Impact of Form Fields on Primordial gravitational Waves

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PLANCK released new data

**Fig. 2.** Planck $TT$ (top), high-$\ell$ $TE$ (centre), and high-$\ell$ $EE$ (bottom) angular power spectra. Here $D_{\ell} = \ell(\ell+1)C_{\ell}/(2\pi)$.

**Fig. 12.** Marginalized joint 68% and 95% CL regions for $n_s$ and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models.
How is the statistical anisotropy constrained?

Planck collaboration 2015


trum. We found that a modulated curvaton model proposed to explain the observed large-scale dipolar power asymmetry cannot account for all of the asymmetry, and hence is not preferred over statistically isotropic base ΛCDM. The full mission temperature data place the tightest constraints to date on a quadrupolar modulation of curvature perturbations.

\[ P(k) = P(k) \left[ 1 + g(k)(k \cdot \hat{n})^2 \right] \]
History of anisotropic inflation

Imprints of a Primordial Preferred Direction on the Microwave Background
L. Ackerman, S. M. Carroll, M. B. Wise 2007

Inflationary Universe with Anisotropic Hair

The Nature of Primordial Fluctuations from Anisotropic Inflation
M. Watanabe, S. Kanno, J. Soda 2010

Primordial Power Spectra from Anisotropic Inflation
T. R. Dulaney, M. I. Gresham 2010

Scalar-Scalar, Scalar-Tensor, and Tensor-Tensor Correlators from Anisotropic Inflation
A.E. Gumrukcuoglu, B. Himmetoglu, M. Pelosi 2010

Anisotropic power spectrum and bispectrum in the f(\phi)F2 mechanism
N. Bartolo, S. Matarrese, M. Pelosi, A. Ricciardone 2013

Anisotropic inflation reexamined:
upper bound on broken rotational invariance during inflation
A. Naruko, E. Komatsu, M. Yamaguchi 2014
Impact of Form Fields on inflation

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Symmetry in inflation

The nature of primordial fluctuations is determined by symmetry in conventional inflation.

- First of all, in order to have inflation, we need to assume spatial translation symmetry. (initial homogeneity)

- In addition to this initial condition, we need to provide a sufficiently flat potential to realize the slow roll inflation. Hence, we have shift symmetry $\phi \rightarrow \phi + c$

- Once the slow roll inflation occurs, the cosmic no-hair conjecture suggests that the exponential expansion erases any classical anisotropy and leads to isotropic universe. This is nothing but the rotational symmetry.

- de Sitter spacetime
  
  $$ds^2 = -dt^2 + e^{2Ht} \left[ dx^2 + dy^2 + dz^2 \right]$$

  has the temporal de Sitter symmetry
  
  $$t \rightarrow t + c, \quad x^i \rightarrow e^{-2Hc} x^i$$
The nature of primordial fluctuations

Now, the symmetries determine the statistical nature of the fluctuations:

First of all, rotational symmetry tells us fluctuations are classified into scalar $\zeta$, vector $h$ perturbations and these are statistically independent

$$\langle \zeta(k_1) h(k_2) \rangle = 0$$

Next, shift symmetry implies Gaussian statistics

$$\langle \zeta(k_1) \zeta(k_2) \rangle = P_\zeta(k_1, k_2) \quad \langle h(k_1) h(k_2) \rangle = P_h(k_1, k_2)$$

Moreover, spatial translation symmetry implies statistical homogeneity

$$\langle \zeta(k_1) \zeta(k_2) \rangle = \delta(k_1 + k_2) P_\zeta(k_1) \quad \langle h(k_1) h(k_2) \rangle = \delta(k_1 + k_2) P_h(k_1)$$

And, rotational symmetry accounts for statistical isotropy

$$\langle \zeta(k_1) \zeta(k_2) \rangle = \delta(k_1 + k_2) P_\zeta(k_1 = |k_1|) \quad \langle h(k_1) h(k_2) \rangle = \delta(k_1 + k_2) P_h(k_1 = |k_1|)$$

Finally, temporal de Sitter symmetry yields scale invariant spectrum

$$P_\zeta(k) \approx \text{const.} \quad P_h(k) \approx \text{const.}$$

The above 0-th order predictions are model independent and robust.
Precision cosmology and violation of symmetry

Precision cosmology forces us to look at fine structures of fluctuations!

