Large-scale magnetic fields, non-Gaussianity, and primordial gravitational waves from inflation

Reference: **Physical Review D 91, 043509 (2015)**
[arXiv:1411.4335 [astro-ph.CO]]

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I. Introduction

- **Galactic magnetic fields**

$$\sim \mu\mathrm{G}$$

[Sofue *et al*., Annu. Rev. Astron. Astrophys. 24, 459 (1986)]

- **Magnetic fields in clusters of galaxies**

$$0.1 - 10\mu\mathrm{G}$$

$$10\mathrm{kpc} - \underline{1\mathrm{Mpc}}$$

[Clarke *et al*., Astrophys. J. 547, L111 (2001)]

$\ast$ **Recent reviews (examples)**

[Kandus *et al*., Phys. Rep. 505, 1 (2011)] **[Yamazaki *et al*., Phys. Rep. 517, 141 (2012)]**

[Maleknejad *et al*., Phys. Rep. (2013)]

[Durrer and Neronov, Astron. Astrophys. Rev. 21, 62 (2013)]

Origin of cosmic magnetic fields

1. Astrophysical process: Plasma instability

[Biermann and Schlüter, Phys. Rev. 82, 863(1951)]

[Weibel, Phys. Rev. Lett. 2, 83 (1959)]

[Hanayama *et al*., Astrophys. J. 633, 941 (2005)]

[Fujita and Kato, Mon. Not. R. Astron. Soc. 364, 247 (2005)]

2. Cosmolosical processes:

- **Electroweak/Quark-hadron phase transitions**

[Baym, Bödeker and McLerran, Phys. Rev. D 53, 662 (1996)]

[Quashnock, Loeb and Spergel, Astrophys. J. 344, L49 (1989)]

- **Density perturbations** **[Ichiki *et al*., Science 311, 827 (2006)]**

[Kobayashi, *et al*, Phys. Rev. D 75, 103501 (2007)]

Origin of cosmic magnetic fields (2)

- Coherence scale
- Strength

→ It is difficult to obtain the results to explain the observations.

⇨ The most natural origin of large-scale magnetic fields:

Electromagnetic quantum fluctuations generated at the inflationary stage

Obstacle

- **Friedmann-Lemaître-Robertson-Walker (FLRW) metric is conformally flat.**

- **The Maxwell theory is conformally invariat.**

The conformal invariance has to be broken at the inflationary stage.

Breaking mechanisms

1. **Coupling of a scalar field to electromagnetic fields**

[Ratra, Astrophys. J. 391, L1 (1992)]

[KB and Yokoyama, Phys. Rev. D 69, 043507 (2004); 70, 083508 (2004)]

2. **Non-minimal coupling of electromagnetic fields to gravity**

[Turner and Widrow, Phys. Rev. D 37, 2743 (1988)]

[KB and Sasaki, JCAP 0702, 030 (2007)]

3. **Trace anomaly**

[Dolgov, Phys. Rev. D 48, 2499 (1993)]

Motivation and Purpose (1)

⇨ **By comparing the theoretical predictions of a toy model with observations, we obtain phenomenological implications on moduli inflation.**

Motivation and Purpose (2)

Model (a) **Coupling of the electromagnetic field to a scalar field**

(b) **That to a pseudo scalar field (inflaton)**

Both couplings: Novel point

∗ **In the past works: Only (b)**

E.g.: [Barnaby and Peloso, Phys. Rev. Lett. 106, 181301 (2011)]
[Barnaby, Namba and Peloso, JCAP 1104, 009 (2011)]

↕

Observables

(i) **Large-scale magnetic fields**

(ii) **Non-Gaussianity**

(iii) **Tensor-to-scalar ratio**

II. Model

$$\mathcal{L} = \frac{M_P^2}{2}R - \frac{1}{4}XF_{\mu\nu}F^{\mu\nu} - \frac{1}{4}g_{\rm ps}\frac{Y}{M}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu Y\partial_\nu Y - V(Y)$$

Gravity term

Breaking of the conformal invariance

$M_{\rm P} \simeq 2.4 \times 10^{18}\ {\rm GeV}$: Reduced Planck mass

R : Scalar curvature

Model (2)

$$\mathcal{L} = \frac{M_P^2}{2} R - \frac{1}{4} X F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\rm ps} \frac{Y}{M} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$- \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) - \frac{1}{2} g^{\mu\nu} \partial_\mu Y \partial_\nu Y - V(Y)$$

Coupling of a scalar field Φ to the electromagnetic field

$X \equiv \exp\left(-\lambda \Phi / M_P\right)$, λ : Normalization constant

$U(\Phi)$: Potential of Φ

$F_{\mu\nu} = \nabla_\mu F_\nu - \nabla_\nu F_\mu$, F_μ : $\mathrm{U}(1)_Y$ gauge field

Model (3)

$$\mathcal{L} = \frac{M_P^2}{2}R - \frac{1}{4}XF_{\mu\nu}F^{\mu\nu} - \frac{1}{4}g_{\mathrm{ps}}\frac{Y}{M}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu Y\partial_\nu Y - V(Y)$$

