The Effects of Radiative Corrections on Inflation Observables

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Kari Enqvist & MK, JCAP02 (2014) [arXiv:1312.5944]

Introduction

1. Single field inflation - effective particle physics theory with heavy degrees and interactions integrated out;
2. Reheating cannot be integrated out $\Rightarrow$ radiative corrections;
3. Cosmological observables and renormalization scale;

$$V(\phi, \chi, \psi) = \frac{1}{2}\lambda_b m_{\mathrm{Pl}}^4 \left(\frac{\phi}{m_{\mathrm{Pl}}}\right)^n - \frac{1}{2} g_b^2 \phi^2 \chi^2 - h_b \phi \bar{\psi}\psi + \dots$$

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$$V_{\rm eff}\left(\phi\right) = \frac{1}{2}\lambda m_{\rm Pl}^4 \left(\frac{\phi}{m_{\rm Pl}}\right)^n + \frac{g^2 - 4h^4}{64\pi^2}\phi^4 \ln\frac{\phi^2}{\mu^2}$$

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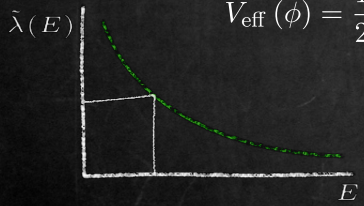

$$V_{\text{eff}}(\phi) = \frac{1}{2}\lambda m_{\text{Pl}}^4 \left(\frac{\phi}{m_{\text{Pl}}}\right)^n + \tilde{\lambda}(\phi)\,\phi^4$$

Inflation Observables

$$\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta *}\left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}n' \ln(k/k_*) + \ldots}$$

$$\mathcal{P}_t(k) = \mathcal{P}_{t *}\left(\frac{k}{k_*}\right)^{n_t + \ldots}$$

$$\mathcal{P}_{t*} = \frac{2}{3\pi^2}\frac{V}{m_{\rm Pl}^4} \qquad \Rightarrow \qquad r \equiv \frac{\mathcal{P}_{t*}}{\mathcal{P}_{s*}} = 16\epsilon$$

$$\mathcal{P}_{\zeta*} = \frac{1}{24\pi^2 m_{\rm Pl}^4}\frac{V}{\epsilon}$$

$$n_s = 1 + 2\eta - 6\epsilon$$

$$n' \equiv \frac{\mathrm{d} n_s}{\mathrm{d}\ln k} = -24\epsilon^2 + 16\epsilon\eta - 2\xi$$

$$\epsilon \equiv \frac{m_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta \equiv m_{\rm Pl}^2 \frac{V''}{V}$$

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Parameters

$$V_{\text{eff}}(\phi) = \frac{1}{2}\lambda m_{\text{Pl}}^4 \left(\frac{\phi}{m_{\text{Pl}}}\right)^n + \frac{g^2 - 4h^4}{64\pi^2}\phi^4 \ln\frac{\phi^2}{\mu^2}$$

where

$$\kappa \equiv \frac{g^4 - 4h^4}{16\pi^2\lambda}$$

Two cases of n:

1. $n = 2$
2. $n = 4$

Two cases of κ:

1. $g \gg h \quad \Rightarrow \quad \kappa > 0$
2. $h \ll g \quad \Rightarrow \quad \kappa < 0$

Two cases of μ:

1. $\mu = \varphi_*$
2. $\mu = \varphi_{\text{end}}$

Parameters

$$V_{\text{eff}}(\varphi) = \frac{1}{2}\lambda m_{\text{Pl}}^4 \left(\varphi^n + \kappa\varphi^4 \ln\frac{\varphi}{\mu}\right)$$

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Equations

For a given (κ, μ, n)

$$\epsilon\left(\varphi_{\mathrm{end}}\right) = 1$$

$$\int_{\varphi_*}^{\varphi_{\mathrm{end}}} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon\left(\varphi\right)}} = -N_*\left(T_{\mathrm{reh}}\left(\kappa\right)\right)$$

$$\mathcal{P}_{\zeta *} = \frac{1}{24\pi^2 m_{\mathrm{Pl}}^4}\frac{V\left(\lambda\right)}{\epsilon}$$

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Slow-roll parameter:

$$\epsilon = \frac{1}{2\varphi}\left[\frac{n + \kappa\varphi^{4-n}\left(1 + 4\ln\frac{\varphi}{\mu}\right)}{1 + \kappa\varphi^{4-n}\ln\frac{\varphi}{\mu}}\right]^2$$

