Lattice Study of $B \to \pi$ Semileptonic Decay using Nonrelativistic QCD

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CONTENTS

$B \to \pi$ Semi-leptonic Decay

$$\langle \pi | V_\mu | B \rangle = \left(p + k - q \frac{m_B^2 - m_\pi^2}{q^2} \right)_\mu f^+(q^2) + q_\mu \frac{m_B^2 - m_\pi^2}{q^2} f^0(q^2)$$

f^+, f^0 : form factors $\qquad f^+(q^2 = 0) = f^0(q^2 = 0)$

$$\frac{d\Gamma}{dq^2} = (\text{known factor}) \cdot |V_{ub}|^2 \cdot |f^+(q^2)|^2$$

Lattice calculation of f^+, f^0

$\Rightarrow$ Determination of V_{ub}
Comparison with HQET prediction

Lattice NRQCD

Heavy quark $\Rightarrow$ Lattice Non-relativistic QCD
with $O(1/m_Q)$ corrections

Evolution equation

$$G_\varphi(t=1) = \left(1-\frac{1}{2n}H_0\right)^n U_4^\dagger \left(1-\frac{1}{2n}H_0\right)^n G_\varphi(t=0)$$
$$G_\varphi(t+1) = \left(1-\frac{1}{2n}H_0\right)^n U_4^\dagger \left(1-\frac{1}{2n}H_0\right)^n (1-\delta H)\, G_\varphi(t)$$

$$H_0 = -\frac{1}{2m_Q}\vec{D}^2 \qquad \cdots \text{ kinetic term}$$
$$\delta H = -\frac{1}{2m_Q}\,\vec{\sigma}\cdot\vec{B} \qquad \cdots \text{ spin-magnetic interaction}$$

n : stabilization parameter $\left(n < 3/2m_Q\right)$

$Q(x)$ in QCD $\leftrightarrow$ $\varphi(x)$ (2 spinor) in NRQCD

$$Q(x) = \begin{pmatrix} \varphi(x) \\ -\frac{i}{2\,m_Q}\,\vec{\sigma}\cdot\vec{D}\,\varphi(x) \end{pmatrix}.$$

Mean-field improvement

$$U_\mu \to U_\mu/u_0, \qquad u_0 = \langle\frac{1}{3}\mathrm{Tr}U_{plaq}\rangle^{1/4}$$

Correlation Functions

Solve the evolution eq. for $t \geq t_s$
with the source $V^\dagger(x_s) S_q(x_s)$

3-point correlation function

$$
\begin{aligned}
& C^{(3)}_\mu(p,k;t_f,t_s,t_i) \\
&= \sum_{\vec{x}_f}\sum_{\vec{x}_s} e^{-i\vec{p}\cdot\vec{x}_f}\, e^{-i(\vec{k}-\vec{p})\cdot\vec{x}_s}\, \langle O_B(t_f,\vec{x}_f) V^\dagger_\mu(t_s,\vec{x}_s) O_\pi(t_i,0)\rangle \\
&\to \tilde{Z}_\pi(k)\, e^{-E_\pi(k)\cdot(t_s-t_i)}\, \tilde{Z}_B(p)\, e^{-E_{\bar{q}Q}(p)\cdot(t_f-t_s)} \times \frac{\langle B(p)|V^\dagger_\mu|\pi(k)\rangle}{\sqrt{2E_\pi(k)}\sqrt{2E_B(k)}}
\end{aligned}
$$

$O_\pi(x)$, $O_B(x)$: Local interpolating operators

2-point correlation function

$$
\begin{aligned}
C^{(2)}_\pi(k;t,t_i) &= \sum_{\vec{x}} e^{-i\vec{k}\cdot\vec{x}} \langle O_\pi(x) O_\pi(x_i)\rangle \to \tilde{Z}^2_\pi(k)\, e^{-E_\pi(k)(t-t_i)} \\
C^{(2)}_B(p;t,t_i) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle O_B(x) O_B(x_i)\rangle \to \tilde{Z}^2_B(p)\, e^{-E_{\bar{q}Q}(p)(t-t_i)}
\end{aligned}
$$

Simulation Parameters

$16^3 \times 32$,$\beta = 5.8$, quenched, #conf. $= 120$

- $a^{-1} \sim 1.7$ GeV (from m_ρ)
- On Intel Paragon XP/S (56 nodes)
  in INSAM, Hiroshima Univ.

Light quark = Wilson fermion

- $\kappa = 0.1570,\ 0.1585,\ 0.1600$ ($m_q \sim 2m_s$ - m_s)
- Diriclet boundary condition for temporal ($U_4(31) = 0$)
- KLM normalization $(u_0^{(\kappa_c)} = 1/8\kappa_c)$

