

# Multi-Dimensional Starobinsky Influm in Maximal Gauged Supergavity

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*Based on the work in collaboration with Masato Nozawa*

- HK, M Nozawa: arXiv: 1502.01378
- HK, M Nozawa: JHEP1301(2013)045 [arXiv:1210.4238]

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# Introduction

# 4D Maximal Supergravity

**11D SUGRA**

*Cremmer-Julia-Scherk 1978*

$(g_{MN}, C_{MNL}, ; \Psi_M)$

N=8 local susies

$T^7$

$S^7$

*Cremmer-Julia, 1979*

*de Wit-Nicolai 1987*

**D=4 N=8  
ungauged SUGRA**

$(g_{\mu\nu}, A_\mu^M, \mathcal{V}; \psi_\mu^a, \chi^{abc})$

$\mathcal{V} \in E_{7(7)}/SU(8) \approx \mathbb{R}^{70}$

**D=4 N=8, SO(8)  
gauged SUGRA**

*de Wit-Nicolai 1982*

**gauging**

$\nabla = \partial - gA^M X_M$

$G \subset E_{7(7)}$

**D=4 N=8, SL8-type  
gauged SUGRAs**

Hull, Werner  
1985

...

# SL8-type Vacua in Gauged Maximal SUGRA

All special SL8-type dyonic gaugings are classified:

[HK, M Nozawa (2013); G Dall'Agata, G Inverso (2012)]

$$(A^M) = (A_e^{[ab]}, A_m[cd]) \mapsto A^M X_M = - \begin{pmatrix} (\theta A_e + A_m \xi) \wedge 1 & 0 \\ 0 & (A_e \theta + \xi A_m) \wedge 1 \end{pmatrix}$$
$$\theta, \xi \in \mathfrak{sl}_s(8, \mathbb{R}); \quad \theta \xi = c I_8$$

$$\theta \xi = 1: \quad \text{SO}(8), \quad \text{SO}(7, 1), \quad \text{SO}(6, 2), \quad \text{SO}(5, 3), \quad \text{SO}(4, 4)$$

$$\theta \xi = 0: \quad \text{SO}(4) \times \text{SO}(2, 2) \ltimes \mathbb{R}^{16}, \quad \text{SO}(6) \times \text{SO}(1, 1) \ltimes \mathbb{R}^{12},$$
$$\text{SO}(2)^2 \ltimes \mathbb{R}^{20}, \quad \text{SO}(7) \ltimes \mathbb{R}^7$$

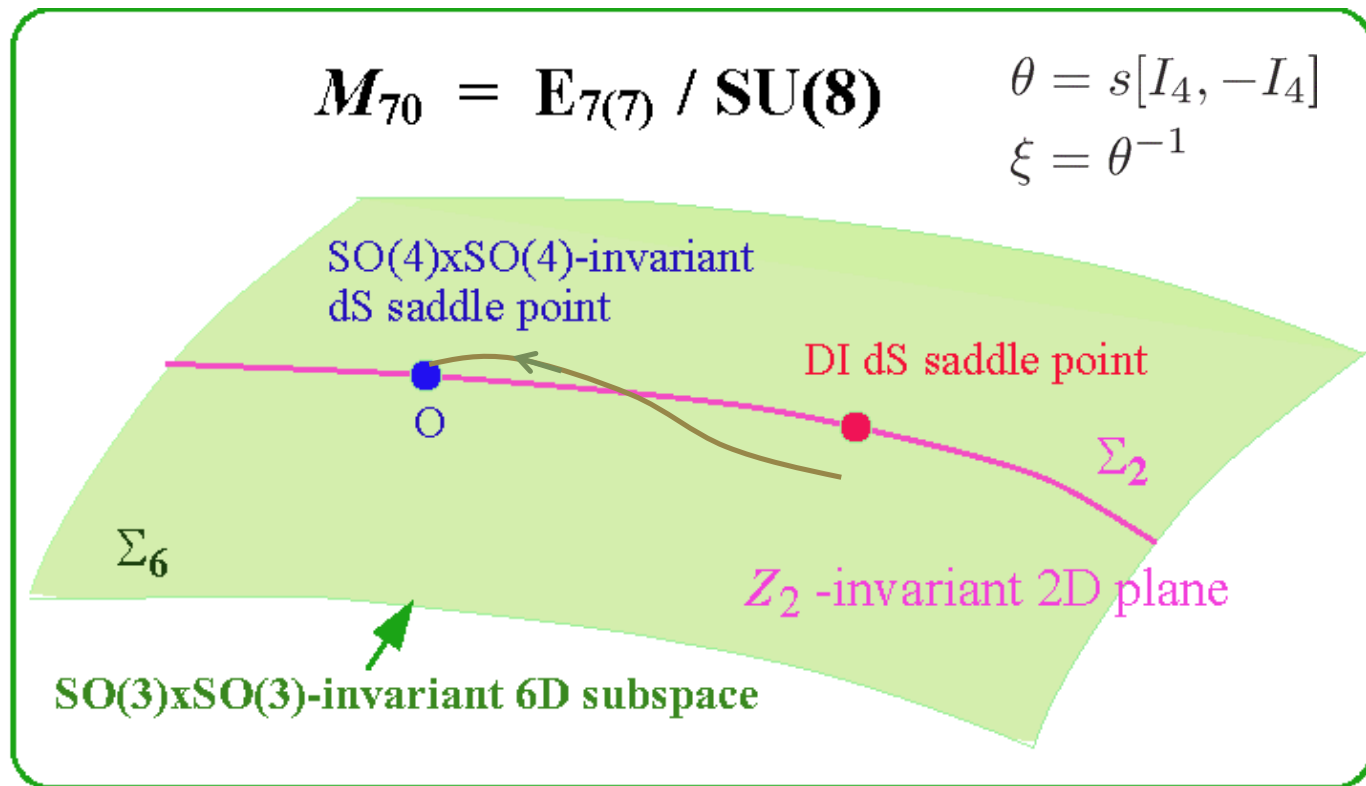
- ◆ (unstable) dS vacua at the origin only for  $\text{SO}(4,4)$  and  $\text{SO}(5,3)$ .
- ◆ There exists a new deformation parameter  $s$  for all semi-simple gaugings in addition to the gauge coupling constant  $g$ .

