# Parton distribution functions in nuclei 

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- Contents -


## Introduction

- parton distributions in the nucleon
- parton distributions in nuclei EMC effect, shadowing


## Parametrization of

## nuclear parton distributions

1. functional form of $f_{i}{ }^{A}(x)$
2. constraints: A dependence, conservation laws
3. $\chi^{2}$ fitting to experimental data 4. results: optimum nuclear parton distributions

## Purposes

## Determination of parton distributions

- unpolarized distributions in the nucleon 3 major groups (CTEQ, GRV, MRS)
- polarized distributions in the nucleon several groups (GS, GRSV, ..., AAC)
- distributions in nuclei
no $\chi^{2}$ analysis! (see Eskola, Kolhinen, Ruuskanen)
$\rightarrow$ for understanding nuclear mechanisms in the high-enegy region
$\rightarrow$ for heavy-ion physics


## - parton distributions in nuclei

Nuclear modification of $\mathrm{F}_{2}{ }^{\mathrm{A}} / \mathrm{F}_{2}{ }^{\mathrm{D}}$ is well known in electron/muon scattering.

$$
\mathrm{F}_{2}^{\mathrm{A}}=\frac{1}{9} \mathrm{x}\left[4 \mathrm{u}_{\mathrm{v}}(\mathrm{x})+\mathrm{d}_{\mathrm{v}}(\mathrm{x})\right]_{\mathrm{A}}+\frac{2}{9} \mathrm{x} \mathrm{~S}_{\mathrm{A}}(\mathrm{x})
$$

Fermi motion

shadowing
original EMC finding
(binding, subnucleon?)

## Parametrization of

## nuclear parton distributions

without relying on models!

- Nuclear parton distributions (per nucleon) if there were no modification

$$
\mathrm{Al}^{\mathrm{A}}=\mathrm{Zu}^{\mathrm{p}}+\mathrm{Nu}^{\mathrm{n}}, \quad \mathrm{Ad}^{\mathrm{A}}=\mathrm{Zd}^{\mathrm{p}}+\mathrm{Nd} \mathrm{~d}^{\mathrm{n}}
$$

Isospin symmetry: $\mathrm{u}^{\mathrm{n}}=\mathrm{d}^{\mathrm{p}} \equiv \mathrm{d}, \mathrm{d}^{\mathrm{n}}=\mathrm{u}^{\mathrm{p}} \equiv \mathrm{u}$

$$
\rightarrow \mathrm{u}^{\mathrm{A}}=\frac{\mathrm{Zu}+\mathrm{Nd}}{\mathrm{~A}}, \quad \mathrm{~d}^{\mathrm{A}}=\frac{\mathrm{Zd}+\mathrm{Nu}}{\mathrm{~A}}
$$

Take into accont the nuclear modification by the factors $\mathrm{w}_{\mathrm{i}}(\mathrm{x}, \mathrm{A})$

$$
\begin{aligned}
& u_{v}^{A}(x)=w_{u_{v}}(x, A) \frac{Z u_{v}(x)+N d_{v}(x)}{A} \\
& d_{v}^{A}(x)=w_{d_{v}}(x, A) \frac{Z d_{v}(x)+N u_{v}(x)}{A} \\
& \bar{q}^{A}(x)=w_{\bar{q}}(x, A) \bar{q}(x) \\
& g^{A}(x)=w_{g}(x, A) g(x)
\end{aligned}
$$

## A dependence

Ref. I. Sick and D. Day, Phys. Lett. B 274 (1992)

roughly speaking $\sigma_{\mathrm{A}}=\mathrm{A} \sigma_{\mathrm{V}}+\mathrm{A}^{2 / 3} \sigma_{\mathrm{S}}$

$$
\begin{aligned}
\rightarrow \frac{\sigma_{\mathrm{A}}}{\mathrm{~A}} & =\sigma_{\mathrm{V}}+\frac{1}{\mathrm{~A}^{1 / 3}} \sigma_{\mathrm{S}} \\
& \sim \frac{1}{\mathrm{~A}^{1 / 3}} \text { dependence }
\end{aligned}
$$



## Functional form of $\mathbf{w}_{\mathbf{i}}(\mathbf{x}, \mathbf{A})$

$$
f_{i}^{A}(x)=W_{i}(x, A) f_{i}(x), \quad i=u_{v}, d_{v}, \bar{q}, g
$$

first, assume the A dependence as $1 / \mathrm{A}^{1 / 3}$
then, use

$$
\mathrm{w}_{\mathrm{i}}(\mathrm{x}, \mathrm{~A})=1+\left(1-1 / \mathrm{A}^{1 / 3}\right) \frac{\mathrm{a}_{\mathrm{i}}+\mathrm{b}_{\mathrm{i}} \mathrm{x}+\mathrm{c}_{\mathrm{i}} \mathrm{x}^{2}+\mathrm{d}_{\mathrm{i}} \mathrm{x}^{3}}{(1-\mathrm{x})^{\beta_{i}}}
$$