Coupling of a pseudo scalar field Y (inflaton) to the electromagnetic field

g_{ps} : Dimensionless constant

$\tilde{F}_{\mu\nu}$: Dual tensor of $F_{\mu\nu}$

M : Mass scale

$V(Y) \approx \bar{V} - \frac{1}{2}m^2Y^2$: Potential of Y

$\bar{V}$: Normalization constant

(Feature of moduli inflation)

m : Mass of Y

Equations of motion (EoM) (1)

Flat FLRW space-time

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2$$

$a(t)$: Scale factor

EoM for Φ

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU(\Phi)}{d\Phi} = 0$$

$H = \dot{a}/a$: Hubble parameter

$\dot{} = \partial/\partial t$

EoM for Y (inflaton)

$$\ddot{Y} + 3H\dot{Y} + \frac{dV(Y)}{dY} = 0$$

Equations of motion (EoM) (1)

EoM for the $\mathrm{U}(1)_Y$ gauge field

* $F_0 = \partial_j F^j(t, \boldsymbol{x}) = 0$: Coulomb gauge

$$\ddot{F}_i(t, \boldsymbol{x}) + \left(H + \frac{\dot{X}}{X} \right) \dot{F}_i(t, \boldsymbol{x}) - \frac{1}{a^2} \partial_j \partial_j F_i(t, \boldsymbol{x}) - \frac{g_{\rm ps}}{M} \frac{1}{aX} \dot{Y} \epsilon^{ijk} \partial_j F_k(t, \boldsymbol{x}) = 0$$

Breaking of the conformal invariance

ϵ_{ijk} : Totally antisymmetric tensor

$(\epsilon_{123} = +1)$

Inflation

$$a(t) = a_k \exp\left[H_{\text{inf}}(t - t_k)\right] \longleftarrow$$ **Slow-roll inflation**

$$\left(\dot{Y}^2/2 \ll V(Y) \right)$$

$$Y = Y_k \exp\left\{ \frac{3}{2}\left[-1 \pm \sqrt{1 + \left(\frac{2m}{3H_{\text{inf}}}\right)^2} \right] H_{\text{inf}}(t - t_k) \right\}$$

$$k/(a_k H_{\text{inf}}) = 1$$

H_{inf} : Hubble parameter at the inflationary stage

a_k : Value of the scale factor at $t = t_k$

t_k : Time when a comoving wavelength $2\pi/k$ of the $\mathrm{U}(1)_Y$ gauge field first crosses the horizon during inflation

Quantization of $\mathrm{U}(1)_Y$ gauge field

Canonical momenta $\pi_0 = 0\,, \quad \pi_i = Xa\dot{F}_i(t,\boldsymbol{x})$

Canonical commutation relation

$\boldsymbol{k}$: Comoving wavenumber

$$[F_i(t,\boldsymbol{x}), \pi_j(t,\boldsymbol{y})] = i\int \frac{d^3k}{(2\pi)^{3/2}} \mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}\left[\delta_{ij} - \left(k_i k_j/k^2\right)\right]$$

$k = |\boldsymbol{k}|$

$$F_i(t,\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[\hat{b}(\boldsymbol{k})\underline{F_i(t,\boldsymbol{k})}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}^\dagger(\boldsymbol{k})\underline{F_i^*(t,\boldsymbol{k})}\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right]$$

$$\left[\hat{b}(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{k}')\right] = \delta^3(\boldsymbol{k}-\boldsymbol{k}'), \quad \left[\hat{b}(\boldsymbol{k}), \hat{b}(\boldsymbol{k}')\right] = \left[\hat{b}^\dagger(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{k}')\right] = 0$$

$\hat{b}(\boldsymbol{k})\,, \quad \hat{b}^\dagger(\boldsymbol{k})$: Annihilation and creation operators

Normalization condition

$$F_i(k,t)\dot{F}_j^*(k,t) - \dot{F}_j(k,t)F_i^*(k,t) = \frac{i}{Xa}\left[\delta_{ij} - \left(k_i k_j/k^2\right)\right]$$

