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Two-loop anomalous dimensions for the structure function h_1

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Abstract

Chiral-odd structure function h_1 is expected to be measured in polarized Drell-Yan process. We calculate two-loop anomalous dimensions for h_1 in the minimal subtraction scheme. Dimensional regularization and Feynman gauge are used for calculating the two-loop contributions. Our results are important in studying Q^2 dependence of h_1.

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Internal spin structure of the proton has been a popular topic since the discovery of a serious proton-spin problem at high energies. In spite of a naive quark-model prediction, it was suggested that the major part of proton spin is not carried by quarks by measuring the structure function g_1. The discovery inspired many theoretical and experimental studies on the spin structure. In order to understand how the proton spin consists of quark and gluon spins, it is important to learn about sea-quark and gluon polarizations. As an experimental project to investigate these issues, the RHIC (Relativistic Heavy Ion Collider) Spin project was proposed.

In this project, the internal spin structure will be investigated by polarized proton-proton reactions. Another virtue of this project is that a chiral-odd structure function h_1 could be measured in the Drell-Yan process with transversely polarized protons [1–4]. It could be also measured in semi-inclusive electron scattering [5]. It provides us another important test of proton spin structure. In particular, the structure function h_1 probes different aspects of spin. Furthermore, a nice point is that it has a clear parton-model interpretation. It measures the transversity distribution, namely the probability to find a quark with spin polarized along the transverse spin of a polarized proton minus the probability to find it polarized oppositely. Its gross properties have been studied recently [6] in particular by Jaffe and Ji [3]. Here, we would like to focus on Q^2 dependence. Leading-order Q^2 evolution was already studied [2]. However, it is our standard to use next-to-leading-order (NLO) results in obtaining unpolarized and polarized parton distributions. Recently, the NLO evolution equations are established for the structure function g_1 [7]. Therefore, it is important to investigate the NLO Q^2 evolution of h_1 for future analyses of polarized Drell-Yan data in connection with the h_1 structure function. Our research purpose is to calculate two-loop anomalous dimensions, which determine the NLO Q^2 evolution of h_1.

In order to study h_1 in perturbative Quantum Chromodynamics (QCD), we need to introduce a set of local operators [3]

$$O^{\nu\mu_1\cdots\mu_n} = S_n \,\overline{\psi}\, i\,\gamma_5\,\sigma^{\nu\mu_1}\, iD^{\mu_2}\cdots iD^{\mu_n}\,\psi - \text{ trace terms}\ , \quad n=1,2,... \quad . \tag{1}$$

Here, S_n symmetrizes the Lorentz indices μ_1, ..., μ_n, and $iD^\mu = i\partial^\mu + gt^a A^\mu_a$ is the covariant derivative. We calculate two-loop anomalous dimensions of these twist-two operators. Because gluon field cannot contribute to the anomalous dimensions of the chiral-odd operators even in the NLO, our calculation procedure becomes a rather simple one which is similar to the nonsinglet formalism for example by Floratos, Ross, and Sachrajda [8]. Renormalization of the $\overline{\psi}\gamma^{\mu_1}(1 \pm \gamma_5) iD^{\mu_2} \cdots iD^{\mu_n}\psi$ type operators was studied in Ref. [8], but the same method can be applied to the operators in Eq. (1). First, Feynman rules at the operator vertices should be provided. In order to satisfy the symmetrization condition and to remove the trace terms, the tensor $\Delta_{\mu_1}\Delta_{\mu_2}\cdots\Delta_{\mu_n}$ with the constraint $\Delta^2 = 0$ is usually multiplied. However, the operators in Eq. (1) are associated with one-more Lorentz index ν, so that it is convenient to introduce another vector Ω_ν with the constraint $\Omega \cdot \Delta$=0. Then, Feynman rules with zero, one, and two gluon vertices become those in Fig. 1.

Next, we give a brief outline how anomalous dimensions are calculated [8]. The bare operator is defined by $O^n_B = \overline{\psi}_B Q^n_B \psi_B = Z_O O^n_R$ with the renormalized one O^n_R. Then, the renormalization constant Z_O is given by $Z_O = Z_F/Z_Q$, where Z_F and Z_Q are wave-function and operator renormalization constants. Anomalous dimensions for O^n, Q^n, and ψ are given by these renormalization constants as $\gamma_{_{O^n}} = \mu\,\partial(\ln Z_{O^n})/\partial\mu$, $\gamma_{_{Q^n}} = \mu\,\partial(\ln Z_{Q^n})/\partial\mu$, and $\gamma_{_F} = (\mu/2)\,\partial(\ln Z_F)/\partial\mu$. Using the relation among the renormalization constants, we express the anomalous dimension $\gamma_{_{O^n}}$ in terms of $\gamma_{_{Q^n}}$ and $\gamma_{_F}$ as $\gamma_{_{O^n}} = 2\gamma_{_F} - \gamma_{_{Q^n}}$. In dimensional regularization with the dimension $d = 4 - \epsilon$, Z_{Q^n} can be obtained in the form $Z_{Q^n} = 1 + \sum_{k=1}^{\infty} Z_k^{Q^n}(g_{_R})/\epsilon^k$. As it is shown in Ref. [8], the anomalous dimension is related to the first coefficient $Z^{Q^n}_{k=1}$ by $\gamma_{_{Q^n}} = -g^2_{_R}\,\partial Z^{Q^n}_1/\partial g^2_{_R}$. In the two-loop calculation, it becomes $\gamma_{_{Q^n}} = -2\,Z^{Q^n}_1$. The anomalous dimension $\gamma_{_{Q^n}}$ is simply given by minus twice the $1/\epsilon$ coefficient in Z_{Q^n}. The same equation can be also applied to the anomalous dimension $\gamma_{_F}$. In this way, we try to find $1/\epsilon$ singularities in the renormalization constants for calculating the anomalous dimensions.

