Hadrons in finite temperature lattice QCD

Hideo Matsufuru (RCNP, Osaka Univ.)

for QCD-TARO Collaboration:

Ph. de Forcrand[a], M. García Pérez[b], T. Hashimoto[c], S. Hioki[d], R. Katayama[e], H. Matsufuru, O. Miyamura[e], A. Nakamura[f], I.-O. Stamatescu[g,h], T. Takaishi[i] and T. Umeda[e]

TMU-Yale Simposium

13-15 Dec 1999, in Tokyo

[a] SCSC, ETH-Zürich, Switzerland
[b] Dept. Física Teórica, Univ. Autónoma de Madrid, Spain
[c] Dept. Appl. Phys., Fac. Engineering, Fukui Univ., Japan
[d] Dept. of Physics, Tezukayama Univ., Nara, Japan
[e] Dept. of Physics, Hiroshima Univ., Japan
[f] Res. Inst. for Inform. Sci. and Education, Hiroshima Univ., Japan
[g] Inst. Theor. Physik, Univ. of Heidelberg, Germany
[h] FEST, Heidelberg, Germany
[i] Hiroshima University of Economics, Japan

References:

Lattice 98, 420-425; hep-lat/9809173
hep-lat/9911001 (Lattice 99)

Introduction

Finite temperature hadron properties:

- What happens on hadrons
  or what new effect above T_c ?
- Change of masses and width near T_c

Example of effective theory:

NJL model analysis

c.f. Hatsuda and Kunihiro, Phys. Rep. 247 (1994) 221

∘ Chiral restoration

∘ parity partner degeneracy

∘ Above T_c : Soft modes

Our Goals

Study of hadronic correlators at finite temperature using lattice QCD

Temporal correlator ;

$\Rightarrow$Pole mass (mass of temporal direction)

$\Updownarrow$

Screening mass (mass of spatial direction)

$\Rightarrow$Wave function

Is there bound state at $T > T_c$?

How can we show that ?

$\Rightarrow$Spectral function

□ Mass shift near the critical temperature

$\rightarrow$*heavy ion collision experiment*

□ Chiral symmetry restoration and deconfinement

— Relation of two phenomena

□ Relation between hadronic correlators and topological quantities

Lattice QCD (1)

Lattice QCD
— SU(3) gauge theory on Euclidean 4-D lattice
∘ Gauge field: link var. $U_\mu(x) \sim e^{igaA_\mu(x)}$
∘ Quark field: $\bar{\psi}(x)$, $\psi(x)$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi O[U, \bar{\psi}, \psi] \exp(-S[U, \bar{\psi}, \psi])$$

S: lattice action
→continuum QCD action for $a \to 0$ limit
(a : lattice spacing)

⇒Monte Carlo simulation

Quark field (Grassmann field)
– integrated out:

$$Z = \int \mathcal{D}U \mathcal{D} \mathrm{det} K[U] \exp(-S[U, \bar{\psi}, \psi])$$

where

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(x)$$

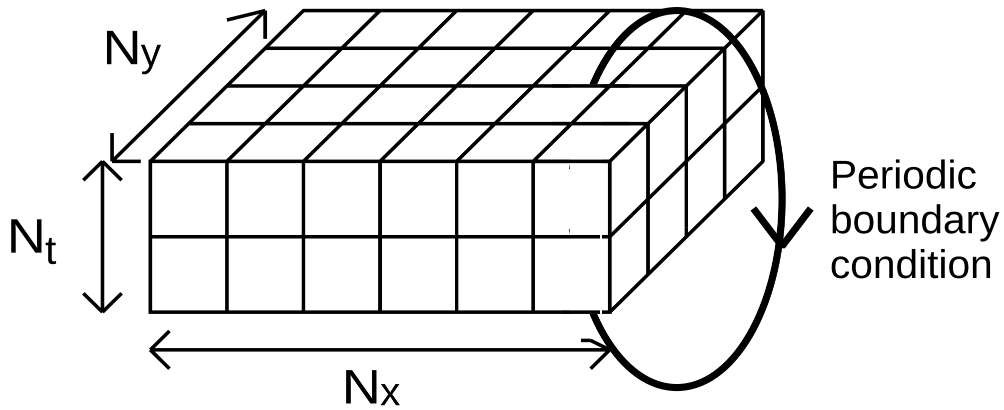
Quenched approximation: set det$K = 1$
– neglect quark loop effect ($m_q \to \infty$ limit)

Lattice QCD (2)

Finite temperature lattice QCD

$$Z = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(-\int_0^\tau d\tau \int d^3x \mathcal{L}\right)$$

Periodic/anti-periodic boundary condition for boson/fermion in temporal direction



