Mesons in Finite Temperature Lattice QCD

Hideo Matsufuru

(TIP, Univ. Heidelberg and Hiroshima Univ.)

for *QCD-TARO Collaboration*

26 November 1998
Teilchen-Tee, ITP, Univ. Heidelberg

— *CONTENTS* —

Introduction
Our Approach and Strategy
Anisotropic Lattice
Results of Simulation:
(1) t-correlators
(2) Wave functions
Outlook on Further study
Conclusion

The QCD-TARO Collaboration

Ph. de Forcrand[a], *M. García Pérez*[b], *T. Hashimoto*[c], *S. Hioki*[d], *H. Matsufuru*[e,f], *O. Miyamura*[e], *A. Nakamura*[g], *I.-O. Stamatescu*[f,h], *T. Takaishi*[i] and *T. Umeda*[e]

[a] SCSC, ETH-Zürich, Switzerland
[b] Dept. Física Teórica, Univ. Autónoma de Madrid, Spain
[c] Dept. Appl. Phys., Fac. Engineering, Fukui Univ., Japan
[d] Dept. of Physics, Tezukayama Univ., Nara, Japan
[e] Dept. of Physics, Hiroshima Univ., Japan
[f] Inst. Theor. Physik, Univ. of Heidelberg, Germany
[g] Res. Inst. for Inform. Sci. and Education, Hiroshima Univ., Japan
[h] FEST, Heidelberg, Germany
[i] Hiroshima University of Economics, Japan

Physical goals:

- Finite Temperature QCD
- Finite Chemical Potential
- MCRG and Improved Actions

Introduction

Finite temperature hadron properties:

Change of masses and width below T_c
What happens on hadrons or what new effect above T_c ?

Cntinuum theories:

- ∘ RG + Universality ($N_f = 2, 2+1; \sim T_c$)
  Rajagopal and Wilczek (1984)
- ∘ Chiral perturbation (small T)
  Leutweyler and Smilga (1990)
- ∘ QCD sum rule
  Hatsuda, Koike and Lee (1993)
- ∘ NJL model (Not closed to T_c)
  Hatsuda and Kunihiro (1984)
- ∘ Soft modes above T_c ?
  If the chiral transition is weak first order or continuum,
  Fluctuation of $\langle \bar{q}q \rangle \to \pi, \sigma$ modes
- ∘ DeTar's conjecture (1985)
  Above T_c, confining feature for scale $> 1/g^2T$
- ∘ J/ψ suppression above T_c ?
  Hashimoto et al (1986), Matsui and Satz (1986)

NJL model

NJL model analysis

c.f. Hatsuda and Kunihiro, Phys. Rep. 247 (1994) 221

- ∘ Mean-field theory
- ∘ $N_f = 2 + 1$
- ∘ Abobe T_c : Soft modes

Figure

Lattice Results

At $T > T_c$:

- Spacial correlators ($\leftrightarrow$ Screening masses)
  Correlation in π, σ sectors is large ?
  Weak in other sectors ($\sim$ free quarks)
  Born et al. (1991) ($N_f = 4$)
  No bosonic pole
  Gupta (1992) (quenched: $N_f = 0$)

- Baryon number susceptibility
  Large, $\chi_S \sim \chi_{NS}$ (isosinglet and nonsinglet)
  $\Rightarrow$ Fundamental excitations are quarks ?
  Gottlieb et al. (1987) ($N_f = 2$)

- Temporal correlators ($\leftrightarrow$ Pole masses)
  - Temperature Green function
    $\Leftarrow$ correlator in Euclidean time direction
  - On anisotropic lattice
    Hashimoto, Nakamura and Stamatescu (1993)

Our Approach

From correlators in Euclidean time direction;
- ∘ Pole masses (compared with screening masses)
- ∘ Wave function
- ∘ Spectral function

Need detailed information in t- (temperature) direction

$\Rightarrow$ *Anisotropic lattice*

[Karsch (1982),
Burgers, Karsch, Nakamura and Stamatescu (1988)]

This work:
- ∘ coarse lattice, ($a_s^{-1} = 0.85$ GeV)
- ∘ unimproved actions

$\longrightarrow$ Qualitative result, Development of procedures

Strategy

Difficulty:

Mass is extracted at $t \gg 1$.

However, at $T > 0$, temporal extent is short.

$\Rightarrow$ Choice of hadronic operator is significant.

Investigate following questions.

(1) Define the "hadronic operator" as one which has sufficiently large overlap with corresponding states. Then, what happens on this operator at $T > 0$?

(2) Is there bound state at $T > T_c$?
Haw can we show that ?

(3) Develop reliable proceedure to extract the pole masses with short extent in t-direction.

Anisotropic lattice

Gauge field action:

$$S_G(\beta,\gamma) = \sum_{\vec{x},i,j} \left[\frac{\beta}{\gamma} U_{ij}(x) + \beta\gamma U_{i4}(x) \right]$$

where

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x).$$

Quark fiels: Anisotropic Wilson action

$$S_F(\kappa_s,\gamma_F) = \sum_x \bar{q}(x) K[U](x,y) q(y),$$

$$K[U](x,y) = \delta_{x,y} - \sum_\mu \kappa_\mu \left[\ (1-\gamma_\mu)\ U_\mu(x)\,\delta_{x+\hat{\mu},y} + (1+\gamma_\mu)\ U_\mu^\dagger(x-\hat{\mu})\,\delta_{x-\hat{\mu},y} \ \right]$$

$$\kappa_i = \kappa_s, \qquad \kappa_4 = \kappa_t = \gamma_F \kappa_s$$

In quenched simulation,

Gauge field with certain β, γ

$\Downarrow$

gluonic observable $\Rightarrow$ $\xi = a_s/a_t$
$e.g.$ Wilson loop

Quark propagator with (κ, γ_F)

calibration: find γ_F which gives $\xi_F = \xi$
$e.g.$ $\xi_F = m^{(t)}/m^{(s)}$

Simulation Parameters

Lattice:
$12^3 \times N_t$,$\beta = 5.68$, $\gamma = 4.0$, quenched
$N_t = 72$ $(T \simeq 0)$, 20 $(T < T_c)$, 16, 12 $(T > T_c)$: $T = 1/N_t a_t$