$\mathrm{a}_{\mathrm{i}}, \mathrm{b}_{\mathrm{i}}, \mathrm{c}_{\mathrm{i}}, \mathrm{d}_{\mathrm{i}}, \beta_{\mathrm{i}}$ : parameters to be determined by $\chi^{2}$ analysis

Fermi motion: $\frac{1}{(1-x)^{\beta_{i}}} \rightarrow \infty$ as $x \rightarrow 1$ if $\beta_{i}>0$
Shadowing: $\quad w_{i}(x \rightarrow 0, A)=1+\left(1-1 / A^{1 / 3}\right) a_{i}<1$
Fine tuning:
$\mathrm{b}_{\mathrm{i}}, \mathrm{c}_{\mathrm{i}}, \mathrm{d}_{\mathrm{i}}$

## Constraints

- Nuclear charge

$$
\begin{aligned}
\mathrm{Z} & =\mathrm{A} \int \mathrm{dx}\left[\frac{2}{3}\left(\mathrm{u}^{\mathrm{A}}-\overline{\mathrm{u}}^{\mathrm{A}}\right)-\frac{1}{3}\left(\mathrm{~d}^{\mathrm{A}}-\overline{\mathrm{d}}^{\mathrm{A}}\right)-\frac{1}{3}\left(\mathrm{~s}^{\mathrm{A}}-\overline{\mathrm{s}}^{\mathrm{A}}\right)\right] \\
& =\mathrm{A} \int \mathrm{dx}\left(\frac{2}{3} \mathrm{u}_{\mathrm{v}}^{\mathrm{A}}-\frac{1}{3} \mathrm{~d}_{\mathrm{v}}^{\mathrm{A}}\right)
\end{aligned}
$$

- Baryon number

$$
\mathrm{A}=\mathrm{A} \int \mathrm{dx} \frac{1}{3}\left(\mathrm{u}_{\mathrm{v}}^{\mathrm{A}}+\mathrm{d}_{\mathrm{v}}^{\mathrm{A}}\right)
$$

- Momentum

$$
A=A \int d x x\left(u_{v}^{A}+d_{v}^{A}+6 \bar{q}^{A}+g^{A}\right)
$$

Three parameters can be determined by these conditions.

## Structure function in leading order

$$
\begin{gathered}
\mathrm{F}_{2}^{\mathrm{A}}=\sum_{\mathrm{i}} \mathrm{e}_{\mathrm{i}}^{2} \mathrm{x}\left(\mathrm{q}_{\mathrm{i}}^{\mathrm{A}}+\overline{\mathrm{q}}_{\mathrm{i}}^{\mathrm{A}}\right)=\mathrm{x}\left(\frac{4}{9} \mathrm{u}_{\mathrm{v}}^{\mathrm{A}}+\frac{1}{9} \mathrm{~d}_{\mathrm{v}}^{\mathrm{A}}+\frac{12}{9} \overline{\mathrm{q}}\right) \\
\mathrm{f}_{\mathrm{i}}^{\mathrm{A}}(\mathrm{x})=\mathrm{w}_{\mathrm{i}}(\mathrm{x}, \mathrm{~A}) \cdot \mathrm{f}_{\mathrm{i}}(\mathrm{x}) \\
\mathrm{w}_{\mathrm{i}}(\mathrm{x}, \mathrm{~A})=1+\left(1-1 / \mathrm{A}^{1 / 3}\right) \frac{\mathrm{a}_{i}+\mathrm{b}_{\mathrm{i}} \mathrm{x}+\mathrm{c}_{\mathrm{i}} \mathrm{x}^{2}+\mathrm{d}_{\mathrm{i}} \mathrm{x}^{3}}{(1-\mathrm{x})^{\beta_{i}}}
\end{gathered}
$$

parameters: $\alpha / \mu_{v}, a / e_{v}, b, c_{V}, d \nsim \beta_{V}$,

$$
a_{\bar{q}}, b_{\bar{q}}, c_{\bar{q}}, d / \alpha, \beta / \bar{q}, a_{g}, b_{g}, c_{g}, d_{\&} \beta_{/}
$$

/f: nuclear charge, baryon \#, momentum
$X: d_{v}=d_{\bar{q}}=d_{g}=0$ in the quadratic fit
$: d_{g}=0$ in the cubic fit
$<: \beta_{\overline{\mathrm{q}}}=\beta_{\mathrm{g}}=1: \overline{\mathrm{q}} \& \mathrm{~g}$ cannot be determined in the large $x$ region at this stage
$\searrow: \mathrm{b}_{\mathrm{g}}=-2 \mathrm{c}_{\mathrm{g}}, \mathrm{g}(\mathrm{x})$ shape cannot be determined at medium and large $x$

