: $\xi \equiv a_s/a_t = 5.3(1)$
  from the ratio of Wilson loops
  Engels, Karsch and Scheideler (1997), Klassen (1998)
- Cutoff: $a_s^{-1} = 0.85$ GeV, $a_t^{-1} = 4.5(2)$ GeV
  from heavy quark potential

Quark: Anisotropic Wilson action

- Hopping parameter and bare anisotropy:

κ_s	γ_F	m_q	m_{PS}	m_V [GeV]
0.0810	4.05	0.17	0.81	0.90
0.0840	3.89	0.12	0.68	0.80
0.0860	3.78	0.10	0.61	0.75

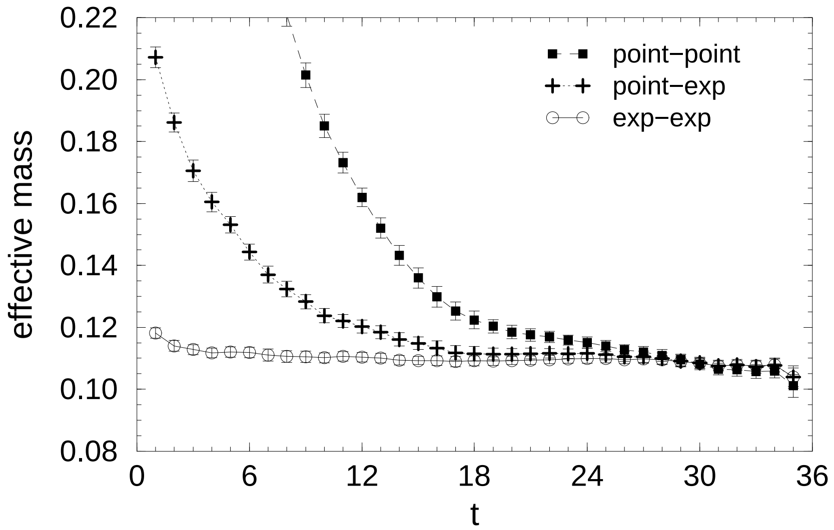
- γ_F determined by calibration
- Periodic b. c. for spatial direction

Effective mass

Effective mass : m_{eff}

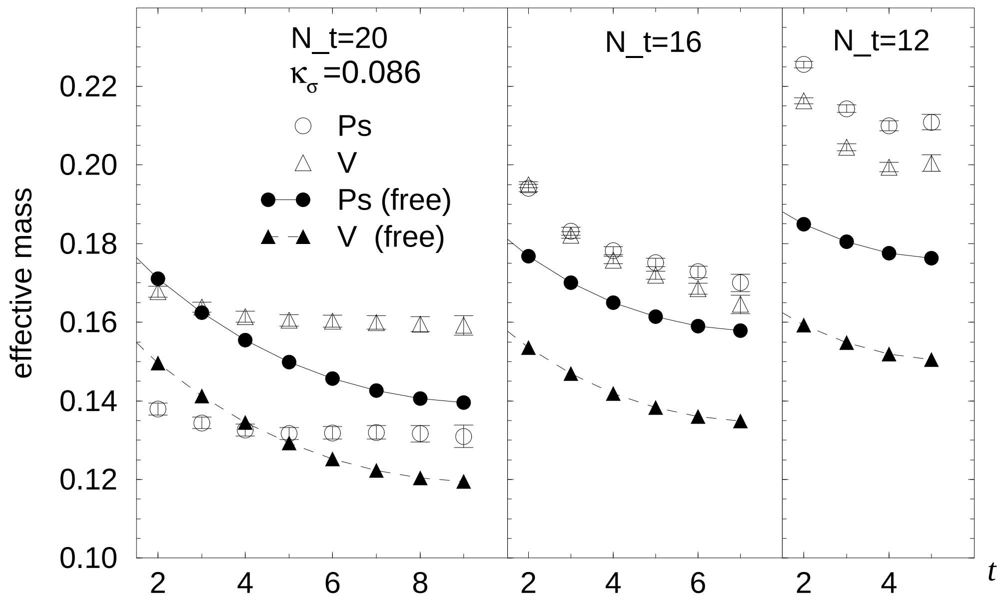
$$\frac{G_M(\vec{x}=0,t)}{G_M(\vec{x}=0,t+1)} = \frac{\cosh\left[m_{\text{eff}}(N_t/2-t)\right]}{\cosh\left[m_{\text{eff}}(N_t/2-t-1)\right]}$$