Temperature Green function
$\Rightarrow$information on spectral function at finite temperature
Hashimoto, Nakamura and Stamatescu (1993)

For detailed investigation of hadronic properties at $T > 0$, sufficient resolution in temporal direction is significant (need large temporal cutoff)

$\Rightarrow$ *Anisotropic lattice*

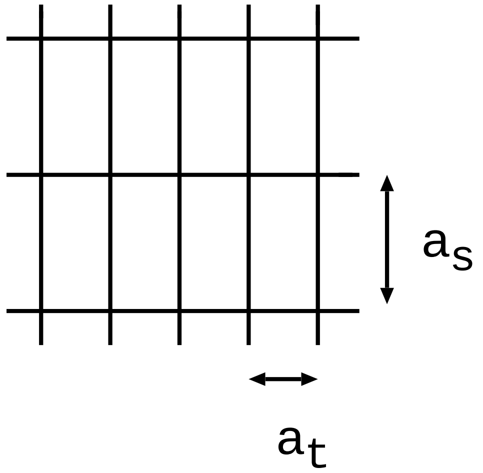
Lattice QCD (3)

Anisotropic lattice: Karsch ('82)

Burgers, Karsch, Nakamura and Stamatescu ('88)

Different cutoffs in spatial (a_σ^{-1}) and in temporal (a_σ^{-1}) directions: $a_\sigma > a_\sigma$

→Enough information in t (Euclidean temporal direction) keeping spatial lattice size not quite large



anisotropy ξ

$\xi = a_s / a_t$

$\xi_F = m_s / m_t \, (= \xi)$

↑ calibration

Bare anisotropy γ in the action and the renormalized anisotropy ξ is different

→ "Calibration" is necessary

(tuning of parameters for desired physical anisotropy and cutoffs)

Strategy

Difficulty;
Mass is extracted at $t \gg 1$.
However, at $T > 0$, temporal extent is short.
$\Longrightarrow$ Choice of hadronic operator is significant.

∘ Investigate following questions.

- Define the "hadronic operator" as one which has sufficiently large overlap with corresponding states.
  Then, what happens on this operator at $T > 0$?

- Is there bound state at $T > T_c$?
  How can we show that ?

- Develop reliable procedure to extract the pole masses with short extent in t-direction.

Simulation

Present simulation: corase, unimproved lattice
$\Rightarrow$Qualitative results, deveropment of procedures

□ *Lattice:*

$12^3 \times N_t$,$\beta = 5.68$, $\gamma = 4.0$, quenched

$N_t = 72$ ($T \simeq 0$), 20 ($T < T_c$), 16, 12 ($T > T_c$) : $T = 1/N_t a_t$

- #conf. = 60
- Anisotropy: $\xi \equiv a_s/a_t = 5.3(1)$
  from the ratio of Wilson loops
  Engels, Karsch and Scheideler (1997), Klassen (1998)
- Cutoff: a_σ^{-1}=0.85(3) GeV ($a_\sigma \sim 0.24$ fm), a_τ^{-1}=4.5(2) GeV
  from heavy quark potential

□ *Light quark:* Anisotropic Wilson action

- Hopping parameter and bare anisotropy:

κ_s	γ_F	m_q	m_{PS}	m_V [GeV]
0.0810	4.05	0.17	0.81	0.90
0.0840	3.89	0.12	0.68	0.80
0.0860	3.78	0.10	0.61	0.75

(m_q is with naive estimate)

- γ_F determined by calibration
  using masses in spatial and temporal direction
- Scalar channel: only connected part is evaluated
- Chiral extrapolation: $m_q \to 0$ limit

□ *Heavy quark:* Anisotropic Fermilab action

- Quark mass around charm quark
- Calibration using the dispersion relation

Light Hadrons (1)

Correlators:

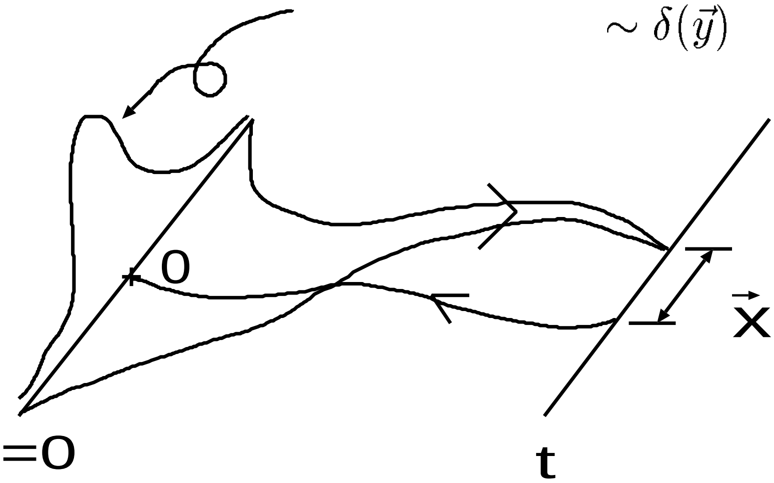
$$G_M(\vec{x},t) = \sum_{z,y_1,y_2} \omega_1(\vec{y}_1)\omega_2(\vec{y}_2)$$
$$\times\langle Tr[S(\vec{y}_1,0;\vec{z},t)\gamma_M\gamma_5 S^\dagger(\vec{y}_2,0;\vec{z}+\vec{x},t)\gamma_5\gamma_M^\dagger]\rangle$$