- #conf. = 60
- Anisotropy: $\xi \equiv a_s/a_t = 5.3(1)$
  from the ratio of Wilson loops
  Engels, Karsch and Scheideler (1997), Klassen (1998)
- Cutoff: $a_s^{-1} = 0.85$ GeV, $a_t^{-1} = 4.5(2)$ GeV
  from heavy quark potential

Quark: Anisotropic Wilson action

- Hopping parameter and bare anisotropy:

κ_s	γ_F	m_q	m_{PS}	m_V [GeV]
0.0810	4.05	0.17	0.81	0.90
0.0840	3.89	0.12	0.68	0.80
0.0860	3.78	0.10	0.61	0.75

- γ_F determined by calibration
- Periodic b. c. for spatial direction

Correlators

Measure the wave function

$$w_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t) O_\Gamma^\dagger \rangle$$

where, $$O_\Gamma(y) = \bar{q}^{(\varphi')}(y)\Gamma q^{(\varphi)}(y),$$

$$q^{(\varphi)}(y) = \sum_{\vec{r}} \varphi(\vec{r}) q(\vec{y}+\vec{r}, y_4)$$

$\varphi(\vec{r})$: smearing function

We use "exp" smearing function:

$$\varphi(\vec{r}) = \exp(-ar^p)$$

with a, p extracted from observed wave function as well as "point" source: $\varphi(\vec{r}) = \delta_{\vec{r},0}$.

$\Rightarrow$ three sets of source smearing:

"point-point", "point-exp", "exp-exp"

Smearing Function

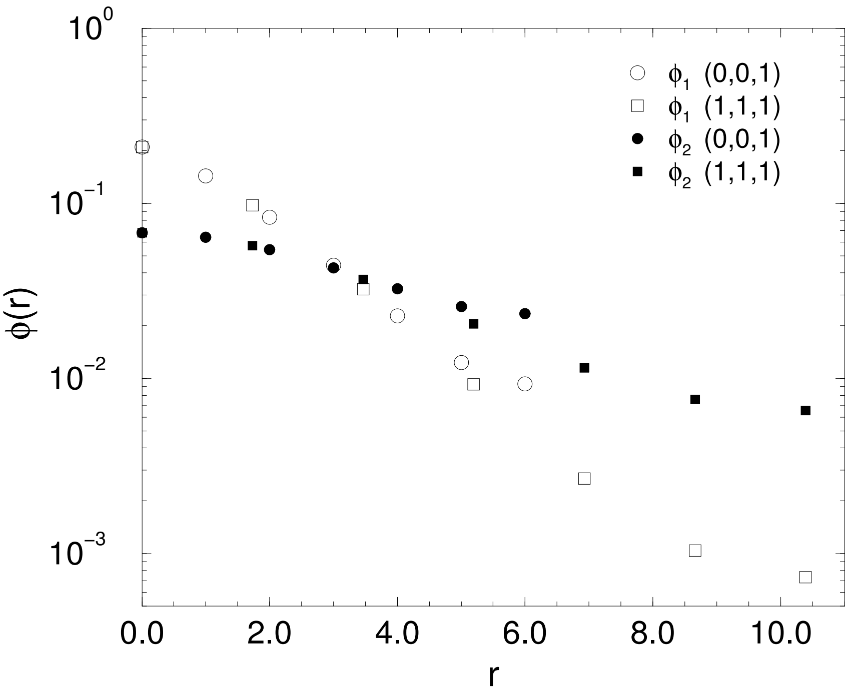
"exp" smearing function:

$$\varphi(\vec{r}) = \exp(-ar^p)$$

$\rightarrow$ "exp-exp" source smearing corresponds to the convolution of two $\varphi(r)$

$$\varphi^{(2)}(\vec{r}) = \int d^3x \varphi(\vec{r}+\vec{x})\varphi(\vec{r})$$

$\kappa_s = 0.081$ $(m_q \sim 2m_s)$



t-correlators

Correlators in Euclidean time direction:

$$C_\Gamma(t) = w(\vec{r}=0,t) = \sum_{\vec{x}} \langle O_\Gamma(\vec{x},t) O_\Gamma^{(\varphi)\dagger}(0,0) \rangle$$

Effective mass: $m^{(eff)}$ s.t.

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m^{(eff)}(N_t/2 - t)]}{\cosh[m^{(eff)}(N_t/2 - t - 1)]}$$