Violation of temporal de Sitter symmetry → spectral tilt

There should be a slight tilt because the expansion is not exactly deSitter. The deviation from deSitter can be characterized by the slow roll parameter. Hence, the tilt should be of the order of the slow roll parameter.

Violation of shift symmetry → non-Gaussianity

Maldacena 2003

There should be small non-gaussianity of the order of the slow roll parameter because the shift symmetry is not exact.

Along this line of thought, one may expect violation of rotational symmetry of the order of the slow roll parameter. which must yield the statistical anisotropy.
What kind of statistical anisotropy can we have?

The violation of rotational symmetry implies a specific direction $\vec{n}$.

Then, we expect direction dependent power spectrum

$$P(k) = P(k)\left[1 + \epsilon_1(k)k \cdot \vec{n} + \epsilon_2(k)(k \cdot \vec{n})^2 + \cdots\right]$$

However, the dipole term cannot be produced when the spatial translation symmetry is kept. Actually, we have the following symmetry

$$P(k) = \left|\zeta(k)\right|^2 = \zeta(k)\zeta^*(k) = \zeta(k)\zeta(-k) = P(-k)$$

Hence, we expect quadratic statistical anisotropy

$$P(k) = P(k)\left[1 + \epsilon_2(k)(k \cdot \vec{n})^2 + \cdots\right]$$

Moreover, we can expect cross correlation between scalar and tensor perturbations

$$\langle \zeta(k_1)h(k_2) \rangle \neq 0$$

which would produce the TB correlation.
What kind of fields can make a specific direction?

There are 5 type of form fields in 4-dimensions. \( \phi, A_\mu, B_{\mu\nu}, C^{(3)}_{\mu\nu\lambda}, C^{(4)}_{\mu\nu\lambda\rho} \)

- Zero-form fields (scalar) cannot violate rotational symmetry in a natural way.
- One-form fields (gauge field) are natural candidates.
- Two-form fields are also candidates. Its field strength is given by the three-form
  \[ H = dB \]
  \[ F^{(4)} = dC^{(3)} = \text{const.} \quad F^{(4)} = \text{const.} \times \text{volume} \quad \Rightarrow \quad * d * F^{(4)} = 0 \]
- Three-form fields have no dynamics.
- Four-form fields are irrelevant in 4-dimensions.
  \[ F^{(5)} = dC^{(4)} = 0 \]

In this talk, I will discuss possible roles of one and two-form fields in inflation.
How does the specific direction arise?

How are matter fluctuations generated?

\[ \int d^4 x \sqrt{-g} \ g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \]

\[ \frac{d^2 \phi_k(\eta)}{d\eta^2} + \left[ k^2 - \frac{2}{\eta^2} \right] \phi_k(\eta) = 0 \]

We can expect the same for form fields in the presence of a kinetic function

\[ \int d^4 x \sqrt{-g} f(\phi) g^{\mu_1 \nu_1} \cdots g^{\mu_p \nu_p} F_{\mu_1 \cdots \mu_p} F_{\nu_1 \cdots \nu_p} \]

\[ \frac{d^2 A_k(\eta)}{d\eta^2} + \left[ k^2 - \frac{2}{\eta^2} \right] A_k(\eta) = 0 \]

\[ F = dA \]

Quantum fluctuations generate classical anisotropic hair

\[ \frac{\rho_{\text{quantum}}}{V} \approx \frac{H^4}{M^4} \approx \left( \frac{M^4}{M_{\text{pl}}^2} \right)^2 \frac{1}{M^4} \approx \left( \frac{M}{M_{\text{pl}}} \right)^4 \approx 10^{-8} \]

This create small anisotropy in the expansion

\[ \frac{\Sigma}{H} = \frac{\text{anisotropic expansion}}{\text{isotropic expansion}} = \frac{\rho_{\text{quantum}}}{V} = 10^{-8} \]

This should be regarded as a minimum value. Apparently, this is quite small. It turns out that, however, even this small value gives rise to observable statistical anisotropy.
How does the anisotropy evolve after classicalization?

