

Static 3-Quark potential in the SU(3) lattice QCD simulation

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1. Introduction

Quark Confinement Feature in Hadrons

- Regge trajectory for hadrons
- Heavy Quarkonium Features
- Lattice QCD Monte Carlo simulations

Quark Confinement Potential for
Quark-Antiquark($Q-\bar{Q}$)System

Linear Potential

$$V_{Q\bar{Q}}(r) \sim \sigma r$$

$$\sigma = 0.89 \sim 1.0 \text{ GeV/fm}$$

\sim String tension

Measurement of $Q-\bar{Q}$ Potential

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$$W \equiv \text{tr} \prod_i U_{\mu_i}(s_i)$$

$$\begin{aligned} \langle W \rangle &\equiv \langle \text{tr} \prod_i U_{\mu_i}(s_i) \rangle = \langle \exp(-\int j_\mu^a(x) A_\mu^a(x) d^4x) \rangle \\ &= \sum_{n=0}^{\infty} C_n(R) \exp(-V_n(R)T) \end{aligned}$$

If we take the limit $T \rightarrow \infty$,

$$\langle W \rangle \sim C_0 \exp(-V_0(R)T)$$

Lattice QCD Result of $Q\text{-}\bar{Q}$ Potential

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The solid curve denotes a fitting function of

$$V(R) = -\frac{A}{R} + \sigma R + C$$

The string tension is estimated as $\sigma \sim 0.85\text{GeV/fm}$

Color-Electric Flux-Tube Picture for Hadrons

- Color-Electric flux is squeezed as a one-dimensional tube or string
- Strong-Coupling QCD
(Expansion on $\frac{1}{g^2}$ in the lattice formalism)
- Dual Superconductor picture
- Lattice QCD simulation

Then, how about 3Q potential V_{QQQ} ??

$$V_{QQQ} = \sigma(a + b + c) ?$$

Triangle Flux configuration

$$V_{QQQ} = \sigma L_{\text{total}} ?$$

Minimal Flux Length configuration
including 'junction point'

Minimal Length Configuration of Flux Tubes for 3 Quark System

In the flux-tube picture, the total tube length is minimized in the ground-state configuration.

For fixed 3 quarks, there are 2 categories for minimal-length flux.

Type-1 configuration

Type-2 configuration

- (a) If each angle of the 'quark triangle' does not exceed 120° , this 'Y-type' flux configuration has minimal length of total flux. Here, a junction appears at the 'Fermat point', and total flux-tube length is given as

$$L(a, b, c) = \left[\frac{a^2 + b^2 + c^2}{2} + \frac{\sqrt{3}}{2} \sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)} \right]^{\frac{1}{2}}$$

- (b) If an angle of the quark triangle exceeds 120° , the flux tubes appear along two shorter sides.

$$L(a, b, c) = a + b + c - \text{Min}(a, b, c)$$

Static 3Q Potential

- Long-distance behavior
 \sim Quark Confinement

3Q potential is expected to obey the Flux-Tube picture

$$V_{QQQ}^{\text{conf}} = \sigma L_{\text{min}}$$

L_{min} : minimal length of Flux-tubes linking 3 quarks

- Short-distance behavior
 \sim Perturbative QCD

QCD exhibits the asymptotic freedom

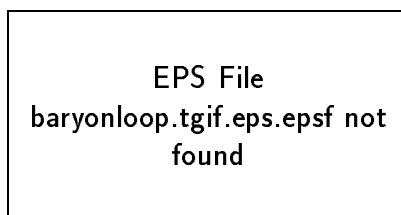
At short distances, perturbative QCD is applicable. At the leading order of P-QCD, one-gluon-exchange is dominant and the inter-quark potential becomes sum of 2-body Coulomb-type as

$$V_{\text{coulomb}} = \sum_{i < j} \frac{-A}{|\vec{r}_i - \vec{r}_j|}$$

To summarize, the 3Q potential is expected to take the form of

$$V_{QQQ}^{\text{total}} = -A \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) + \sigma L_{\text{min}} + C$$

Measurement of 3Q Potential



Then, how can we get the 3Q potential ? For this purpose, we consider the 3Q operator W_{3Q} defined on the contour of 3 large staples as shown in the above figure. We denote the product of link-variables along each staple by U_1 , U_2 and U_3 , which are SU(3) matrices.

Similar to Q- \bar{Q} case, the 3Q potential can be obtained from the expectation value of the 3Q operator $\langle W_{3Q} \rangle$.

$$\begin{aligned} \langle W_{3Q} \rangle &= \langle \epsilon^{a_1 a_2 a_3} \epsilon^{b_1 b_2 b_3} U_1^{a_1 b_1} U_2^{a_2 b_2} U_3^{a_3 b_3} \rangle \\ &= \sum_{n=0} C_n \exp(-V_n(i, j, k)T) \end{aligned}$$

By taking the limit $T \rightarrow \infty$, the ground-state potential $V_0(i, j, k)$ can be extracted as

$$\langle W_{3Q} \rangle \sim C_0 \exp(-V_0(i, j, k)T)$$

To estimate the ground-state potential, it is desired to take large T .

Lattice QCD Result for 3Q Potential

(a) Y-type ansatz

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$$V_{QQQ}^{\text{total}} = -A \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) + \sigma L_{\text{min}} + C$$

(b) Delta-type ansatz

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$$V_{\text{QQQ}}^{\text{total}} = -A \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) + \sigma(a + b + c) + C$$