Monte Carlo Renormalization Group Analysis of QCD in two dimensional coupling space

QCD-TARO Collaboration

Lattice '99 / 1 July 1999 / Pisa, Italy

Abstract: We report our recent result of the Monte Carlo Renormalization Group analysis in two dimensional coupling space. The qualitative feature of the RG flow is described with a phenomenological RG equation. The data on the lattice spacing for various actions serve the condition to determine the parameters.

— *CONTENTS* —

The QCD-TARO Collaboration

Ph. de Forcrand[a], *M. García Pérez*[b], *T. Hashimoto*[c], *S. Hioki*[d], *H. Matsufuru*[e], *O. Miyamura*[f], *A. Nakamura*[g], *I.-O. Stamatescu*[h,i], *T. Takaishi*[j] and *T. Umeda*[f]

[a] SCSC, ETH-Zürich, Switzerland
[b] Dept. Física Teórica, Univ. Autónoma de Madrid, Spain
[c] Dept. Appl. Phys., Fac. Engineering, Fukui Univ., Japan
[d] Dept. of Physics, Tezukayama Univ., Nara, Japan
[e] Research Center for Nuclear Physics, Osaka Univ., Japan
[f] Dept. of Physics, Hiroshima Univ., Japan
[g] Res. Inst. for Inform. Sci. and Education, Hiroshima Univ., Japan
[h] Inst. Theor. Physik, Univ. of Heidelberg, Germany
[i] FEST, Heidelberg, Germany
[j] Hiroshima University of Economics, Japan

Physical goals:

- Finite Temperature QCD
  → Umeda's poster
- Finite Chemical Potential
  → Takaishi's talk
- MCRG and Improved Actions
  – this presentation

Blocking and MCRG

Action: $S = \beta \, \Sigma_i \, c_i O_i$

given configuration with $\{\beta^{(0)}, c_i^{(0)}\}$

$\downarrow$

blocking $a \to 2a$

[Swendsen '81]

$\downarrow$

blocked conf. $\to$ coupling $\{\beta^{(1)}, c_i^{(1)}\}$

- ∘ Schwinger-Dyson equation
- ∘ Demon method

– Improved in comparison with directly generated conf.

Perfect Action

Blocking trajectory
$\rightarrow$ Renormalization Trajectory (RT)

Action on RT – Perfect action:
Same physics as in continuum QCD

Finite number of operators
$\rightarrow$ truncation effect

Strategy

First we investigate GR flow in 2-D coupling space

- ∘ Determination of RT on 2-D space
- ∘ Is truncation effect large ?
  - – comparison with 3-D coupling space
- ∘ Description of flow dynamics
  - – phenomenological RG equation

Quantitative determination of RT
$\rightarrow$ Perfect action in 2-D coupling space