Heavy quark : NRQCD with $O(1/m_Q)$ terms

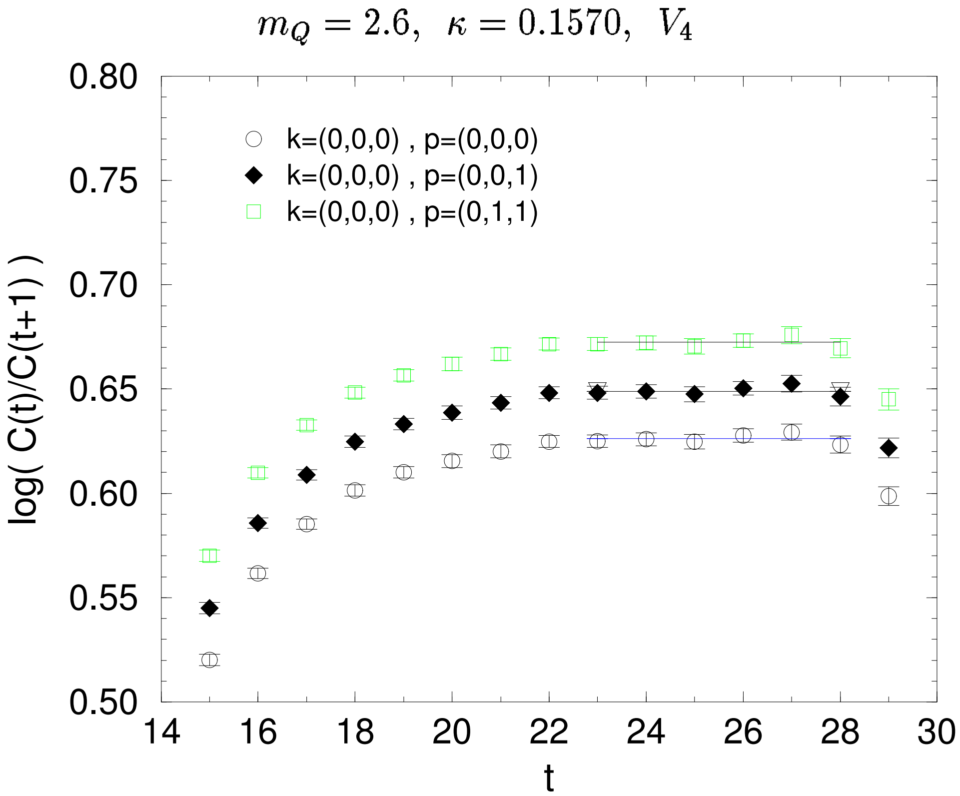
m_Q	5.0	2.6	2.1	2.1	1.5	1.2	1.2	0.9
n	1	1	1	2	2	2	3	2

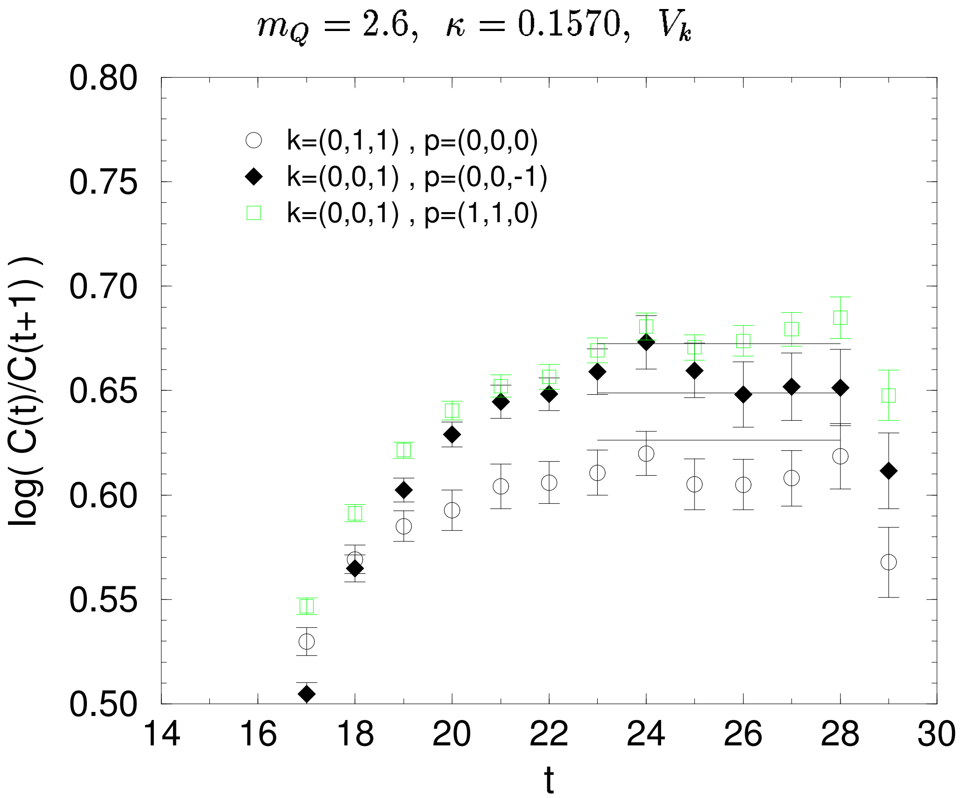
- Roughly $m_Q = 2.6 \sim b$-quark, $0.9 \sim c$-quark
- Mean-field improved evolution equation (with $u_0^{(plaq)}$)

3-point correlation functions

- 20 sets of ($\vec{p}, \vec{k}$) $\cdots$ $|\vec{p}|^2, |\vec{k}|^2 \leq 3 \cdot 2\pi/16$
- Source : $O_\pi^\dagger(x_i)$ – Current : $V^\dagger(x_s)$ – Sink : $O_B(x_f)$
  $t_i = 4$ $t_s = 14$ $t_f = 23 - 28$

Effective plots





Matrix Elements

Raw number from simullation :

$$\hat{V}_\mu(\,\vec{p},\vec{k}\,) \equiv \frac{\tilde{Z}_\pi(k)}{C_\pi^{(2)}(k;t_s,t_i)} \frac{e^{E_{\bar{q}Q}(t_f-t_s)}}{\tilde{Z}_B(p)} C_\mu^{(3)}(p,k;t_f,t_s,t_i)$$

: expressed in terms of numerical quantities only

(No assumption such as dispersion relation)

$$\hat{V}_\mu(\,\vec{p},\vec{k}\,) \;=\; \frac{\langle\,\pi\,(\,\vec{k}\,)\,|\,V_\mu\,|\,B\,(\,\vec{p}\,)\,\rangle}{\sqrt{2\,E_\pi(k)}\,\sqrt{2\,E_B(p)}}$$

Heavy quark effective theory (HQET)

is realized at small momentum region

($|\vec{k}|, |\vec{p}| \ll m_Q$, $q^2 \sim q^2_{max}$)