Two-loop contributions are calculated by using the dimensional regularization with the Feynman gauge. Here, minimal subtraction (MS) scheme is used. Possible Feynman diagrams contributing to the one-particle irreducible matrix elements are shown in Fig. 2. Furthermore, there are two-loop diagrams contributing to the quark self-energy as shown in Fig. 3. However, the quark-field renormalization is already discussed in Ref. [8], so that we do not have to repeat the same calculations. In order to explain our calculation procedure for the operator part, we pick a simple case. In Fig. 4, Lorentz and color indices are shown with momenta for the diagram (d) in Fig. 2. The diagram is calculated with a Feynman rule in Fig. 1 as

$$I_d = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \, igt^a \gamma^\mu \, \frac{i\not{k}_1}{k_1^2} \, igt^b \gamma^\nu \, \frac{i\not{k}_2}{k_2^2} \, \frac{-ig_{\nu\rho}\delta_{bc}}{(k_1-k_2)^2} \, igt^c \gamma^\rho \, \frac{i\not{k}_1}{k_1^2} \, \frac{-ig_{\mu\sigma}\delta_{ad}}{(p-k_1)^2} \\ \times \sum_{j=1}^{n-1} \gamma_5 \not{\Delta} \not{n} \, (\Delta\cdot p)^{j-1} \, gt^d \Delta^\sigma \, (\Delta\cdot k_1)^{n-1-j} \quad . \tag{2}$$

Calculating the above integral, we obtain

$$I_d = \gamma_5 \not{\Delta} \not{n} \, (\Delta\cdot p)^{n-1} \, 2(2-\epsilon) \, g^4 C_F^2 \, \frac{1}{(4\pi)^4} \, \frac{\Gamma(\epsilon/2)\Gamma(\epsilon)}{\Gamma(1+\epsilon/2)} \, B(2-\epsilon/2, 1-\epsilon/2) \\ \times \sum_{j=1}^{n-1} B(j+1-\epsilon, 1-\epsilon/2) \left(\frac{-p^2}{4\pi}\right)^{-\epsilon} \quad , \tag{3}$$

where $\Gamma(x)$ and $B(x,y)$ are gamma and beta functions, and C_F is given by $C_F = (N_c^2 - 1)/(2N_c)$ with the number of color N_c. Subtracting a one-loop counter term from the above equation and evaluating singular terms, we have

$$I_d' = \gamma_5 \not{\Delta} \not{n} \, (\Delta\cdot p)^{n-1} \, 2 \, \frac{g^4}{(4\pi)^4} \, C_F^2 \sum_{j=1}^{n-1} \frac{1}{j+1} \left[-\frac{2}{\epsilon^2} + \frac{1}{\epsilon} \{ 1 - S_1(j+1) \} \right] \quad , \tag{4}$$

where $S_1(n) = \sum_{k=1}^{n} 1/k$. From the $1/\epsilon$ singularity in the above equation, we obtain the diagram (d) contribution to the anomalous dimension as $\gamma_n^{(2d)} = 8\, g^4 \, C_F^2 \, [\, G_1(n) - S_1(n)\,]/(4\pi)^4$ with $G_1(n) = \sum_{j=1}^{n} (1/j) \sum_{i=1}^{j} (1/i)$. The factor of two is included by considering a similar diagram with gluons attached to the initial quark line. We find that the obtained anomalous dimension is exactly the same with the one in Ref. [8]. The reason is the following. Because the operator vertex part $\gamma_5 \not{\Delta} \not{n}$ can be separated from the k_1 and k_2 integrals in Eq. (2), the integrals are independent of the operator form. The vertex $\not{\Delta}$ in Ref. [8] is simply replaced by the present one $\gamma_5 \not{\Delta} \not{n}$. Therefore, the renormalization calculations are the same in both cases, so that the obtained anomalous dimensions are exactly the same.

We notice that several diagrams in Fig. 2 have the same property in the sense that the operator part can be separated from the integrals. These are cases with gluon propagators attached only to the final-quark or initial-quark line. In addition to the diagram (d), these are diagrams (g), (h), (l), (m), (q), and (r). We do not have to repeat the same calculations. Furthermore, there are certain diagrams which do not contribute to the anomalous dimension. Calculating diagrams (a), (b), (c), and (p), we find that there exist no $1/\epsilon$ type singularity. Therefore, there is no contribution to the anomalous dimension from these diagrams. From the above discussions, the problem reduces to calculations of remaining diagrams (e), (f), (i), (j), (k), (n), and (o).

