2015@KEK-B

Scale dependence from additional degree of freedom

References

K. Kohri and T. Matsuda, arXiv:1405.6769
"Ambiguity in running spectral index with an extra light field during inflation",

And

K. Kohri, Chia-Min Lin and T. Matsuda,
to appear

Easiest way to catch the point is..

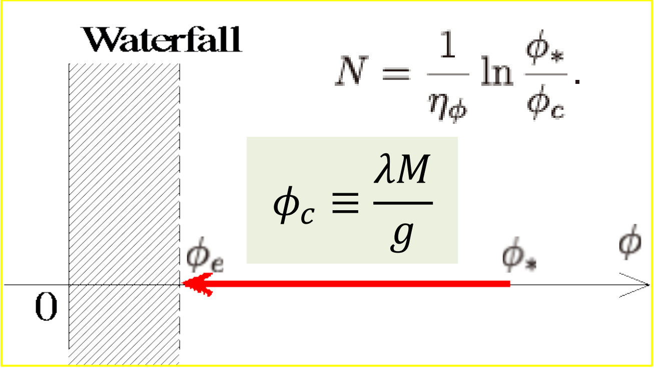
Hybrid Inflation + trivial field

First consider the inflaton ϕ and the waterfall field σ with the hybrid-type potential given by

$$V(\phi,\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - M^2\right)^2 + \frac{g^2}{2}\sigma^2\phi^2 + \frac{1}{2}m_\phi^2\phi^2.$$

Then, what happens if a trivial field is added ?

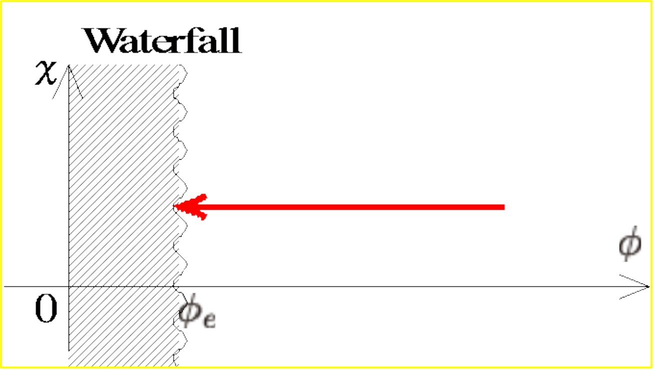
$+\frac{1}{2}m_\chi^2\chi^2$ $m_\chi \sim H$ is possible



To avoid confusion, remember that Lyth's model of "generating curvature perturbation at the end" uses $\sigma^2\chi^2$-interaction to create $\delta\phi_e \propto \delta\chi$

If one introduces "light χ"
with the same interaction
one will find $\phi_e^2 + \chi^2 = \left(\frac{\lambda M}{g}\right)^2$
Then the end ϕ_e is modulated as

$$\delta\phi_e = -\frac{g\chi}{\lambda M}\delta\chi$$



This is not my case. My question is
"what happens if a trivial (non-interacting) field is added to the model?"

Simple example : Hybrid inflation plus one

T.Matsuda JCAP **1204**, 020 (2012) arXiv:1204.0303

Just for the simplest example, we consider

$$V(\phi,\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - M^2\right)^2 + \frac{g^2}{2}\sigma^2\phi^2 + \frac{1}{2}m^2(\phi^2 + \chi^2).$$

Note that **unlike** the usual multi-field extension of the hybrid-type potential we are omitting interaction $\sim \sigma^2\chi^2$. Degeneracy of the mass term ($m_\chi = m_\phi \equiv m$) makes the trajectory straight. *Equal mass is just for simplicity