What is the effect of form fields on the spacetime?
Anisotropic inflation driven by one-form
One-form (Gauge field) and inflation

power-law inflation

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \left( \partial_\mu \phi \right)^2 - V(\phi) \right] \]

In this case, it is well known that there exists an isotropic power law inflation

\[ ds^2 = -dt^2 + t^{4/\lambda^2} \left( dx^2 + dy^2 + dz^2 \right) \]

\[ \frac{\phi}{M_p} = -\frac{2}{\lambda} \log t \]

What happens if one-form exists with a non-trivial kinetic term?

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \left( \partial_\mu \phi \right)^2 - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right] \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

\[ f = f_0 e^{\frac{\rho}{M_p}} \]

We already know quantum fluctuations can generate a coherent one-form field.

Let the direction of the one-form to be \( x \)-axis.

\[ A_\mu = (0, A_x(t), 0, 0) \]

Then, the metric should be Bianchi Type-I

\[ ds^2 = -dt^2 + e^{2\alpha(t)} \left[ e^{-4\sigma(t)} dx^2 + e^{2\sigma(t)} (dy^2 + dz^2) \right] \]
Exact Anisotropic inflation

For the parameter region \( \lambda^2 + 2\rho\lambda - 4 > 0 \), we found the following new solution

\[
\begin{align*}
\begin{align*}
\text{ds}^2 &= -dt^2 + t^{2\omega} \left[ t^{-4\zeta} dx^2 + t^{2\zeta} \left( dy^2 + dz^2 \right) \right] \\
\phi &= \frac{2}{\lambda} \log t \\
\dot{A}_x(t) &= C t^\gamma
\end{align*}
\end{align*}
\]

\[
\omega = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)}
\]

\[
\zeta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)}
\]

\[
\gamma = 4 \frac{\rho}{\lambda} - \omega - 4\zeta
\]

Apparently, the expansion is anisotropic and its degree of anisotropy is given by

\[
\begin{align*}
\Sigma = \frac{\dot{\phi}}{\dot{t}} = \frac{1}{3} I \varepsilon \\
I &= \frac{\lambda^2 + 2\rho\lambda - 4}{\lambda^2 + 2\rho\lambda} \\
\varepsilon &= -\frac{\dot{H}}{H^2} = \frac{6\lambda(\lambda + 2\rho)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}
\end{align*}
\]

\(0 \leq I < 1\)  \hspace{1cm} \text{slow roll parameter}

oblate
The phase space structure

Quantum fluctuations generate seeds of coherent vector fields.

$\lambda^2 + 2\rho\lambda - 4 > 0$

The result universally holds for other set of potential and gauge kinetic functions.
Anisotropic inflation driven by two-form
Anisotropic Inflation with a two-form field

Ohashi, Tsujikawa, Soda, PRD, 2013

Chaotic inflation

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \left( \partial_\mu \phi \right)^2 - V(\phi) - \frac{1}{12} f^2(\phi) H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right] \]

\[ H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} \]

We consider a homogeneous background \( \phi = \phi(t) \)

For the two-form field, we have polarization. Without losing generality, one can put the ansatz

\[ \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu = \nu(t) dy \wedge dz \]

Thus, we need to consider anisotropic background.

\[ ds^2 = -dt^2 + e^{2\alpha(t)} \left[ e^{-4\alpha(t)} dx^2 + e^{2\sigma(t)} \left( dy^2 + dz^2 \right) \right] \]
Basic equations

\[ M_p = 1 \]

Hamiltonian Constraint

\[ \dot{\alpha}^2 = \dot{o}^2 + \frac{1}{3} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{A^2}{2} f^{-2}(\phi) e^{-2\alpha+4\sigma} \right] \]

Scale factor

\[ \ddot{\alpha} = -3\dot{\alpha}^2 + V(\phi) + \frac{A^2}{3} f^{-2}(\phi) e^{-2\alpha+4\sigma} \]

Anisotropy

\[ \ddot{\sigma} = -3\dot{\alpha} \dot{\sigma} - \frac{A^2}{3} f^{-2}(\phi) e^{-2\alpha+4\sigma} \]

Scalar field

\[ \ddot{\phi} = -3\dot{\alpha} \dot{\phi} - V'(\phi) + A^2 f^{-3}(\phi) f''(\phi) e^{-2\alpha+4\sigma} \]

const. of integration
Behavior of the two-form field

Suppose 2–form field is negligible initially. Then, we obtain conventional slow roll equations.