Circular polarization of $\mathrm{U}(1)_Y$ gauge field

- x^3 **axis to lie along the spatial momentum *k* direction**

  $\longrightarrow$ **Transverse directions** x^I **(** $I = 1, 2$ **)**

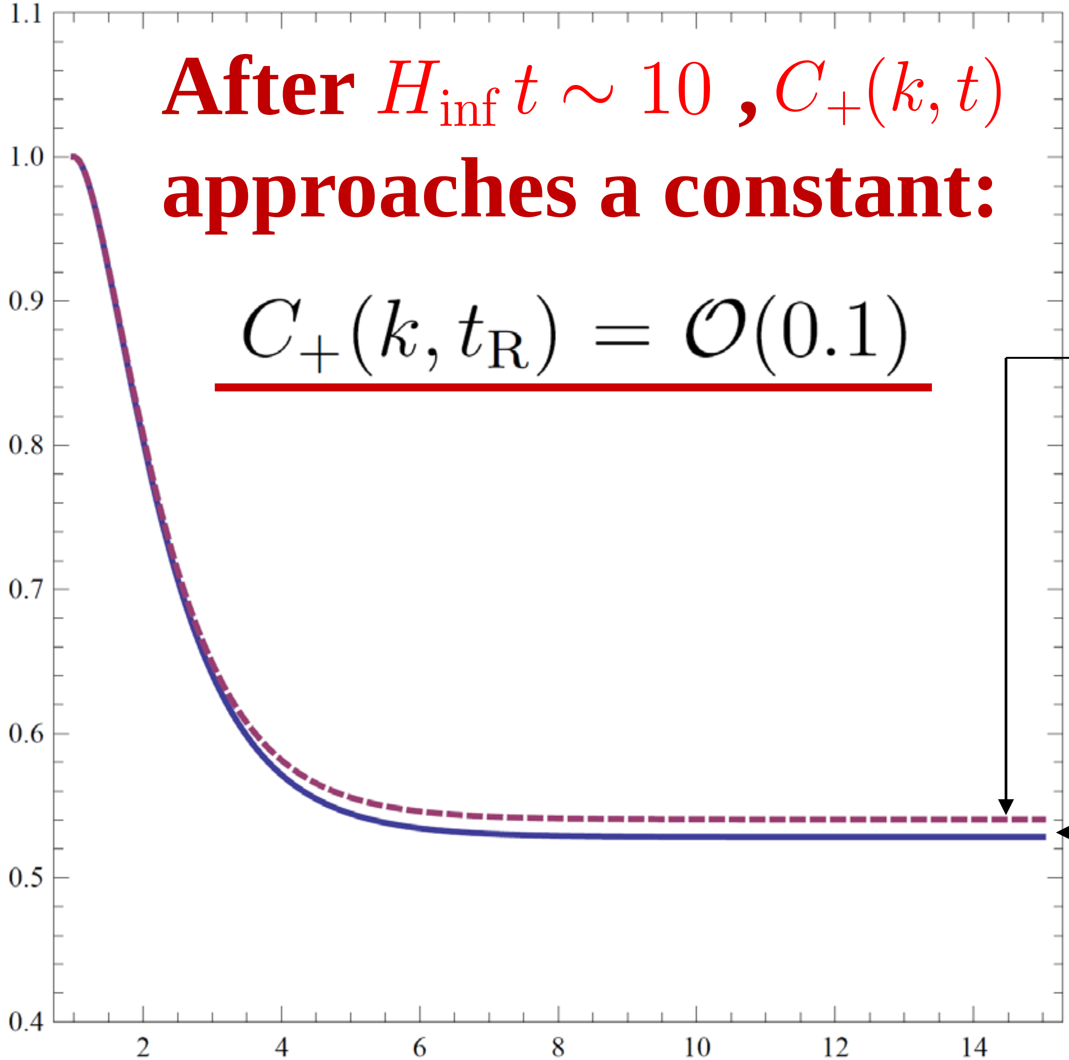
- **Circular polarization:** $F_\pm(k,t) \equiv F_1(k,t) \pm i F_2(k,t)$

$$\ddot{F}_\pm(k,t) + \left(H_{\mathrm{inf}} + \frac{\dot{X}}{X} \right) \dot{F}_\pm(k,t) + \left[1 \pm \frac{g_{\mathrm{ps}}}{M} \frac{\dot{Y}}{X} \left(\frac{k}{a} \right)^{-1} \right] \left(\frac{k}{a} \right)^2 F_\pm(k,t) = 0$$

$$C_+(k,t) \equiv \frac{F_+(k,t)}{F_+(k,t_k)}$$: **Normalized amplitude**

Evolution of $C_+(k,t)$ (1)

$C_+(k,t)$



After $H_{\rm inf}\, t \sim 10$, $C_+(k,t)$ approaches a constant:

$$C_+(k, t_{\rm R}) = \mathcal{O}(0.1)$$

$t_{\rm R}$: End of inflation

For the case of $\dot{Y} = 0$ (Dotted line)

For the case of dynamical Y (Solid line)

$H_{\rm inf}\, t$

Fig. 1

Evolution of $C_+(k,t)$ (2)

Values of parameters for Fig.1

$$X(t_k) \equiv \exp(\chi_k), \qquad \chi_k = -0.940$$

$$H_{\rm inf} = 1.0 \times 10^{10}{\rm GeV}, \qquad m = 2.44 \times 10^{9}{\rm GeV}$$

$$Y_k = 7.70 \times 10^{-2} M_{\rm P}, \qquad M = 1.0 \times 10^{-1} M_{\rm P}$$

$$\bar{V} = 5.07 \times 10^{-17} M_{\rm P}^4, \qquad g_{\rm ps} = 1.0$$

Evolution of $C_+(k,t)$ (3)

- **COBE normalization**

(Amplitude of power spectrum of the curvature perturbations)

$$\Delta^2_{\mathcal{R}}(k) \;=\; 2.4 \times 10^{-9} \quad \textbf{at} \quad k = 0.002\mathrm{Mpc}^{-1}$$

- **Planck result**

(Spectral index of power spectrum of the curvature perturbations)

$$n_{\mathrm{s}} \;=\; 0.9603 \pm 0.0073 \;(68\%\,\mathrm{CL}) \quad \textbf{at} \quad k = 0.05\mathrm{Mpc}^{-1}$$