We need to calculate integrals which appear in evaluating these diagrams. Fortunately, many of them are already listed in Appendix of Ref. [8]. All divergent subintegrals are renormalized by subtracting the pole part of the subintegral. However, special care has to be taken in some diagrams, for example the one in Fig. 2(i). Contracting γ matrices in the numerator, we find a term proportional to ϵ. In this case, the $1/\epsilon$ contributions come from the double-pole terms, in which it is known physically that there is no counterterm contribution [8]. Therefore, when we write the double integral results, it is more convenient to express the $1/\epsilon^2$ terms without subtracting counterterms even though the $1/\epsilon$ terms are calculated with the counterterms. This is actually done in Appendix of Ref. [8] for writing the integral results. We also express extra integrals in this way in Appendix B. Once we have all the integral information, it is straightforward calculation for obtaining the anomalous dimensions. Because calculation procedure is essentially the same with the one for the diagram (d) and the details are already described in Ref. [8], each evaluation is not explained. We simply list our results in Appendix A.

We summarize our study. Two-loop anomalous dimensions for the chiral-odd structure function h_1 are calculated in the MS scheme. Dimensional regularization and Feynman gauge are used for calculating the possible diagrams. Our results are important for studying the details of Q^2 dependence in h_1, which is expected to be measured at RHIC.

Note added: After we submitted this paper (hep-ph/9706420), two preprints appeared in the Los Alamos archive on the same topic. Our anomalous dimensions agree not only with the Vogelsang's result (hep-ph/9706511) but also with the Hayashigaki, Kanazawa, and Koike's (hep-ph/9707208) although the integral expressions are different in some diagrams.

Appendix A: List of anomalous dimensions

Contributions from the diagrams in Figs. 2 and 3 to the anomalous dimension are listed. In the following equations, $\hat{H}_k$ is defined by $\hat{H}_k = (\not{\Delta}\not{p})^{-1}[H_k(n, b=0)\not{p} - H_k(n-1, b=1)\not{\Delta}]$, and DP (SP) denotes double-pole (simple-pole) part of the integral. The integrals H_k are listed in Appendix B, and I_k are the integrals given in Appendix of Ref. [8]. The function $\widetilde{F}$ is defined by $\widetilde{F}(a,c) = \sum_{J=0}^{c}(-1)^{a+J+1}\, c!\, F(a+J, c-J)/[J!\,(c-J)!]$. Some results are exactly the same with those in Ref. [8] as explained in the text:

$$\begin{aligned}&\gamma_n^{(2d)},\ \gamma_n^{(2g)},\ \gamma_n^{(2h)},\ \gamma_n^{(2l)},\ \gamma_n^{(2m)},\ \gamma_n^{(2q)},\ \gamma_n^{(2r)},\\ &\gamma_n^{(3a)},\ \gamma_n^{(3b)},\ \gamma_n^{(3c)},\ \gamma_n^{(3d)},\ \gamma_n^{(3e)} \quad \text{given in Ref. [8]} \quad .\end{aligned} \tag{5}$$

Other anomalous dimensions are calculated as

$$\gamma_n^{(2a)} = \gamma_n^{(2b)} = \gamma_n^{(2c)} = \gamma_n^{(2p)} = 0 \quad , \tag{6}$$

$$\gamma_n^{(2e)} = -\frac{g^4}{(4\pi)^4}\, C_F\, T_R\, \frac{16}{9} \quad , \quad \gamma_n^{(2f)} = +\frac{g^4}{(4\pi)^4}\, C_F\, C_G\, \frac{32}{9} \quad , \tag{7}$$

$$\begin{aligned}\gamma_n^{(2i)} = &+ g^4\,(C_F^2 - C_F C_G/2)\, 4\, (\Delta\cdot p)^{-(n-1)} \Big[4\,\hat{H}_1^{DP}(n) - 6\,\hat{H}_2^{DP}(n) + 4\,\hat{H}_3^{DP}(n) - 4\,\hat{H}_4^{DP}(n)\\
&-2\, I_2^{DP}(0,n-1) - I_{11}^{DP}(0,n-1) + 2\, I_{12}^{DP}(0,n-1) - I_{13}^{DP}(0,n-1) + 3\, I_{14}^{DP}(0,n-1)\\
&+(\Delta\cdot p)^{-1}\Big\{ I_2^{DP}(0,n) - 2\, I_2^{DP}(1,n-1) - I_{12}^{DP}(0,n) + I_{12}^{DP}(1,n-1) + I_{13}^{DP}(0,n)\\
&-I_{13}^{DP}(1,n-1) - I_{14}^{DP}(0,n) + 2\, I_{14}^{DP}(1,n-1)\Big\} + 4\,\hat{H}_2^{SP}(n) - 2 I_3^{SP}(n-1,0)\\
&+(\Delta\cdot p)^{-1}\Big\{ -2\, I_1^{SP}(0,n) + 2\, I_2^{SP}(0,n) - 4\, I_2^{SP}(1,n-1) + 2\, I_3^{SP}(n,0)\\
&+2\, I_3^{SP}(n-1,1) + 4\, I_6^{SP}(n) - 2\, I_{12}^{SP}(0,n) + 2\, I_{12}^{SP}(1,n-1) + 2\, I_{13}^{SP}(0,n)\\
&-2\, I_{13}^{SP}(1,n-1) - 2\, I_{14}^{SP}(0,n) + 2\, I_{14}^{SP}(1,n-1)\Big\}\\
&+(\Delta\cdot p)^{-2}\Big\{ 4\, H_6^{SP}(n+1,0) - 8\, H_6^{SP}(n,1) + 2\, I_1^{SP}(0,n+1) - 2\, I_2^{SP}(0,n+1)\end{aligned}$$