Then,

- ∘ Coupling space with $D > 2$
- ∘ Inclusion of fermions

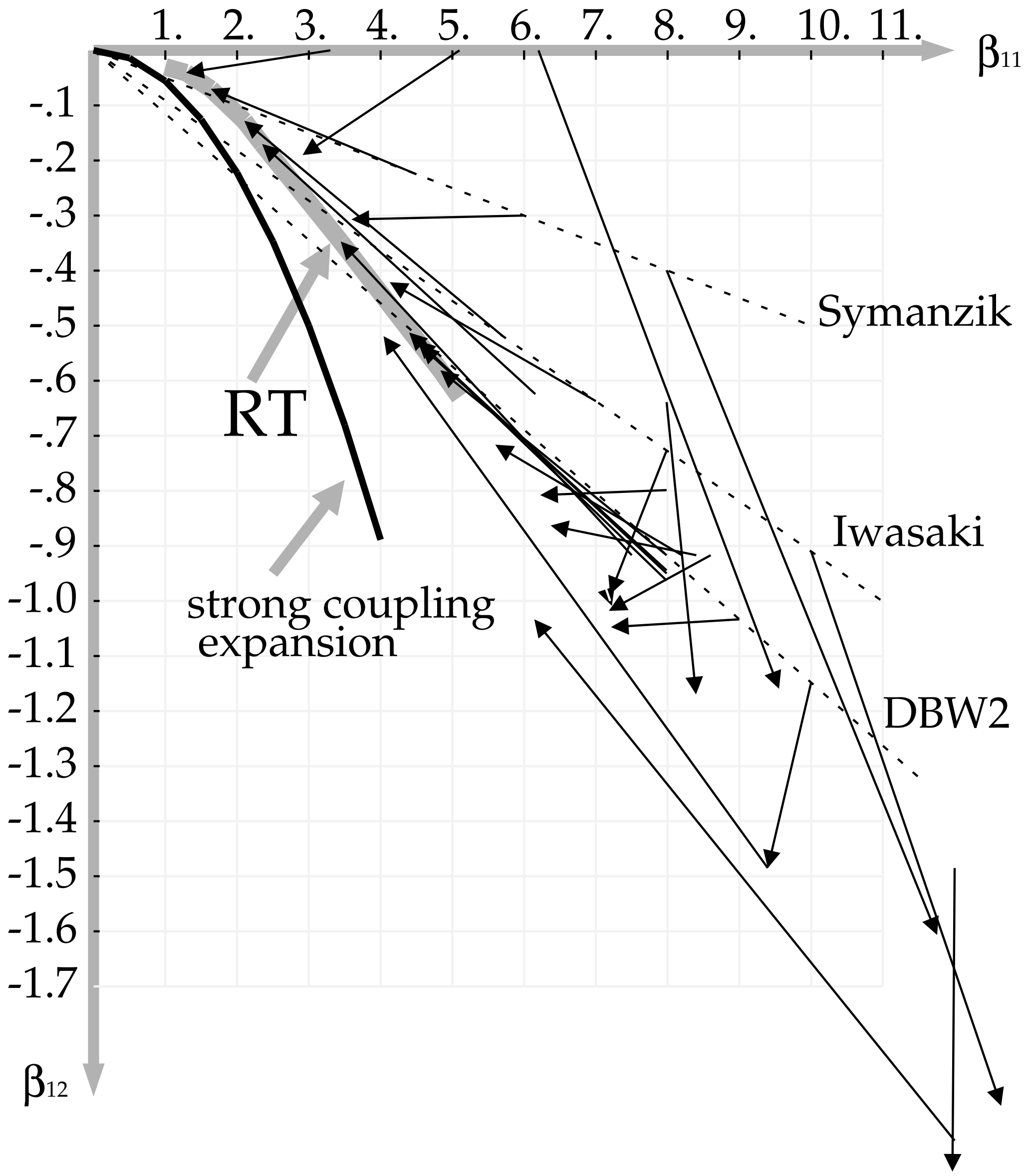
Flow in 2-D coupling space

Action:

$$S = \beta_{11} \sum ReTr(1 - \frac{1}{3}U_{11}) + \beta_{12} \sum ReTr(1 - \frac{1}{3}U_{12})$$

Swendsen blocking → Schwinger-Dyson method

[QCD-TARO, hep-lat/9806008]