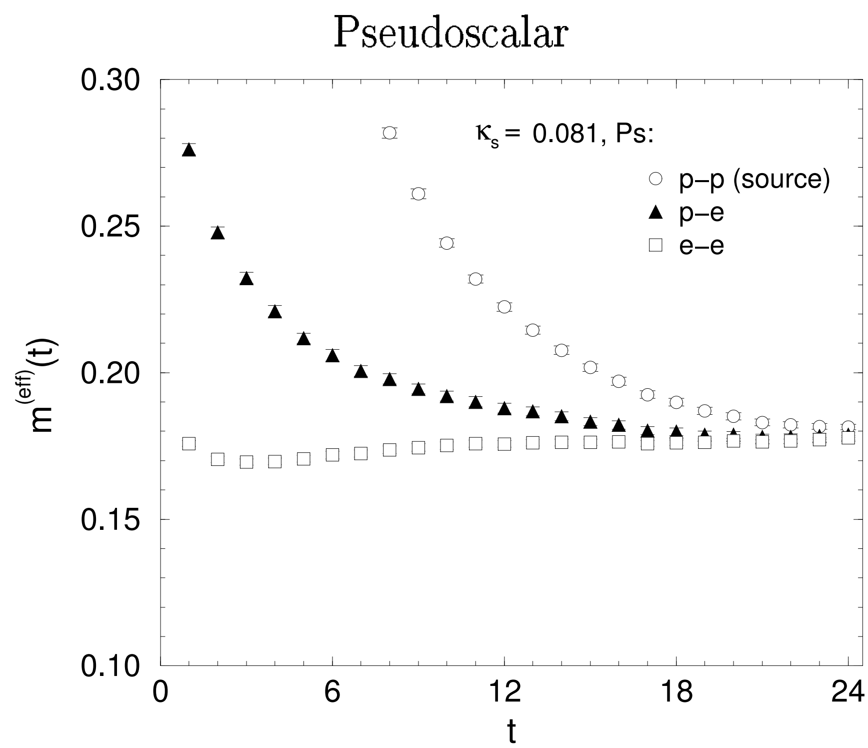
For large t, $m^{(eff)} \to m_\Gamma$

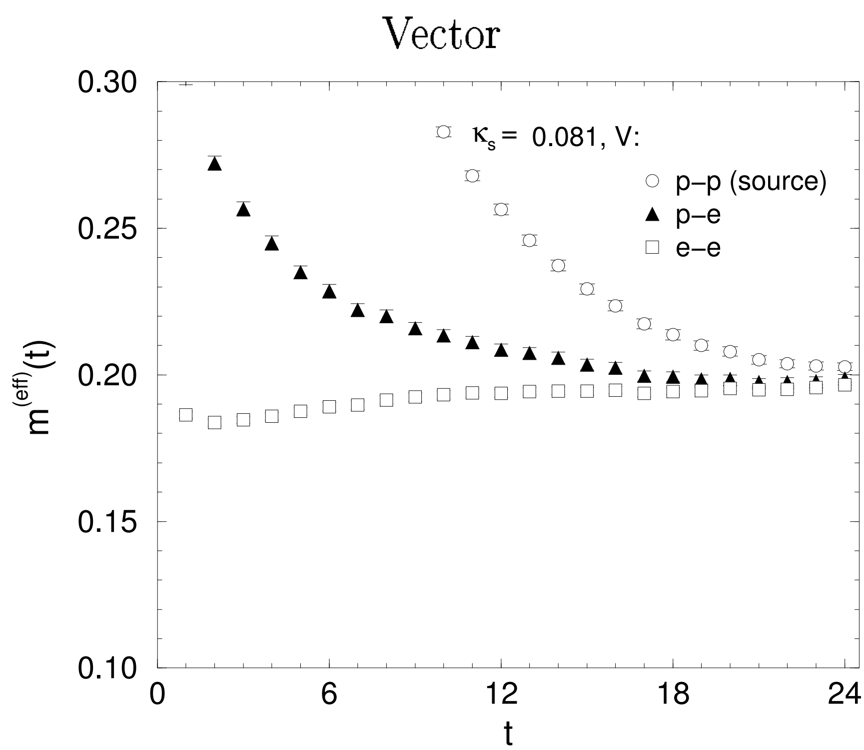
Problem at finite temperature:

- ◦ Temporal extent is not enough for $m^{(eff)} \to m_\Gamma$
- ◦ Even in free quark case, ficticious plateau is observed

Effective Mass Plots at $N_t = 72$

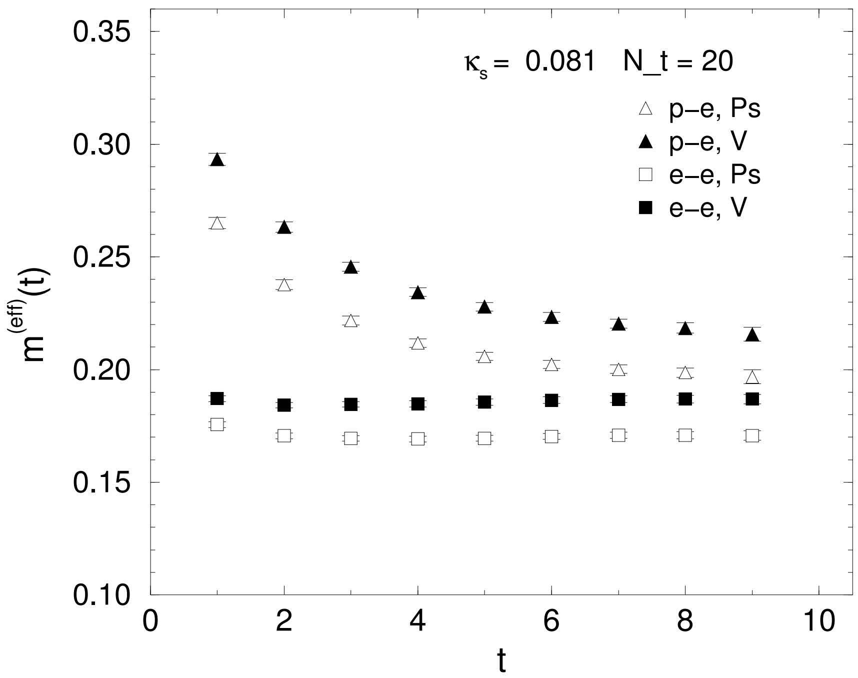
$$\kappa_s = 0.081 \; (m_q \sim 0.17 GeV)$$