[Ade *et al*. [Planck Collaboration], Astron. Astrophys. 571, A22 (2014)]

$$n_{\mathrm{s}} \simeq 1 - 6\epsilon_V + 2\eta_V$$

Slow-roll parameters:

$$\epsilon_V \equiv \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'(Y)}{V(Y)}\right)^2, \quad \eta_V \equiv M_{\mathrm{P}}^2 \frac{V''(Y)}{V(Y)}$$

- $\bar{V} \;=\; 3H_{\mathrm{inf}}^2 M_{\mathrm{P}}^2$ **: Friedmann equation**

III. Current strength of the magnetic fields

Proper magnetic field

$$B_{Y\,i}^{\text{proper}}(t,\boldsymbol{x}) = \frac{1}{a^2} B_{Y\,i}(t,\boldsymbol{x}) = \frac{1}{a^2}\epsilon_{ijk}\partial_j F_k(t,\boldsymbol{x})$$

- **Instantaneous reheating**
- **After reheating,** $\sigma_{\text{c}} \gg H$.

σ_{c} : Cosmic conductivity

$$\Rightarrow \quad B^{\text{proper}} \propto a^{-2}$$

Current strength of the magnetic fields (2)

$$\rho_B(L,t) \simeq \frac{1}{8\pi^2}\frac{1}{X(t_k)}\frac{1}{\sqrt{2\xi_k}}\exp\left[2\left(\pi\xi_k - 2\sqrt{2\xi_k}\right)\right]\left(\frac{k}{a}\right)^4 |C_+(k,t_{\rm R})|^2$$

Tachyon instability

$$\propto (X(t_k))^{-1}$$

$$\xi \equiv \frac{1}{2}\frac{g_{\rm ps}}{M}\frac{1}{X}\frac{\dot{Y}}{H_{\rm inf}}, \qquad \xi_k \equiv \xi(t=t_k)$$

$L = 2\pi/k$: Comoving scale

* $X(t_{\rm R}) = 1$: **After inflation, the Maxwell theory is recovered.**

Current strength of the magnetic fields (3)

	$B(H_0^{-1}, t_0)$ [G]	$H_{\rm inf}$ [GeV]	m [GeV]	$Y_k/M_{\rm P}$	$C_+(k, t_{\rm R})$
(a)	7.15×10^{-64}	1.0×10^{11}	2.44×10^{10}	7.70×10^{-2}	0.528
(b)	7.15×10^{-64}	1.0×10^{10}	2.44×10^{9}	7.70×10^{-2}	0.528
(c)	2.33×10^{-64}	1.0×10^{8}	1.0×10^{7}	1.62×10^{1}	0.172
(d)	2.33×10^{-64}	1.0×10^{6}	1.0×10^{5}	1.62×10^{1}	0.172
(e)	2.85×10^{-64}	1.0×10^{4}	8.0×10^{2}	2.23×10^{1}	0.211
(f)	2.85×10^{-64}	1.0×10^{2}	8.0	2.23×10^{1}	0.211

Current strength of magnetic fields on the Hubble horizon scale H_0^{-1} :

t_0 : Present time

For Fig. 1

$$B(H_0^{-1}, t_0) \sim 10^{-64}\,\mathrm{G}$$

Current strength of the magnetic fields (3)

- **The resultant strength of the magnetic fields satisfies the constraints suggested by the back reaction problem.**

[Demozzi, Mukhanov and Rubinstein, JCAP 0908, 025 (2009)]

Cf. [Kanno, Soda and Watanabe, JCAP 0912, 009 (2009)]

[Suyama and Yokoyama, Phys. Rev. D 86, 023512 (2012)]

[Fujita and Mukohyama, JCAP 1210, 034 (2012)]

[Fujita and Yokoyama, JCAP 1403, 013 (2014)]

* Observational constraints found from Planck

$$B < 4.4 \ \mathrm{nG} \quad \textbf{on} \quad 1\mathrm{Mpc}$$

[Ade *et al*. [Planck Collaboration], arXiv:1502.01594]

IV. Non-Gaussianity of curvature perturbations

- **We suppose that the $\mathrm{U}(1)_Y$ gauge field F_μ couples to another Higgs-like field φ , which develops a vacuum expectation value, through the kinetic term $|D\varphi|^2$.**

[Meerburg and Pajer, JCAP 1302, 017 (2013)]

$D_\mu \equiv \partial_\mu + ig' F_\mu$: Covariant derivative for φ

g' : Gauge coupling

⇨ **The gauge symmetry can spontaneously be broken, so that the gauge field acquires its mass.**

Non-Gaussianity of curvature perturbations (2)

- **The number of *e*-folds of inflation could be changed by the perturbations of φ.**

- **The curvature perturbations can be generated through the perturbations of φ.**

⇒ **The local-type non-Gaussianity in terms of the curvature perturbations is produced.**

$*$ δN **formalism**

[Sasaki and Stewart, Prog. Theor. Phys. 95, 71 (1996)]

[Starobinsky, JETP Lett. 42, 152(1985)]