$S(\vec{x}_1,t_1;\vec{x}_2,t_2)$: quark propagator
$\gamma_M = \gamma_5,\ \gamma_1,\ 1,\ \gamma_1\gamma_5$
$(M = P_S,\ V,\ S,\ A\ )$

◇Gauge fixing : Coulomb gauge

Source $\omega(\vec{y}) \sim \exp(-ay^p)$ (exp)
$\sim \delta(\vec{y})$ (point)

$G_M(\vec{x},t) =$



Parameters a, p :
determined from the wave function at $T=0$

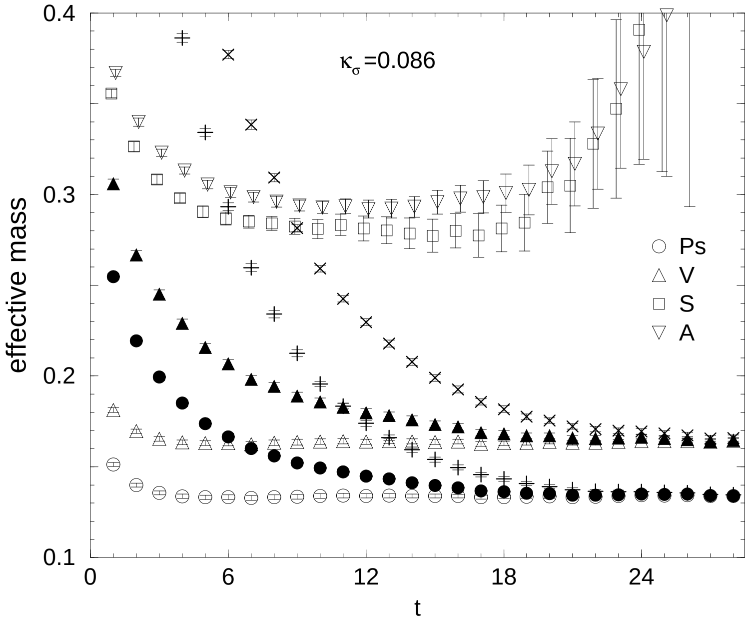
Light Hadrons (2)

Effective mass : m_{eff}

$$\frac{G_M(\vec{x}=0,t)}{G_M(\vec{x}=0,t+1)} = \frac{\cosh\left[m_{\rm eff}(N_t/2-t)\right]}{\cosh\left[m_{\rm eff}(N_t/2-t-1)\right]}$$

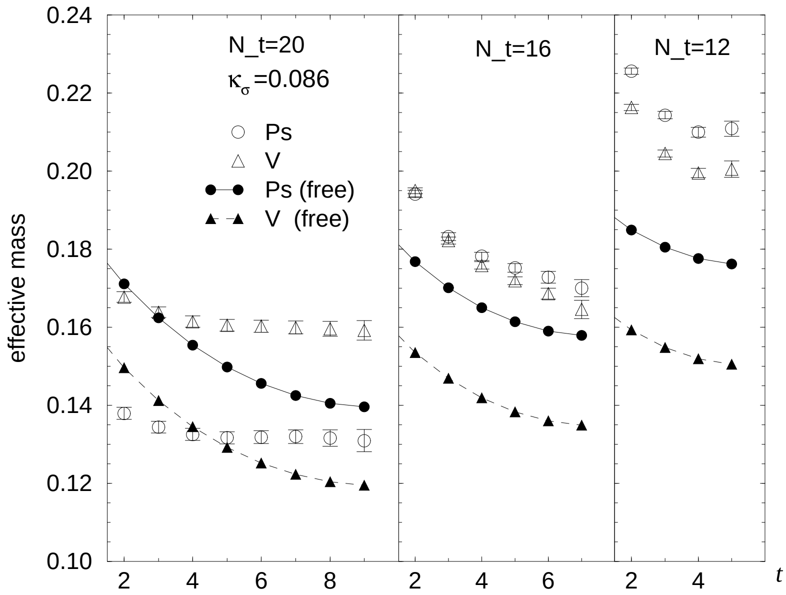
$m_{eff} \to m_M$ for sufficient large t

- Source dependence (at $T = 0$)



$\Rightarrow$exp-exp source seems the best choice (?)