Hamiltonian Constraint

\[
\dot{\alpha}^2 = \frac{1}{3} V(\phi) \quad \quad \quad \frac{d\alpha}{d\phi} = \frac{\dot{\alpha}}{\phi} = -\frac{V(\phi)}{V'(\phi)} \quad \quad \alpha = -\int \frac{V}{V'} d\phi
\]

Scalar field

\[
3\dot{\alpha}\dot{\phi} = -V'(\phi)
\]

Now we can keep the energy density of 2–form constant.

energy density of 2–form = \( \frac{A^2}{2} f^{-2}(\phi) e^{-2\alpha} \)

\[
f(\phi) = e^{-\alpha} = e^{\int \frac{V}{V'} d\phi} \quad \quad V = \frac{m^2}{2} \phi^2 \quad \quad f = e^{\frac{1}{4} \phi^2}
\]

To go beyond the critical case, we generalize the function by introducing a parameter

\[
f = e^{\frac{c}{4} \phi^2} \quad c \begin{cases} > 1 & \text{Two-form grows} \\ = 1 & \text{Two-form remains const.} \\ < 1 & \text{Two-form is negligible} \end{cases}
\]

Will the two–form field keep growing forever?
Inflaton dynamics in the attractor phase

The 2–form field modify slow–roll equations.

Hamiltonian Constraint

\[ \dot{\alpha}^2 = \frac{1}{6} m^2 \phi^2 \]

Scalar field

\[ 3 \dot{\alpha} \dot{\phi} = -m^2 \phi + \frac{c}{2} \phi A^2 e^{-c \phi^2 / 2 - 2 \alpha} \]

\[ \phi \frac{d\phi}{d\alpha} = -2 + \frac{c A^2}{2 m^2} e^{c \phi^2 / 2 - 2 \alpha} \]

\[ e^{c \phi^2 / 2 + 2 \alpha} = \frac{c^2 A^2}{2 m^2 (c - 1)} + D e^{-2(c-1) \alpha} \]

\[ (c - 1) \alpha \to \infty \]

\[ e^{-c \phi^2 / 2 - 2 \alpha} = \frac{2 m^2 (c - 1)}{c^2 A^2} \]

Thus, the energy density of the form field is constant.

Energy density of two–form field

\[ \rho_v = \frac{A^2}{2} e^{-c \phi^2 / 2 - 2 \alpha} = \frac{c - 1}{c} \epsilon V \]

\[ \epsilon = -\frac{\dot{H}}{H^2} \]

Degree of anisotropy

\[ 3 \dot{\alpha} \dot{\sigma} = -\frac{A^2}{3} f^{-2}(\phi) e^{-2 \alpha} \]

\[ \frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} = -\frac{2 \rho_v}{3 V} = -\frac{2}{3} \frac{c - 1}{c} \epsilon \]
Anisotropic Inflation is an attractor

\[ ds^2 = -dt^2 + e^{2Ht} \left[ e^{-4\Sigma t} dx^2 + e^{2\Sigma t} (dy^2 + dz^2) \right] \]

\[
\frac{\Sigma}{H} = -\frac{2}{3} \frac{\rho_v}{V} \approx - \frac{\text{energy density of form field}}{\text{energy density of inflaton field}}
\]

It is true that exponential expansion erases any initial memory. In this sense, we have still the predictability. However, the two-form field generates a slight anisotropy in spacetime.

