Spectral analysis at finite temperature on anisotropic lattices

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Plan of talk

1.Introduction

2.Spectral analysis on the lattice

- spectral function on the lattice
- inversion method

3.Spectral function at finite temperature

- an example of preliminary results

4.Summary

1.Introduction

Hadronic states at finite temperature

- change of masses and width below T_c
- excitation of hadronic modes above T_c

$\Longrightarrow$ Spectral function
at finite temperature

A. Nakamura, I. O. Stamatescu, S. Hioki, O. Miyamura, Y. Osada, K. Hirose, T. Kanki and T. Hashimoto
"Meson masses at finite temperature on an anisotropic lattce"
Nuclear Physics **B (Proc. Suppl.) 9** (1989) 373-377.

T. Hashimoto, A. Nakamura and I.O. Stamatescu
"Temperature-dependent structure in the mesonic channels of QCD"
Nucl. Phys. **B400** (1993) 267-305.

T. Hashimoto, A. Nakamura and I.O. Stamatescu
"QCD with dynamical quarks at finite temperature: spectral structure in the mesonic channels" Nucl. Phys. **B406** (1993) 325-339.

G. Boyd, S. Gupta, F. Karsch and E. Laermann
"Spatial and temporal hadron correlators below and above the chiral phase transition"
Zeitschrift für Physik **C64** (1994) 331-338.

Lattice calculation at finite temperture

- fine measurement of propagator

$\Longrightarrow$ anisotropic lattices

G. Burgers, F. Karsch, A. Nakamura and I. O. Stamatescu
"QCD on anisotropic lattices"
Nuclear Physics **B304** (1988) 587-600.

Spectral function from propagator

- ill-posed problem

$\Longrightarrow$ regularization of the inverse problem

C. W. Groetsch
"Inverse Problem in the Mathematical Sciences"
Friedr. Vieweg & Sohn Verlagsgesellshaft mbH (1993).

2.Spectral analysis on the lattice

Spectral function on the lattice

The spectral function $\sigma(s)$ is defined as :

$$G(\epsilon) = \int_0^\infty \frac{\sigma(s)}{s+\epsilon^2} ds,$$

where $G(\epsilon)$ is a propagator at energy ϵ.

This is an integral equation of the first Fredholm type. We have to solve an inverse problem of the equation of this type. The natural method to invert the equation is the discretization of the equation on a lattice.
On the lattice it takes a form of linear equation:

$$\tilde{G} = \tilde{F}\tilde{\sigma},$$

with

$$\tilde{G} = (G_n), \quad \tilde{\sigma} = (\sigma_m)$$

$$\tilde{F} = (F_{nm}) = \left(\frac{1}{s_m + 4\sin^2(\frac{\pi n}{N})} \right).$$

- various solutions
- not positive definite

This a typical behavior of the ill-posed problem in the inversion of an integral equation.

Inversion method

Ill-poseness $\Longrightarrow$ We need a regularization .

i.e. a technique to turn an ill-posed problem to a well-posed one

To choose a physically meaning full solution from the solutions,

we require followings:

- positivity of the spectral function
- maximal entropy

maximal entropy method

The entropy of a function $\nu(p)$ is defined as:

$$\nu(p) = -\int_a^b p(t) \ln p(t) dt.$$

For a integral equation:

$$f(x) = Kg(x) = \int_a^b k(x,y) g(y) dy,$$

the maximal entropy method requires to minimize the following evaluation function:

$$E(g) \equiv ||Kg - f||^2 + \alpha\nu(g),$$

where $\alpha > 0$ is a regularization parameter.

For an approximately discretized problem, the entropy of a vector is similarly defined as

$$\tilde{\nu}(\tilde{g}) = -\sum_m g_m \ln g_m.$$

The each component of the sum has a minimum at $g_m = 1/e$, so to minimize the evaluation function means to make a solution have a tendency to tend to the natural base of the logarithmic function e .

The modified entropy:

$$\tilde{\nu}(\tilde{g}) = \sum_m \frac{g_m}{\theta} \ln(\frac{g_m}{\theta}),$$

has a same tendency towards θ/e . We adjust θ to the strength of the continuum of a spectral function. It is expected if we have a good resolution of resonances this method will give an approximated spectral function of resonance-continuum type.