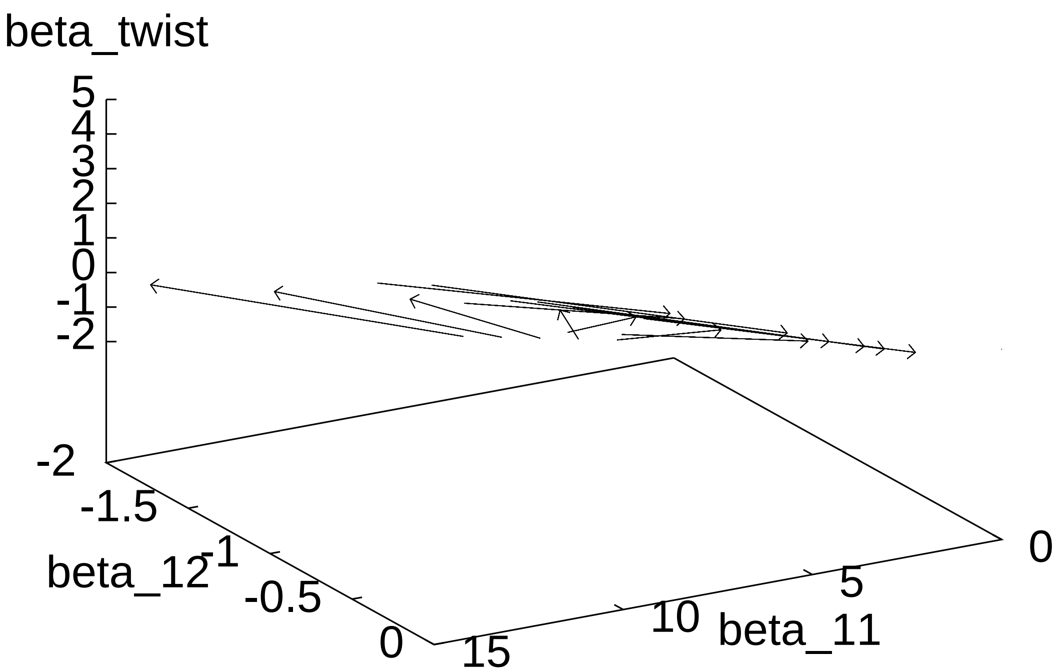


3-D Coupling Space (1)

Flow in three-coupling space:

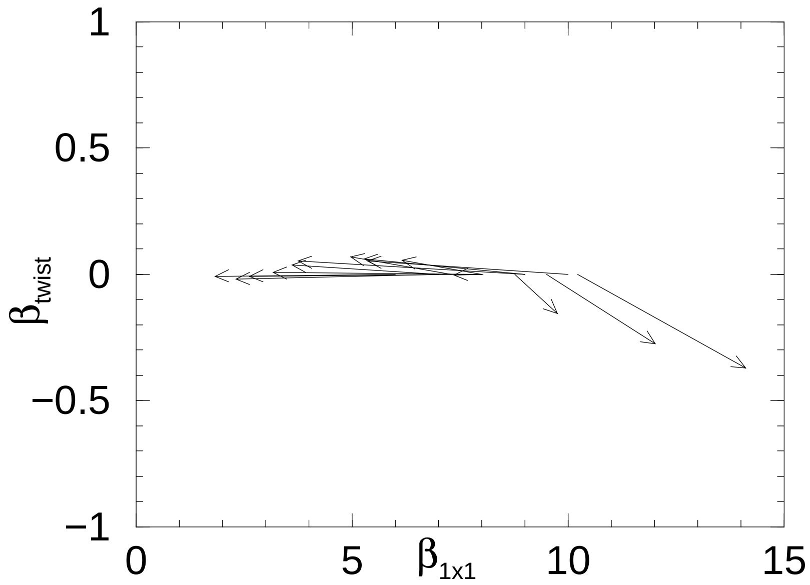
$$\begin{aligned} S = \quad & \beta_{11} \sum ReTr(1 - \frac{1}{3}U_{11}) \\ + \quad & \beta_{12} \sum ReTr(1 - \frac{1}{3}U_{12}) \\ + \quad & \beta_{twist} \sum ReTr(1 - \frac{1}{3}U_{twist}) \end{aligned}$$