$$+4\, I_2^{SP}(1,n) - 2\, I_3^{SP}(n+1,0) - 4\, I_4^{SP}(n+1) - 4\, I_6^{SP}(n+1) + (-1)^n 4\, \widetilde{I}_8^{SP}(n-1,2)$$
$$+2\, I_{12}^{SP}(0,n+1) + 2\, I_{12}^{SP}(2,n-1) - 4\, I_{12}^{SP}(1,n) - 2\, I_{13}^{SP}(0,n+1)$$
$$+4\, I_{13}^{SP}(1,n) - 2\, I_{13}^{SP}(2,n-1) + 2\, I_{14}^{SP}(0,n+1) - 4\, I_{14}^{SP}(1,n) \Big\} \Big]$$
$$= -\frac{g^4}{(4\pi)^4}\,(C_F^2 - C_F C_G/2)\, 8\, \frac{S_1(n)}{n} \quad , \tag{8}$$

$$\gamma_n^{(2j)} = +g^4\, C_F C_G\, (\Delta\cdot p)^{-(n-1)} \Big[8\, \hat{H}_1^{DP}(n) - 4\, \hat{H}_2^{DP}(n) + 8\, \hat{H}_3^{DP}(n) + I_1^{DP}(0,n-1)$$
$$+I_1^{DP}(n-1,0) - 4\, I_2^{DP}(0,n-1) - 5\, I_9^{DP}(n-1,0) + (\Delta\cdot p)^{-1} \Big\{ -3\, I_1^{DP}(0,n)$$
$$+4\, I_1^{DP}(1,n-1) - I_1^{DP}(n-1,1) + 3\, I_2^{DP}(0,n) - 4\, I_2^{DP}(1,n-1) + I_2^{DP}(n-1,1) \Big\}$$
$$-8\, \hat{H}_2^{SP}(n) + 2\, I_1^{SP}(0,n-1) + 2\, I_1^{SP}(n-1,0) + (\Delta\cdot p)^{-1} \Big\{ 6\, I_1^{SP}(0,n)$$
$$-6\, I_1^{SP}(n-1,1) + 4\, I_1^{SP}(1,n-1) + 2\, I_2^{SP}(0,n) + 6\, I_2^{SP}(n-1,1)$$
$$-8\, I_2^{SP}(1,n-1) - 8\, I_3^{SP}(n,0) + 4\, I_3^{SP}(n-1,1) - 8\, I_6^{SP}(n) \Big\}$$
$$+(\Delta\cdot p)^{-2} \Big\{ (-1)^{n+1} 16\, \widetilde{H}_6^{SP}(n,1) + 4\, I_1^{SP}(n-1,2) - 8\, I_1^{SP}(1,n) - 4\, I_2^{SP}(n-1,2)$$
$$+8\, I_2^{SP}(1,n) - 8\, I_4^{SP}(n+1) + 8\, I_6^{SP}(n+1) - 8\, I_8^{SP}(n-1,2) + 16\, I_8^{SP}(n,1) \Big\} \Big]$$
$$= 0 \quad , \tag{9}$$

$$\gamma_n^{(2k)} = +g^4\, (C_F^2 - C_F C_G/2)\, 2\, (\Delta\cdot p)^{-(n-1)} \Big[(-1)^n\, 8\, \hat{H}_4^{DP}(n) - I_{11}^{DP}(n-1,0) + 2\, I_{12}^{DP}(n-1,0)$$
$$+I_{13}^{DP}(n-1,0) + 3\, I_{14}^{DP}(n-1,0) + (\Delta\cdot p)^{-1} \Big\{ I_2^{DP}(n-1,1) - I_{12}^{DP}(n,0)$$
$$+I_{12}^{DP}(n-1,1) + I_{13}^{DP}(n,0) - I_{13}^{DP}(n-1,1) - I_{14}^{DP}(n-1,1) \Big\}$$
$$+8\,\{1-(-1)^n\}\,\{\hat{H}_4^{SP}(n) - \hat{H}_5^{SP}(n)\} + (\Delta\cdot p)^{-1} \Big\{ 4\, H_7^{SP}(n-1,1) - 4\, H_7^{SP}(n,0)$$
$$-4\, \widetilde{H}_7^{SP}(n-1,1) + 2\, I_2^{SP}(n-1,1) - 2\, I_3^{SP}(n,0) - 2\, \widetilde{I}_3^{SP}(n,0) + 2\, I_{12}^{SP}(n-1,1)$$
$$-2\, I_{12}^{SP}(n,0) - 2\, I_{13}^{SP}(n-1,1) + 2\, I_{13}^{SP}(n,0) - 2\, I_{14}^{SP}(n-1,1) \Big\}$$
$$+(\Delta\cdot p)^{-2} \Big\{ (-1)^n\, 4\, H_6^{SP}(n+1,0) + 8\, H_7^{SP}(n,1) - 4\, H_7^{SP}(n+1,0)$$
$$-4\, H_7^{SP}(n-1,2) + 4\, \widetilde{H}_7^{SP}(n-1,2) + 2\, I_3^{SP}(n+1,0) - 4\, I_8^{SP}(n-1,2) - 8\, \widetilde{I}_8^{SP}(n,1)$$
$$+4\, \widetilde{I}_8^{SP}(n-1,2) \Big\} \Big]$$
$$= -\frac{g^4}{(4\pi)^4}\,(C_F^2 - C_F C_G/2)\, 8\, \{1-(-1)^n\}\, \frac{1}{n(n+1)} \quad , \tag{10}$$