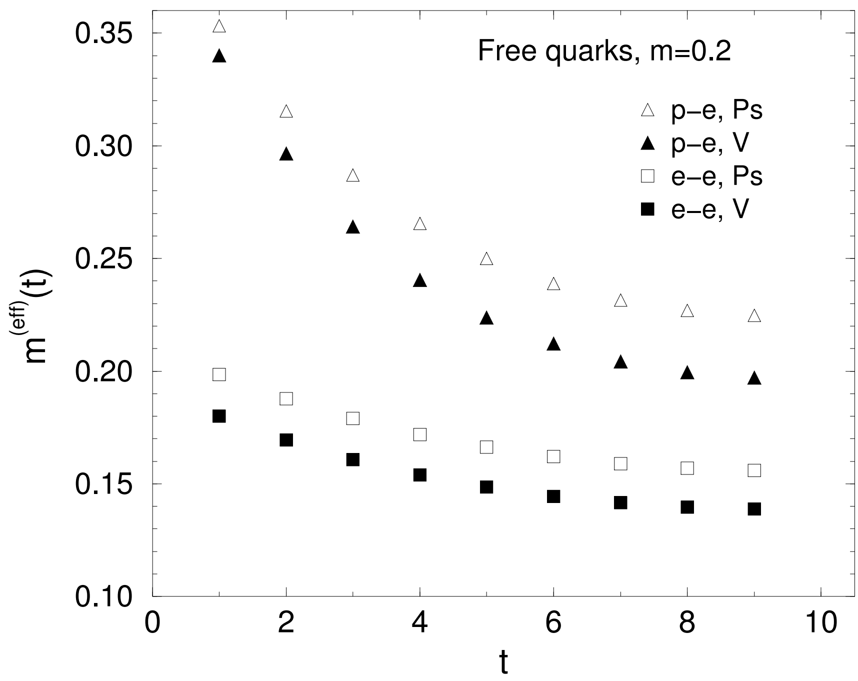


Effective Mass Plots at $N_t = 20$

$\kappa_s = 0.081\ (m_q \sim 0.17GeV)$

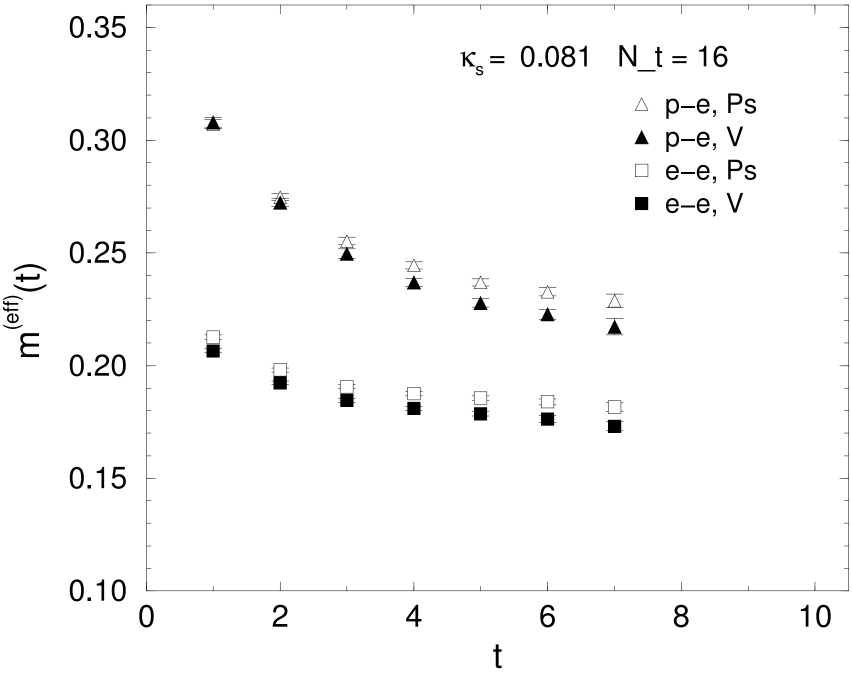


Free quarks ($m = 0.2$ in lattice unit)