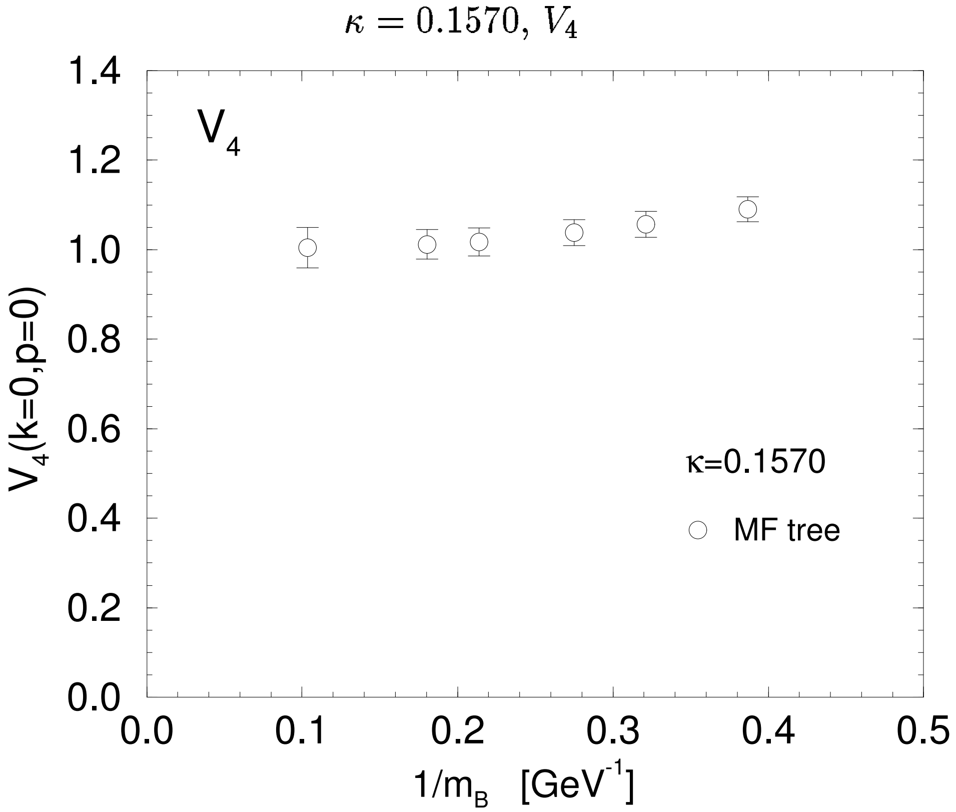
HQET tells us,

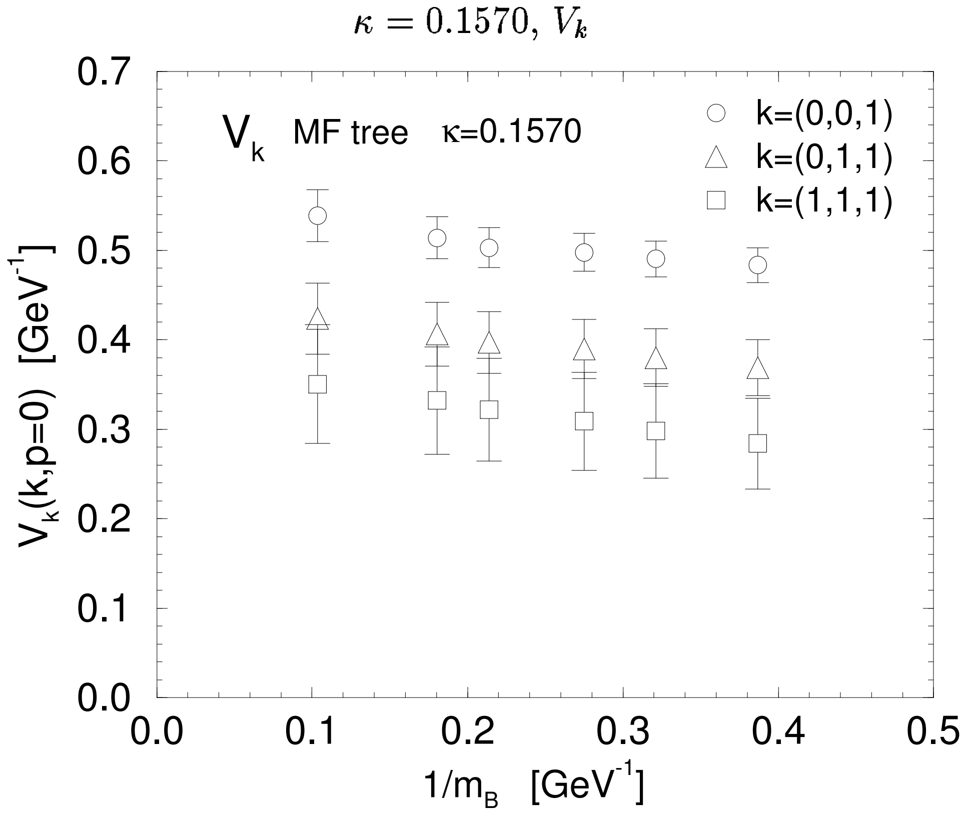
$$\hat{V}_4(\,\vec{p}=0,\vec{k}=0\,) \;=\; V_4^\infty\left(1+\frac{c_1}{m_B}+\cdots\right)$$

$$\hat{V}_k(k) \;\equiv\; \frac{\sum_i k_i\cdot\tilde{V}_i(\,\vec{p},\vec{k}=0\,)}{\vec{k}^2} \;=\; V_k^\infty\left(1+\frac{c_1'}{m_B}+\cdots\right)$$

$$\hat{V}_p \;\equiv\; \lim_{\vec{p}\to 0}\frac{\sum_i p_i\cdot\tilde{V}_i(\,\vec{p},\vec{k}=0\,)}{\vec{p}^{\,2}} \;=\; \frac{1}{m_B}V_p^\infty\left(1+\frac{c_1''}{m_B}+\cdots\right)$$