∘ Source dependence



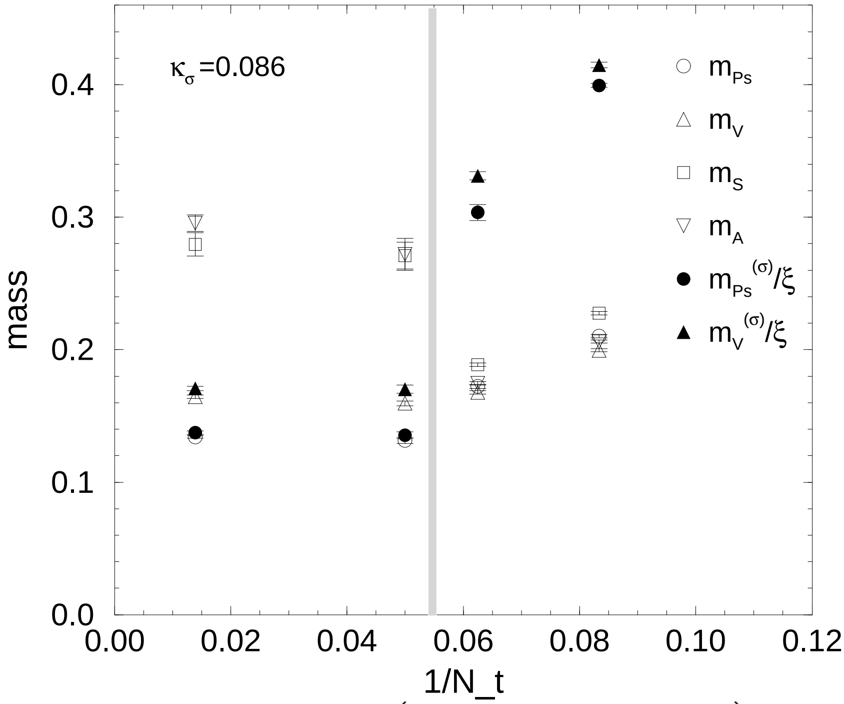
$\Longrightarrow$ We adopt the exp-exp source

∘ effective mass at $T > 0$

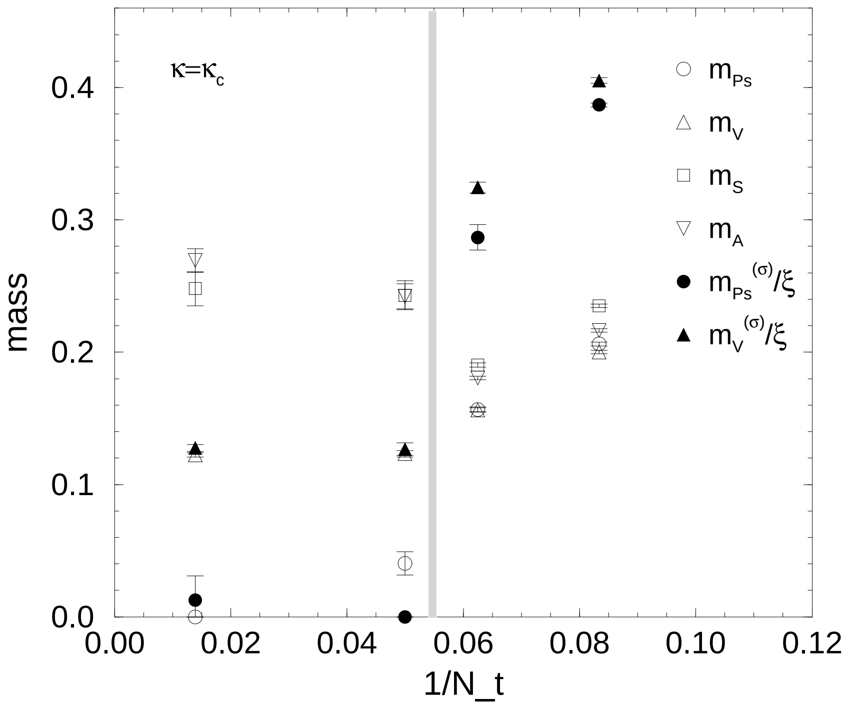


Temperature dependence of mass

$$\kappa_s = 0.086(m_q \sim 0.10\text{GeV})$$



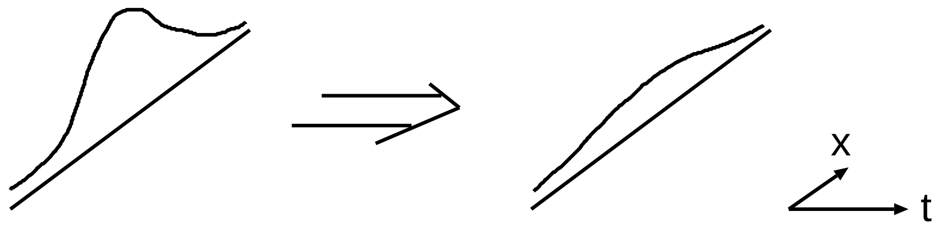
$$\kappa = \kappa_c(m_q \to 0 \quad \text{limit})$$