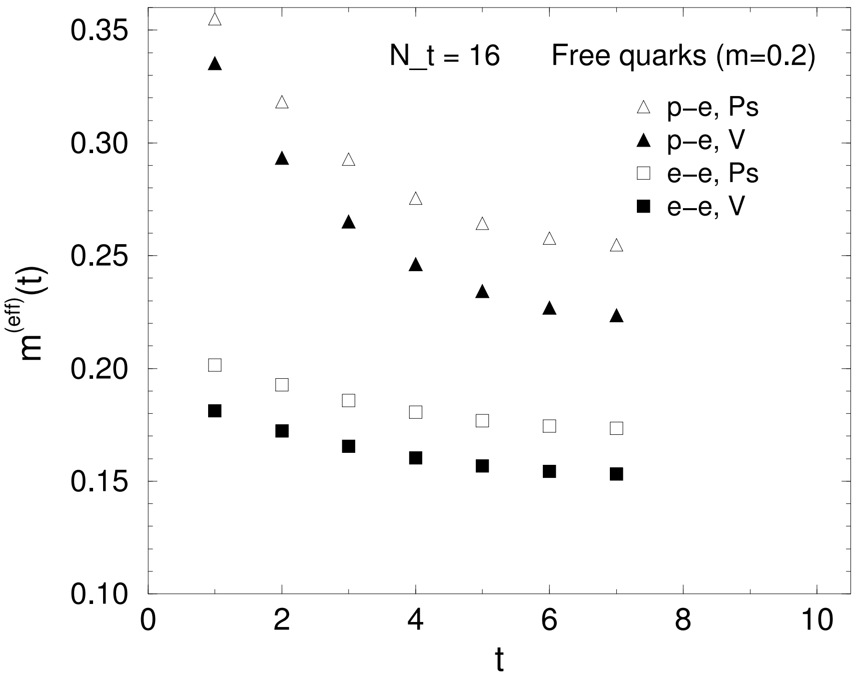


Effective Mass Plots at $N_t = 16$

$\kappa_s = 0.081 \ (m_q \sim 0.17 GeV)$

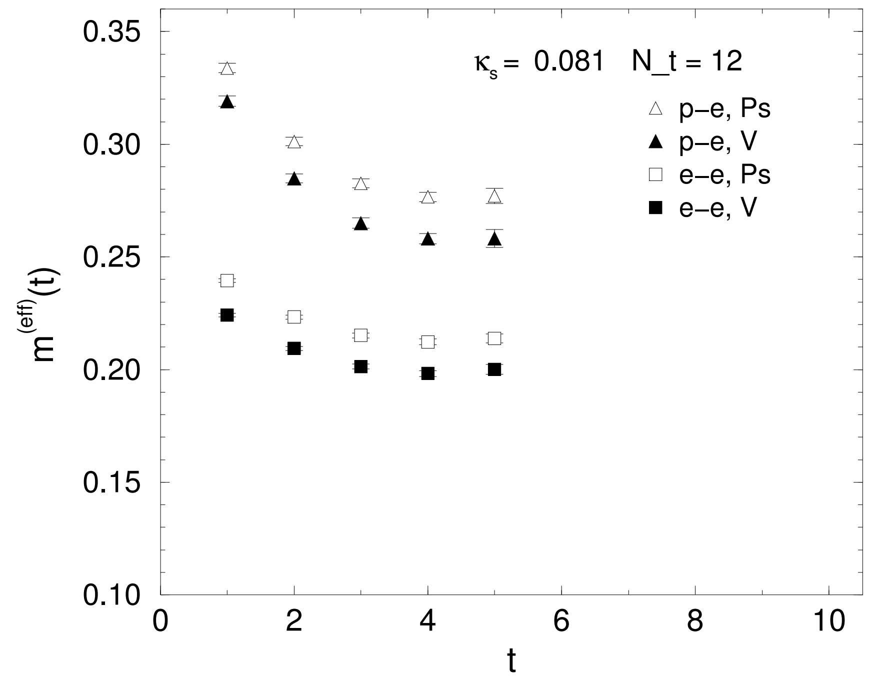


Free quarks ($m = 0.2$ in lattice unit)

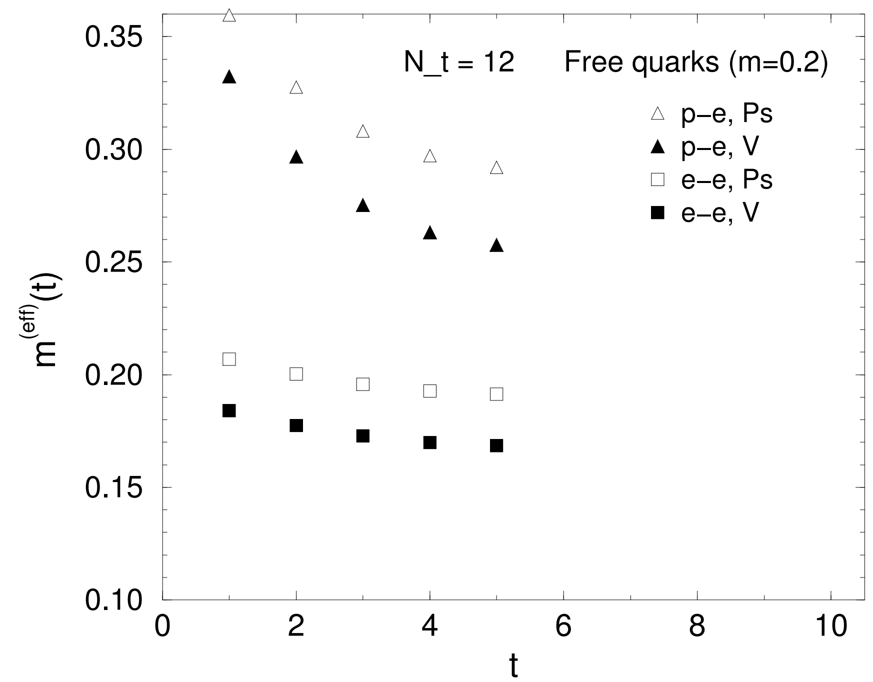


Effective Mass Plots at $N_t = 12$

$\kappa_s = 0.081 \ (m_q \sim 0.17 GeV)$



Free quarks ($m = 0.2$ in lattice unit)



Answer to (1)

At $N_t = 72$, "exp-exp" correlator almost have required property: large overlap with meson states
(More detailed analysis — e.g. Variational analysis)

$\Downarrow$

Observe the temperature dependence of them:
"masses" extracted from exp-exp correlators

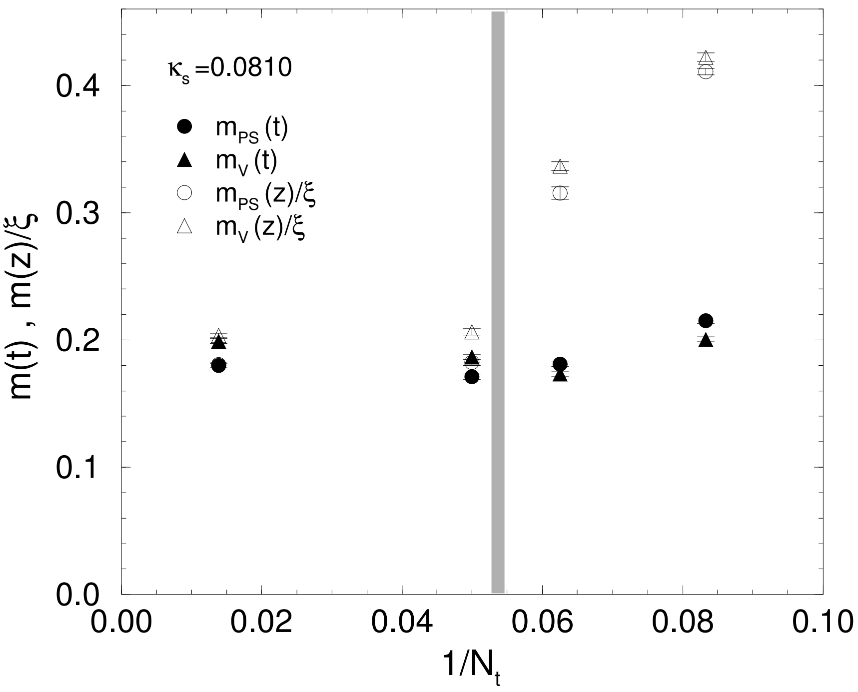
At finite temperature,

- ∘ "exp-exp" and "point-exp" correlators do not give the same effective mass
- ∘ Difficult to find clear plateau