$$\gamma_n^{(2n)} = +4\,g^4\,C_F C_G\,(\Delta\cdot p)^{-(n-1)}\sum_{j=1}^{n-1}\Big[\,I_1^{SP}(j,n-1-j)+(\Delta\cdot p)^{-1}\Big\{-I_1^{SP}(n-1-j,j+1)$$

$$+I_1^{SP}(j,n-j)+I_2^{SP}(n-1-j,j+1)-I_2^{SP}(j,n-j)\Big\}\Big]$$

$$= +\frac{g^4}{(4\pi)^4}\,C_F C_G\,4\left[\frac{1}{2n}\left\{4S_1(n)-S_1^2(n)-S_2(n)\right\}\,+\,S_3(n)\,-\,2\,\hat{S}(n)\right]\;, \tag{11}$$

$$\gamma_n^{(2o)} = +g^4\,(C_F^2-C_F C_G/2)\,8\,(\Delta\cdot p)^{-(n-1)}\sum_{j=1}^{n-1}\Big[-I_{13}^{SP}(n-1-j,j)+I_{13}^{SP}(n-j,j-1)$$

$$+I_{14}^{SP}(n-1-j,j)+I_{14}^{SP}(n-j,j-1)+(\Delta\cdot p)^{-1}\Big\{+I_2^{SP}(n-1-j,j+1)$$

$$-I_2^{SP}(n-j,j)-I_{12}^{SP}(n-1-j,j+1)+2\,I_{12}^{SP}(n-j,j)-I_{12}^{SP}(n-j+1,j-1)$$

$$+I_{13}^{SP}(n-1-j,j+1)-2\,I_{13}^{SP}(n-j,j)+I_{13}^{SP}(n-j+1,j-1)$$

$$-I_{14}^{SP}(n-1-j,j+1)+I_{14}^{SP}(n-j,j)\Big\}\Big]$$

$$= +\frac{g^4}{(4\pi)^4}\,(C_F^2-C_F C_G/2)\,8\Big[2\,S_1(n)\,\Big\{\frac{1}{n}-2\,S_2'(n/2)\,+2\,S_2(n)\Big\}$$

$$-2\,S_3(n)\,-\,S_3'(n/2)\,+\,4\,\hat{S}(n)\,+\,8\,\widetilde{S}(n)\Big]\;. \tag{12}$$

In these equations, the color constants are $C_G = N_c$, $C_F = (N_c^2-1)/(2N_c)$, and $T_R = N_f/2$, where N_c is the number of color and N_f is the number of flavor. The functions S_1, S_2, and others are defined by $S_j(n)=\sum_{k=1}^n 1/k^j$, $S_j'(n/2)=[\{1+(-1)^n\}S_j(n/2)+\{1-(-1)^n\}S_j((n-1)/2)]/2$, $\widetilde{S}(n)=\sum_{k=1}^n(-1)^k S_1(k)/k^2$, and $\hat{S}(n)=\sum_{k=1}^n S_1(k)/k^2$. We have to be careful in handling the $I_3(0,c)$, $I_8(0,c)$, and $H_7(0,c)$ terms; however, the above results are valid also for $n=1$. From the above expressions together with other diagram results including the quark-field renormalization part in Refs. [8] and [9], we obtain the anomalous dimension for the structure function h_1:

$$\gamma_n^{total} = C_F^2\Big[32\,S_1(n)\,\{\,S_2(n)\,-\,S_2'(n/2)\,\}\,+\,24\,S_2(n)\,-\,8\,S_3'(n/2)\,+\,64\,\widetilde{S}(n)$$

$$-8\,\frac{1-(-1)^n}{n(n+1)}\,-\,3\,\Big]+\,C_F\,T_R\Big[-\,\frac{160}{9}\,S_1(n)\,+\,\frac{32}{3}\,S_2(n)\,+\,\frac{4}{3}\,\Big]$$

$$+\,C_F\,C_G\Big[\frac{536}{9}\,S_1(n)\,-\,\frac{88}{3}\,S_2(n)\,+\,4\,S_3'(n/2)\,-\,32\,\hat{S}(n)$$

$$-\,16\,S_1(n)\,\{\,2S_2(n)\,-\,S_2'(n/2)\,\}\,+\,4\,\frac{1-(-1)^n}{n(n+1)}\,-\,\frac{17}{3}\,\Big]\;, \tag{13}$$

where the factor $g^4/(4\pi)^4$ is taken out.