- effective mass at $T > 0$



Light Hadrons (3)

Variational analysis ($T = 0$):

$$C_M^{(i,j)}(t) = \sum_{\vec{x}} \langle O_M^{(i)}(\vec{x}, t) O_M^{(j)\dagger}(0) \rangle$$

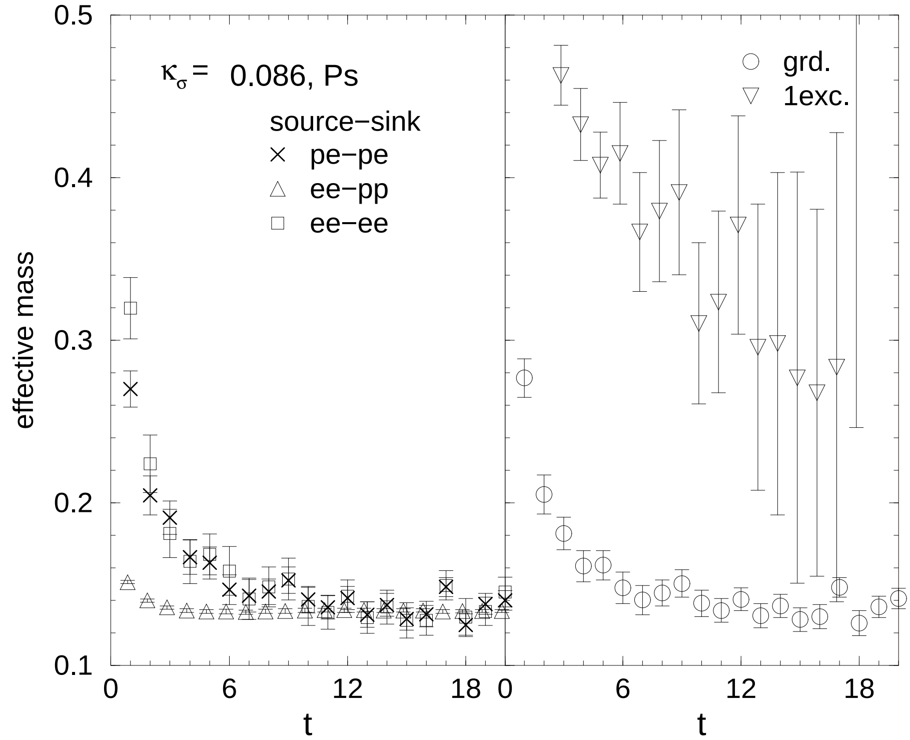
$$O_M^{(i)}(\vec{x}, t) = \sum_{\vec{y}} \phi(\vec{y}) \bar{q}(\vec{x} + \vec{y}, t) \Gamma_M q(\vec{x}, t)$$

$\phi(\vec{y})$: point, exp, exp-exp (convoluted)functions

Diagonalize 3×3 correlator matrix $C_M^{(i,j)}(t)$

$\rightarrow D^{(k)}(t) \quad (k = 1, 2, 3)$ (diagonal components)

: large overlap with each state is expected



Contribution to ground state:

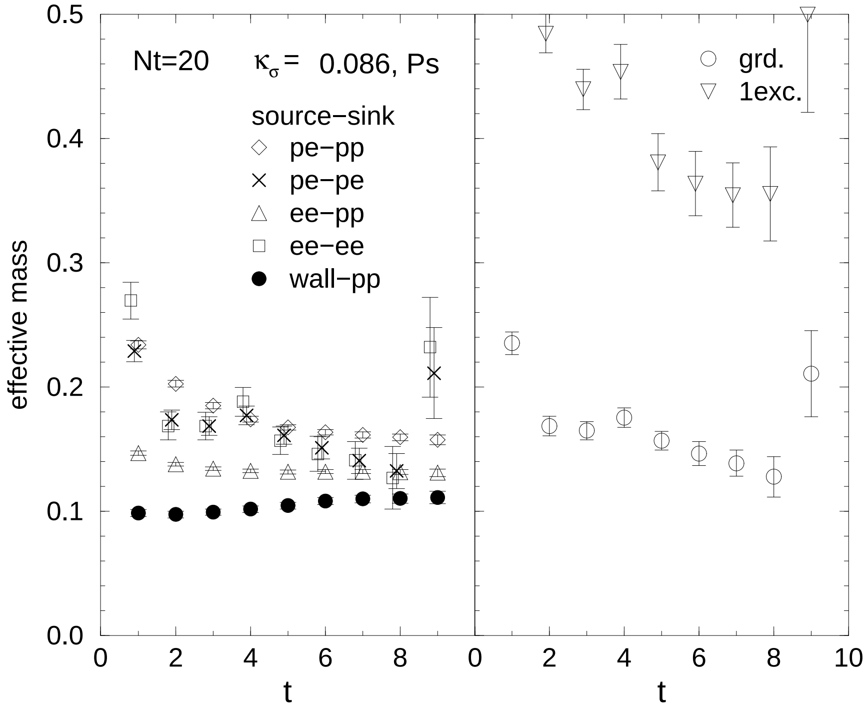
point-exp $\gtrsim$ exp-exp $\gg$ point-point