The minimization is a highly non linear problem . We propose a next iterative method.

Table of adjustable parameters in our method

- convergence criterion
- iteration number
- the theta parameter
- step size to the maximal entropy

3.Spectral function at finite temperature

Anisotropic lattices

We calculated mesonic correlation function on anisotropic lattices.

lattice size: $12^3 \times 72, 20, 18, 16$

coupling: $\beta = 5.68$

spatial hopping parameter: $\kappa_s = 0.068$

number of configurations: 20

The anisotropic factor is the ratio of the spatial lattice size to the temporal one.

anisotropic factor : $\gamma = 4$

effective anisotropic factor : $\xi = 5.9$

Preliminary relusts

An example for

lattice size: $N_t = 20$

pseudo scalar channel

point source

Effective mass analysis show the resion of s is from 0 to 0.5.

region of s : $0 < s < 0.5$

results for s_m and σ_m

m	s_m	σ_m
0	0.02	0.03
1	0.04	0.03
2	0.06	0.03
3	0.08	0.07
4	0.10	0.015
5	0.12	0.015
6	0.20	0.01
7	0.30	0.01
8	0.40	0.60
9	0.50	1.00

4.Summary

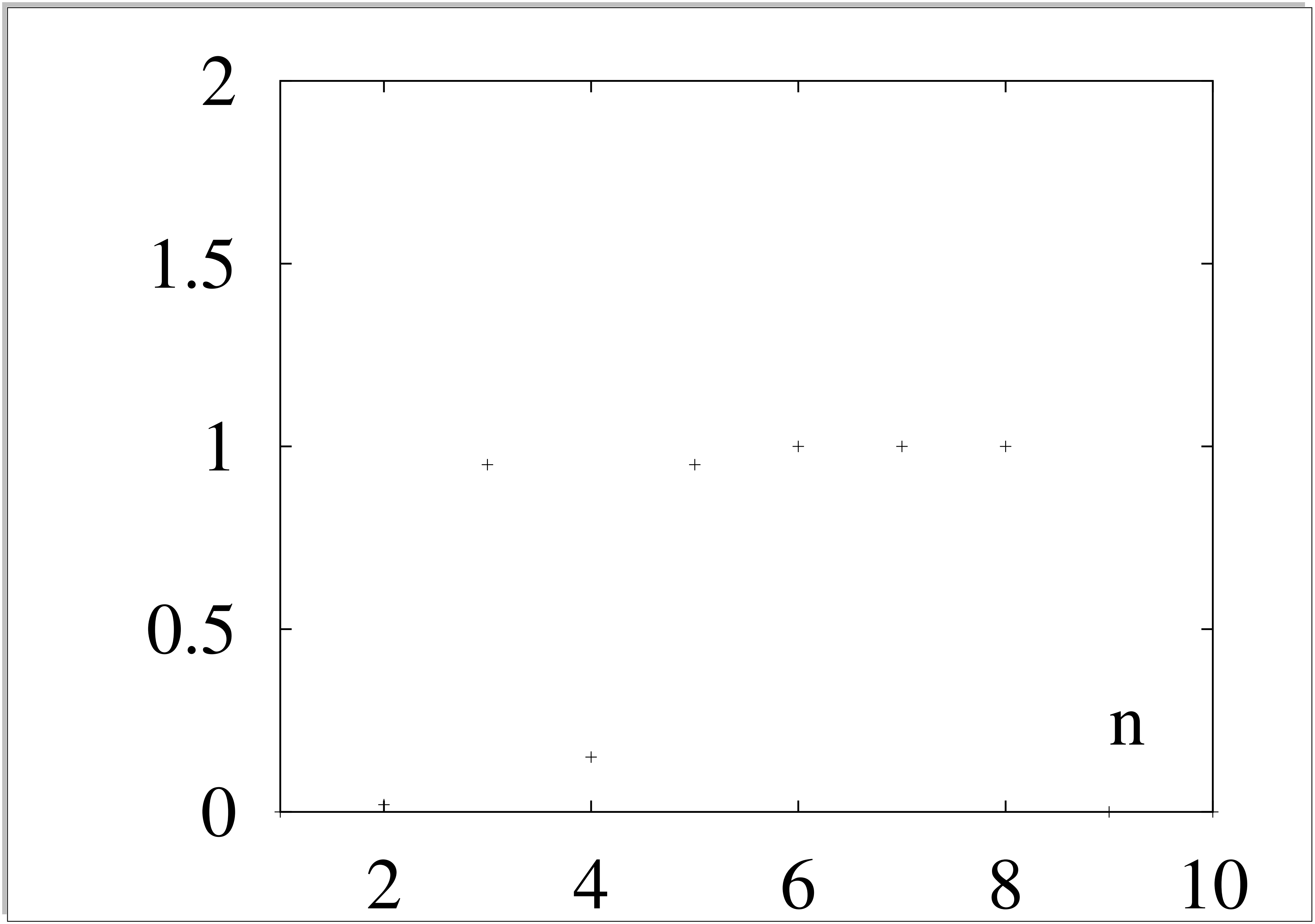
Spectral analysis on anisotropic lattices