Appendix B: Integrals

We list integrals H_k. The simple-pole terms are written with counter terms and the double-pole terms are without them. Finite contributions are neglected. Because the $\hat{H}$ functions are used for H_{1-5} in Appendix A, we express their results in the $\hat{H}$ form:

$$H_1(a,b) = \int \frac{d^d k_1}{(2\pi)^d} \int \frac{d^d k_2}{(2\pi)^d} \frac{(\Delta \cdot k_1)^a \, (\Omega \cdot k_1)^b \, \not{k}_1}{(k_1^2)^2 \, (k_1 - p)^2 \, (k_2 - k_1)^2 \, (k_2 - k_1 + p)^2} \quad ,$$

$$\hat{H}_1(a) = +\frac{1}{(4\pi)^4} \, (\Delta \cdot p)^{a-1} \, \frac{1}{2a} \, \hat{S}_1(\epsilon, a) \quad , \tag{14}$$

where $\hat{S}_1(\epsilon,a) = -4/\epsilon^2 - (2/\epsilon)/(a+1)$. The following integrals are given by the above expression with some replacements:

$\hat{H}_2$: $(k_2 - k_1)^2 \to k_2^2$ and $\hat{S}_1(\epsilon,a) = -2/\epsilon^2 + (1/\epsilon)[-1/(a+1) + 1/a - 2]$,

$\hat{H}_3$: $(k_2 - k_1 + p)^2 \to k_2^2$ and $\hat{S}_1(\epsilon,a) = -2/\epsilon^2 + (1/\epsilon)[S_1(a) - 1/(a+1) - 2]$,

$\hat{H}_4$: $(k_1 - p)^2 (k_2 - k_1 + p)^2 \to (k_2 - p)^2 (p + k_1 - k_2)^2$

and $\hat{S}_1(\epsilon,a) = -(2/\epsilon^2)/a + (1/\epsilon)[1/(a+1) - 1/a - 1/a^2]$,

$\hat{H}_5$: $\not{k}_1/[(k_1-p)^2(k_2-k_1+p)^2] \to \not{k}_2/[(k_2-p)^2(p+k_1-k_2)^2]$. $\hat{H}_5 - \hat{H}_4$ is expressed by Eq. (14) with $\hat{S}_1(\epsilon,a) = +(1/\epsilon)/a$.

Different integral types are

$$\begin{aligned} H_6(a,c) &= \int \frac{d^d k_1}{(2\pi)^d} \int \frac{d^d k_2}{(2\pi)^d} \frac{(\Delta \cdot k_1)^a \, (\Delta \cdot k_2)^c \, k_2 \cdot p}{(k_1^2)^2 \, (k_1 - k_2)^2 \, k_2^2 \, (k_2 - k_1 + p)^2} \\ &= \frac{1}{(4\pi)^4} \frac{(-1)^c \, a! \, c!}{(a+c+1)! \, (a+c)} (\Delta \cdot p)^{a+c} \Big[-\frac{1}{\epsilon^2} + \frac{1}{2\epsilon} \Big\{ -S_1(a+c) + S_1(a) \\ &\qquad + S_1(c) - \frac{1}{a+c+1} - \frac{1}{a} + 1 \Big\} \Big] \quad , \end{aligned} \tag{15}$$

$$\begin{aligned} H_7(a,c) &= \int \frac{d^d k_1}{(2\pi)^d} \int \frac{d^d k_2}{(2\pi)^d} \frac{(\Delta \cdot k_1)^a \, (\Delta \cdot k_2)^c \, k_1 \cdot p}{(k_1^2)^2 \, (k_2 - p)^2 \, (k_1 - k_2)^2 \, (p + k_1 - k_2)^2} \\ &= \frac{1}{(4\pi)^4} (\Delta \cdot p)^{a+c} \sum_{J=0}^{c} \frac{(a+c-J-1)! \, (J+1)! \, a! \, c!}{(c-J)! \, (a+c)! \, (a+J+1)!} \Big[-\frac{1}{\epsilon^2} \\ &\quad + \frac{1}{2\epsilon} \Big\{ -2S_1(a+c+1) + S_1(a+c-J-1) + S_1(a+J+1) \Big\} \Big] \quad . \end{aligned} \tag{16}$$