∘ Cancelation of 1st and 2nd excited states in exp-exp operator

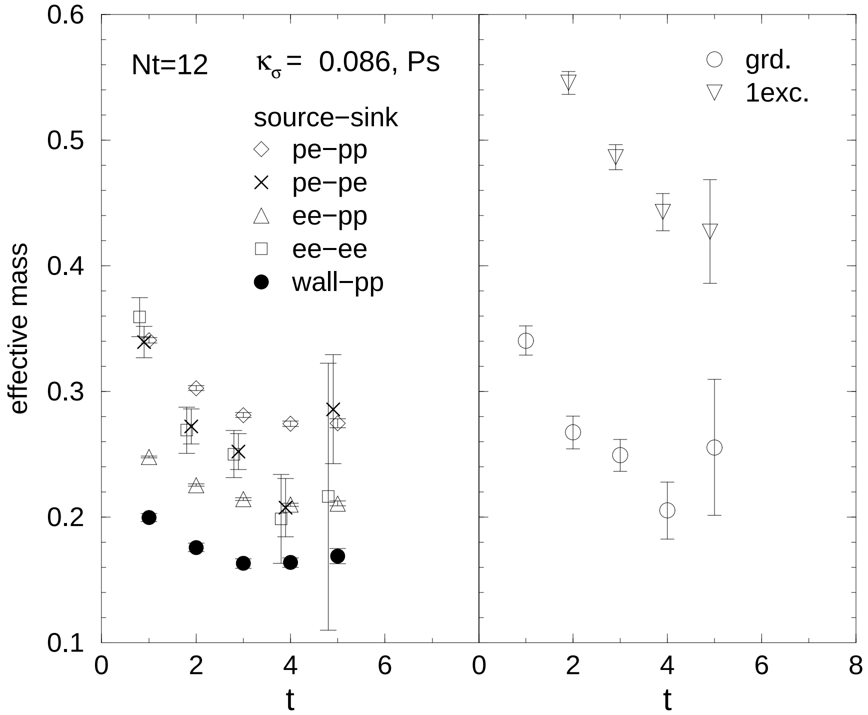
Light Hadrons (4)

Variational analysis ($T > 0$):

$N_t = 20 \;\; (T < T_c) \;\; Ps$



$N_t = 12 \;\; (T > T_c) \;\; Ps$



Light Hadrons (5)

□ Result of the variational analysis:

- ∘ Diagonalized correlator with the lowest effective mass
  — consistent with exp-exp correlator
- ∘ Large statistical fluctuation
- ∘ Different behavior from "fake" meson which is composed with free quark propagators even above T_c

— Bound state in the deconfining phase ?

□ t-dependence of the wave function (next page) also suggests that quark and anti-quark tend to stay together

We assume the existence of mesonic modes at all T
and extract temporal (pole) masses from exp-exp correlators

- ∘ Fit range: largest 3 points
- ∘ Large systematic uncertainties

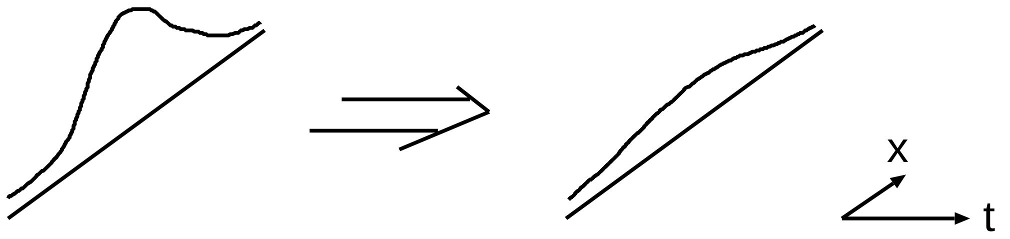
$$\Downarrow$$

- Temperature dependence of mass
- Comparison with screening mass

t-dependence of the wave function:

$$w_\Gamma(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t) O^\dagger(0)\rangle$$