The adiabatic field is defined as $\sigma^2 \equiv \phi^2 + \chi^2$, which gives

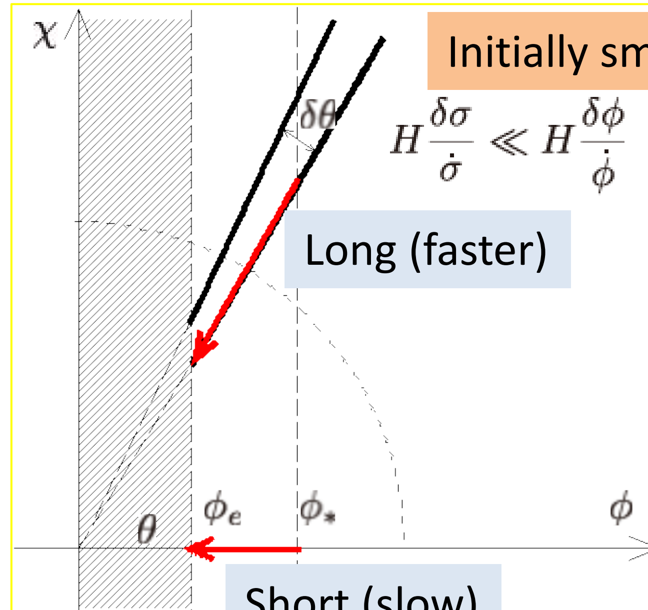
$$\phi = \sigma\cos\theta$$
$$\chi = \sigma\sin\theta.$$

$$\sigma_e(\theta) \equiv \frac{\phi_c}{\cos\theta} = \frac{\lambda M}{g\cos\theta};$$

$\delta\theta$ generates δN_e at the end.

$$\delta N_e \simeq H\frac{\delta\sigma_e}{\dot{\sigma}_e} \simeq H\frac{\delta\phi}{\dot{\phi}}$$

compensated



$$N = \frac{1}{\eta_\sigma}\ln\frac{\sigma_*}{\sigma_c} = \frac{1}{\eta_\phi}\ln\frac{\phi_*}{\phi_c}.$$

Identical because of the similar triangle
Intuitively no change in δN.

Changed Nothing ?

No! Difference appears in the scale dependence

Hybrid inflation plus one: Scale dependence

1. Spectral index is different (The same reason as the original Lyth's)

$n_s - 1 \simeq -2\epsilon_H + 2\eta_\phi$ for "fast χ" and $n_s - 1 \simeq -6\epsilon_H + 2\eta_\phi$ for "slow χ"

2. $\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \epsilon_\phi + \epsilon_\chi$ is different because $\epsilon_\chi \neq 0$

$\epsilon_\phi \ll \epsilon_\chi$ for $\theta \sim \pi/2 \rightarrow \epsilon_\chi$ can dominate n_s

3. Original Hybrid inflation predicts blue spectrum $n_s - 1 \simeq 2\eta_\phi > 0$
   This can be cured if $\epsilon_\chi > \eta_\phi$....???

"replacement" could be possible, but it requires

$\epsilon_\chi > \eta_\phi$ requires $\frac{\chi}{M_p} > \frac{2\sqrt{\eta_\phi}}{\eta_\chi}$

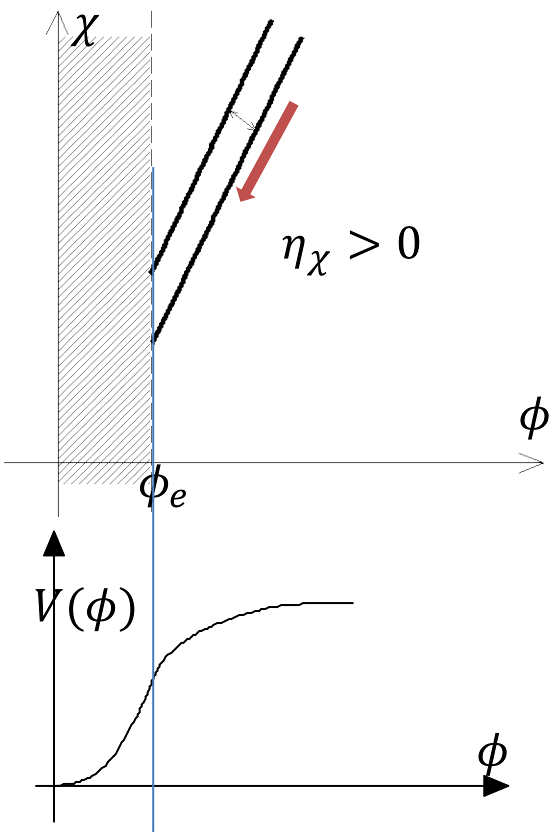
Note:
Trivial scalar field cannot change the spectrum
but it can change the scale-dependence