New reguralization scheme with maximal entropy method

Inversion test

A very preliminary result for pseudo-scalar channel with $N_t = 20$

- still ill-posed
- full automatic search
- choice of the position of s_m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n
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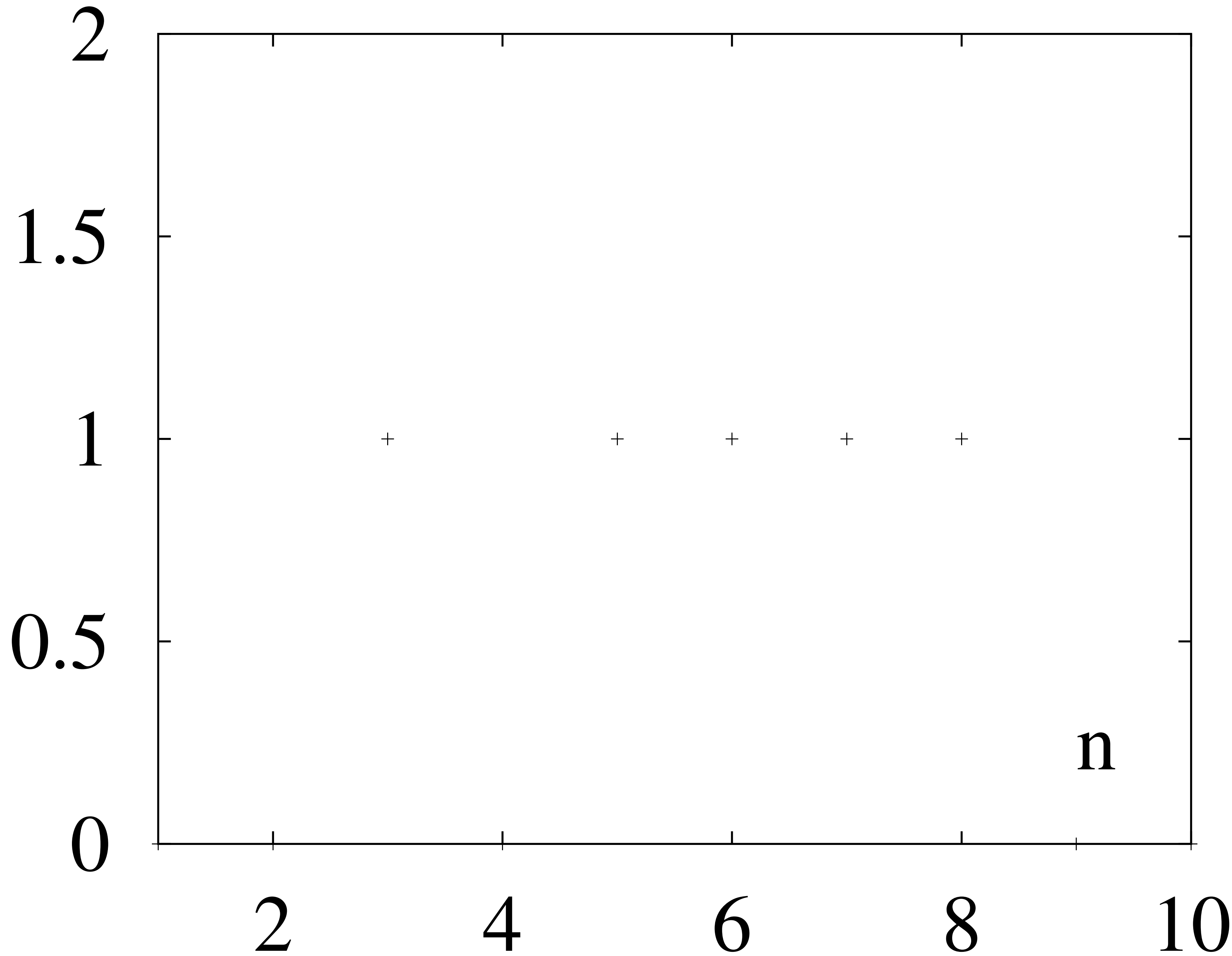
10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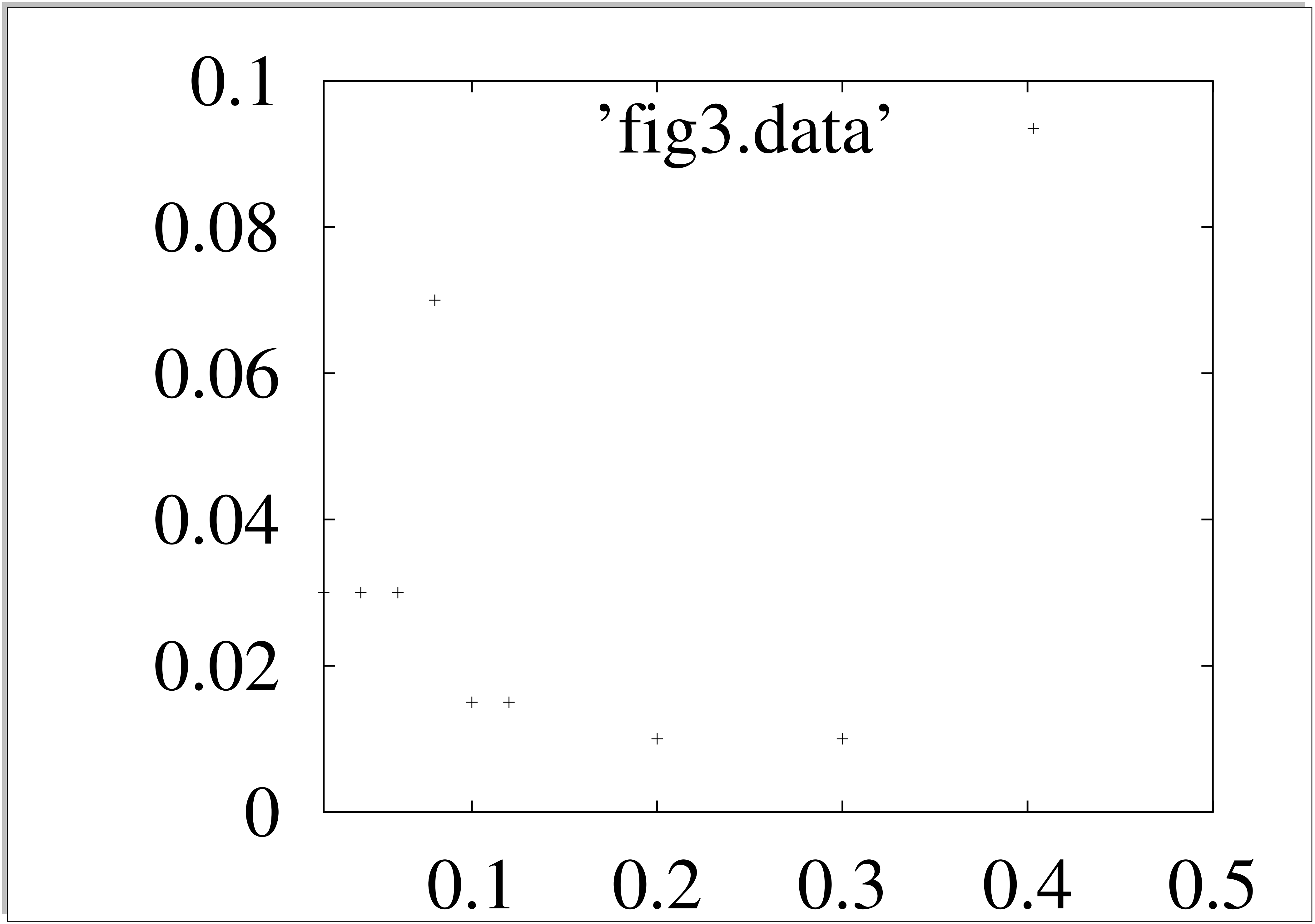