[cf.  $\text{SO}(8)$  gauging: G Dall'Agata, G Inverso, M Trigiante (2012) ]

# dS vacua in the $SO(4,4)$ and $SO(5,3)$ gaugings.

- New vacua in  $SO(4,4)$  gaugings avoiding the  $\eta$ -problem.

[ G Dall'Agata, G Inverso (2013)]



# Can realistic inflation be realised around the new DI saddle point in the $SO(4,4)$ gauging ?

The full potential in the  $SO(3) \times SO(3)$ -invariant 6D sector for  $SO(4,4)$  &  $SO(5,3)$  gaugings. No new vacuum.  
[HK, M Nozawa (2015)]

**Note: The potential has no lower bound in gauged maximal supergravities. So, we cannot discuss reheating and the post-inflation evolution of the universe in this framework.**



# **Cosmology in the 6D sector**

# System Description

## ● Fundamental variables

- spacetime metric:  $a(t) = \exp(\alpha(t))$ :

$$ds^2 = -dt^2 + a(t)^2 dx^2$$

- scalar fields:  $(x, y, z, \theta_1, \theta_2, \theta_3) \leftarrow (x_1, y_1, x_2, y_2, w_1, w_2)$

$$x = x_2(t), \quad y = \frac{1}{2}(\mu_2(t) - \mu_3(t)), \quad z = \frac{1}{2}(\mu_2(t) + \mu_3(t))$$

- SO(3) Gauge fields:  $\psi_1(t), \psi_2(t)$

$$A^{(1)I} = a(t)\psi_1(t)dx^I, \quad A^{(2)I} = a(t)\psi_2(t)dx^I$$

$$F^{(\epsilon)I} = a(t) \left( \dot{\psi}_\epsilon + \dot{\alpha}\psi_\epsilon \right) dt \wedge dx^I + \frac{g}{2} a(t)^2 \psi_\epsilon(t)^2 \epsilon_{IJK} dx^J \wedge dx^K$$



# Basic Equations

## ● Action

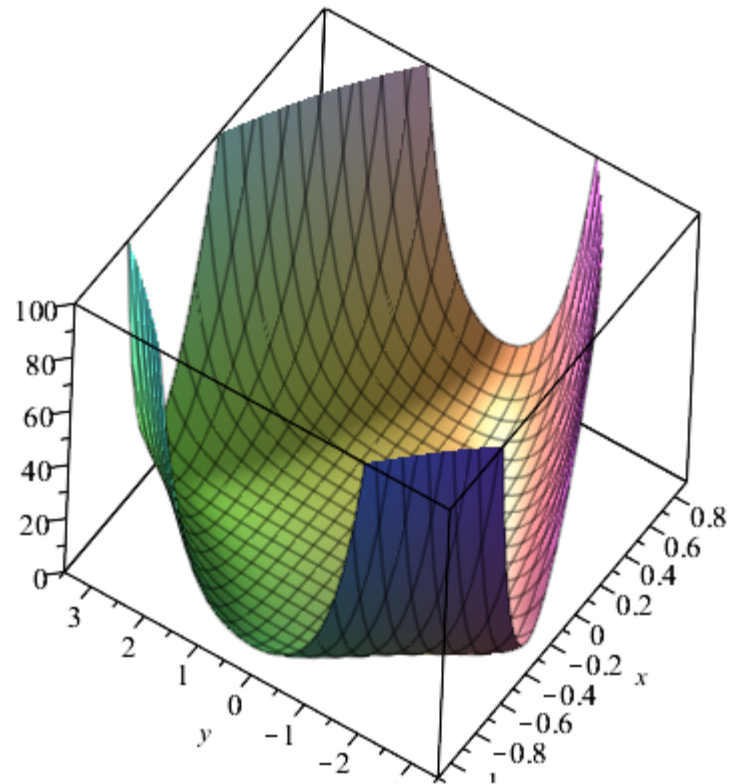
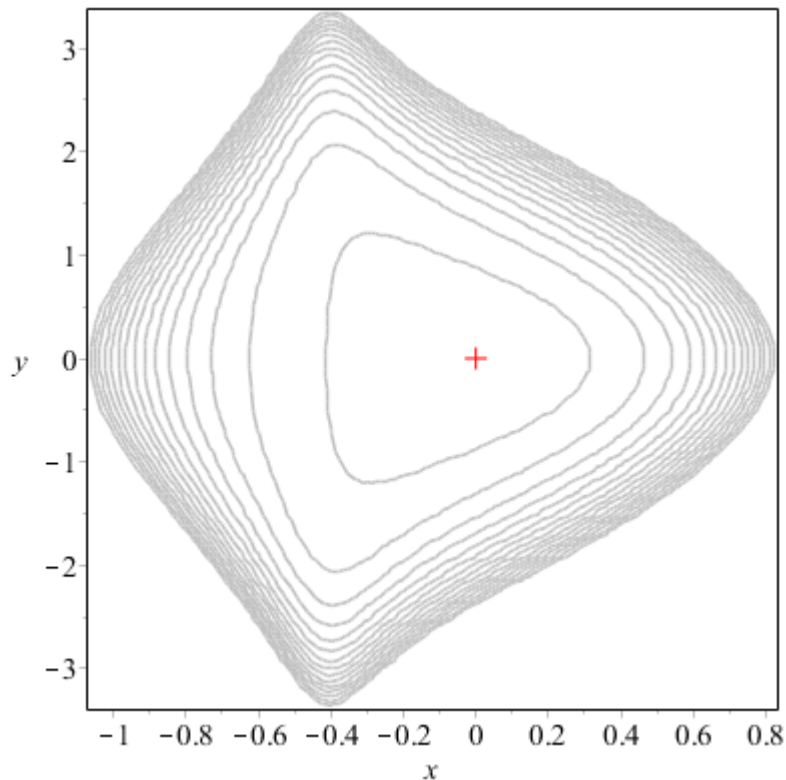
$$\begin{aligned}L &= L_G + L_K - a^3 NV + L_G; \\ \kappa^2 L_G &= -\frac{3a^3}{N} \dot{\alpha}^2, \\ \kappa^2 L_K &= \frac{3a^3}{N} \left[ 3\dot{x}_2^2 + \frac{1}{2} \sum_i \dot{\mu}_i^2 \right. \\ &\quad \left. + \sinh^2(\mu_2 - \mu_3) \dot{\chi}_1^2 + \sinh^2(\mu_1 - \mu_3) \dot{\chi}_2^2 + \sinh^2(\mu_1 - \mu_2) \dot{\chi}_3^2 \right], \\ \kappa^2 L_F &= a^3 \sum_\epsilon \left[ \frac{3}{2} \text{Im}(-\mathcal{N}_\epsilon) \left\{ \frac{1}{N} \left( \dot{\psi}_\epsilon + \dot{\alpha} \psi_\epsilon \right)^2 - g^2 N \psi_\epsilon^4 \right\} \right. \\ &\quad \left. + \frac{3g}{2} N \text{Re}(\mathcal{N}_\epsilon) \psi_\epsilon^2 (\dot{\psi}_\epsilon + \dot{\alpha} \epsilon) \right].\end{aligned}$$