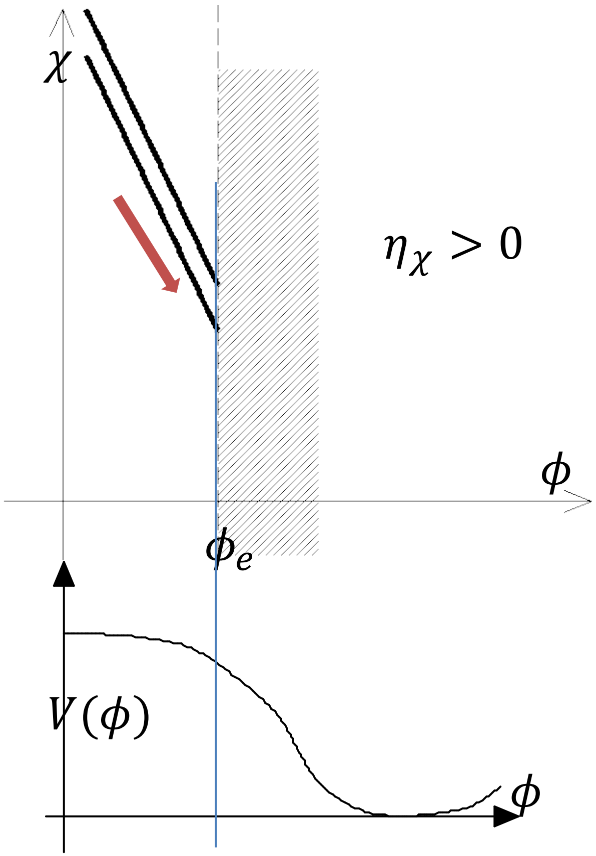
Previous “hybrid inflation plus one” is indeed a multi-field model of inflation

Now we consider **Single field inflation plus one**

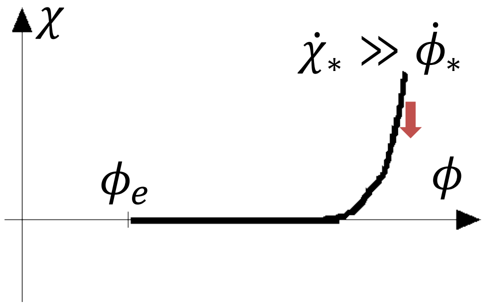
You may easily find that other inflationary models may have similar property when the original model uses $V = V_0 + V(\phi)$

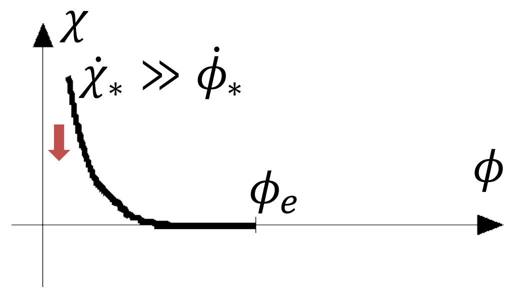
Initial curvature perturbation is smaller than the original if $\dot{\chi} > \dot{\phi}$.
It can be compensated at the end





Single field inflation plus one (another possibility)





χ decays fast
No effect on ϕ_e

Intuitively N is determined by the motion of ϕ

We define the instantaneous adiabatic and entropy perturbations as

$$\delta\sigma = \delta\phi\cos\varphi + \delta\chi\sin\varphi$$
$$\delta s = -\delta\phi\sin\varphi + \delta\chi\cos\varphi,$$

C. T. Byrnes and D. Wands,
"Curvature and isocurvature perturbations from two-field inflation in a slow-roll expansion,"
Phys. Rev. D **74**, 043529 (2006) [astro-ph/0605679].

where $\tan\varphi = \dot{\chi}/\dot{\phi}$. Also we define the curvature and entropy perturbations

$$R = H\frac{\delta\sigma}{\dot{\sigma}}$$
$$S = H\frac{\delta s}{\dot{\sigma}}.$$