[Lyth and Rodriguez, Phys. Rev. Lett. 95, 121302 (2005)]

Non-Gaussianity of curvature perturbations (3)

$$\Phi(\boldsymbol{x}) = \Phi_L(\boldsymbol{x}) + \underline{f_{\mathrm{NL}}^{\mathrm{local}}}(\Phi_L^2(\boldsymbol{x}) - \langle\Phi_L^2(\boldsymbol{x})\rangle)$$

↑

Quantity showing the non-Gaussianity

[Ade *et al*. [Planck Collaboration], Astron. Astrophys. 571, A24 (2014)]

$\Phi_L(\boldsymbol{x})$ **: Gravitational potential with the Gaussian statistical property**

Non-Gaussianity of curvature perturbations (4)

$$f_{\rm NL}^{\rm local} \approx 1.0 \times 10^{14} \frac{g'^4}{\xi^6} \frac{m^2}{H_{\rm inf}^2}$$

g' : Gauge coupling **[Meerburg and Pajer, JCAP 1302, 017 (2013)]**

	$f_{\rm NL}^{\rm local}$	g'^2	$H_{\rm inf}$ [GeV]	m [GeV]	$Y_k/M_{\rm P}$	$B(H_0^{-1}, t_0)$ [G]
(A)	2.70	1.13×10^{-5}	1.0×10^{13}	2.44×10^{12}	7.70×10^{-2}	7.15×10^{-64}
(B)	2.12×10^{8}	1.0×10^{-1}	1.0×10^{12}	2.44×10^{11}	7.70×10^{-2}	7.15×10^{-64}

$\xi_k = 2.5590616$

Planck result

$$f_{\rm NL}^{\rm local} = 2.5 \pm 5.7 \quad (68\% \ {\rm CL})$$

[Ade *et al*. [Planck Collaboration], arXiv:1502.01592]

V. Tensor-to-scalar ratio

$$r_{\mathrm{TS}} = 16\epsilon_V(t_k), \qquad \epsilon(t_k) = \frac{2M_{\mathrm{P}}^2 m^4 Y_k^2}{\left(2\bar{V} - m^2 Y_k\right)^2}$$

[Barnaby and Peloso, Phys. Rev. Lett. 106, 181301 (2011)]

[Barnaby, Namba and Peloso, JCAP 1104, 009 (2011)]

	r_{TS}	H_{inf} [GeV]	m [GeV]	Y_k/M_{P}
(A)	1.87×10^{-5}	1.0×10^{13}	2.44×10^{12}	7.70×10^{-2}
(B)	1.87×10^{-5}	1.0×10^{12}	2.44×10^{11}	7.70×10^{-2}

Planck result

$$r_{\mathrm{TS}} < 0.11\ (95\%\ \mathrm{CL}) \quad \textbf{at} \quad k = 0.002\mathrm{Mpc}^{-1}$$

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

Cf. [Ade *et al*. [Planck Collaboration], arXiv:1502.00612]

VI. Conclusions

- We have studied the generation of the large-scale magnetic fields from a kind of moduli inflation.

- We have first estimated the explicit values of three cosmological observables such as the current magnetic fields on the Hubble horizon, local non-Gaussianity, and the tensor-to-scalar ratio.

- The local non-Gaussianity and tensor-to-scalar ratio obtained in this model are consistent with the Planck results.

Acknowledgments

- The presenter would like to sincerely appreciate the significant discussions with Professor Tatsuo Kobayashi and Professor Osamu Seto and their kind important suggestions and comments.

Back up slides

Non-Gaussianity of curvature perturbations (4)

- **If $\sigma_{\rm c} \gg H$, the effect of the coupling between F_μ and φ on the resultant amplitude of the large-scale magnetic fields can be negligible.**

[Bassett, Pollifrone, Tsujikawa and Viniegra, Phys. Rev. D 63, 103515 (2001)]

Axion monodromy inflation

$$V(Y) = AY^q\,, \quad q = 1$$

$X = 1$, (i.e., $\Phi = 0$)

[Kobayashi, Seto and Yamaguchi, PTEP 2014, 103C01 (2014)]

[Higaki, Kobayashi, Seto and Yamaguchi, JCAP 1410, 025 (2014)]

$$Y = \bar{Y}t\,, \quad \bar{Y} = -\frac{A}{3H_{\rm inf}}$$

$$\ddot{F}_i(t,\boldsymbol{x}) + H\dot{F}_i(t,\boldsymbol{x}) - \frac{1}{a^2}\partial_j\partial_j F_i(t,\boldsymbol{x}) + \frac{g_{\rm ps}}{M}\frac{1}{a}\frac{A}{3H_{\rm inf}}\epsilon^{ijk}\partial_j F_k(t,\boldsymbol{x}) = 0$$

$$\xi = -\frac{1}{6}\frac{g_{\rm ps}}{M}\frac{A}{H_{\rm inf}^2}$$

Axion monodromy inflation (2)

Example

$$M = 3.55 \times 10^{18}\mathrm{GeV}\,, \quad g_{\mathrm{ps}} = 1.0$$

$$|\xi| = 2.98\,, \qquad H_{\mathrm{inf}} = 6.51 \times 10^{15}\mathrm{GeV}$$

$$n_{\mathrm{s}} \;=\; 0.9603$$

$$\longrightarrow \quad r_{\mathrm{TS}} = 16\epsilon_V = 0.106$$

This is consistent with BICEP2 result.