t-dependence of the wave function

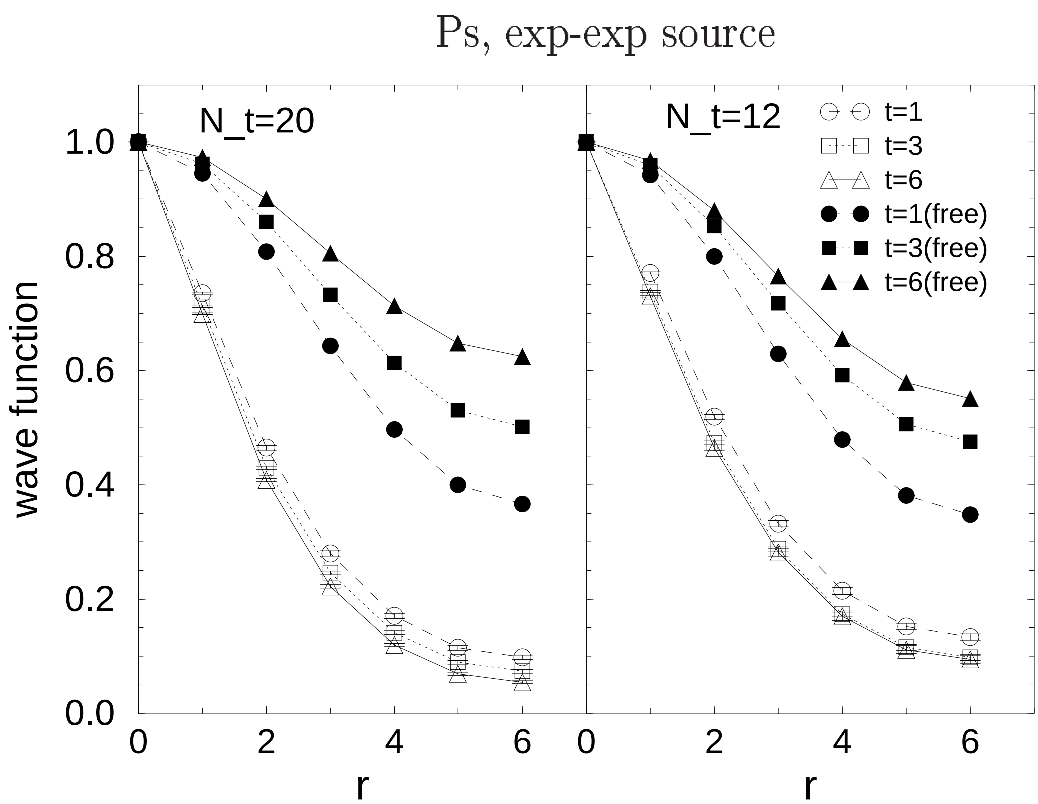
$$w_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t) O^\dagger(0)\rangle$$

If there is no bound state (like free quark case), wave function becomes broader as t.



In this case, $\phi_\Gamma(r,t) = w_\Gamma(r,t)/w_\Gamma(r=0,t)$ (normalized at spatial origin)

increase as t.



Summary and Conclusion (I)

(1) Masses

- Ground state survives at $N_t = 20 (T \sim 0.93 T_c)$
  No significant change of mass.
- Significant change at $N_t = 16, 12 (T \sim 1.15 T_c, 1.5 T_c)$

(2) Wave functions

- No significant change at $N_t = 20$.
- Narrower than free $q - \bar{q}$ even at $N_t = 16, 12$.
  Still prefer to stay together.

(3) s-mass (screening mass) vs t-mass (pole mass)

t-mass is not well identified in the whole t.
$\Rightarrow$ Take largest 3 point for cosh-fitting.

- Grows above T_c.
- Stronger growing of s-masses.
- Consistent with NJL model.