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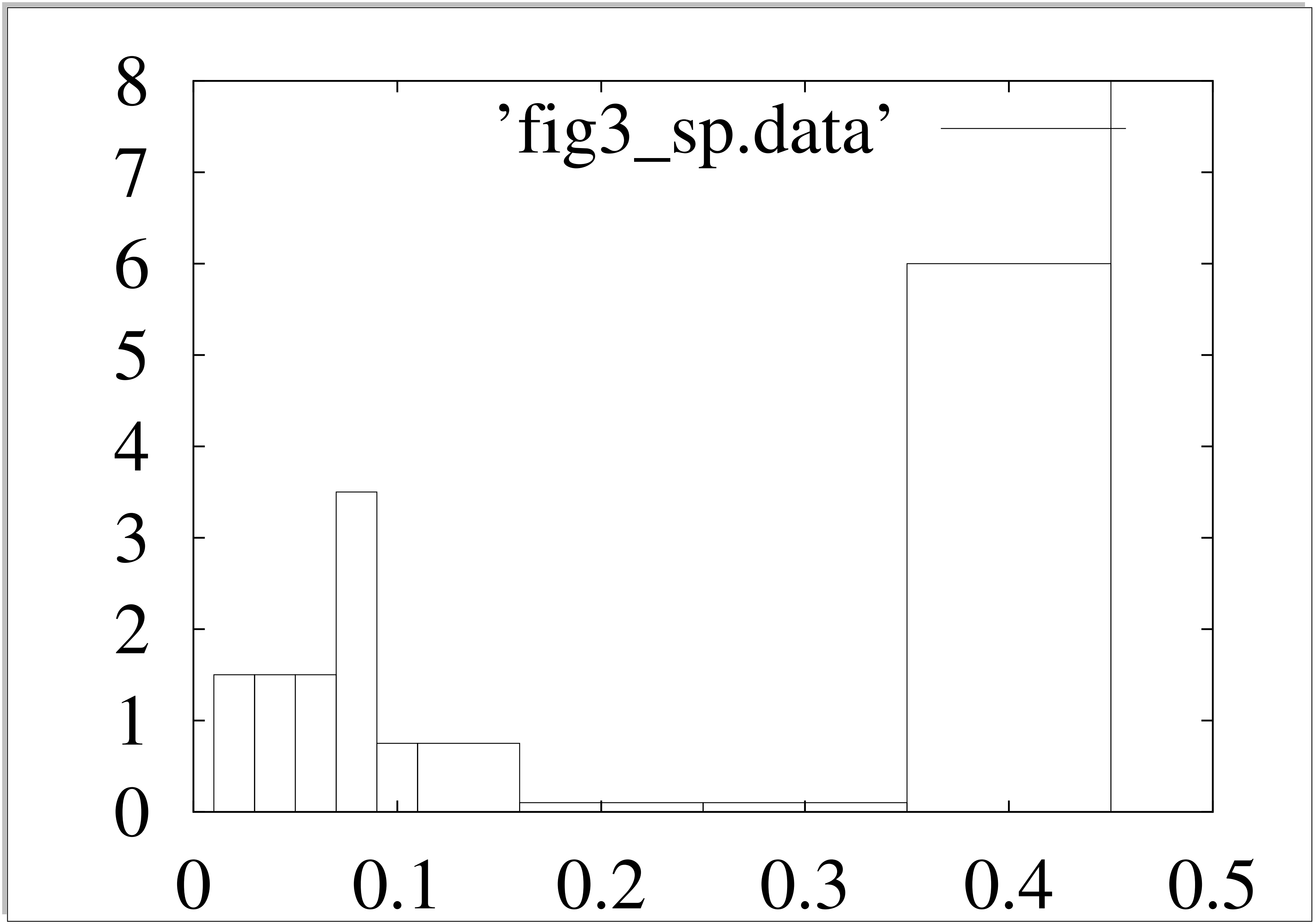