Power spectrum of the curvature perturbations

Evolution equation of quantum fluctuations of Y

$$\frac{\partial^2 \delta Y(t,\boldsymbol{x})}{\partial t^2} + 3H\frac{\partial \delta Y(t,\boldsymbol{x})}{\partial t} - \frac{\nabla^2 \delta Y(t,\boldsymbol{x})}{a^2} = \frac{g_{\rm ps}}{M} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Two-point correlation function in the Fourier space

$$< \mathcal{R}_{\boldsymbol{k}} \mathcal{R}_{\boldsymbol{k}'} > \equiv (2\pi^2/k^3)\, P_{\mathcal{R}}(k) \delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}')$$

$$\mathcal{R} = -\left(H/\dot{Y}\right)\delta Y$$

: Curvature perturbations

$$P_{\mathcal{R}}(k) \simeq \Delta^2_{\mathcal{R}} \left(\frac{k}{k_*}\right)^{n_{\rm s}-1} \left(1 + \Delta^2_{\mathcal{R}} f_{\rm S}(\xi) \exp(4\pi\xi)\right)$$

$$\Delta^2_{\mathcal{R}} = \left(\frac{H_{\rm inf}}{2\pi}\right)^2 \frac{H^2_{\rm inf}}{|\dot{Y}|^2} \quad , \qquad k_* = 0.002\,{\rm Mpc}^{-1}$$

$$f_{\rm S}(\xi) \cong 3.0\times 10^{-5}\,\xi^{-5.4} \quad \text{for } 2 \leq \xi \leq 3$$

[Barnaby and Peloso, Phys. Rev. Lett. 106, 181301 (2011)]

[Barnaby, Namba and Peloso, JCAP 1104, 009 (2011)]

[Meerburg and Pajer, JCAP 1302, 017 (2013)]

Normalization with COBE and Planck data

Slow-roll parameters: $\epsilon_V \equiv \frac{M_\mathrm{P}^2}{2}\left(\frac{V'(Y)}{V(Y)}\right)^2, \quad \eta_V \equiv M_\mathrm{P}^2 \frac{V''(Y)}{V(Y)}$

Spectral index: $n_\mathrm{s} \simeq 1 + 6\epsilon_V + 2\eta_V$

With COBE and Planck data, we find

$$f_\mathrm{S}(\xi)\exp(4\pi\xi) = \frac{25}{144}\times 10^8$$

$$Y_k = \pm \frac{M_\mathrm{P}}{\sqrt{2}\beta}\left[4\frac{\bar{V}}{M_\mathrm{P}^2 m^2 \beta^{-2}} + 1 \pm \sqrt{12\frac{\bar{V}}{M_\mathrm{P}^2 m^2 \beta^{-2}} + 1}\right]^{1/2}$$

$$\beta \equiv \sqrt{\frac{-(n_\mathrm{s}-1)}{8}}$$

Normalization with COBE and Planck data (2)

For $m/H_{\rm inf} \ll 1$ **, we obtain**

$$m \approx \sqrt{6}\xi_k \frac{M}{M_{\rm P}} X(t_k) H_{\rm inf}$$

$$Y_k \approx \sqrt{6} M_{\rm P} \frac{H_{\rm inf}}{m} = \frac{M_{\rm P}^2}{\xi_k X(t_k) M}$$

$$\xi_k \approx \left[Y_k / \left(6 M X(t_k)\right)\right] \left(m^2 / H_{\rm inf}^2\right)$$

Cf.

$$\dot{Y}(t_{\rm R}) = \frac{3}{2}\left[-1+\sqrt{1+\left(\frac{2m}{3H_{\rm inf}}\right)^2}\right] H_{\rm inf} Y_k \exp\left\{\frac{3}{2}\left[-1+\sqrt{1+\left(\frac{2m}{3H_{\rm inf}}\right)^2}\right](N-1)\right\}$$

Tensor-to-scalar ratio (4)

BICEP2 experiment

$$r_{\mathrm{TS}} = 0.20^{+0.07}_{-0.05} \qquad (68\%\ \mathrm{CL})$$

[Ade *et al*. [BICEP2 Collaboration], PRL (2014)]

∗ **Inflationary models (Examples)**

[KB, Cognola, Odintsov, Zerbini, arXiv:1404.4311]

[KB, Myrzakulov, Odintsov, Sebastiani, arXiv:1403.6649]

[Kobayashi, Seto, PRD (2014); 1404.3102]

[Kobayashi, Seto, Yamaguchi, arXiv:1404.5518]

[Higaki, Kobayashi, Seto, Yamaguchi, arXiv:1405.0775]

[Nakayama, Takahashi, PLB (2014)]

[Harigaya, Ibe, Schmitz, Yanagida, PLB (2014)]

[Bonvin, Durrer, Maartens, GRG (2014)]

Evolution of $C_+(k,t)$ (3)