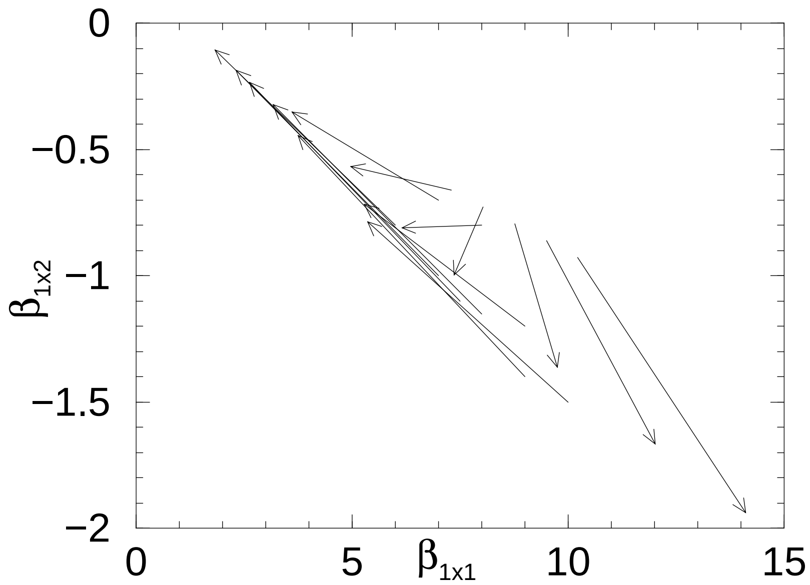
[QCD-TARO, in Lattice '97; hep-lat/9802006]



3-D Coupling Space (2)

Projection onto 2-D planes





- ◦ Flow projected on (β_{11}, β_{12}) are similar to 2-D result.
- ◦ Coupling β_{twist} is small
  - – does not change coupling flow very much.

DBW2 action (1)

DBW2 action – midway to perfect action
(Doubly Blocked Wilson action in 2 coupling space)
[QCD-TARO, in Lattice '98; hep-lat/9809086]

Near RT, actions really improved ?

Fixed c_{12}/c_{11} ($\beta_{11} = \beta c_{11}$, $\beta_{12} = \beta c_{12}$)
with $c_{11} + 8c_{12} = 1$

Twice blocked lattice from Wilson action
$\rightarrow$ Couplings determined with Demon method

$$c_{12} = -1.4088$$

c.f. Symanzik action: $c_{12} = -1/12$
Iwasaki action : $c_{12} = -0.331$

Result

Fig.1: Restration of rotational symmetry in $Q\bar{Q}$potential

$$\delta_V^2 \equiv \sum_{\text{off}} \frac{[V(R) - V_{on}(R)]^2}{V(R)^2 \delta V(R)^2} \left(\sum_{\text{off}} \frac{1}{\delta V(R)^2} \right)^{-1}$$

$V_{on}(R)$: fitting function from only on-axis data
Σ_{off} implies summation over only off-axis data

Fig.2: Scaling of $T_c/\sqrt{\sigma}$

Improvement is actually found near RT.

DBW2 action (2)