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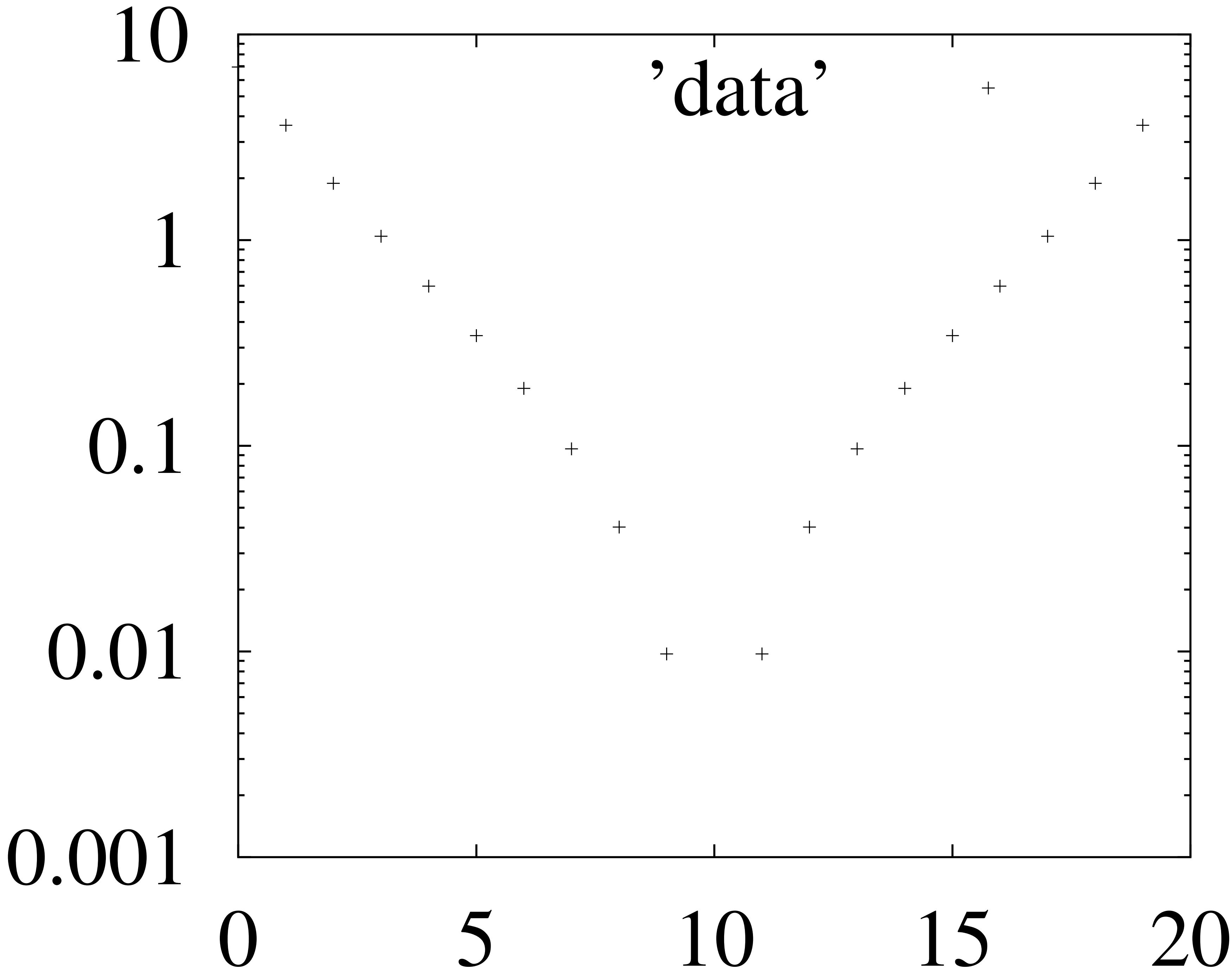
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