$$(\mu_1 + \mu_2 + \mu_3 = 0)$$

# $Z_2$ -invariant 2D sector: $(x_2, y_2)$

- Subcritical case:  $(\sqrt{2} - 1)^{1/2} < s < (\sqrt{2} + 1)^{1/2}$ .

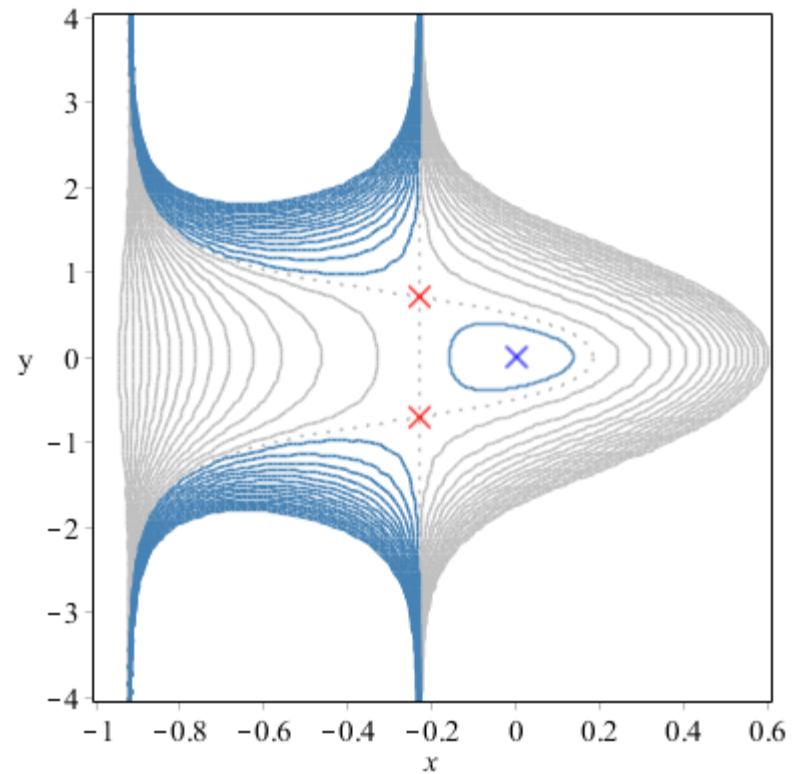
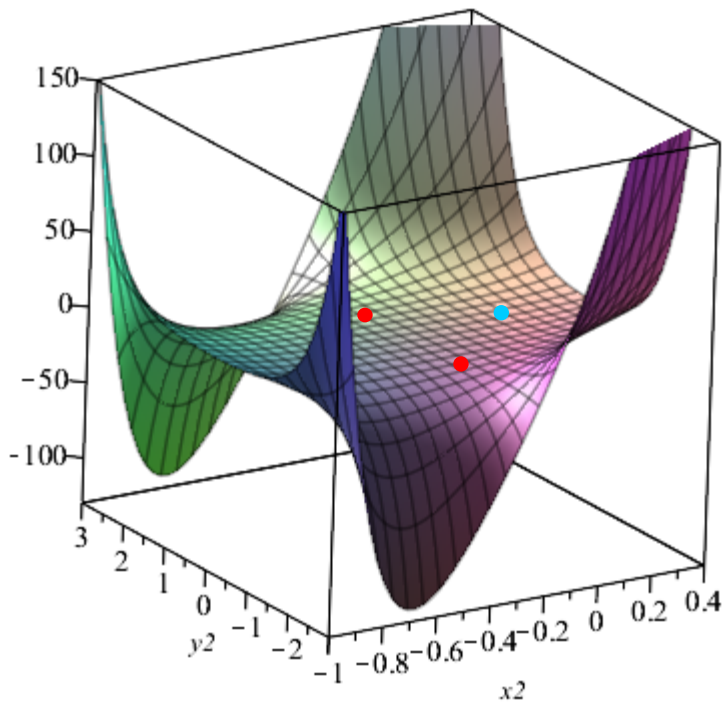
Unique dS vacuum at  $(x,y)=(0,0)$



$s=1.5$

- Supercritical case:  $s < (\sqrt{2} - 1)^{1/2}$ , or  $s > (\sqrt{2} + 1)^{1/2}$ .