Statistical Symmetry Breaking in the CMB
Phenomenology
How is the statistical anisotropy constrained?

\[ P(k) = P(k) \left[ 1 + g_\ast(k) (\mathbf{k} \cdot \mathbf{n})^2 \right] \]

**Table 16.** Constraints on \( g_\ast \), the quadrupolar asymmetry amplitude, determined from the SMICA, NILC, SEVEM, and Commander foreground-cleaned maps. Error bars are 68% CL.

<table>
<thead>
<tr>
<th>( g_\ast \times 10^2 )</th>
<th>( q = 0 )</th>
<th>( q = 1 )</th>
<th>( q = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commander ...</td>
<td>0.19^{+1.97}_{-1.40}</td>
<td>-0.09^{+1.85}_{-1.83}</td>
<td>-0.27^{+1.14}_{-1.10}</td>
</tr>
<tr>
<td>NILC ............</td>
<td>0.60^{+1.73}_{-1.62}</td>
<td>0.16^{+1.51}_{-0.99}</td>
<td>-0.04^{+0.73}_{-0.66}</td>
</tr>
<tr>
<td>SEVEM ............</td>
<td>0.13^{+1.55}_{-1.26}</td>
<td>0.03^{+1.38}_{-1.04}</td>
<td>-0.01^{+0.98}_{-0.71}</td>
</tr>
<tr>
<td>SMICA ............</td>
<td>0.23^{+1.70}_{-1.24}</td>
<td>0.16^{+1.47}_{-1.00}</td>
<td>0.13^{+1.01}_{-0.61}</td>
</tr>
</tbody>
</table>

Cf. WMAP constraint \( g_\ast \leq 0.01 \quad \text{Kim & Komatsu 2013} \)
Statistical anisotropy in the power spectrum

One form

\[ \bar{P}(k) = P_\xi(k) \left[ 1 - g_s (k \cdot \hat{n})^2 \right] \]

Here, we have an enhancement factor.

Typical order

\[ g_s \approx \frac{24}{0.01} \times 10^{-8} \times 50^2 \approx 6.0 \times 10^{-2} \]

Two form

\[ \bar{P} = P_\xi(k) \left[ 1 + g_s (k \cdot \hat{n})^2 \right] \]

\[ g_s = \frac{6 E_{xy}^2}{\varepsilon V} N_k^2 \]

Watanabe, Kanno, Soda 2010
Dulaney, Gresham, PRD, 2010
Gumrukcuoglu, et al. PRD, 2010

Ohashi, Tsujikawa, Soda, PRD, 2013
Statistically anisotropic Non-gaussianity

Bispectrum

\[
B = \frac{3}{10} (2\pi)^{5/2} f_{NL} P^2 \sum_{i=1}^{3} \frac{k_i^3}{\prod_{i=1}^{3} k_i^3}
\]

One form

\[
f_{NL, local}^{\text{local}} \approx 2.6 \left( \frac{g_s}{0.01} \right) \left( \frac{N_{CMB}}{60} \right)
\]

Bartolo et al. 2012

There was no large local type non-gaussianity in Planck data!

\[
f_{NL}^{\text{local}} = 0.8 \pm 5.0 \quad f_{NL}^{\text{equil}} = -4 \pm 43
\]

Two form

Ohashi, Tsujikawa, Soda, PRD, 2013

\[
f_{NL} = 60 I P^{(0)2} \frac{N_{k_1} N_{k_2} N_{k_3}}{P^2} \left[ r_3^3 \cos \theta_{k_1,k_2} \cos \theta_{k_1,k_3} \cos \theta_{k_2,k_3} + r_3^3 \cos \theta_{k_1,k_3} \cos \theta_{k_2,k_3} + r_3^3 \cos \theta_{k_1,k_1} \cos \theta_{k_2,k_2} \cos \theta_{k_3,k_3} \right]
\]

\[
f_{NL}^{\text{local}} \approx 0.33 \left( \frac{g_s}{0.01} \right) \left( \frac{N_{CMB}}{60} \right) \quad f_{NL}^{\text{equil}} \approx 0.37 \left( \frac{g_s}{0.01} \right) \left( \frac{N_{CMB}}{60} \right)
\]
What is the impact of form fields on primordial GW?