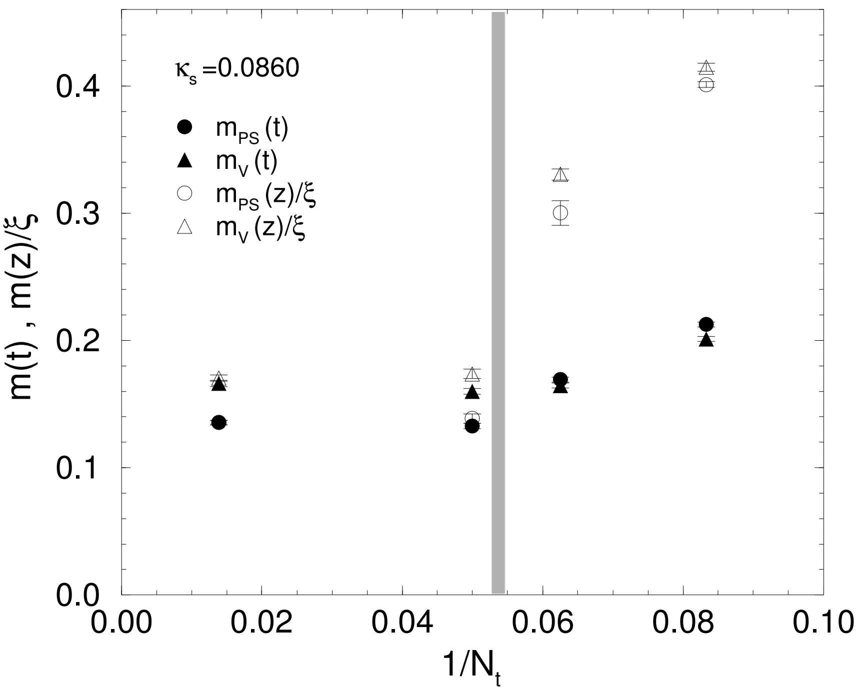
Uncertainty of extracted "mass" values are large (20–30 %)

Temperature dependence of "masses"

$\kappa_s = 0.081 \ (m_q \sim 0.17GeV)$



$\kappa_s = 0.086 \ (m_q \sim 0.10GeV)$



Chiral extrapolation

Naive relation

$$\frac{1}{\kappa} \equiv \frac{1}{\kappa_s} - 2(\gamma_F - 1) = 2(m_0 + 4)$$

$\Downarrow$

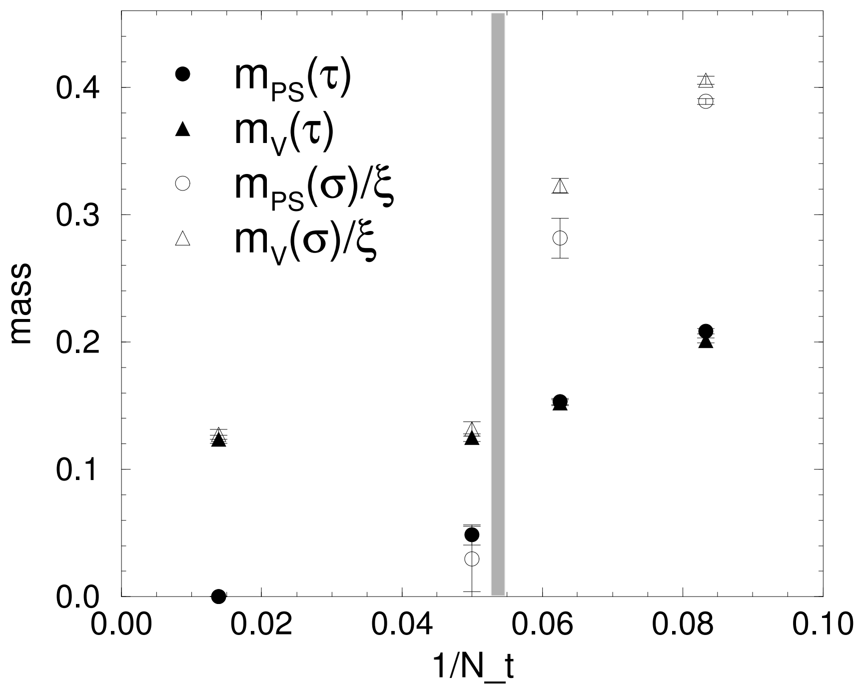
Extrapolation in $1/\kappa$

Ps meson:

$T = 0$ — m_{PS}^2 linear in $1/\kappa \Rightarrow \kappa_c$
$T < T_c$ — m_{PS}^2 linear in $1/\kappa$
$T > T_c$ — m_{PS} linear in $1/\kappa$

V meson: linear in $1/\kappa$

$\kappa = \kappa_c$



t-dependence of the wave function

$$w_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)O^\dagger(0)\rangle$$

If there is no bound state (like free quark case), wave function become broader as t.

In this case,

$$\phi_\Gamma(r,t) = w_\Gamma(r,t)/w_\Gamma(r=0,t)$$

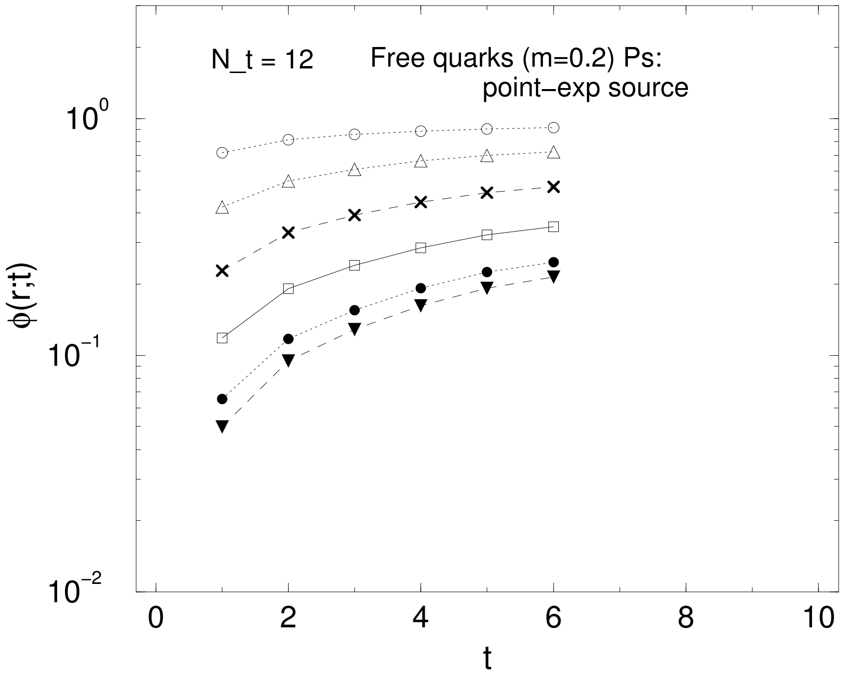
(normalized at spatial origin)

increase as t.