- **COBE normalization**

(Amplitude of power spectrum of the curvature perturbations)

$$\Delta_{\mathcal{R}}^2(k) \ = \ 2.4 \ \times \ 10^{-9} \quad \textbf{at} \quad k = 0.002\mathrm{Mpc}^{-1}$$

- **Planck result**

(Spectral index of power spectrum of the curvature perturbations)

$$n_{\mathrm{s}} \ = \ 0.9603 \pm 0.0073 \ (68\% \ \mathrm{CL}) \quad \textbf{at} \quad k = 0.05\mathrm{Mpc}^{-1}$$

[Ade *et al*. [Planck Collaboration], Astron. Astrophys. 571, A22 (2014)]

Cf. $n_{\mathrm{s}} = 0.968 \pm 0.006 \ (68\% \ \mathrm{CL}) \quad \textbf{at} \quad k = 0.05\mathrm{Mpc}^{-1}$

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

- $\bar{V} \ = \ 3H_{\mathrm{inf}}^2 M_{\mathrm{P}}^2$ **: Friedmann equation**

Xの変動

$$X \equiv \exp\left(-\lambda\Phi/M_P\right)$$

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU(\Phi)}{d\Phi} = 0$$

$$\longrightarrow \quad 3H_{\rm inf}\dot{\Phi} + dU(\Phi)/d\Phi = 0$$

⇧

$$\left|\ddot{\Phi}/\left(H_{\rm inf}\dot{\Phi}\right)\right| \ll 1$$

→ **解：** $\Phi = \left(\lambda\kappa\right)^{-1} \ln\left[\lambda^2 w H_{\rm inf}\left(t - t_{\rm R}\right) + \exp\left(\lambda\kappa\Phi_{\rm R}\right)\right]$

⇨ **定性的に同様の結果が得られる。**

Contents

Origin of cosmic magnetic fields (1)

1. **Astrophysical process: Plasma instability**

(a) **Biermann battery mechanism**

[Biermann, Schlüter, Phys. Rev. (1951)]

[Hanayama *et al*., ApJ (2005)]

(b) **Weibel instability**

[Weibel, PRL (1959)]

[Fujita, Kato, MNRAS (2005)]

Origin of cosmic magnetic fields (2)

2. Cosmolosical processes:

- **Phase transitions**

  (i) Electroweak phase transition (EWPT)

  [Baym, Bödeker, McLerran, PRD (1996)]

  (ii) Quark-hadron phase transition (QCDPT)

  [Quashnock, Loeb, Spergel, ApJ (1989)]

- **Density perturbations**

  [Ichiki *et al*., Science (2006)]

  [Kobayashi, *et al*, PRD (2007)]

Inflationary cosmology

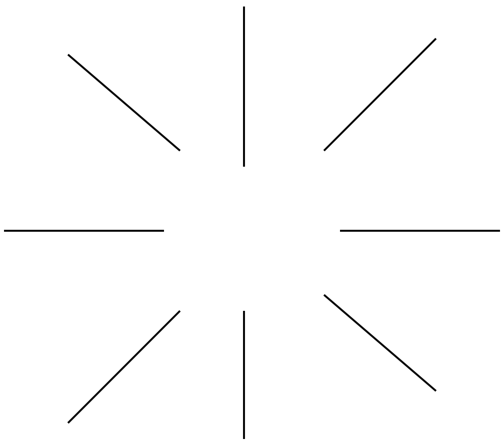
Inflation:

Exponential cosmic acceleration in the early universe

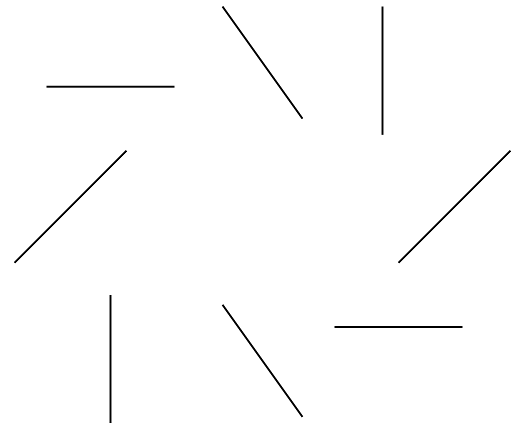
1. **Homogeneity, Isotropy, Flatness**

2. **Primordial density perturbations**

Tensor-to-scalar ratio (2)



E-mode polarization



B-mode polarization

↑

Only from the tensor mode

[E. Komatsu, The astronomical herald,
The Astronomical Society of Japan, 2003]

IV. Non-Gaussianity of curvature perturbations

Einstein equation

→ **Metric perturbation = Matter density perturbation**

(1) Scalar mode (Curvature perturbation)

Temperature perturbation of the cosmic microwave background radiation (CMB): $\frac{\delta T}{T} \simeq 10^{-5}$

(2) Vector mode

(3) Tensor mode (Primordial gravitational wave)

V. Tensor-to-scalar ratio

Power spectrum of the curvature perturbation

$$\propto k^{n_{\rm s}-1}$$

$n_{\rm s}$ **: Spectral index**

Tensor-to-scalar ratio

$$r_{\rm TS} \equiv \frac{\text{Power spectrum of the tensor mode}}{\text{Power spectrum of the scalar mode}}$$