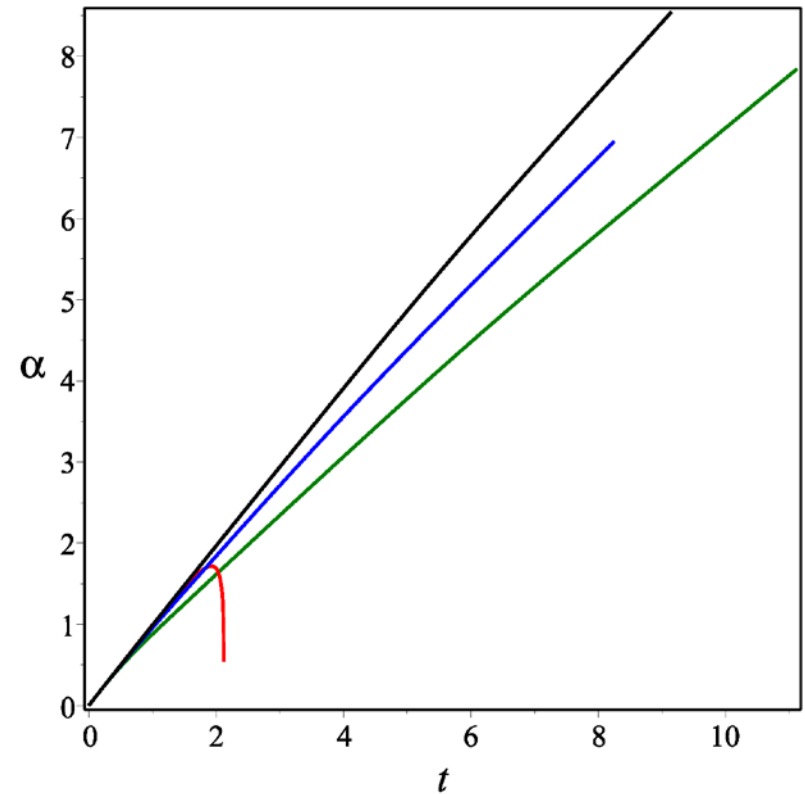
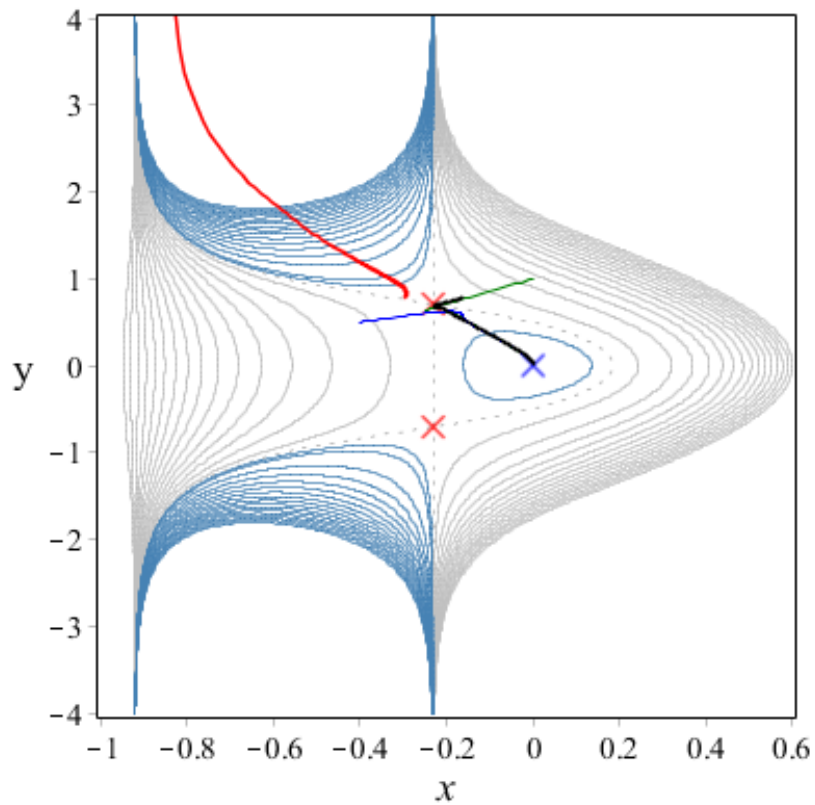
**Two new dS vacua at  $(x_*, +y_*)$  and  $(x_*, -y_*)$**



$s=2$

# Trajectories on the $Z_2$ -inv plane

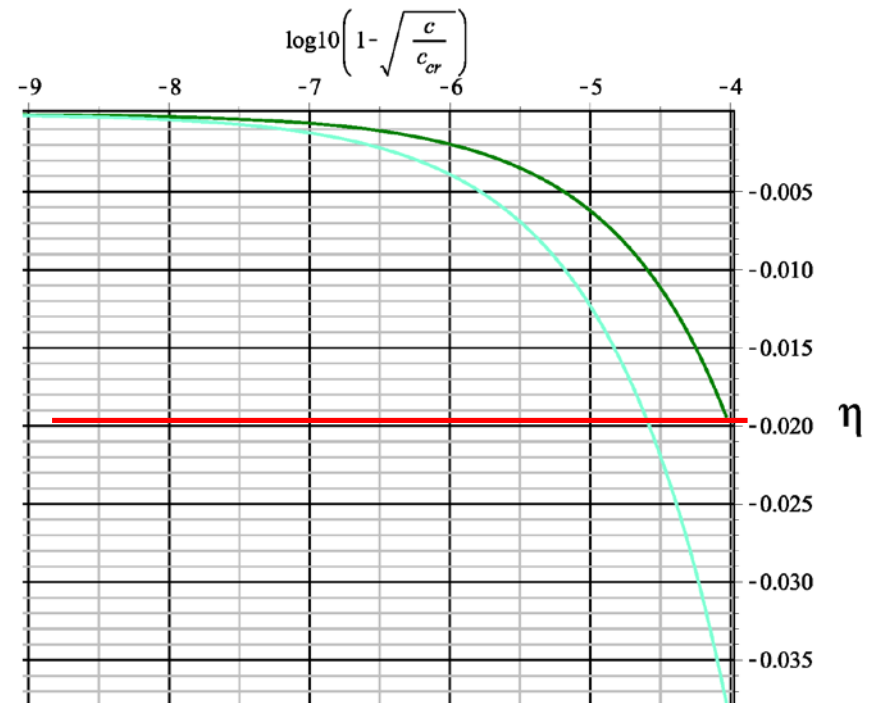
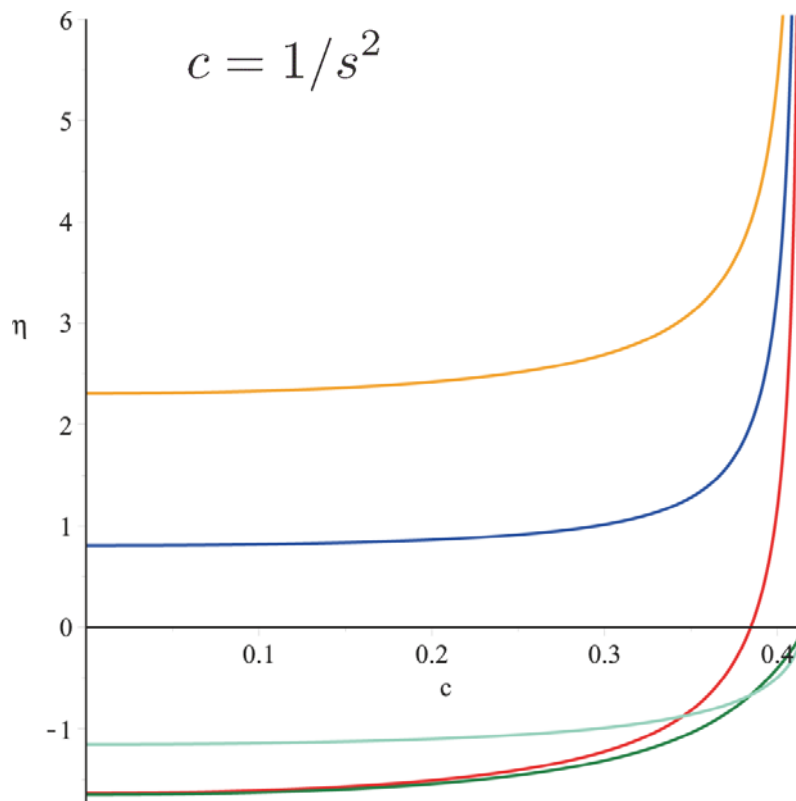
$s=2$