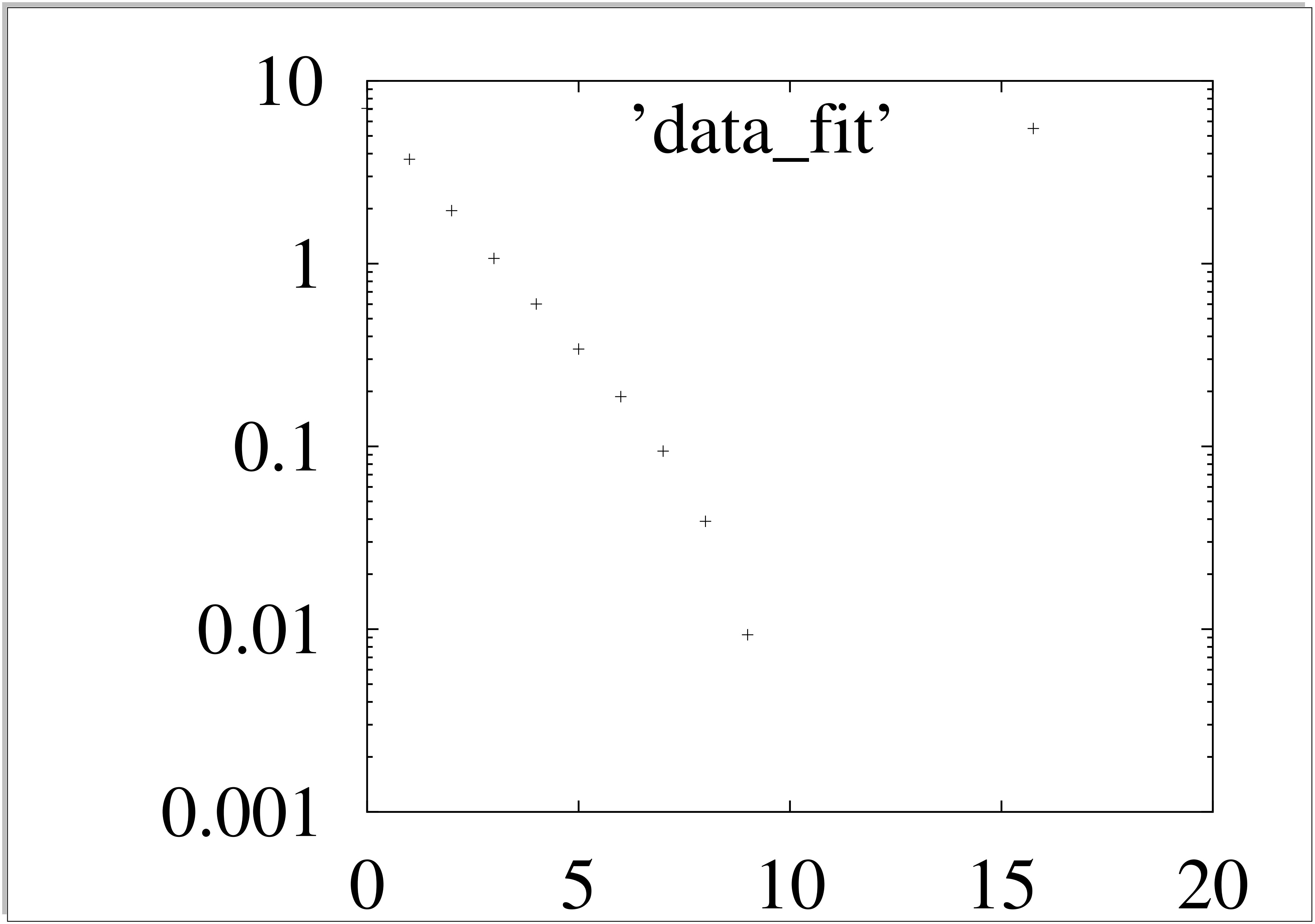

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