7 or 9 parameters for 309 data points

## Experimental data on $\mathrm{F}_{2}{ }^{\mathrm{A}} / \mathbf{F}_{2}{ }^{\mathrm{D}}$

NMC: $\quad \mathrm{He}, \mathrm{Li}, \mathrm{C}, \mathrm{Ca}$
SLAC: $\mathrm{He}, \mathrm{Be}, \mathrm{C}, \mathrm{Al}, \mathrm{Ca}, \mathrm{Fe}, \mathrm{Ag}, \mathrm{Au}$
EMC: $\quad \mathrm{C}, \mathrm{Ca}, \mathrm{Cu}, \mathrm{Sn}$
E665: $\mathrm{C}, \mathrm{Ca}, \mathrm{Xe}, \mathrm{Pb}$
BCDMS: N, Fe


## Analysis conditions

- parton distributions in the nucleon

$$
\text { MRST98-LO ( } \left.\Lambda_{\text {QCD }}=174 \mathrm{MeV}\right)
$$

- $\mathrm{Q}^{2}$ point at which the parametrized distributions are defined: $\mathrm{Q}^{2}=1 \mathrm{GeV}^{2}$
- used experimental data: $\mathrm{Q}^{2} \geq 1 \mathrm{GeV}^{2}$
- total number of data: 309
- number of flavor: $\mathrm{n}_{\mathrm{f}}=3$
- subroutine for the $\chi^{2}$ analysis: CERN - Minuit

$$
\begin{aligned}
\chi^{2}= & \sum_{\mathrm{i}} \frac{\left(\mathrm{R}_{\mathrm{i}}^{\text {data }}-\mathrm{R}_{\mathrm{i}}^{\text {calc }}\right)^{2}}{\left(\sigma_{\mathrm{i}}^{\text {data }}\right)^{2}} \\
& \mathrm{R}=\frac{\mathrm{F}_{2}^{\mathrm{A}}}{\mathrm{~F}_{2}^{\mathrm{D}}}, \quad \sigma_{\mathrm{i}}^{\text {data }}=\sqrt{\left(\sigma_{\mathrm{i}}^{\text {sis }}\right)^{2}+\left(\sigma_{\mathrm{i}}^{\text {tata }}\right)^{2}}
\end{aligned}
$$

$\rightarrow$ obtained $\chi_{\text {min }}^{2} /$ d.o.f. $=583.7 / 302$ (quadratic)

$$
=546.6 / 300 \text { (cubic) }
$$

## $\chi^{2}$ contributions

A \# of data | $\chi 2$ | $\chi 2$ |
| ---: | ---: | ---: |
|  | quad. cubic |

| He | 35 | 55.6 | 54.5 |
| :--- | ---: | ---: | ---: |
| Li | 17 | 45.6 | 49.2 |
| Be | 17 | 39.7 | 38.4 |
| C | 43 | 97.8 | 88.2 |
| N | 9 | 10.5 | 10.4 |
| Al | 35 | 38.8 | 41.4 |
| Ca | 33 | 72.3 | 69.7 |
| Fe | 57 | 115.7 | 92.7 |
| Cu | 19 | 13.7 | 13.6 |
| Ag | 7 | 12.7 | 11.5 |
| Sn | 8 | 14.8 | 17.7 |
| Xe | 5 | 3.2 | 2.4 |
| Au | 19 | 55.5 | 49.2 |
| Pb | 5 | 7.9 | 7.6 |

$\begin{array}{llll}\text { total } & 309 & 583.7 & 546.6\end{array}$

## Analysis results






## Weight functions $\mathbf{w}_{\mathbf{i}}(\mathbf{x}, \mathbf{A})$

for $\mathrm{He}, \mathrm{Ca}, \mathrm{Au}$



## Weight functions and

## parton distributions for $\mathbf{C a}$




## Summary

- first $\chi^{2}$ analysis for the nuclear parton distributions
- reasonably good fit with

$$
\chi_{\min }^{2}=583.7 \text { (quad.), } 546.6 \text { (cubic) }
$$

- $g^{A}(x)$ cannot be well determined
- $\bar{q}^{\mathrm{A}}(\mathrm{x})$ at medium x ?
- need analysis refinement
- need a variety of data
$\rightarrow$ Drell-Yan, direct $\gamma, v$ scattering, $\ldots$
- relation to heavy-ion physics
- computer codes could be obtained from http://www-hs.phys.saga-u.ac.jp