Fig.1:Rotational symmetry breaking of $Q\bar{Q}$potential

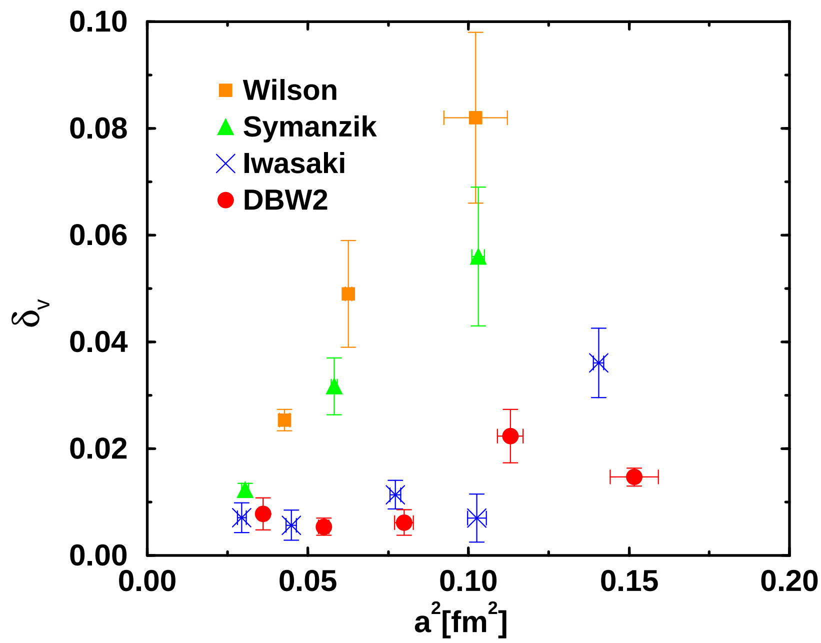
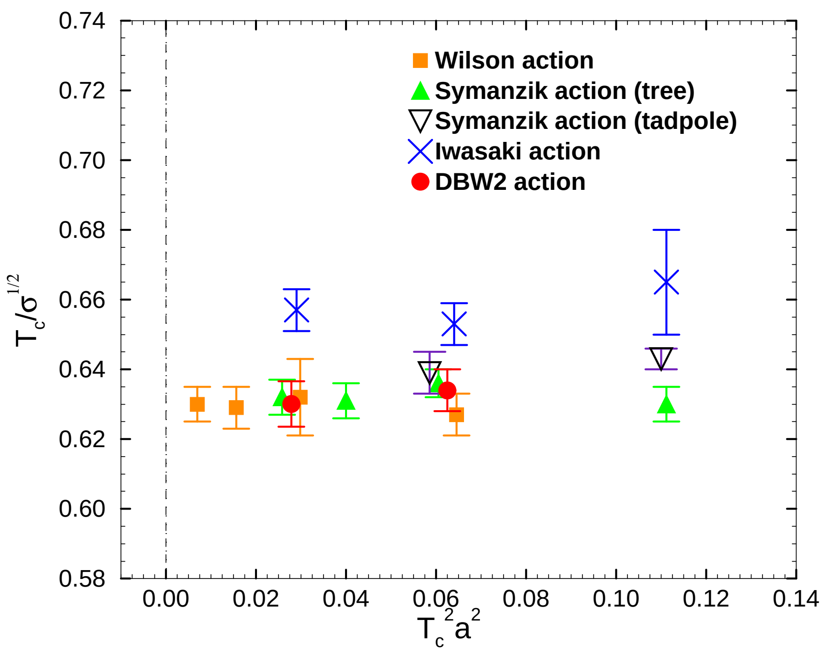


Fig.2: Scaling of $T_c/\sqrt{\sigma}$



with Iwasaki et al., Phys.Rev.D56 (1997) 151,
Beinlich et al., Eur.Phys.J. C6 (1999) 133

Flow dynamics

Phenomenological RG equation in 2-D coupling space

$$\frac{d\vec{\beta}}{da} = -2a \begin{pmatrix} \sigma + \zeta_1/a & 0 \\ 0 & 2\sigma + \zeta_2/a \end{pmatrix} \vec{\beta} + \frac{1}{a}\left[\mathbf{A}\vec{\beta} - B(\beta_n)\frac{\vec{n}}{|\vec{n}|^2}\right]$$

— (1)

where,

$$\vec{\beta} = \begin{pmatrix} \beta_{11} \\ \beta_{12} \end{pmatrix}$$

$\mathbf{A}$: 2×2 constant matrix with eigenvalues 0 and λ,

$B(x) = 12b_0 + 72b_1/x$,

with $b_0 = 33/(48\pi^2)$, $b_1 = (102/121)b_0^2$

$\vec{n}$: constant vector – set to $(1, 8)$

(from weak coupling expansion of action)

σ : string tension

ζ_1, ζ_2, components of $\mathbf{A}$ are parameters

First term describes flow in strong coupling region

Second term is for weak coupling region

Weak coupling region (1)

The first term of (1):

$$\frac{d\vec{\beta}}{dlna} = \mathbf{A}\vec{\beta} - B(\beta_n)\frac{\vec{n}}{|\vec{n}|^2}$$

Asymptotic scaling of SU(3) gauge theory
$\Rightarrow$ one of eigenvalue of A is zero,
the other is irrelevant: $\lambda < 0$.

Features described by this equation are as follows:

If $A = 0$,

$$\vec{\beta} = \beta_n \frac{\vec{n}}{|\vec{n}|^2} + c\vec{n}_\perp$$

$$\text{with} \quad a\Lambda = (\frac{\beta_n/6 + b_1/b_0}{b_0})^{b_1/2b_0^2} \exp(-\beta_n/12b_0)$$

($\beta_n = (\vec{n} \cdot \vec{\beta})$, $\vec{n}_\perp$ is orthogonal to $\vec{n}$.)