Cross correlation

One form \[ P_{\xi h}(k) = \sqrt{2} P_{\xi}(k) \varepsilon g s \sin^2 \theta \]

Two form \[ P_{\xi h}(k) = 0 \]

Angular power spectrum of X and Y reads

\[ C_{\ell \ell'}^{XY} \propto \int d\Omega_k P_{XY}(k) Y_{\ell m}^*(\hat{k}) Y_{\ell' m'}(\hat{k}) \]

For isotropic spectrum, \( P(k) = P(\hat{k}) \), we have \( C_{\ell \ell'}^{XY} \propto \delta_{\ell \ell'} \)

For anisotropic spectrum, there are off–diagonal components.

For example,

\[ C_{\ell \ell'}^{TB} \propto \int d\Omega_k P_{TB}(k) Y_{\ell m}^*(\hat{k}) Y_{\ell' m'}(\hat{k}) \propto \delta_{\ell, \ell' \pm 1} \]

The off–diagonal part of the angular power spectrum tells us if the gauge kinetic function plays a role in inflation.
The impact of form fields on primordial gravitational waves

When we assume the tensor to the scalar ratio

\[ r = 0.3 \]

and scalar anisotropy

\[ g_s = 0.3 \]

The off-diagonal spectrum becomes


The anisotropic inflation can be tested through the CMB observation!
Summary

- Anisotropic inflation can be realized in the presence of one and two-forms.
- This can be regarded as the counterexample to the cosmic no-hair conjecture.
- We have shown that anisotropic inflation with one and two-form fields induce the statistical symmetry breaking in the CMB.

More precisely, we have

- the statistical anisotropy in scalar and tensor fluctuations
- the cross correlation between scalar and tensor (not for two-forms)
- the sizable non-gaussianity

- The local type non-gaussianity is small for two-form, which is consistent with the Planck data.
- The shape of bi-spectrum is different for one and two-form cases.
Axion and a dual transformation

It is well known that 2-form fields are equivalent to pseudo scalar fields, the so-called axions.

Actually, one can define an axion by the dual transformation of the field strength

\[ *H = d\chi \quad \leftrightarrow \quad \frac{1}{3!} \varepsilon_{\mu \nu \lambda \rho} H^{\nu \lambda \rho} = \partial_\mu \chi \]

Does this mean two-form fields are redundant?

The answer is yes and no. The point is that there is a polarization of the two form field.

It is possible to give this polarization information to the scalar field if you allow the inhomogeneity of the scalar field

\[ H_{0,xy}(t) = \partial_z \chi \quad \chi = H_{0,yz}(t) z + c(t) \]

However, one may think this is unnatural. Hence, we use two-form language from now on.
Inflaton dynamics in the attractor phase

The 2-form field modify slow-roll equations.

Hamiltonian Constraint
\[
\dot{\alpha}^2 = \frac{1}{6} m^2 \phi^2
\]

Scalar field
\[
3\dot{\alpha} \dot{\phi} = -m^2 \phi + \frac{c}{2} \phi A^2 e^{-c\phi^2/2-2\alpha}
\]

\[
\phi \frac{d\phi}{d\alpha} = -2 + \frac{cA^2}{2m^2} e^{-c\phi^2/2-2\alpha}
\]

For \(c=1\), we obtain
\[
e^{-\phi^2/2-2\alpha} = \frac{2m^2}{A^2} \frac{1}{\alpha + \alpha_0} \approx \frac{2m^2}{A^2} \frac{1}{\alpha_0}
\]

In any case, the energy density of the form field is constant.

Energy density of two-form field
\[
\rho_v = \frac{A^2}{2} e^{-c\phi^2/2-2\alpha} = \frac{m^2}{\alpha_0}
\]

Degree of anisotropy
\[
3\dot{\alpha} \dot{\sigma} = -\frac{A^2}{3} f^{-2}(\phi)e^{-2\alpha}
\]
\[
\frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} = -\frac{2}{3} \frac{\rho_v}{V} = -\frac{4}{3} \frac{1}{\alpha_0 \phi^2}
\]
Form fields in inflationary background