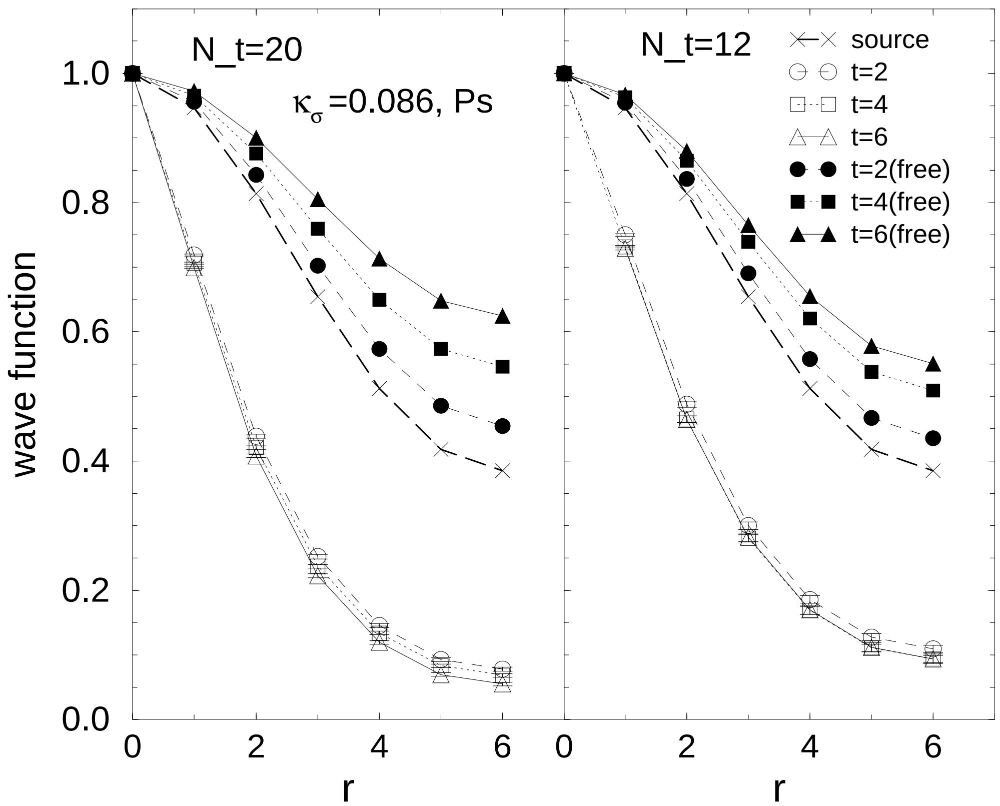
If there is no bound state (like free quark case), wave function becomes broader as t.



In this case, $\phi_\Gamma(\vec{r},t) = w_\Gamma(\vec{r},t)/w_\Gamma(\vec{r}=0,t)$ (normalized at spatial origin)

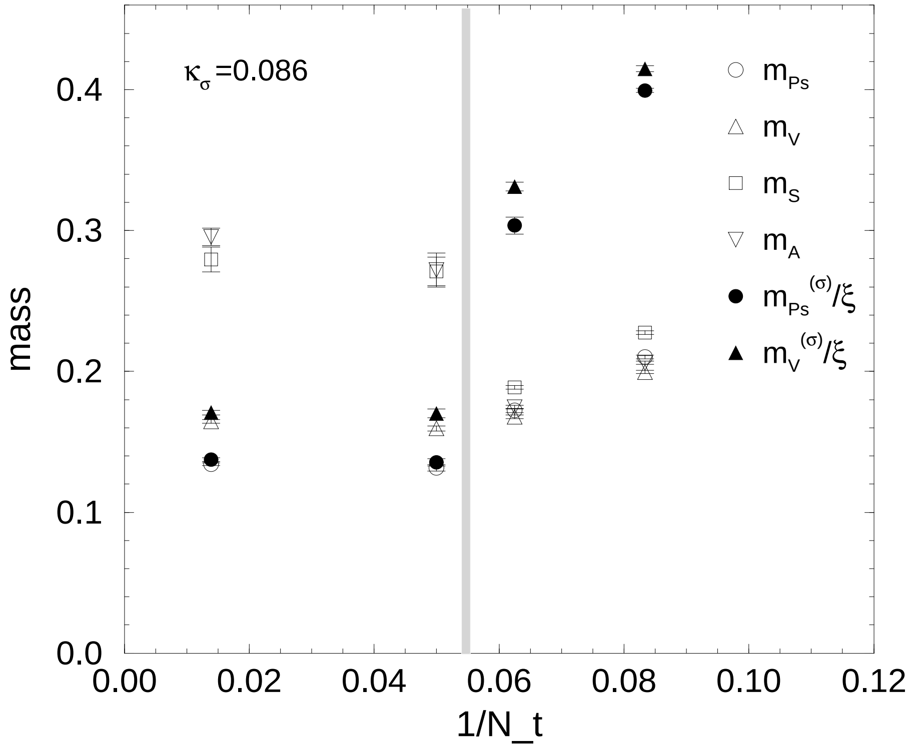
increases as t.