Matrix elements (mean-field tree)





Form Factors

$$\langle\, \pi\,(\,\vec{k}\,)\,|\, V_\mu \,|\, B\,(\,\vec{p}\,)\,\rangle = \sqrt{2\,E_\pi(k)}\,\sqrt{2\,E_B(p)}\;\hat{V}_\mu(\,\vec{p},\vec{k}\,)$$

Dispersion relation :

We assume,

$$E_\pi = \sqrt{m_\pi^2 + \Sigma_i\, 4\sin^2(k_i/2)}$$

$$E_B = \sqrt{m_B^2 + \Sigma_i\, 4\sin^2(p_i/2)}$$

where, $m_B = m_Q + E_{\bar{q}Q}(\vec{p}=0)$

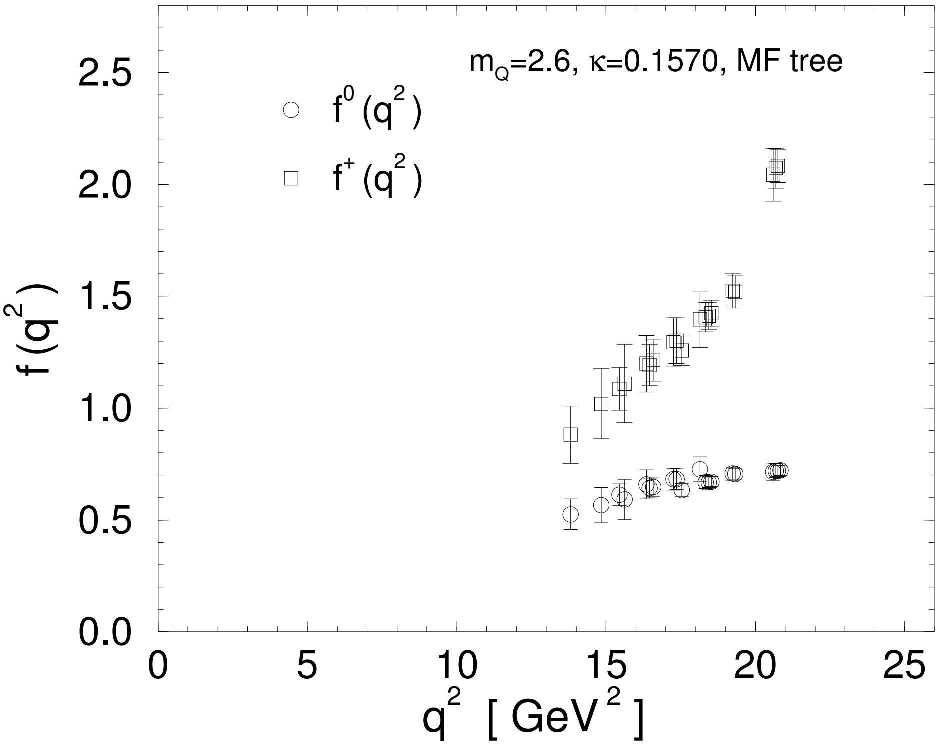
$\Rightarrow$ Form factors $f^+(q^2),\ f^0(q^2)$

$$q^\mu\langle\, \pi \,|\, V_\mu \,|\, B\,\rangle = (m_B^2 - m_\pi^2)\; f^0(q^2)$$

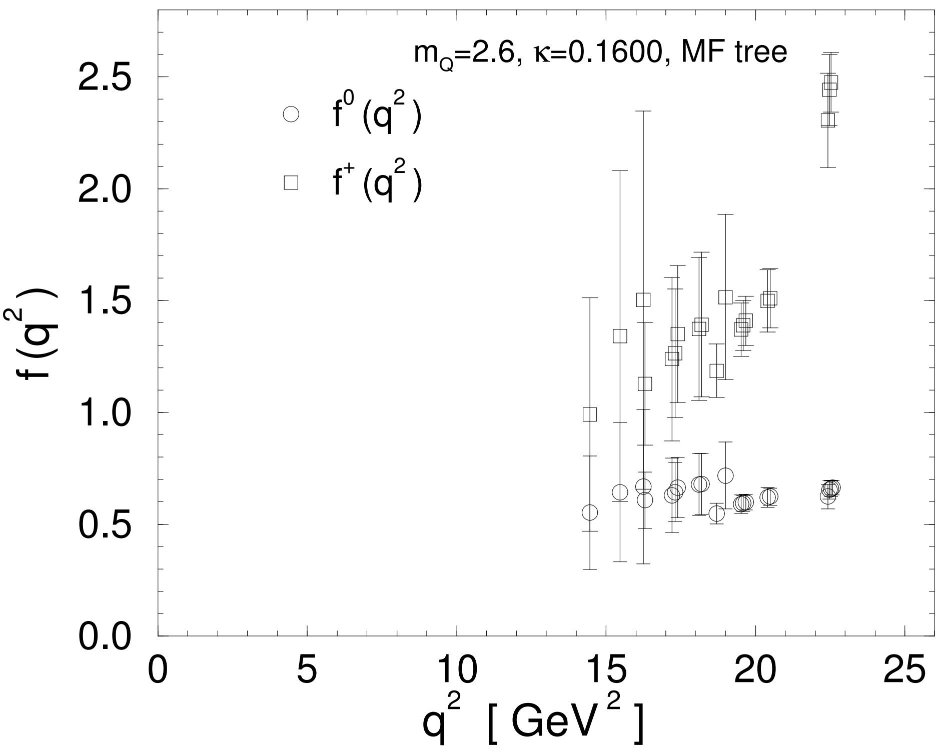
$$(p+k)^\mu\langle\, \pi \,|\, V_\mu \,|\, B\,\rangle \text{ and } f^0 \rightarrow\ \ f^+(q^2)$$

Form factors (mean-field tree)

$m_Q = 2.6, \ \ \kappa = 0.1570$



$m_Q = 2.6, \ \ \kappa = 0.1600$



Perturbative corrections

Heavy quark action

$$S_\phi = \sum \varphi(x) \left[\left(1 - \frac{1}{2n} H_0\right)^{-n} U_4(x) \left(1 - \frac{1}{2n} H_0\right)^{-n} \varphi(x+\hat{4}) - (1 - \delta H)\varphi(x) \right]$$

$\Rightarrow$ Perturbative expansion (c.f. K. Ishikawa's talk)

Self energy

$\Sigma(p) =$

$\rightarrow E_0$ (energy shift), Z_m (mass renorm.), Z_ϕ (field renorm.)