: 2-loop asymptotic scaling

Weak coupling region (2)

$A \neq 0$ case:
consider a simple case with $b_1 = 0$.
$\rightarrow$ inhomogeneous linear diff. eq.

$$\frac{d\vec{\beta}}{dlna} = \mathbf{A}\vec{\beta} - 12b_0\frac{\vec{n}}{|\vec{n}|^2} = \mathbf{A}\vec{\beta} + h_1\vec{v_1} + h_2\vec{v_2}$$

where $\vec{v_1}$ and $\vec{v_2}$ are normalized eigenvectors
for eigenvalues λ and 0,

$$\begin{aligned} h_1 &= -(12b_0/\lambda)(\vec{v_1}\cdot\mathbf{A}\vec{n})/|\vec{n}|^2, \\ h_2 &= -12b_0[(\vec{v_2}\cdot\vec{n}) - (\vec{v_2}\cdot\vec{v_1})(\vec{v_1}\cdot\mathbf{A}\vec{n})/\lambda]/|\vec{n}|^2. \end{aligned}$$

$\Downarrow$

General solution is

$$\vec{\beta} = (c_1 a^{\lambda} - h_1/\lambda)\vec{v_1} + (c_2 + h_2\ln a)\vec{v_2}$$

With increasing a,
$\vec{\beta}$ firstly approach to a degenerate line,

$$(\vec{v}_{2\perp}\cdot\vec{\beta}) = -\frac{h_1}{\lambda}(\vec{v}_{2\perp}\cdot\vec{v_1}) \qquad (\vec{v}_{2\perp} \perp \vec{v_2}, |\vec{v}_{2\perp}| = 1)$$

with power behavior in a,
and successively runs logarithmically along $\vec{v_2}$.

Strong coupling region (1)

Model equation in strong coupling region:

$$\frac{d\vec{\beta}}{da^2} = -\begin{pmatrix} \sigma_1 + \zeta_1/a & 0 \\ 0 & \sigma_2 + \zeta_2/a \end{pmatrix} \vec{\beta} \ ,$$

where σ's and ζ's are parameters.

$\Uparrow$

Strong coupling calculation of Wilson loop

$$< W(N \times M) >= (\beta_1/18)^{NM} P_{NM}(\beta_2/\beta_1^2) \ ,$$

$P_{NM}(x)$: Polynomial whose coefficients are tiling weights for filling the area by 1×1 and 1×2 actions

Requiring the area law with physical string tension σ,

$$(\beta_1/18)^{NM} P_{NM}(\beta_2/\beta_1^2) = e^{-a^2\sigma NM}.$$

Guess of $P_{NM}(\beta_2/\beta_1^2)$ for large NM:

$P_{NM} \sim e^{sNM}$ with entropy density s corresponding to choice of 1×1 or 1×2 tiles

$$\rightarrow \qquad ln(\beta_1/18) + s = -a^2\sigma$$

In a differential form, $d\beta_1/da^2 = -\sigma \quad (\sigma_1 = \sigma)$

Area law $\rightarrow \sigma_2 = 2\sigma$

ζ's control next order correction.

Fit of Numerical Results

In terms of dimensionless variable $a_s = a\sqrt{\sigma}$, (1) becomes

$$\frac{d\vec{\beta}}{da_s} = -2\begin{pmatrix} a_s + \bar{\zeta}_1 & 0 \\ 0 & 2a_s + \bar{\zeta}_2 \end{pmatrix}\vec{\beta} + \frac{1}{a_s}\left[\mathbf{A}\vec{\beta} - B(\beta_n)\frac{\vec{n}}{|\vec{n}|^2}\right]$$

where $\bar{\zeta} = \zeta/\sqrt{\sigma}$.