One can define the evolution as

$$R = [R_* + T_{RS}S_*],$$

In the above cases R finally recovers $\sim H\frac{\delta\phi}{\dot{\phi}}$

Compensated during inflation
Scale dependence changed

The index is calculated as

$$n_s - 1 \simeq -(6 - 4\cos^2\theta)\epsilon_H + 2\eta_{\sigma\sigma}\sin^2\theta + 4\eta_{\sigma s}\sin\theta\cos\theta + 2\eta_{ss}\cos^2\theta.$$

$$\cos\theta = \frac{T_{RS}}{\sqrt{1+T_{RS}^2}}.$$

Think about $\dot{\chi}_* \gg \dot{\phi}_*$.

Initially the adiabatic direction is $\dot{\sigma} \simeq \dot{\chi}$ but it will be $\dot{\sigma} \simeq \dot{\phi}$ before the end.

One will find $\cos\theta \sim 1$ and the spectral index is

$$n_s - 1 \simeq -2\epsilon_H + 2\eta_\phi. \qquad \epsilon_H \simeq \epsilon_\chi \gg \epsilon_\phi$$

Fast χ, Changes are obvious
The same as hybrid

The opposite limit ($\dot{\chi}_* \ll \dot{\phi}_*$) is the standard single-field inflation plus $\epsilon_\chi \neq 0$. The spectral index will be

$$n_s - 1 \simeq -6\epsilon_H + 2\eta_\phi. \qquad \epsilon_H \simeq \epsilon_\phi \gg \epsilon_\chi$$

Small χ, Small shift
The same as hybrid

Although the shift of n_s is not significant, slow χ is not trivial since higher runnings can be dominated by χ.

For this scenario running of n_s changes => $\Delta\alpha_s \sim 12\eta_\chi\epsilon_\chi$

This could be large/negative. η_χ is not bounded by the spectral index

Single inflation plus "nothing"

"Deviation from the slow-roll velocity"
may change the scale dependence of the spectrum

Remember a model O. Seto, J. Yokoyama and H. Kodama, "What happens when the inflaton stops during inflation," Phys. Rev. D **61**, 103504 (2000) [astro-ph/9911119].

Even if the inflaton stops during inflation
(velocity deviates from the slow-roll velocity defined at ϕ_*)

$\dot{\phi} \neq \dot{\phi}_s \equiv -\frac{V_\phi(\phi_*)}{3H}$,

the curvature perturbation can converge

$H\frac{\delta\phi}{\dot{\phi}} \to H\frac{\delta\phi}{\dot{\phi}_s}$ before the end of inflation.

Similar to the previous models
What about the scale dependence?

If one uses $d\ln k = Hdt$, the deviation changes the result

$$\dot{\epsilon}_\phi = \dot{\phi}\frac{d\epsilon_\phi}{d\phi} \equiv [1+R_D]\left[\dot{\phi}_s\frac{d\epsilon_\phi}{d\phi_s}\right]$$

Without deviation

$$\mathrm{n_s} - 1 = (1+\mathrm{R_D})(n_s^0 - 1 - 6\epsilon_\phi R_D)$$

Can reverse the sign!