# The $\eta$ matrix and $\eta$ parameter

The minimum eigenvalue of the eta matrix determines the slow-roll trajectory.

$$\eta = (\eta_{ab}) : \quad \eta_{ab} = \frac{1}{V} e_a^\alpha e_b^\beta D_\alpha D_\beta V$$

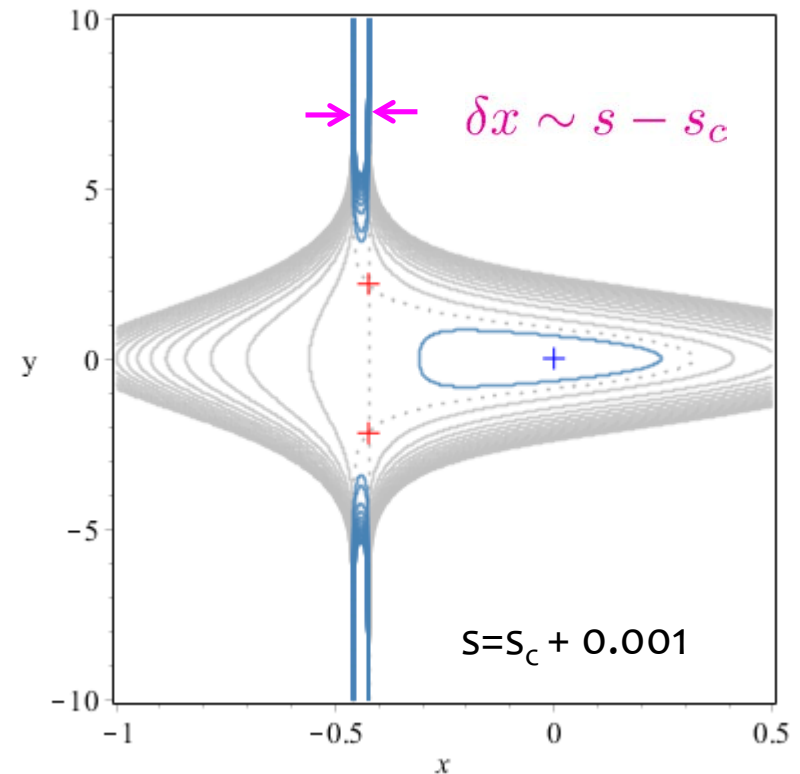
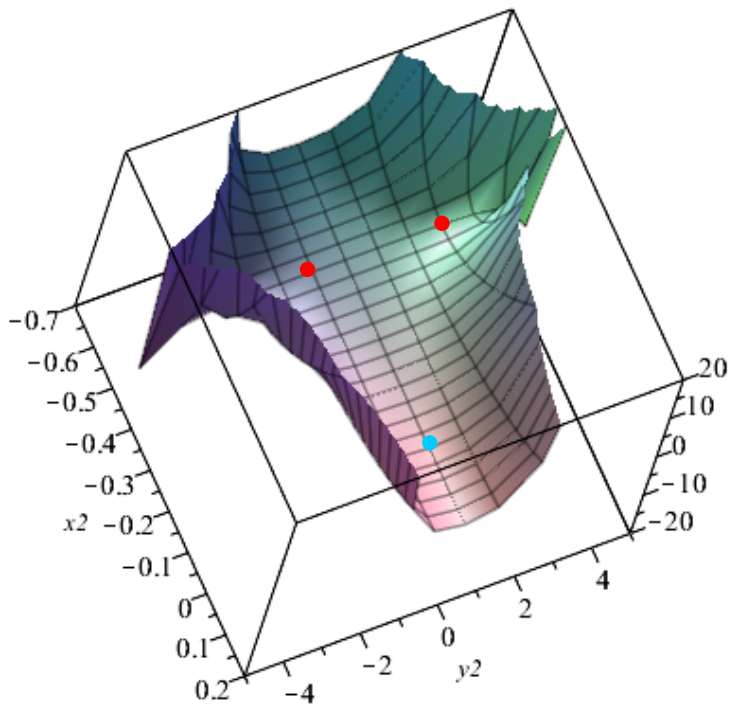


# Critical Limit

- As  $s$  approaches the critical value, the DI saddle point runs away to infinity keeping the potential height finite.

$$s \rightarrow \sqrt{1 + \sqrt{2}} : (x_*, y_*) \rightarrow (-\frac{1}{2} \log(1 + \sqrt{2}), \infty), \quad V_* \rightarrow 3g^2/2$$

- At the same time, the waterfall region becomes narrower and narrower.