B meson mass : $m_B = Z_m m_Q - E_0 + E_{\bar{q}Q}$

Vertex correction

$Z_\mu^{latt}(\vec{p}=0, \vec{k}=0) =$

(with massless Wilson quark)

$\rightarrow Z_{V,\mu}$: $\langle\, \pi \,|\, V_\mu \,|B\, \rangle_{cont} \;=\; Z_{V,\mu} \langle\, \pi \,|\, V_\mu \,|B\, \rangle_{latt}$

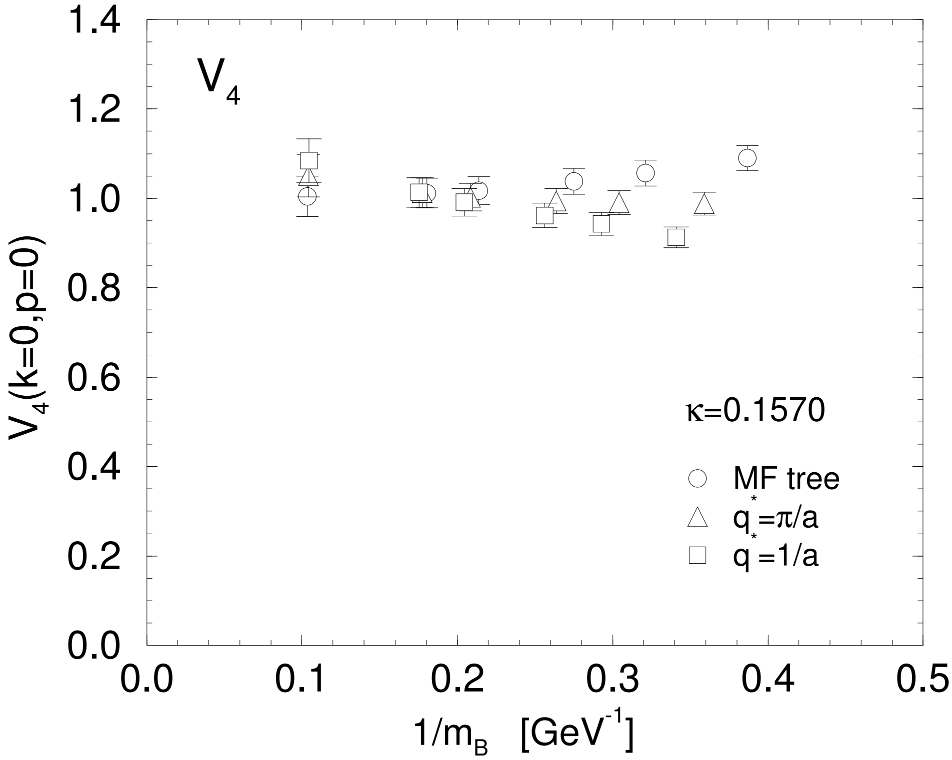
We use $n^{(pert)}$ different from n in simulation

$$\begin{pmatrix} m_Q \\ n^{(pert)} \end{pmatrix} = \begin{pmatrix} 5.0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2.6 \\ 2 \end{pmatrix}, \begin{pmatrix} 2.1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1.5 \\ 3 \end{pmatrix}, \begin{pmatrix} 1.2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0.9 \\ 6 \end{pmatrix}$$

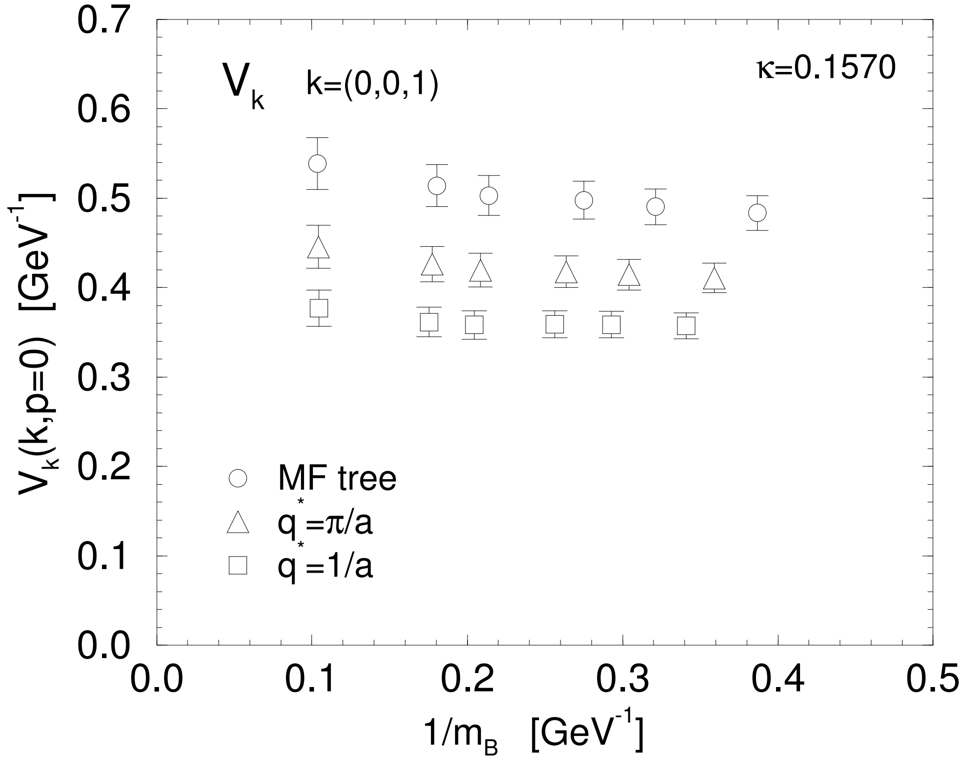
Scale : $q^* \;=\; \pi/a,\ 1/a$

Matrix elements (Renormalized)