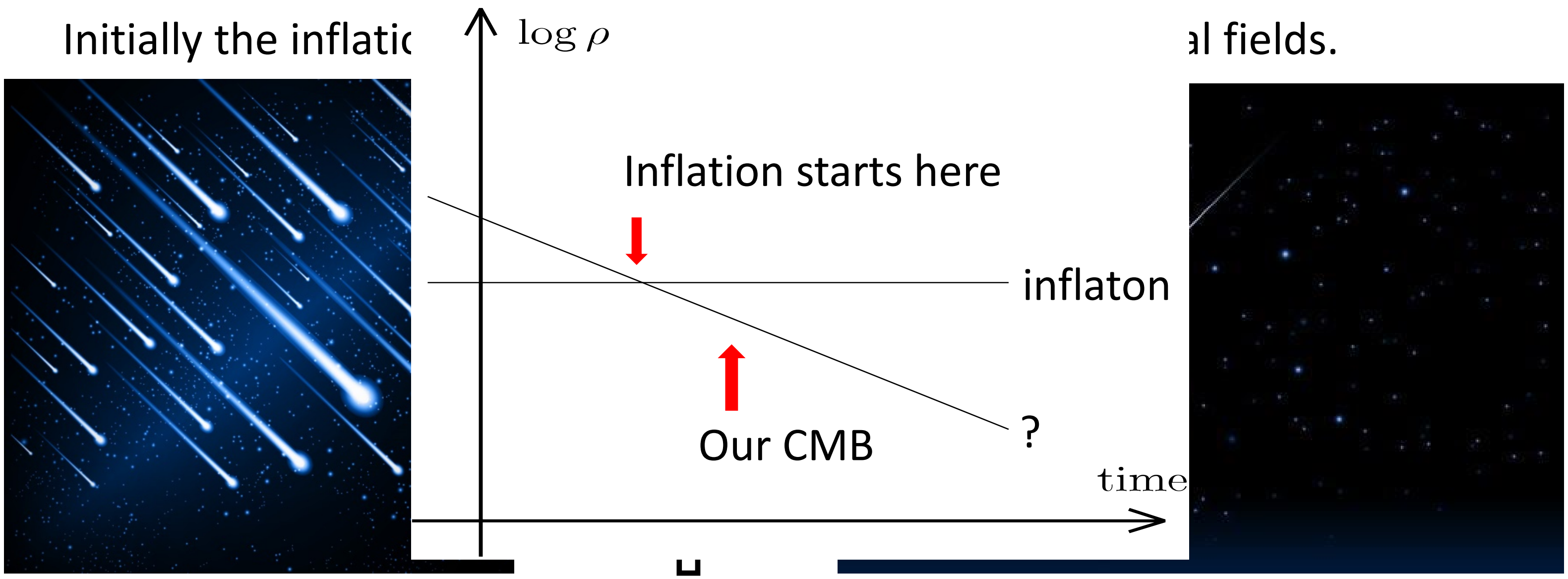
$1 + R_D = 0$ when inflaton stops

Sorry about such a long preamble

Main part of this talk

We have seen that trivial field may not change the spectrum but may change the scale dependence

Initially the inflati … al fields.



Our CMB is generated somewhere between

We have seen that
Just one trivial field can change the scale-dependence of the spectrum,
while the scale dependence is now being used to discriminate inflationary models.

The ambiguity must be removed by future observations

This talk is aimed to explain how to remove this ambiguity

How to measure the "shift"

The spectrum of the tensor perturbation is $P_T^{1/2} = H/(2\pi)$, which gives the scalar to tensor ratio

$$r = \frac{P_T}{P_\mathcal{R}}.$$

Trivial χ does not change this ratio

Usually one will find $r = 16\epsilon_\phi$ for single field inflation.

Think about running of the tensor mode

$$n_t \equiv \frac{d\log P_T}{d\log k} = -2\epsilon_H = -2\epsilon_\phi - 2\epsilon_\chi$$

$\mathrm{n_t}$ shifted when $\epsilon_\chi \neq 0$

Think about $r + 8n_t$

$r + 8n_t = 0$ is exact for standard single-field inflation

$r + 8n_t = -16\epsilon_\chi \neq 0$ is the sign of trivial remnant χ

We need n_t!

More specifically, if $r + 8n_t \simeq 8(n_s - 1) \simeq -0.28$ is found ($n_t \simeq n_s - 1 \simeq -0.035\,, r \simeq 0$) it means χ is initially the adiabatic field. The index is

$$n_s - 1 = -2\epsilon_H + 2\eta_\phi \simeq -2\epsilon_\chi$$

Case with fast $\chi (\dot\chi > \dot\phi)$

If $r + 8n_t \neq 8(n_s - 1)$ but still $r + 8n_t \neq 0$,
$n_s - 1$ is shifted by $\Delta(n_s - 1) \simeq -6\epsilon_\chi \simeq 0.4(r + 8n_t)$

Case with slow $\chi (\dot\chi < \dot\phi)$

In both cases the shift of the index can be fixed by the tensor mode

Other possibilities

*Of course the relation is violated (by definition) when the source of the curvature perturbation is the curvaton (σ) or modulation (e.g. modulated reheating).