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Number of e-folds of observable inflation:

$$N_*\left(T_{\mathrm{reh}}\right) = 68.5 + \frac{1}{2}\ln\frac{V_*}{m_{\mathrm{Pl}}^4} - \frac{1}{3}\ln\frac{V_{\mathrm{end}}}{m_{\mathrm{Pl}}^4} + \frac{1}{12}\ln\frac{\rho_{\mathrm{reh}}}{m_{\mathrm{Pl}}^4}$$

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Perturbative decay

$$\rho_{\mathrm{reh}} \simeq 3 m_{\mathrm{Pl}}^2 \Gamma^2\left(\kappa\right)$$

$$\phi \to \chi + \chi$$

$$\Gamma \simeq \frac{g^4\sigma^2}{8\pi m}$$

$$\kappa > 0$$

$$\phi \to \psi + \bar{\psi}$$

$$\Gamma \simeq \frac{h^2 m}{8\pi}$$

$$\kappa < 0$$

Equations

For a given (κ, μ, n)

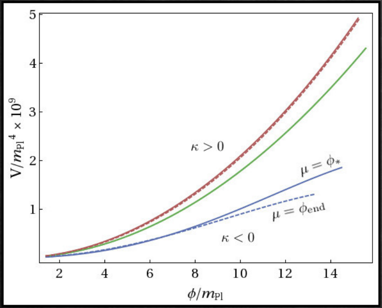
$$\epsilon\left(\varphi_{\rm end}\right) = 1$$

$$\int_{\varphi_*}^{\varphi_{\rm end}} \frac{{\rm d}\varphi}{\sqrt{2\epsilon\left(\varphi\right)}} = -N_*\left(T_{\rm reh}\left(\kappa\right)\right)$$

$$\mathcal{P}_{\zeta*} = \frac{1}{24\pi^2 m_{\rm Pl}^4}\frac{V\left(\lambda\right)}{\epsilon}$$

$$\lambda\left(\mu\right) = 24\pi^2\mathcal{P}_{\zeta*}\frac{\left[n+\kappa\varphi_*^{4-n}\left(1+4\ln\varphi_*/\mu\right)\right]^2}{\varphi_*^{2+n}\left[1+\kappa\varphi_*^{4-n}\ln\varphi_*/\mu\right]^3}$$

Example: $m^2\phi^2$ Inflation



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	$\mu = \phi_*$		
	1	2	3
$h \times 10^3$	1.24	1.05	0.414
$m\left(10^{13}\ \mathrm{GeV}\right)$	1.11	1.33	1.46
ϕ_*/m_{Pl}	14.7	15.2	15.4
N_*	58.7	58.8	58.6
	$\mu = \phi_{\mathrm{end}}$		
$h \times 10^3$	0.886	0.792	0.233
$m\left(10^{13}\ \mathrm{GeV}\right)$	1.18	1.38	1.47
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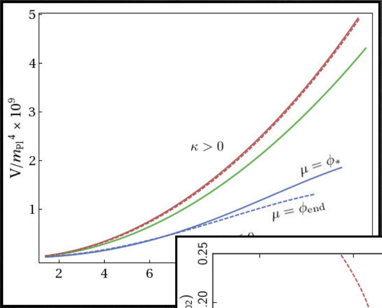