$\kappa = 0.1570,\ V_4$

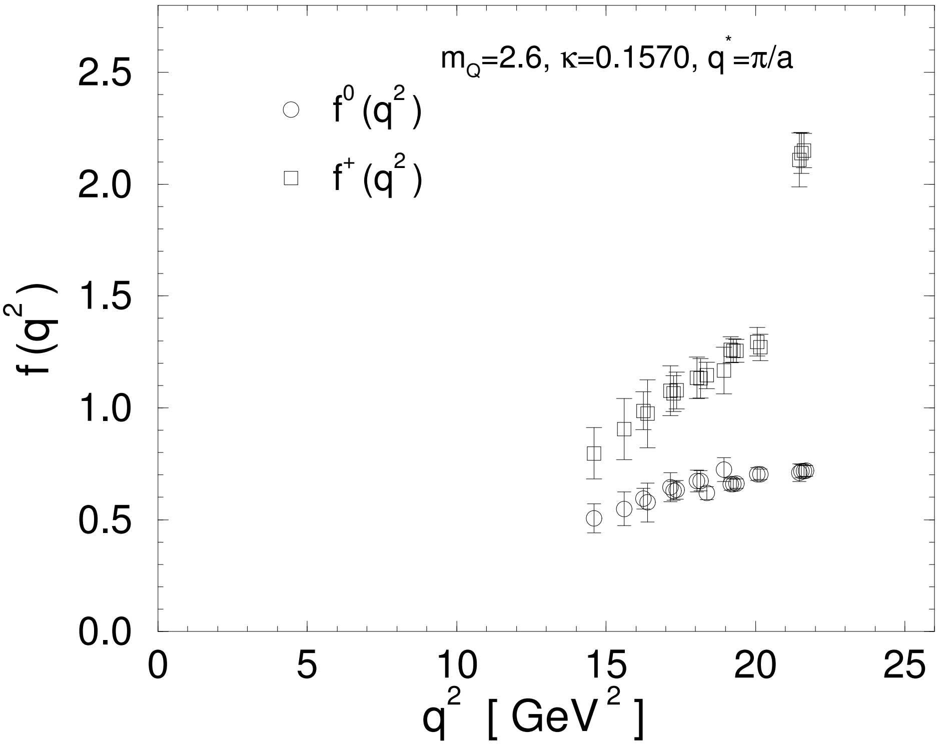


$\kappa = 0.1570,\ V_k$

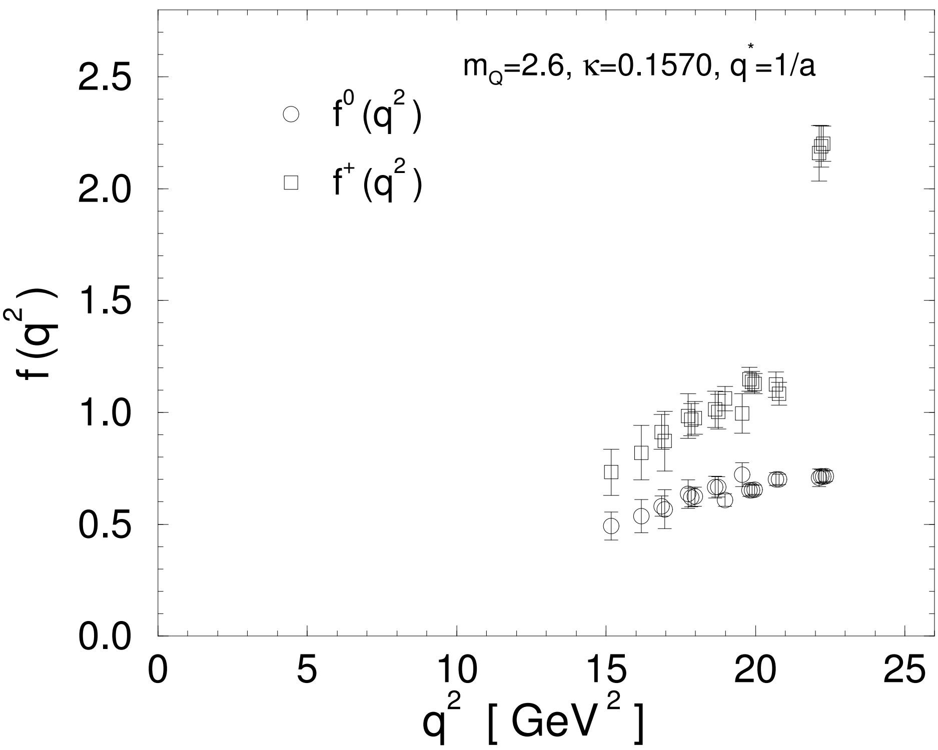


Form factors (Renormalized)

$m_Q = 2.6, \;\; \kappa = 0.1570, \; q^* = \pi/a$



$m_Q = 2.6, \;\; \kappa = 0.1570, \; q^* = 1/a$



Chiral extrapolation

Which quantity should be extrapolated linearly in m_q ($1/\kappa$) ?