磁場の強度に対する制限

1. CMB

$$B < 5 \times 10^{-9}\mathrm{G}$$

$$L = H_0^{-1}$$

[Barrow, Ferreira, Silk, PRL (1997)]

2. BBN

$$B < 10^{-6}\mathrm{G}$$

$h_{70} \equiv h/0.70$

$$L \sim 1.4 \times 10^{-4} h_{70}^{-1}\ \mathrm{Mpc}$$

[Grasso, Rubinstein, PLB (1996)] **[Cheng, Olinto, Schramm, Truran, PRD (1996)]**

3. ファラデー回転の Rotation measure (RM)

$$B < 6 \times 10^{-10} \left(n_{e0}/10^{-7}\,\mathrm{cm}^{-3}\right)^{-1} \mathrm{G}$$

n_{e0} :熱的電子の現在の平均密度 **[Vallée, ApJ (1990)]**

II B. 磁場の現在での強度

<固有磁場>

$$B_i^{\text{proper}}(t,\boldsymbol{x}) = a^{-1}B_i(t,\boldsymbol{x}) = a^{-2}\epsilon_{ijk}\partial_j A_k(t,\boldsymbol{x})$$

<Fourier 空間での磁場のエネルギー密度>

$$\rho_B(k,t) = \frac{1}{2}\left[\left|B_+^{\text{proper}}(k,t)\right|^2 + \left|B_-^{\text{proper}}(k,t)\right|^2\right]$$

$$\left|B_\pm^{\text{proper}}(k,t)\right|^2 = \frac{1}{a^2}\left(\frac{k}{a}\right)^2 |A_\pm(k,t)|^2$$

$$B_\pm^{\text{proper}}(k,t) \equiv B_1^{\text{proper}}(k,t) \pm iB_2^{\text{proper}}(k,t)$$

$\rho_B(k,t)$ に相空間密度: $4\pi k^3/(2\pi)^3$ をかける。

<磁場のエネルギー密度>

$$\rho_B(L,t) = \frac{k^3}{4\pi^2}\left[\left|B_+^{\text{proper}}(k,t)\right|^2 + \left|B_-^{\text{proper}}(k,t)\right|^2\right]$$

⇩

$$\rho_B(L,t) = \frac{1}{8\pi^2}\left(\frac{k}{a}\right)^4 \mathcal{I}(k,t), \quad \mathcal{I}(k,t) \equiv |C_+(k,t)|^2 + |C_-(k,t)|^2$$

インフレーション期での増幅項 ($\mathcal{I}(k,t)$)

現在の磁場の強度

$$B_{Y}{}^{\rm proper}_{i}(t,\boldsymbol{x}) = \frac{1}{a^2} B_{Y\,i}(t,\boldsymbol{x}) = \frac{1}{a^2}\epsilon_{ijk}\partial_j F_k(t,\boldsymbol{x})$$

$$E_{Y}{}^{\rm proper}_{i}(t,\boldsymbol{x}) = a^{-1} E_{Y\,i}(t,\boldsymbol{x}) = -a^{-1}\dot{F}_i(t,\boldsymbol{x})$$

$$\rho_{B_Y}(L,t) = \frac{k^3}{4\pi^2}\left[\left|B_{Y}{}^{\rm proper}_{+}(k,t)\right|^2 + \left|B_{Y}{}^{\rm proper}_{-}(k,t)\right|^2\right] X$$

$$\left|B_{Y}{}^{\rm proper}_{\pm}(k,t)\right|^2 = \left(1/a^2\right)\left(k/a\right)^2 \left|F_{\pm}(k,t)\right|^2$$

$\longrightarrow B(L,t) = \sqrt{2\rho_B(L,t)}$ を用いる。

$$B(L,t_0) = 8.2\times 10^{18}\exp(-2N)\left(\frac{H_{\rm inf}}{[\rm GeV]}\right)^2\left(\frac{a_{\rm R}}{a_0}\right)^2\sqrt{\mathcal{I}(k,t_{\rm R})}\ \ [\rm G]$$

$$N = 45 + \ln\left(\frac{L}{[\rm Mpc]}\right) + \ln\left\{\frac{[30/(\pi^2 g_{\rm R})]^{1/12}\ {\rho_{\rm R}}^{1/4}}{10^{38/3}\ [\rm GeV]}\right\}$$

: t_1 と $t_{\rm R}$ の間の e-folds 数

$\rho_{\rm R} = \dfrac{\pi^2}{30} g_{\rm R} {T_{\rm R}}^4$: $t_{\rm R}$での輻射のエネルギー密度、 $T_{\rm R}$: $t_{\rm R}$での温度

$$\frac{a_{\rm R}}{a_0} = \left(\frac{g_{\rm R}}{3.91}\right)^{-1/3}\frac{T_{\gamma 0}}{T_{\rm R}}$$

$T_{\gamma 0}\,(= 2.73[\rm K])$: 現在の温度

$g_{\rm R}(\approx 100)$: $t_{\rm R}$ での相対論的粒子の全自由度数