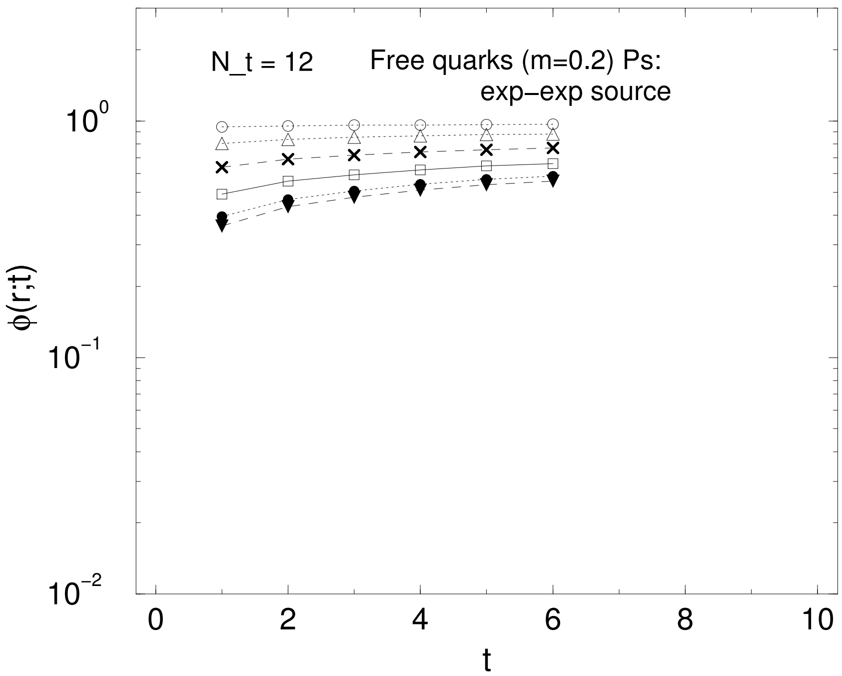
t-dependence of the wave function $(N_t = 12)$

Free quarks ($m - 0.2$ in lattice unit)

point-exp source



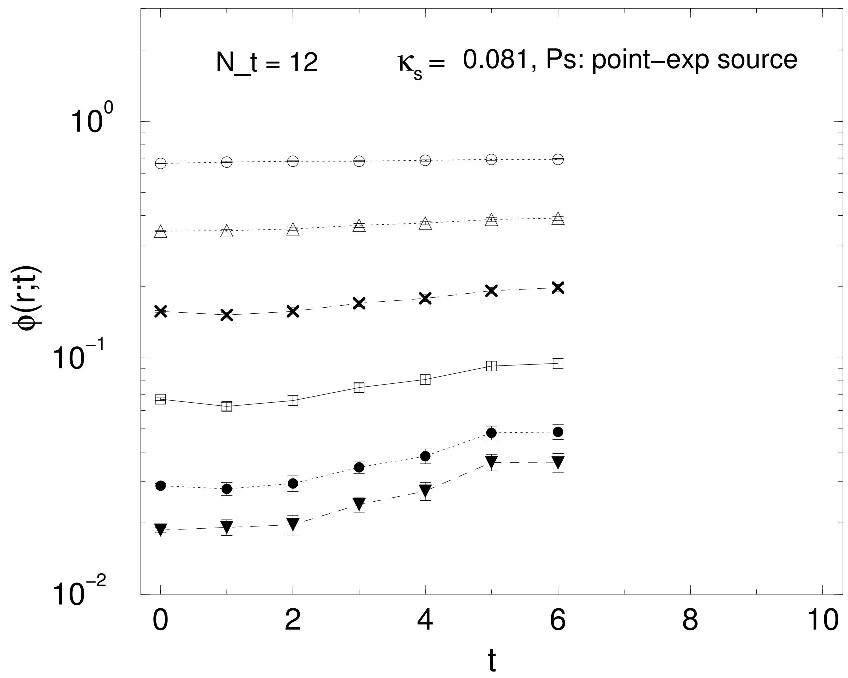
exp-exp source



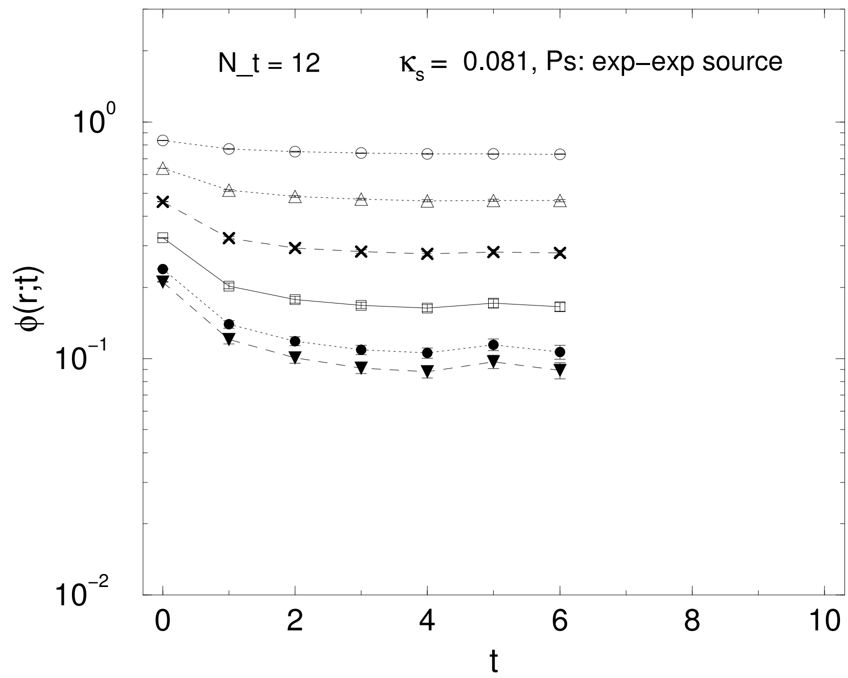
t-dependence of the wave function $(N_t = 12)$

$\kappa_s = 0.081$ $(m_q \sim 2m_s)$

point-exp source



exp-exp source



Answer to (2)

The wave function with "exp-exp" source become narower as t at all T.