(i) Matrix element $\langle \pi \,|\, V_\mu \,|B \,\rangle$

(ii) Form factor $f^0(q^2), f^+(q^2)$

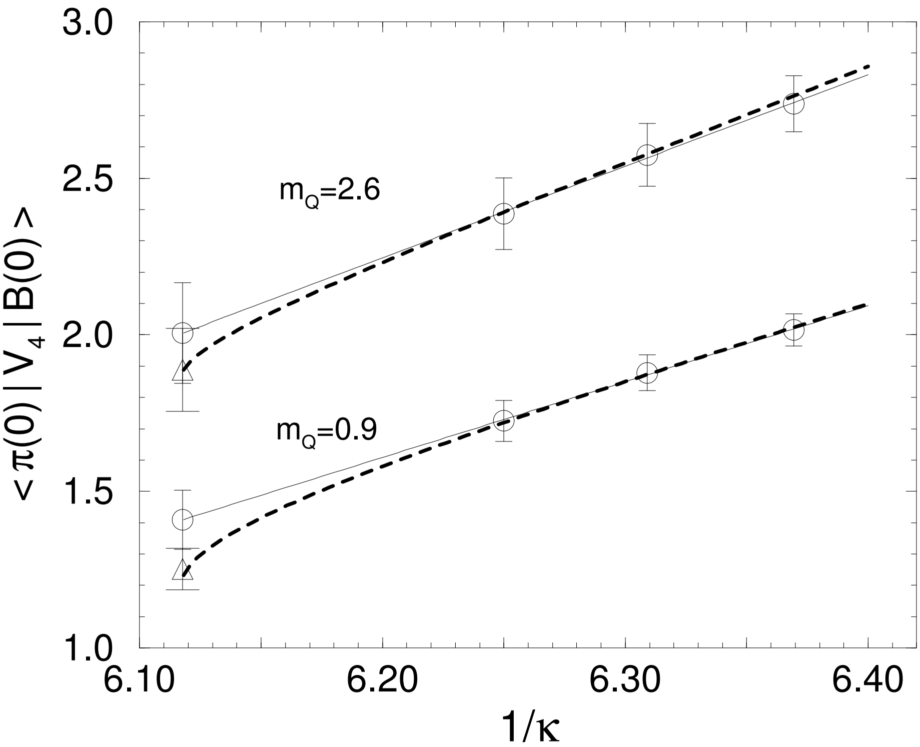
For example, at $q^2 = q^2_{max}$,

$$\langle \, \pi(\vec{k}=0) \,|\, V_\mu \,|B(\vec{p}=0) \,\rangle = (m_B + m_\pi)\, f^0(q^2_{max}) \;,$$

where $\quad m_B = c_0 + c_1 m_q \;, \quad m_\pi \propto \sqrt{m_q}$

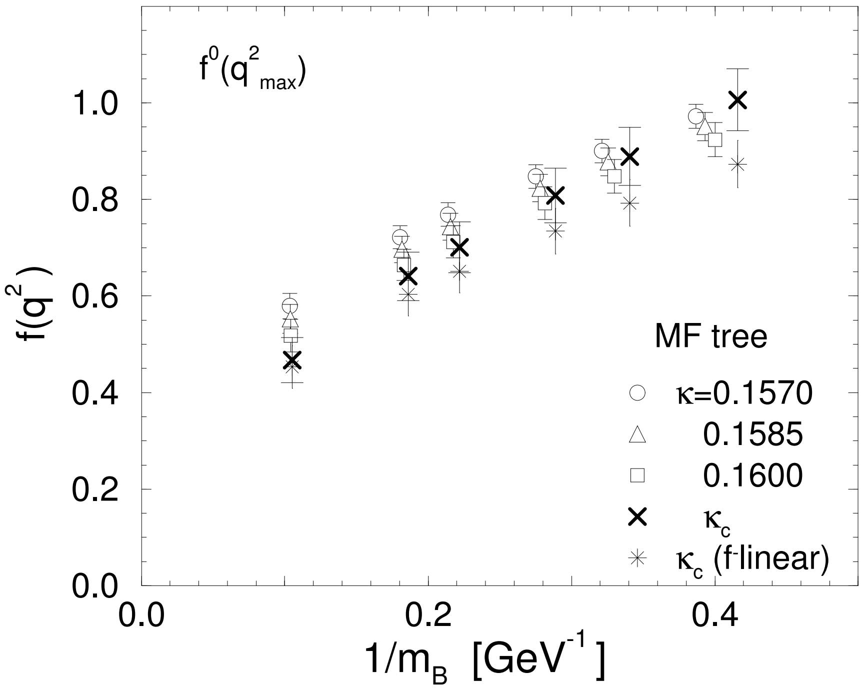
Difference is large for small m_Q.

We extrapolate matrix element linearly in m_q.