**de Sitter background**

\[ ds^2 = -dt^2 + a^2(t) \left( dx^2 + dy^2 + dz^2 \right) = a^2(\eta) \left[ -d\eta^2 + dx^2 + dy^2 + dz^2 \right] \quad a(\eta) = e^{\mu} = \frac{1}{H\eta} \]

**Two-form fields**

\[ S = -\frac{1}{12} \int d^4x \sqrt{-g} f^2(a) H_{\mu\nu\lambda} H^{\mu\nu\lambda} \]

\[ H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]} + \partial_{[\nu} B_{\lambda\mu]} + \partial_{\lambda} B_{\mu\nu} \]

\[ f = \frac{1}{a} \]

**gauge symmetry**

\[ \bar{B}_{\mu\nu} = B_{\mu\nu} + \partial_{\mu} \xi_{\nu} - \partial_{\nu} \xi_{\mu} \]

It is possible to take the gauge \( B_{0i} = 0 \), \( \partial_i B_{ij} = 0 \)

\[ B_{ij} = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ a_k \chi(k,\eta) e_{ij}(k)e^{ik\cdot x} + c.c. \right] \]

\[ k_i e_{ij}(k) = 0, \quad e_{ii}(k) = 0, \quad e_{ij}(-k)e_{ij}(k) = 2 \]

\[ u(k,\tau) = \frac{f}{a} \chi(k,\tau) \]

\[ u'' + \left[ k^2 - \frac{2}{\tau^2} \right] u = 0 \quad \rightarrow \quad u = \frac{Ha}{\sqrt{2k^3}} \left( 1 + ik\tau \right) e^{-ik\tau} \]
Calculational tools

perturbed field strength

\[
\delta E_{ij} = \frac{f}{a^3} \delta H_{ij} = \int \frac{d^3k}{(2\pi)^{3/2}} \delta \mathcal{E}_{ij}(\tau,k) e^{ik \cdot x}
\]

\[
\delta \mathcal{E}_{ij} = \varepsilon_{ij} \left( a_k + a^*_k \right) \frac{3H^2}{\sqrt{2k^3}} \quad \text{on large scales}
\]

curvature perturbations

\[
\zeta = \frac{\delta \phi}{\sqrt{2\varepsilon}} = \int \frac{d^3k}{(2\pi)^{3/2}} \tilde{\zeta}(\tau,k) e^{i k \cdot x}
\]

\[
\tilde{\zeta}(\tau,k) = \zeta^{(0)}(\tau,k) a_k + \zeta^{(0)*}(\tau,k) a^*_k
\]

\[
\zeta^{(0)}(\tau,k) = \frac{H(1 + ik\tau)}{2\sqrt{\varepsilon} k^{3/2}} e^{-ik\tau}
\]

\[
\left\langle \tilde{\zeta}(\tau,k_1) \tilde{\zeta}(\tau,k_2) \right\rangle = \frac{2\pi^2}{k_1^3} \delta(k_1 + k_2) P(k_1)
\]

Interaction Hamiltonian

\[
-H_{\text{int}} = L_{\text{int}} = -\frac{a^4}{12} \frac{\partial \left\langle f^2 \right\rangle}{\partial \phi} \delta \phi \left( H_{\mu \nu \lambda \kappa} + \delta H_{\mu \nu \lambda \kappa} \right) \left( H^{\mu \nu \lambda} + \delta H^{\mu \nu \lambda} \right) = \frac{1}{2} a^4 \left( 4E_{yz} \delta E_{yz} + \delta E_{ij} \delta E_{ij} \right) \zeta
\]

Correction

\[
\delta \left\langle \tilde{\zeta}(\tau,k_1) \tilde{\zeta}(\tau,k_2) \right\rangle = -\int_{\tau_m}^{0} d\tau_1 \int_{\tau_m}^{0} d\tau_2 \left\langle \left[ \left[ \tilde{\zeta}(\tau,k_1) \tilde{\zeta}(\tau,k_2), H_{\text{int}}(\tau_1) \right], H_{\text{int}}(\tau_2) \right] \right\rangle
\]

\[
= \frac{6E_{yz}^2}{\varepsilon V} N_k^2 \cos^2 \theta_{k,x} P(k)
\]