- $\phi(r,t)$ should approaches certain shape
- exp-exp source (convolution of two φ(r)) is too broad

It sugests there are bound states even above T_c.

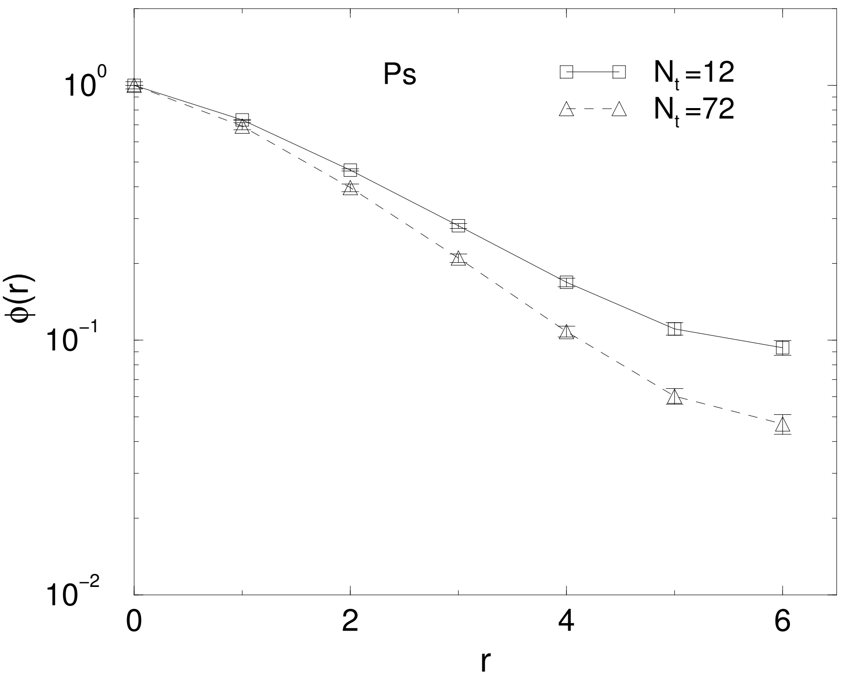
Further analysis:

- Wave functions with various sources
- Mix point-exp and exp-exp correlators to give
  flat t-dependence

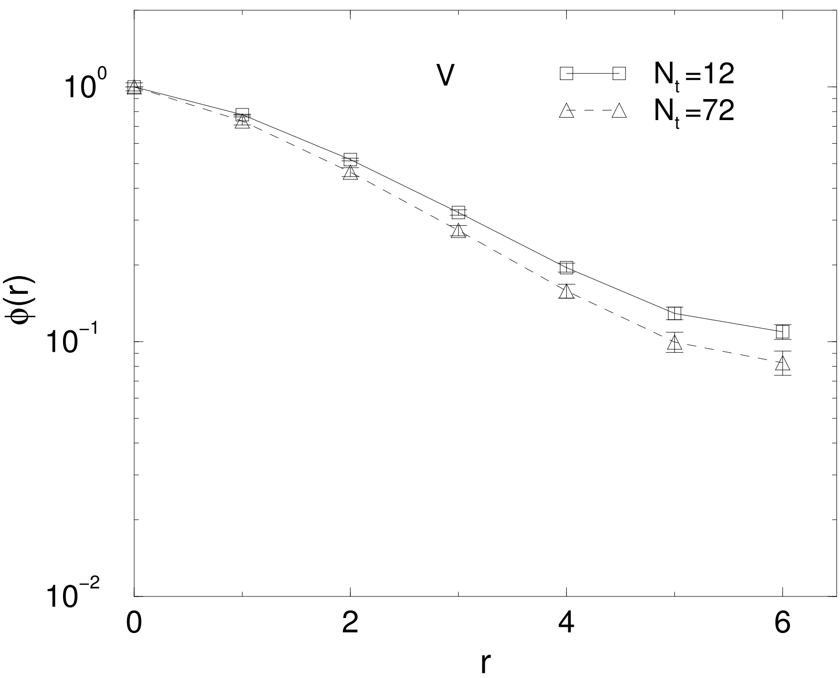
Wave functions (from exp-exp correlators)

$\kappa_s = 0.081 \ (m_q \sim 0.17GeV)$

Pseudoscalar



Vector



Outlook on (3)

Possible analysis:
∘ Variational analysis
∘ using the wave function
— *in progress*

Spectral function may have broad width at $T > 0$
In this case, extraction of mass as the peak position of spectral function is difficult

$$\Downarrow$$

Direct determination of the spectoral function from correlator:
Study in progress
QCD-TARO, in Lattice 97

Conclusion

(1) We observed the temperature dependence of "exp-exp" correlator, as the best operator at $T = 0$. Clear T-dependece of exracted masses were observed. Uncertainty of them are rather large at $T > 0$. ☺

(2) Observed t-dependence of the wave function suggest there is certain bound state up to 1.5 T_c. Further verification is necessary. ☺

(3) Now under progress. For example, direct determination of the spectral function from the correlator is investigated.

Outlook

Other channels: S, A, N
Relation with topological quantities
Precise determination of mass shift
Spectral function

$\Rightarrow$ Larger, improved lattice

With dynamical quarks