(II) Heavy quark

∘ Heavy quark action :Fermilab action
($O(a)$ improved (Clover action))

$\Longrightarrow$ Heavy quarkonium physics

Clover action : Sheikholeslami,Wohlert('85)
Fermilab action : El-Khadra,Kronfeld,Mackenzie('97)

We study the Charmonium at $T \geq 0$

- mass shift near T_c
  T.Hashimoto et al,('86)
- J/ψ suppression above T_c
  T.Matsui and H.Satz,('86)

Simulation parameters

On the same configuration as (I) (see p.5)
Target meson mass is J/ψ mass : $m_V \sim 3.1 GeV$

Heavy quark action

Fermilab (Clover) action on Anisotropic lattice

$$
\begin{aligned}
S_F(\kappa_s,\gamma_F) &= \sum_x \bar{q}(y) K[U](x,y) q(x) \\
K[U](x,y) &= 1 - \kappa_s \sum_i \left[\left(\frac{1}{\xi} - \gamma_i\right) T_{+i} + \left(\frac{1}{\xi} + \gamma_i\right) T_{-i} \right] \\
&- \kappa_t \left[(1-\gamma_4)\, T_{+4} + (1+\gamma_4) T_{-4} \right] \\
&- \kappa_s \frac{1}{2\xi} c_B \vec{\Sigma}\cdot\vec{B} + \kappa_t \frac{1}{2\xi} c_E \vec{\alpha}\cdot\vec{E}
\end{aligned}
$$

$$
\begin{aligned}
\kappa_t &= \gamma_F \kappa_s \\
c_B, c_E &: \text{ Clover coefficients} \\
&\quad\ \text{(Mean-field improved)} \\
T_{+\mu}(x,y) &= U_\mu(x)\delta_{x+\hat{\mu},y} \\
T_{-\mu}(x,y) &= U_\mu(x-\hat{\mu})\delta_{x-\hat{\mu},y}
\end{aligned}
$$

c.f. Klassen hep-lat/9809174

Calibration

∘ Anisotropy of heavy quark field : ξ_F

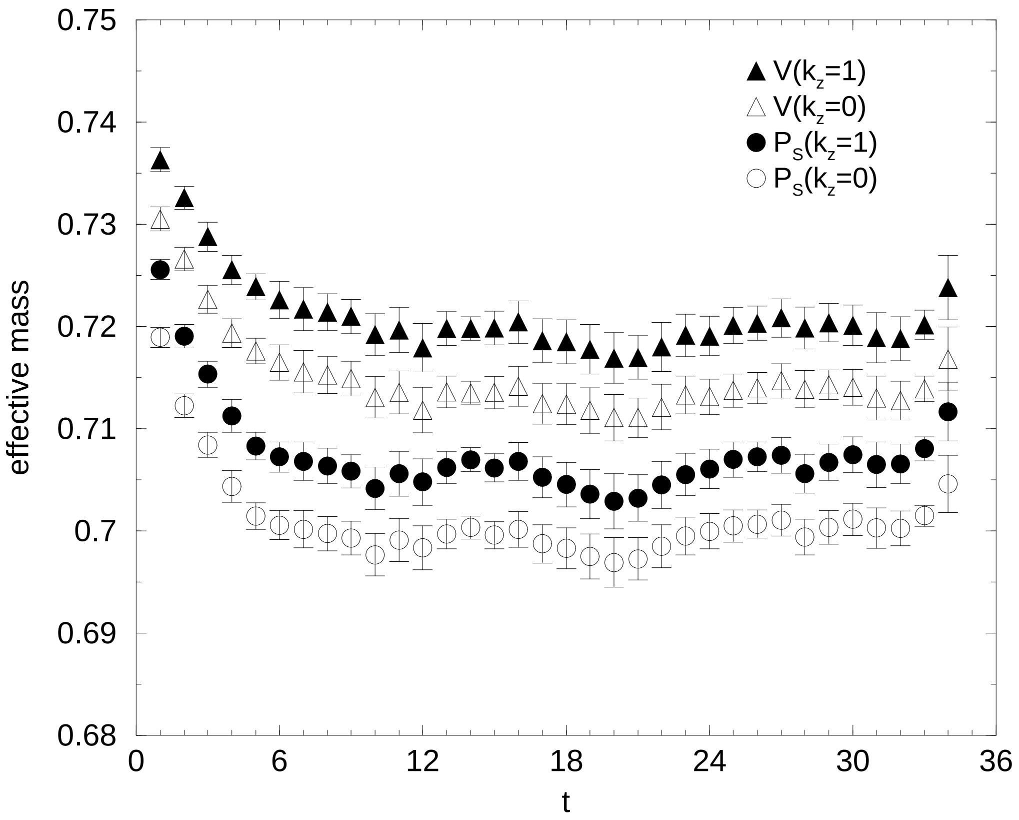
are determined by the dispersion relation of meson

