Static 3-Quark potential in the SU(3) lattice QCD simulation

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1. Introduction

Quark Confinement Feature in Hadrons

- Regge trafectory for hadrons
- Heavy Quarkonium Features
- Lattice QCD Monte Carlo simulations

Quark Confinement Potential for
Quark-Antiquark(Q-$\overline{\mathrm{Q}}$)System

Linear Potential

$$V_{\mathrm{Q}\overline{\mathrm{Q}}}(r) \sim \sigma r$$

$$\sigma = 0.89 \sim 1.0 \text{ GeV/fm}$$

$\sim$String tension

Measurement of Q-$\overline{\mathrm{Q}}$ Potential

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$$W \equiv \mathrm{tr}\, \Pi_i\, U_{\mu_i}(s_i)$$

$$\begin{aligned} < W > \;&\equiv\; < \mathrm{tr} \prod_i U_{\mu_i}(s_i) > = < \exp(-\int j^a_\mu(x) A^a_\mu(x) d^4x) > \\ &= \sum_{n=0}^{\infty} C_n(R) \exp(-V_n(R)T) \end{aligned}$$

If we take the limit $T \to \infty$,

$$< W > \sim C_0 \exp(-V_0(R)T)$$

Lattice QCD Result of Q-$\overline{\mathrm{Q}}$ Potential

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The solid curve denotes a fitting function of

$$V(R) = -\frac{A}{R} + \sigma R + C$$

The string tension is estimated as $\sigma \sim 0.85\mathrm{GeV/fm}$

Color-Electric Flux-Tube Picture for Hadrons

◦ Color-Electric flux is squeezed as a one-dimensional tube or string

- Strong-Coupling QCD
  (Expansion on $\frac{1}{g^2}$ in the lattice formalism)
- Dual Superconductor picture
- Lattice QCD simulation

Then, how about 3Q potential $V_{\rm QQQ}$??

$V_{\rm QQQ} = \sigma(a+b+c)$?

Triangle Flux configuration

$V_{\rm QQQ} = \sigma L_{\rm total}$?

Minimal Flux Length configuration including ' junction point'

Minimal Length Configuration of Flux Tubes for 3 Quark System

In the flux-tube picture, the total tube length is minimized in the ground-state configuration.
For fixed 3 quarks, there are 2 categories for minimal-length flux.

Type-1 configuration Type-2 configuration

(a) If each angle of the ' quark triangle' does not exceed 120°, this ' Y-type' flux configuration has minimal length of total flux. Here, a junction appears at the ' Fermat point', and total flux-tube length is given as

$$L(a,b,c) = \left[\frac{a^2+b^2+c^2}{2} + \frac{\sqrt{3}}{2}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}\right]^{\frac{1}{2}}$$

(b) If an angle of the quark triangle exceeds 120°, the flux tubes appear along two shorter sides.

$$L(a,b,c) = a+b+c-\mathrm{Min}(a,b,c)$$

Static 3Q Potential

- Long-distance behavior

  $\sim$ Quark Confinement

3Q potential is expected to obey the Flux-Tube picture

$$V_{\mathrm{QQQ}}^{\mathrm{conf}} = \sigma L_{\mathrm{min}}$$

L_{min} : minimal length of Flux-tubes linking 3 quarks

- Short-distance behavier

  $\sim$ Perturbative QCD

QCD exhibits the asymptotic freedom

At short distances, perturbative QCD is applicable. At the leading order of P-QCD, one-gluon-excange is dominant and the inter-quark potential becomes sum of 2-body Coulomb-type as

$$V_{\mathrm{coulomb}} = \sum_{i<j} \frac{-A}{|\vec{r_i} - \vec{r_j}|}$$

To summarize,the 3Q potential is expected to take the form of

$$V_{\mathrm{QQQ}}^{\mathrm{total}} = -A\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) + \sigma L_{\mathrm{min}} + C$$

Measurement of 3Q Potential

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Then, how can we get the 3Q potential ? For this purpose, we consider the 3Q operator $W_{\rm 3Q}$ defined on the contour of 3 large staples as shown in the above figure. We denote the product of link-variables along each staple by U_1, U_2 and U_3, which are SU(3) matricies.

Similar to Q-$\overline{\rm Q}$ case, the 3Q potential can be obtained from the expectation value of the 3Q operator $< W_{\rm 3Q} >$.

$$\begin{aligned} < W_{\rm 3Q} > &= < \epsilon^{a_1 a_2 a_3} \epsilon^{b_1 b_2 b_3} U_1^{a_1 b_1} U_2^{a_2 b_2} U_3^{a_3 b_3} > \\ &= \sum_{n=0} C_n \exp(-V_n(i,j,k)T) \end{aligned}$$

By taking the limit $T \to \infty$, the ground-state potential $V_0(i,j,k)$ can be extracted as

$$< W_{\rm 3Q} > \sim C_0 \exp(-V_0(i,j,k)T)$$

To estimate the ground-state potential, it is desired to take large T.

Lattice QCD Result for 3Q Potential

(a) Y-type ansatz

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$$V_{\mathrm{QQQ}}^{\mathrm{total}} = -A\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)+\sigma L_{\min}+C$$

(b) Delta-type ansatz

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$$V_{\rm QQQ}^{\rm total} = -A\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)+\sigma(a+b+c)+C$$