Parameterization of $\mathbf{A}$:

$$\vec{v_1} = (\cos\theta', \sin\theta') \quad \text{and} \quad \vec{v_2} = (\cos\theta, \sin\theta),$$

then

$$\mathbf{A} = \frac{\lambda}{\sin(\theta - \theta')}\begin{pmatrix} \sin\theta\cos\theta' & -\cos\theta\cos\theta' \\ \sin\theta\sin\theta' & -\cos\theta\sin\theta' \end{pmatrix}$$

We fit the following data to this RG equation.
– Initial condition: Wilson action ($\beta_{12} = 0$)
– Results for Symanzik, Iwasaki and DBW2 actions

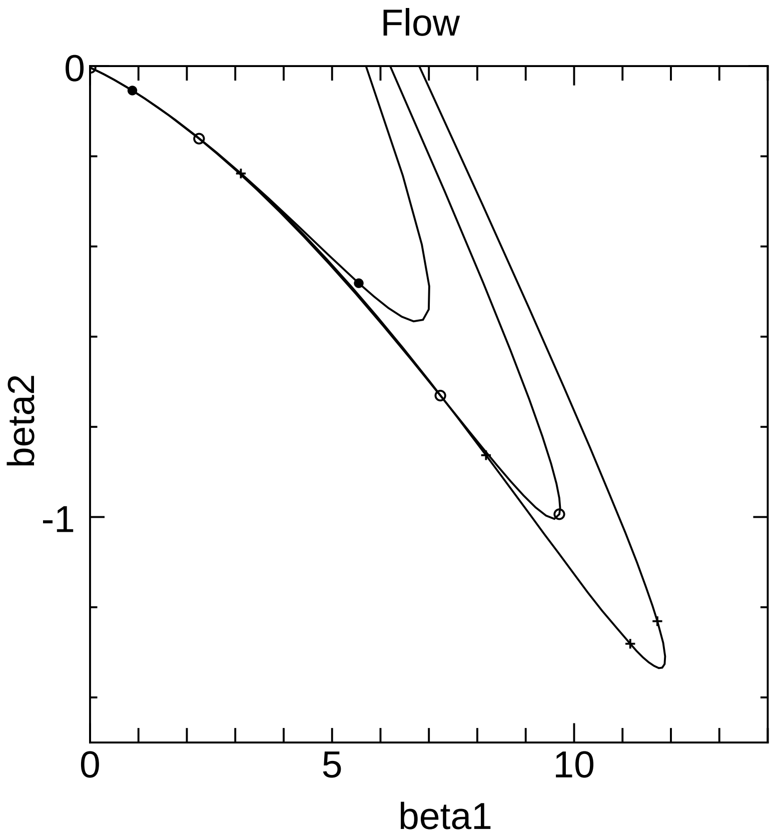
- ◦ QCD-TARO, Phys.Rev.Lett.71(1993)3063
- ◦ Borici and Rosenfelder, hep-lat/9711035 (Lattice '97)
- ◦ QCD-TARO, hep-lat/9809086 (Lattice '98)
- ◦ CP-PACS, hep-lat/9905005

Preliminary Result (1)

Preliminary result of fit:

$$\theta = -0.118, \quad \theta' = -0.218, \lambda = -2.0 \quad \zeta_1 = \zeta_2 = 0.25$$

(to show qualitative feature)