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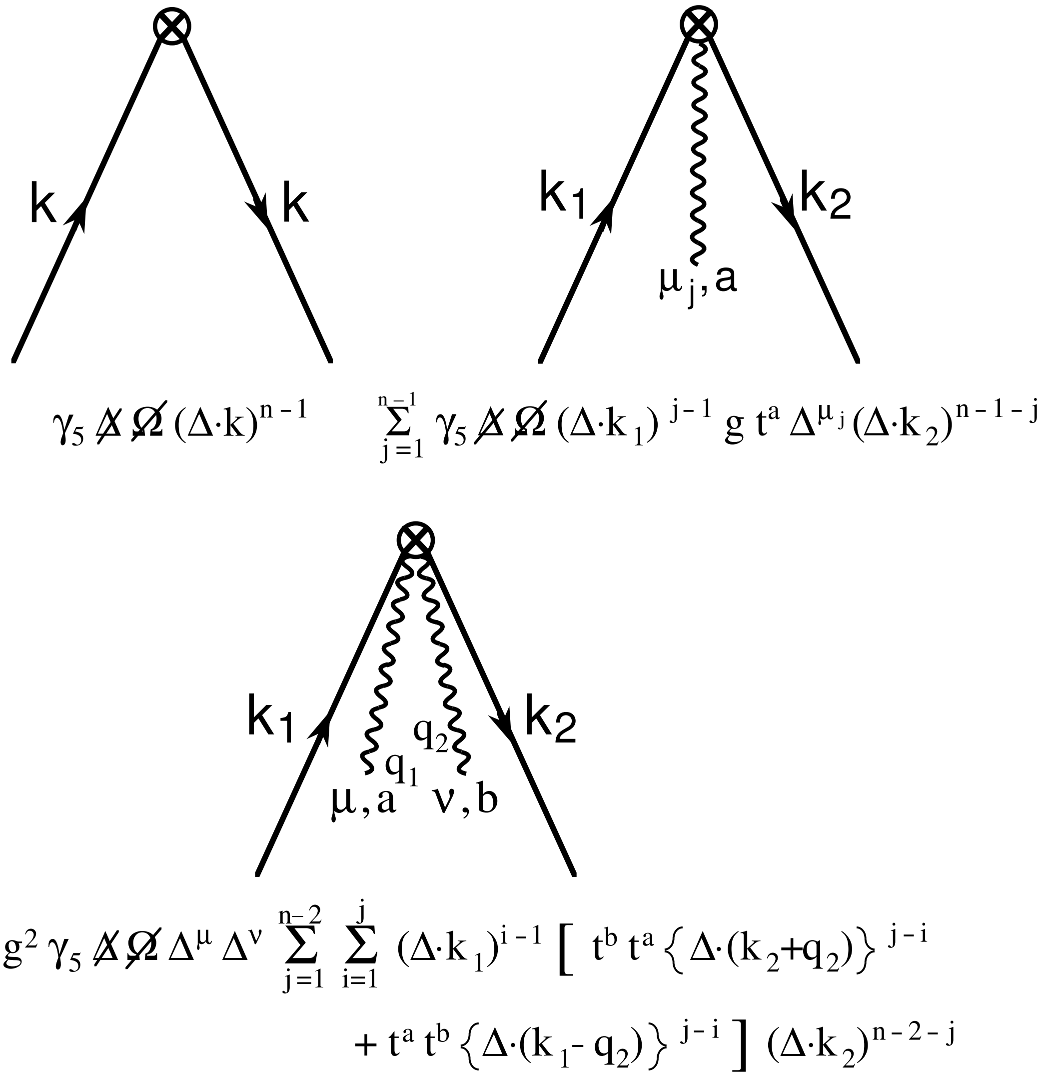

FIG. 1. Feynman rules for zero, one, and two gluon vertices.