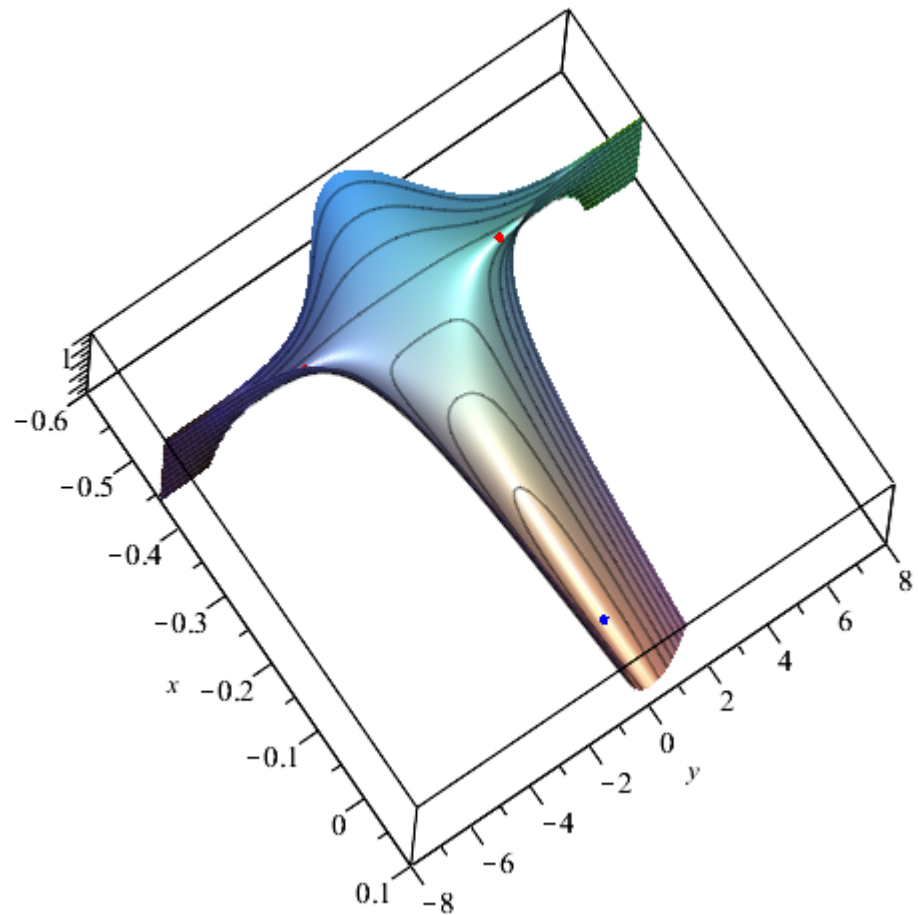
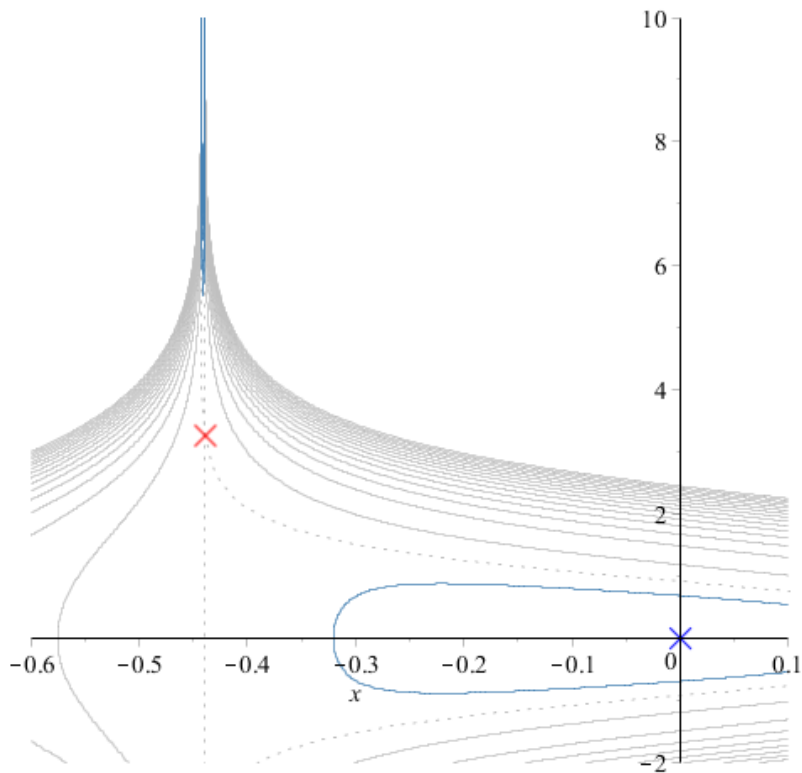