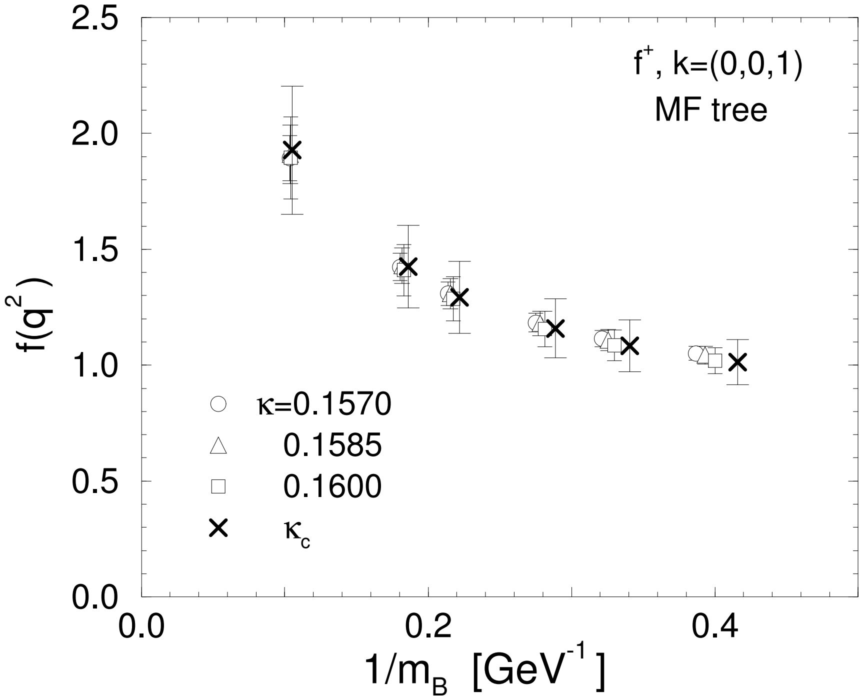


Chiral behavior of Form factors

$f^0(q^2_{max})$

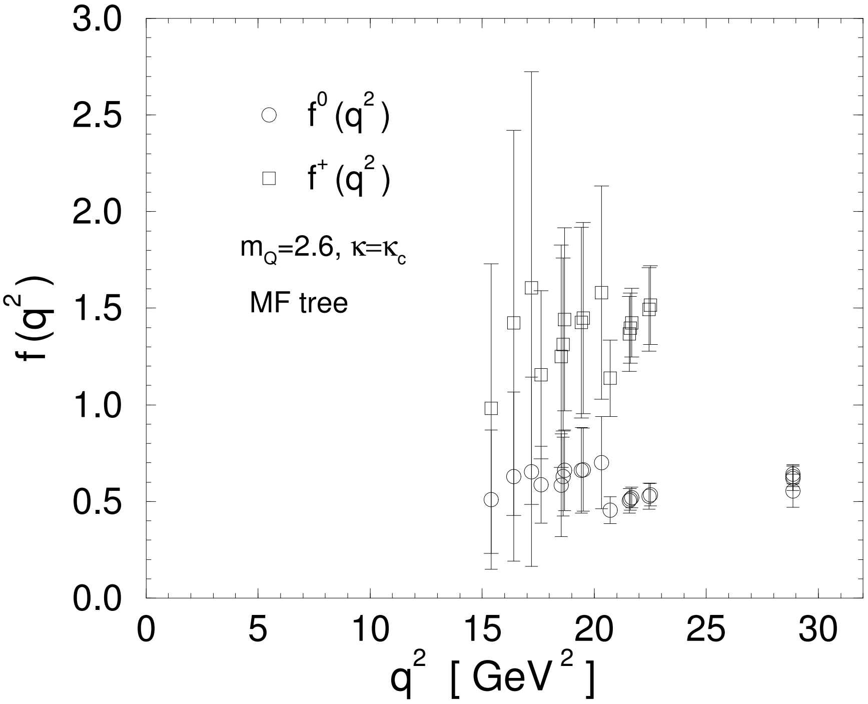


$f^+(q^2),\ \vec{p}=0, \vec{k}=(0,0,1)$

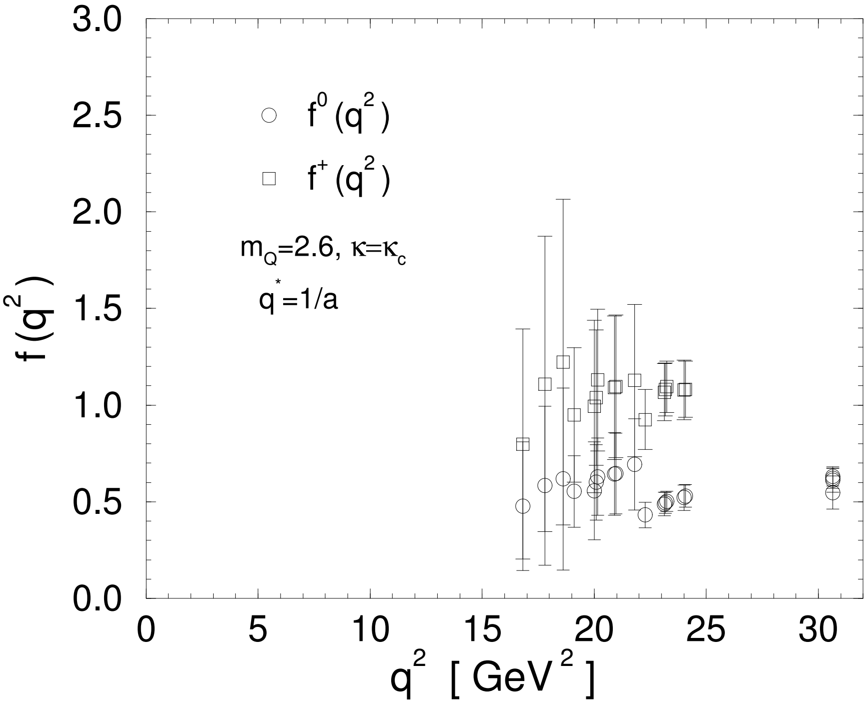


Form factors ($\kappa = \kappa_c$)

$m_Q = 2.6, \;\; \kappa = \kappa_c$, mean-field tree



$m_Q = 2.6, \;\; \kappa = \kappa_c, \; q^* = 1/a$



Conclusion and Discussion

We calculated $B \to \pi$ form factors near $q^2 = q^2_{max}$,
with six m_Q ($\sim 1.5 - 8.5$ GeV) and three κ ($\sim 2m_s - m_s$).
Perturbative correction is incorporated (partially).
Results are extrapolated to chiral limit.

- ◦ HQET prediction appears to work.
- ◦ Consistent with Wilson and Clover results.
- ◦ q^2 dependence of form factors are consistent with pole behavior.
- ◦ Perturbative correction is larger for V_j than for V_4.
- ◦ Chiral extrapolation is uncertain.

Similar calculation is applicable to other processes,
$B \to \rho$, $B \to K^*\gamma$, B-$\bar{B}$ mixing, etc.

With improvement and statistics,
quantitative calculation will be possible.