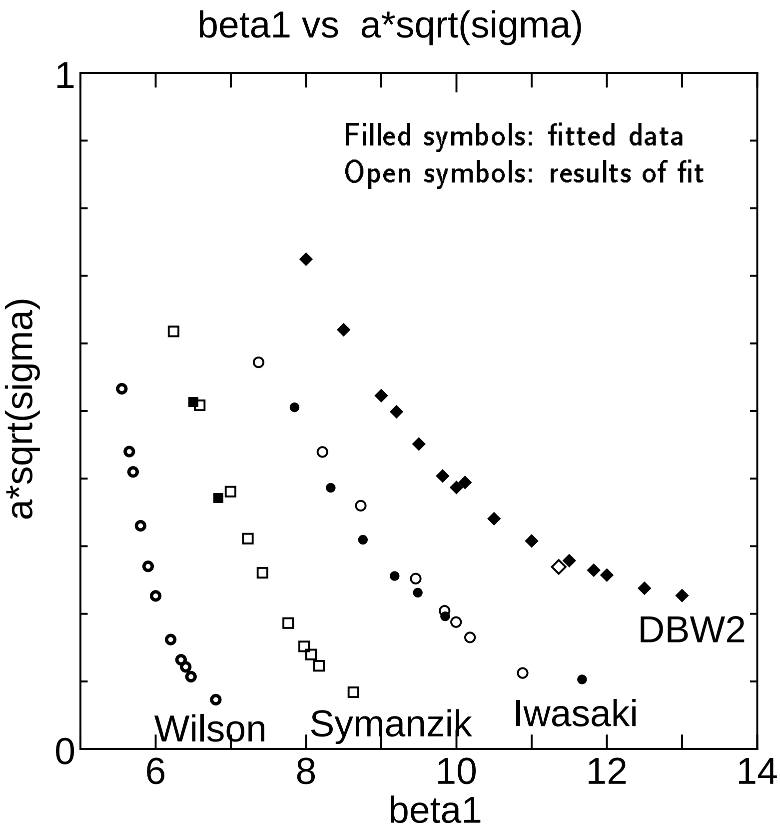
Qualitative feature of flow is well described.

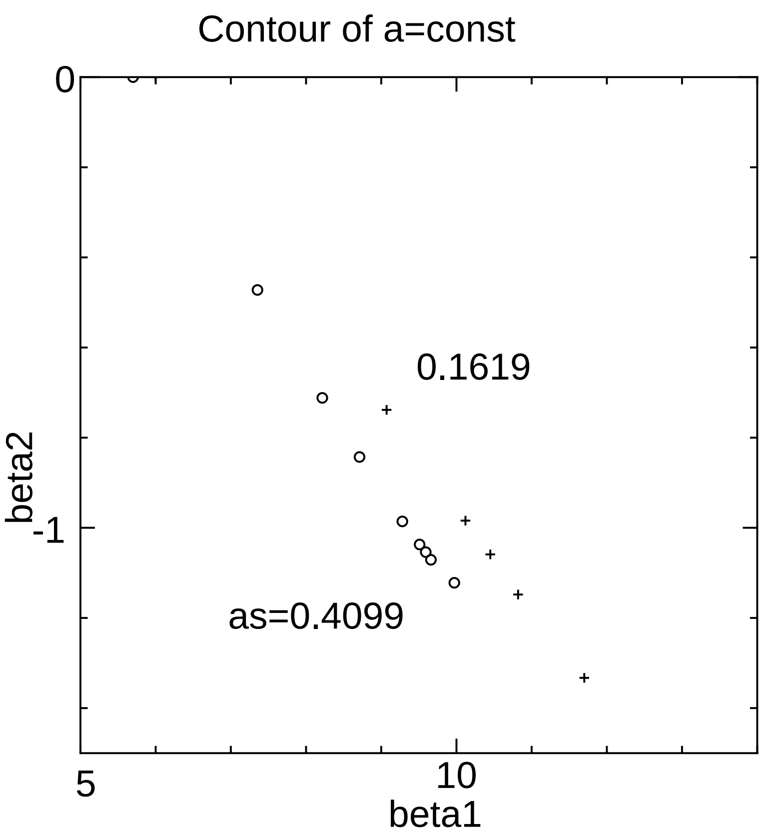
$\Downarrow$

Quantitative fit

- ○ Renormalization trajectory in 2-D space
- ○ Constant a contour

Preliminary Result (2)





Conclusion and Outlook

Summary of our previous works:

- ∘ Determination of flow in 2-D coupling space
- ∘ Comparison with 3-D space (with twist operator)
  - – c_{tist} is small.
- ∘ DBW2 action

  (obtained on twice blocked conf. from Wilson action)
  - – near to RT, rotational symmetry and scaling of $T_c/\sqrt{\sigma}$ are improved

In this work,
We found that phenomenological renormalization group equation well describe the observed feature of flow.
Preliminary fit result is encouraging.

Outlook:

- – Quantitative determination of RT and use as an improved action
- – MCRG with fermions