# $s=s_c + 1.4e-5$ : On $\Sigma_2$

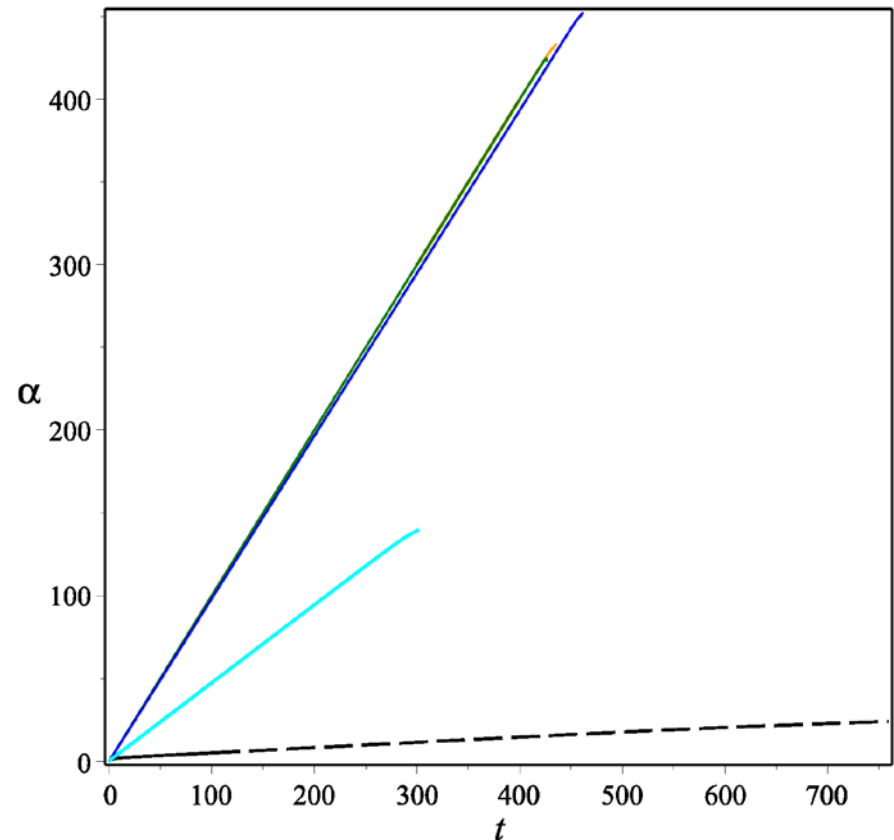
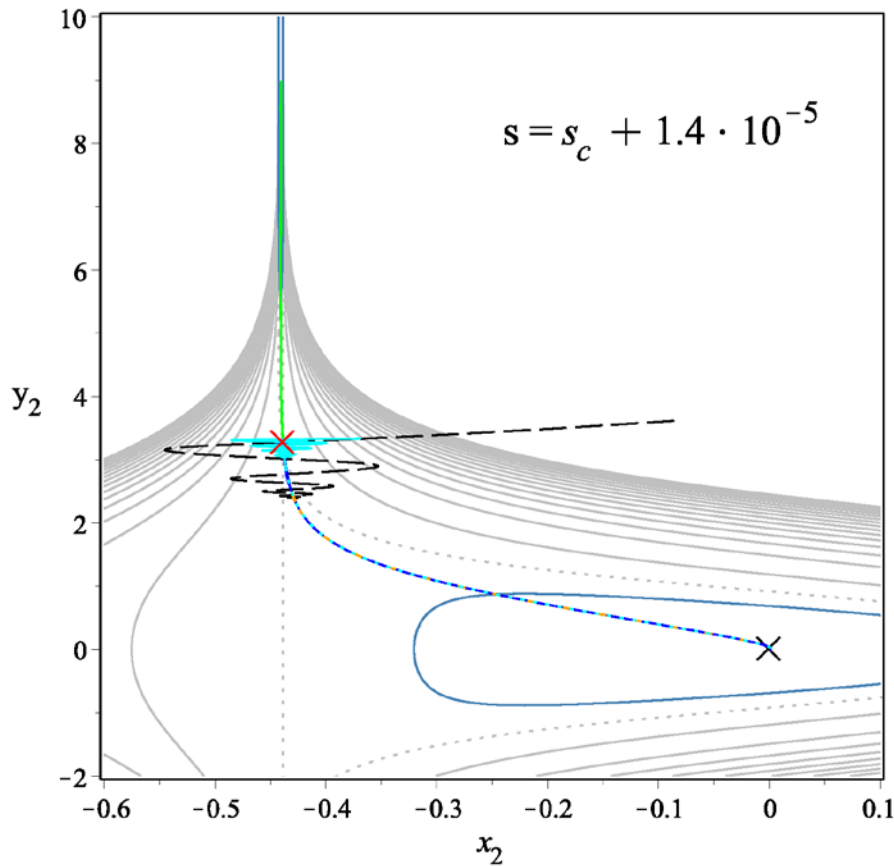
## DI Saddle Point:

$$(x_*, y_*) = (-0.44, \pm 3.26.), \quad H_* = 0.705g$$

The  $\eta$  parameter :  $\eta_* = [-0.01168, 228; -0.005891, 226; 168; 0]$



- For any initial condition, after initial oscillations, all trajectories settle down to one of the two attractor slow roll trajectories.
- If the initial point is within  $O(m_{pl})$  from the DI saddle point, sufficient inflation is realised.



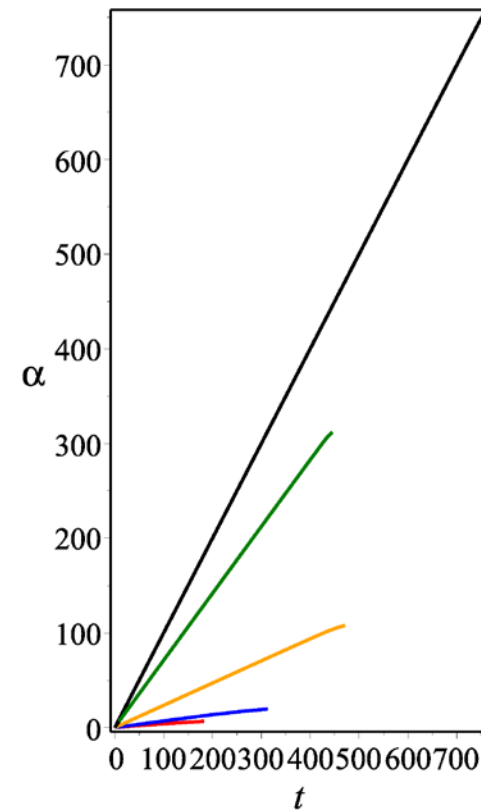
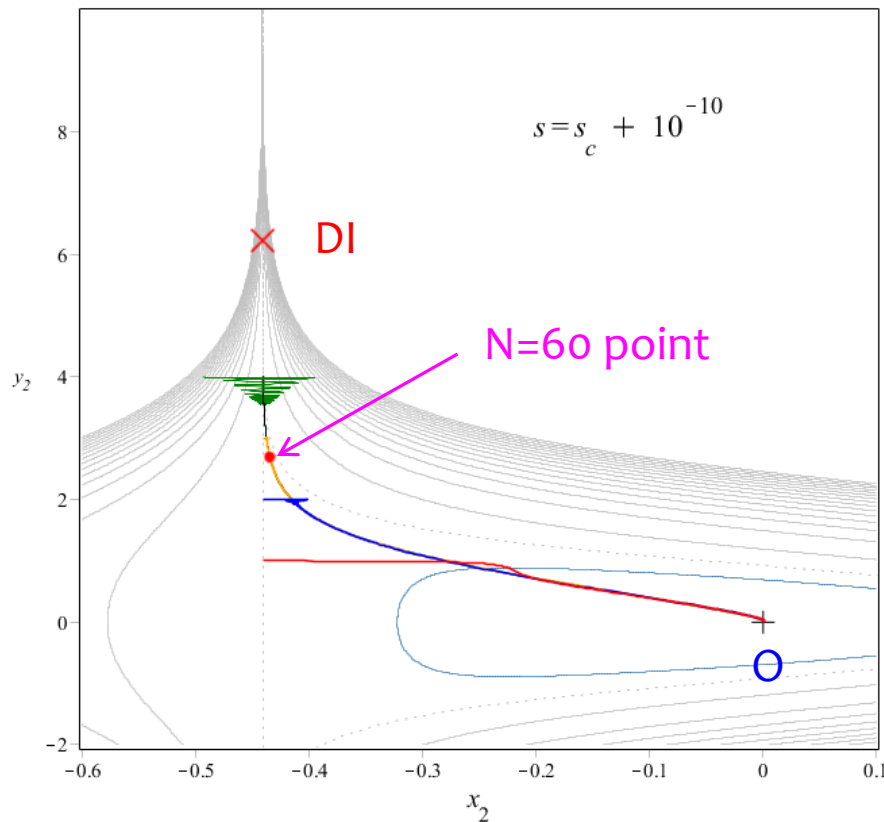