K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, "Non-Gaussianity, Spectral Index and Tensor Modes in Mixed Inflaton and Curvaton Models," Phys. Rev. D **78**, 023513 (2008) [arXiv:0802.4138 [astro-ph]].

The original curvaton expects $r \simeq \epsilon_\sigma \simeq \eta_\sigma \simeq 0$.

In that simplest case one can expect

$$r + 8n_t = -16\epsilon_\phi \simeq 8(n_s - 1) \simeq -0.28$$

i.e.

$$n_t \simeq -0.035$$

(Same with "fast χ")

Of course we cannot see n_t when $r \simeq 0$. The tensor mode is unseen.

* I am not claiming that $n_t \simeq -0.035$ is the sign of the curvaton

Tensor mode is important only when r is observable.

Important for the "mixed" curvaton models.

What if ϵ_χ is coming from radiation?

"χ" could be "gas (radiation)" or "matter"
These can contribute ϵ_χ in the same way

Suppose that
$r + 8n_t = -16\epsilon_\chi \neq 0$
could be observed.
How can we identify the source?

1. If the remnant is radiation or matter, it decays fast

$$\dot{\rho}_\chi = -3(1+w)H\rho_\chi$$

2. Then we find

$$\epsilon_\chi = \frac{3(1+w)}{2}\frac{\rho_\chi}{\rho_\phi}$$

$$\Delta\alpha_s \simeq 18(1+w)\epsilon_\chi$$ ⬅ No suppression

3. Suppose upper bound $\alpha_s < \hat{\alpha}_s$ we find

$$\epsilon_\chi < \frac{1}{18(1+w)}\hat{\alpha}_s \quad \text{and} \quad \frac{\rho_\chi}{\rho_\phi} < \frac{\hat{\alpha}_s}{27(1+w)^2}$$

4. Since $\rho_\chi \leq \rho_\phi$ at the beginning, our CMB exits when

$$N > 5 \qquad \text{for} \qquad \hat{\alpha}_s \simeq 10^{-5}$$

Seeing higher running we can see that
Extra "Gas" or "matter" is a negligible source of the scale dependence

Summary

As far as I know th … disregarded,
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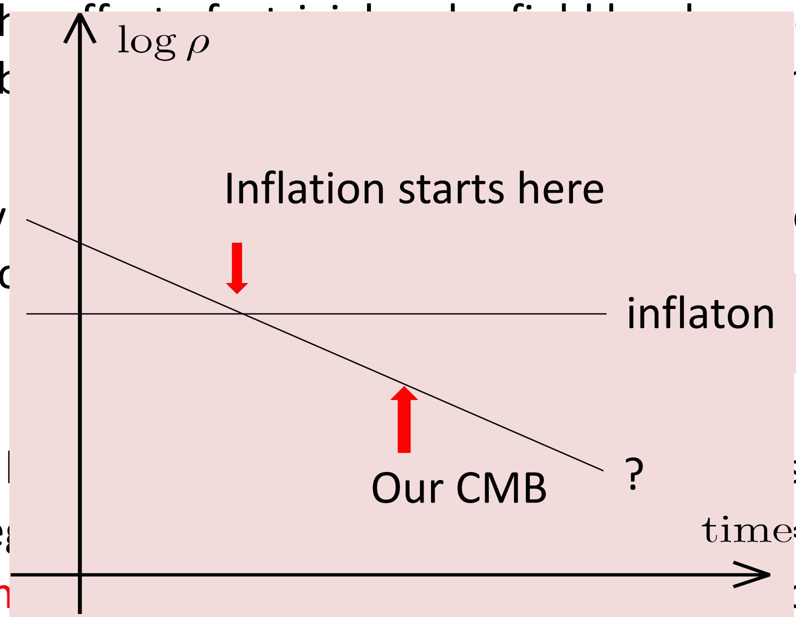

Excluding inflationary models using the current data could be "fast and sloppy". "Something else" could be there at the very beginning of inflation, and it can change the scale-dependence of the inflationary spectrum. We must (1) identify the component or (2) confirm that "something else" is already gone away.