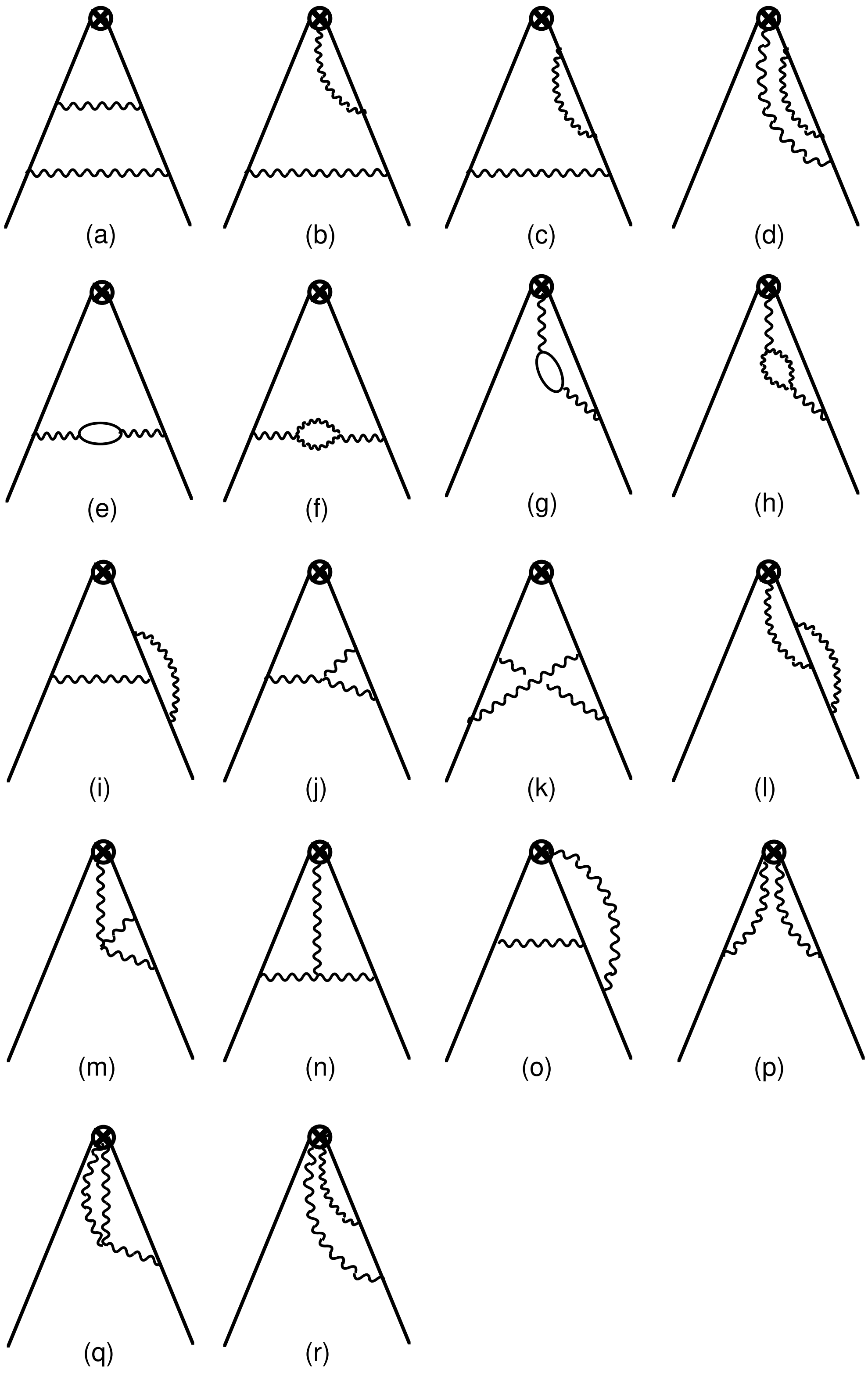

FIG. 2. Contributions to the two-loop anomalous dimension.

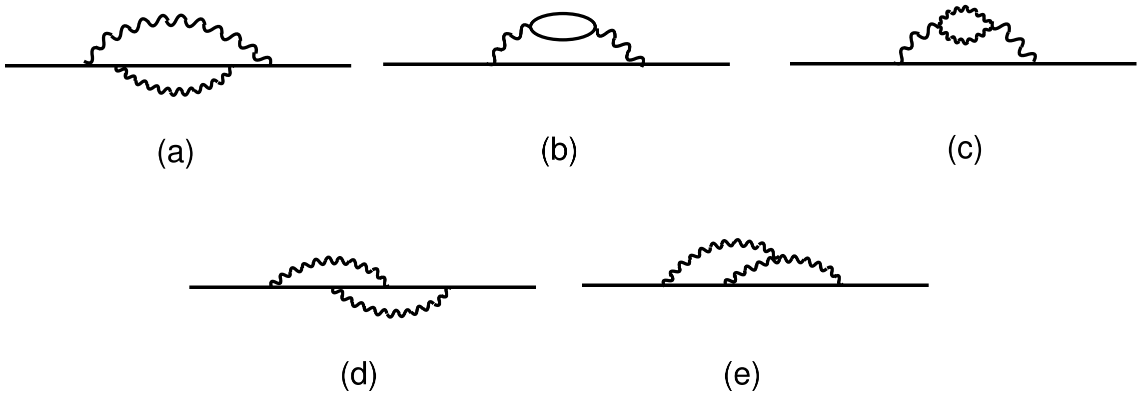

FIG. 3. Two-loop contributions to the quark self-energy.

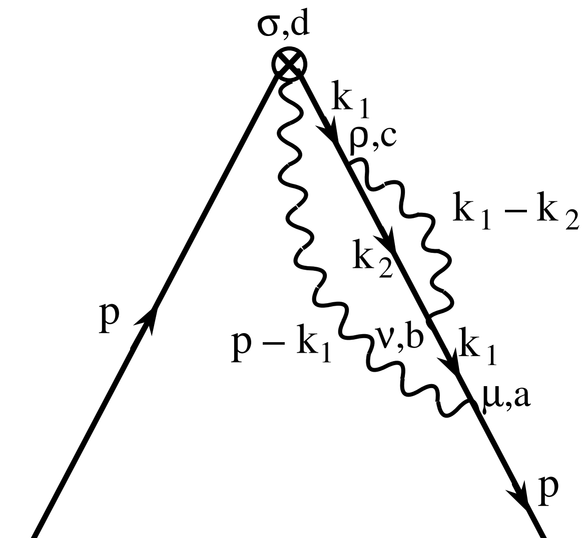

FIG. 4. Lorentz and color indices are shown with momenta for the diagram in Fig. 2(d).