Ps, exp-exp source

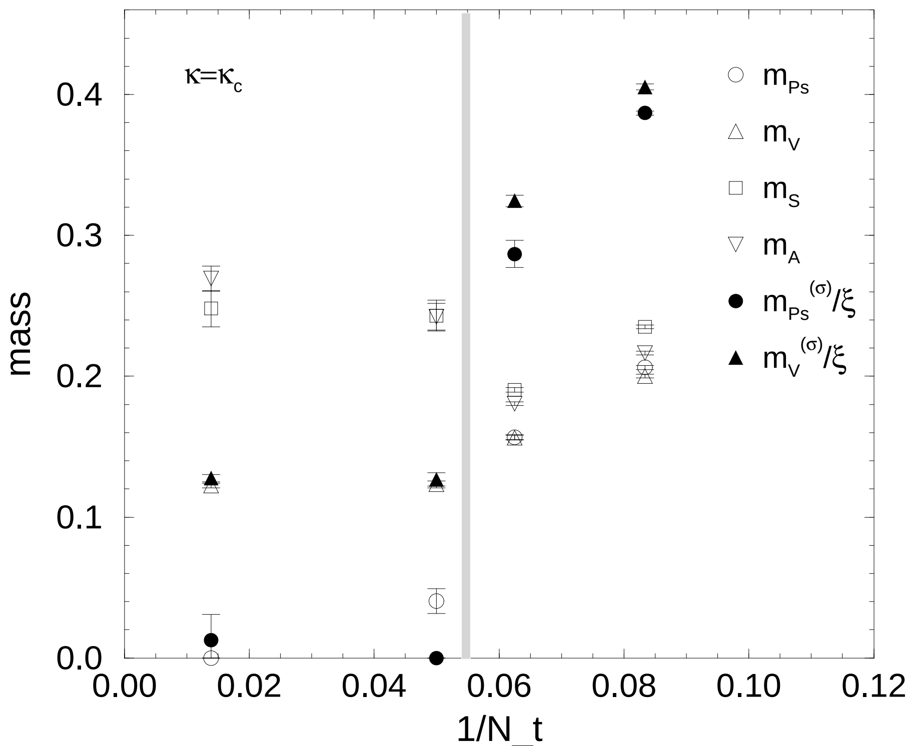


Temperature dependence of masses

$\kappa_s = 0.086$ ($m_q \sim 0.10$ GeV)



$\kappa = \kappa_c$ ($m_q \to 0$ limit)



Light Hadrons (8)

— *Conclusion* —

(1) Masses

- No significant change of mass at $N_t = 20$ ($T \sim 0.93T_c$)
- Significant change at $N_t = 16$, 12 ($T \sim 1.15T_c$, $1.5T_c$)

(2) Wave functions

- No significant change at $N_t = 20$.
- Narrower than free q-$\bar{q}$ even at $N_t = 16, 12$.
  Still prefer to stay together.

(3) s-mass (screening mass) vs t-mass (pole mass)

t-mass is not well identified in the whole t.
$\Rightarrow$ Take largest 3 point for cosh-fitting.

- Grows above T_c.
- Stronger growing of s-masses.
- Consistent behaviour with NJL model.

— *Outlook* —

- ◦ Extraction of spectral function
  QCD-TARO, in Lattice 97
  Nakahara, Asakawa and Hatsuda (1999)
- ◦ Other channels: baryons etc.
- ◦ Relation with topological quantities
- ◦ Improved action – clover quark action
- ◦ With dynamical quarks

Heavy Quarkonium (1)

Heavy quark: $m_Q \gtrsim a_\sigma^{-1}$

$\Rightarrow$Fermilab action – Effective theory approach

With $O(a)$ improvement (with clover term)

Clover action: Sheikholeslami and Wohlert (1985)
Fermilab action: El-Khadra, Kronfeld and Mackenzie (1997)

Anisotropic Fermilab action
— Calibration with dispersion relation

cf. Klassen, hep-lat/9809174

Charmonium at finite temperature

Target meson mass $\sim m_{J/\psi} = 3.1$ GeV

- mass shift near T_c ?

  Hashimoto et al.,(1986)

- J/ψ suppression above T_c ?