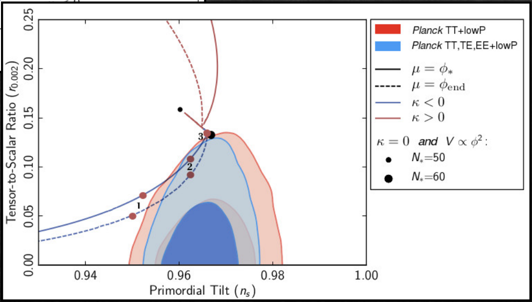

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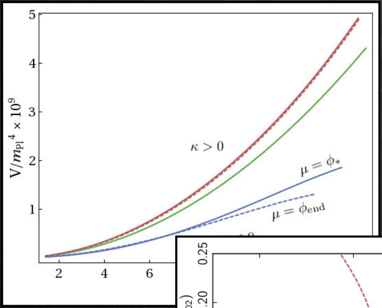
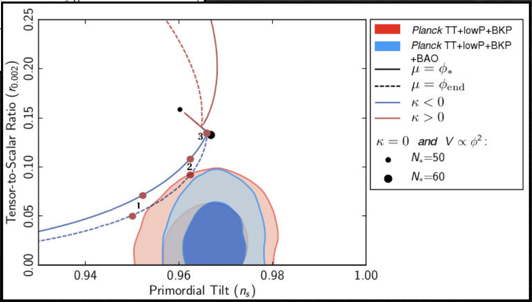
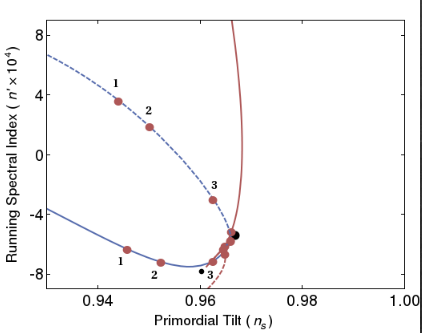
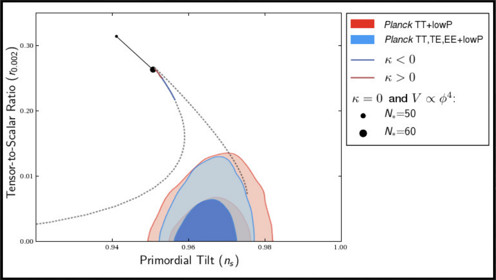




Running of the Spectrum



Example: ϕ^4 Inflation



Conclusions

In the era of precision cosmology:

1. The effects of radiative corrections can be important;
2. Specifying the model of inflation, one must specify the renormalization point;

Slow-Roll Parameters

$$
\begin{aligned}
\epsilon &= \frac{K(\varphi)^2}{2\varphi^2}\left[n + \kappa\varphi^{4-n}\left(1 + 4\ln\frac{\varphi}{\mu}\right)\right]^2 , \\
\eta &= \frac{K(\varphi)}{\varphi^2}\left[n\,(n-1) + \kappa\varphi^{4-n}\left(7 + 12\ln\frac{\varphi}{\mu}\right)\right] , \\
\xi &= \frac{K(\varphi)^2}{\varphi^4}\left[n + \kappa\varphi^{4-n}\left(1 + 4\ln\frac{\varphi}{\mu}\right)\right]\left[n\,(n-1)\,(n-2) + \kappa\varphi^{4-n}\left(14 + 46\ln\frac{\varphi}{\mu}\right)\right]
\end{aligned}
$$

where

$$
K^{-1}(\varphi) \equiv 1 + \kappa\varphi^{4-n}\ln\frac{\varphi}{\mu}
$$

$\mu = \phi_*$

	1	2	3	4		5	6
$\kappa \times 10^4$	-37	-28	-10	-0.2	0	0.3	4
$n_{\rm s}$	0.946	0.952	0.963	0.966	0.967	0.965	0.966
r	0.057	0.072	0.109	0.135	0.132	0.142	0.151
$n' \times 10^4$	-6.35	-7.20	-7.12	-5.77	-5.46	-6.14	-5.17
$h,\ g \times 10^3$	1.27	1.24	1.05	0.414	0	0.662	1.28
$m\left(10^{13}\ \mathrm{GeV}\right)$	1.01	1.11	1.33	1.46	1.01	1.53	1.57
$\phi_*/m_{\rm Pl}$	14.5	14.7	15.2	15.4	15.6	15.1	15.2
N_*	58.7	58.7	58.8	58.6	60	56.2	56.7

$\mu = \phi_{\rm end}$

	1	2	3	4		5	6
$\kappa \times 10^5$	-75	-64	-30	-0.2	0	0.8	7
n_s	0.944	0.950	0.963	0.966	0.967	0.965	0.965
r	0.0403	0.0509	0.0926	0.136	0.132	0.143	0.150
$n' \times 10^4$	3.63	1.88	-2.98	-5.76	-5.46	-6.31	-6.63
$h,\ g \times 10^4$	8.89	8.86	7.92	2.33	0	4.76	8.18
$m\left(10^{13}\ \mathrm{GeV}\right)$	1.10	1.18	1.38	1.47	1.01	1.53	1.53
$\phi_*/m_{\rm Pl}$	13.4	13.7	14.6	15.3	15.6	15.1	15.3
N_*	58.4	58.4	58.6	58.4	60	56.1	56.5