· Dispersion Relation

$$\xi_F^2 = \frac{1}{2} \cdot \frac{\hat{\vec{P}}^2}{\cosh E(\vec{P}) - \cosh E(0)}$$

$$\hat{P}_i = 2 \sin \frac{P_i}{2}, \quad P_i = \frac{2\pi k_i}{N_i}, \quad k_i \in Z$$

· effective energy (smeared source (point-exp))



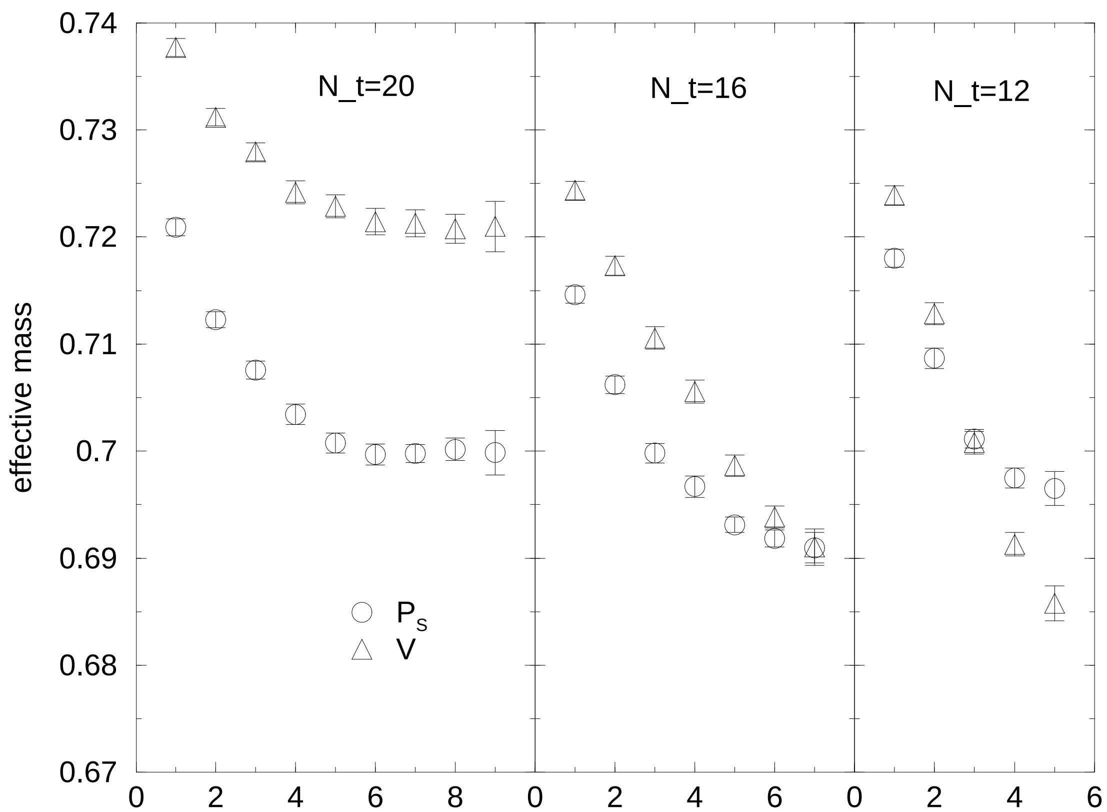
Effective mass at $T > 0$

— Heavy quark field—

- calibration (♯ conf. = 20)
  $\kappa_s = 0.0965, \quad \gamma_F = 3.70 \;\Rightarrow\; \xi_F = 5.269(67)$
- results at $T = 0$: (♯ conf. = 20)
  $m_{Ps} = 3.15(14)\text{GeV}, \quad m_V = 3.21(14)\text{GeV}$

- Finite Temperature (♯ conf. = 40)

effective mass at $T > 0$



Summary and Conclusion (II)

(1) We use the anisotropic Fermilab action for heavy quark system.

(2) Meson correlator

- No significant change at $N_t = 20$
- Significant change at $N_t = 16, 12$

The Temperature dependence is different from that of light mesons .
Especially behavior of vector meson change.

Out look

- Precise determination of mass shift
  - variational analysis
  - Spectral function
  - Fine lattice

    $\Rightarrow$ Now under progress

    Lattice size : $16^2 \times 24 \times N_t$

    $N_t = 96$ (at $T \sim 0$)

    $\beta = 4.56$ (Symanzik action (tree))

    Anisotropy : $\xi \sim 4.0$

    Cutoff : $a_s^{-1} \sim 1.6$ GeV ($a_s \sim 0.12$ fm)

- More detailed analysis of Heavy quarkonium : $J/\psi, \cdots$

- Other channels : baryons etc.

- Relation with topological quantities.

- With dynamical quarks.

Dispersion relation