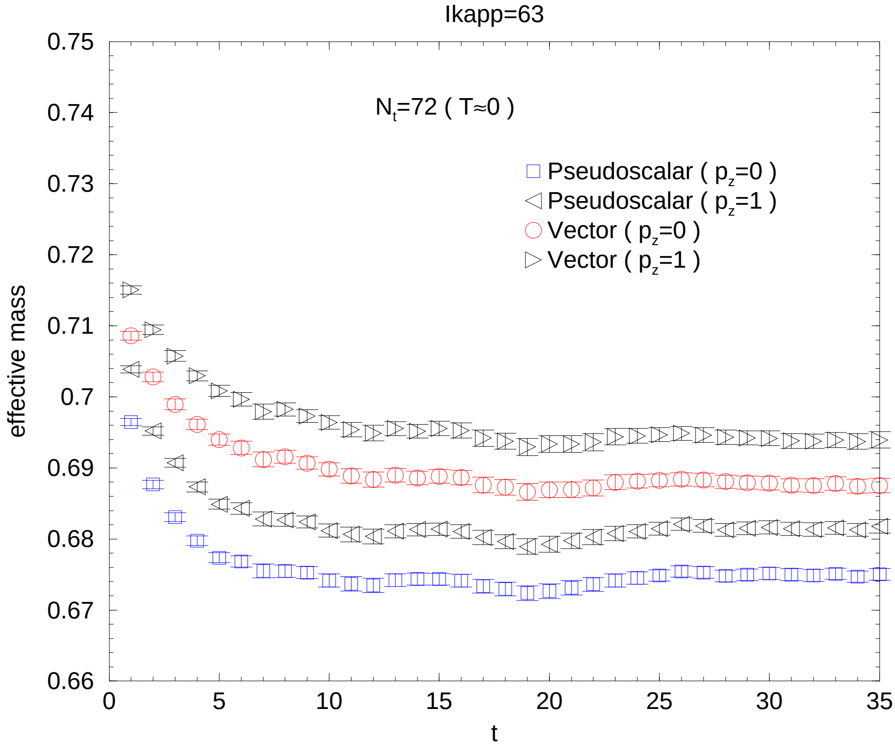
  Matsui and Satz (1986)

Heavy Quarkonium (2)

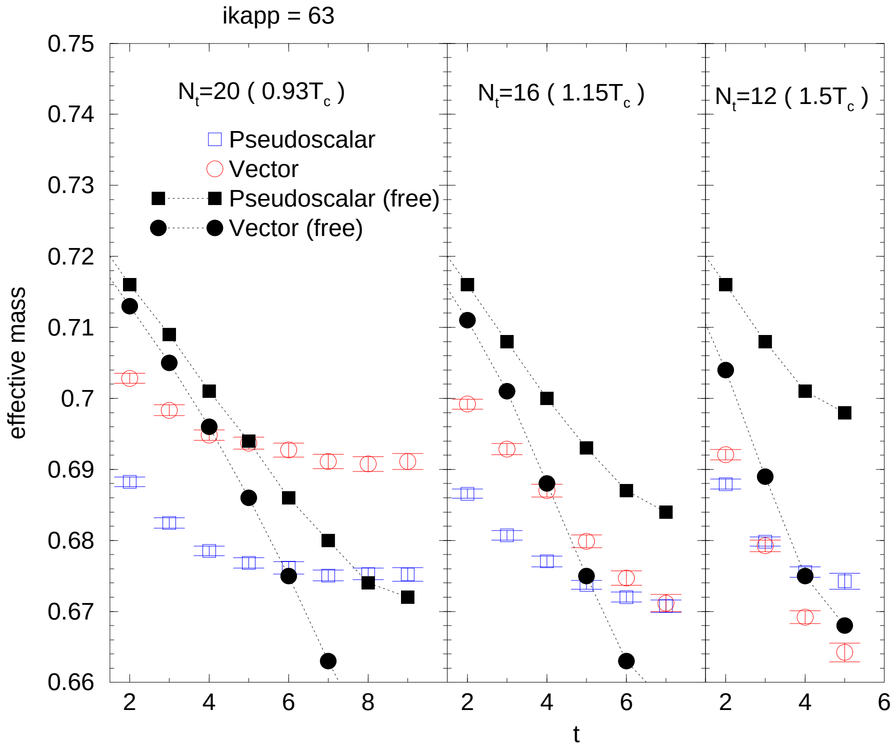
Effective mass

Correlator with exp type smearing, $m_Q \sim m_{charm}$

$T = 0$



$T > 0$



Heavy Quarkonium (3)

— *Conclusion* —

Meson effective mass:

- No significant change at $N_t = 20$
- Significant change at $N_t = 16, 12$ in the vector channel

The Temperature dependence is different from that of light mesons, especially in vector channel.

— *Outlook* —

Need more detailed analysis
 – Variational analysis, and so on

∘ Precise determination of mass shift near T_c
 Comparison with quark potential model
 using measured Q-$\bar{Q}$ potential at $T > 0$

∘ Fate of J/ψ meson above T_c