# $s=s_c + 1e-10$ : On $\Sigma_2$

**DI Saddle Point:**  $(x_*, y_*) = (-0.440, \pm 6.22)$ ,  $H_* = 0.707g$

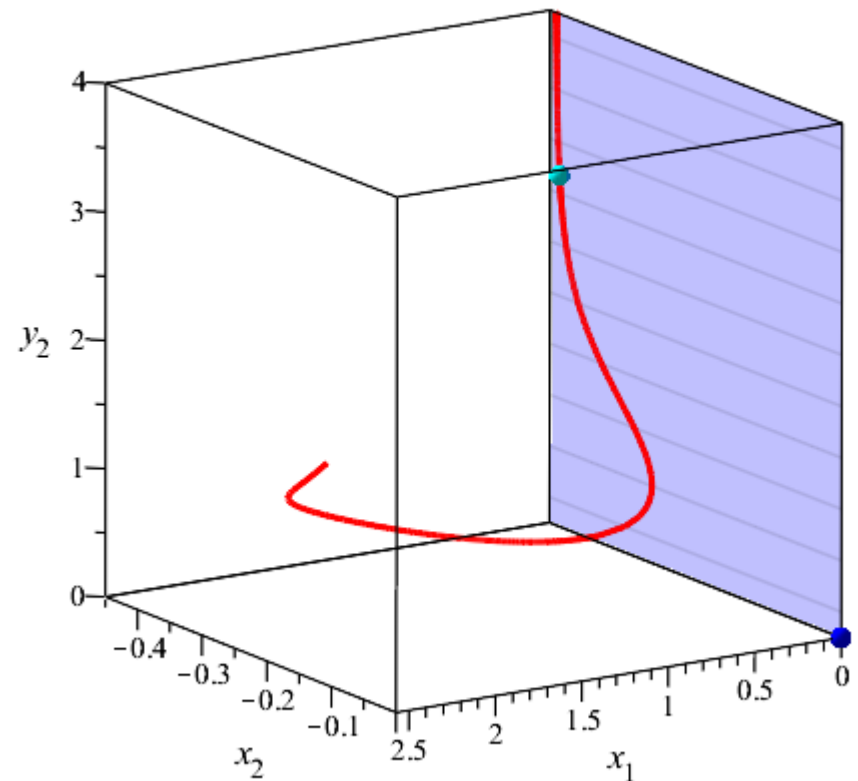
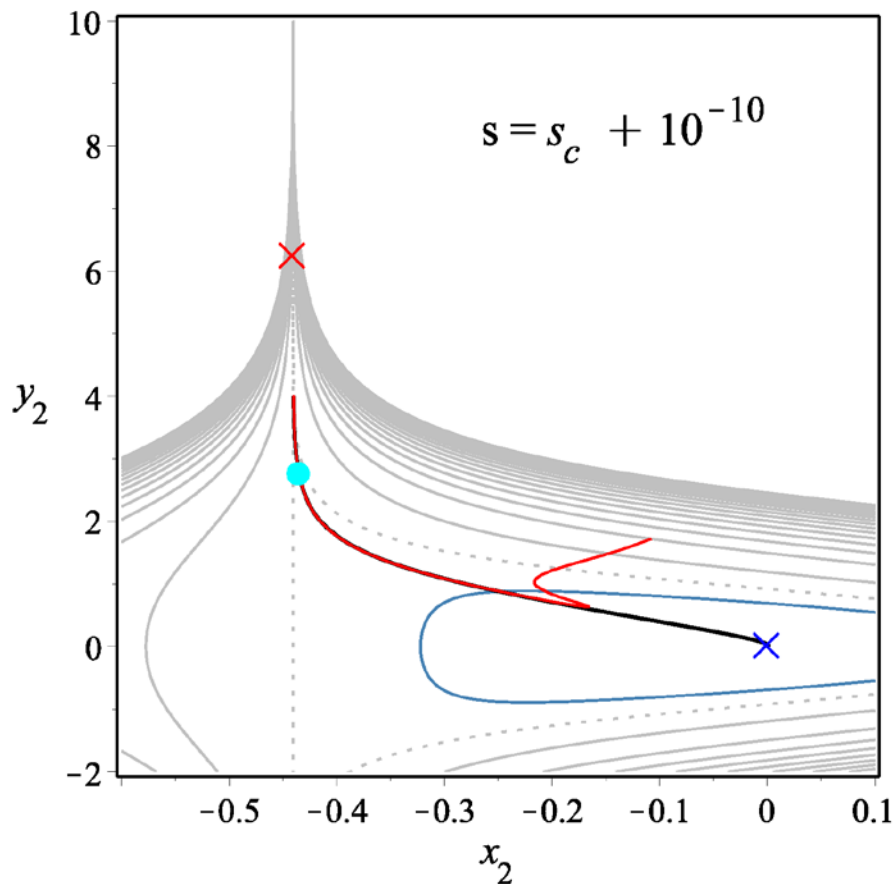
The  $\eta$  parameter :  $\eta_* = [-3.115 \cdot 10^{-5}, 8.44 \cdot 10^4; \dots]$

- If  $y_2 > 3$  and  $V = O(V_*)$  initially, sufficient inflation is realised.



# Off $\Sigma_2$ Trajectories

- If the initial offset from  $\Sigma_2$  is less than  $10^4 H_*(g=4 \times 10^{-6})$ , sufficient inflation is realised, although the final stage of inflation is sensitive to the offset.



# Observational predictions

- Spectral index of the scalar perturbation:

When a sufficient inflation is realised and  $s$  is close to  $s_c$ ,  $n_s$  is miraculously within the  $1\sigma$  range of Planck result:

$$n_s = 0.958 \sim 0.967 \quad \text{Cf. Planck2015: } n_s = 0.9645 \pm 0.0049$$

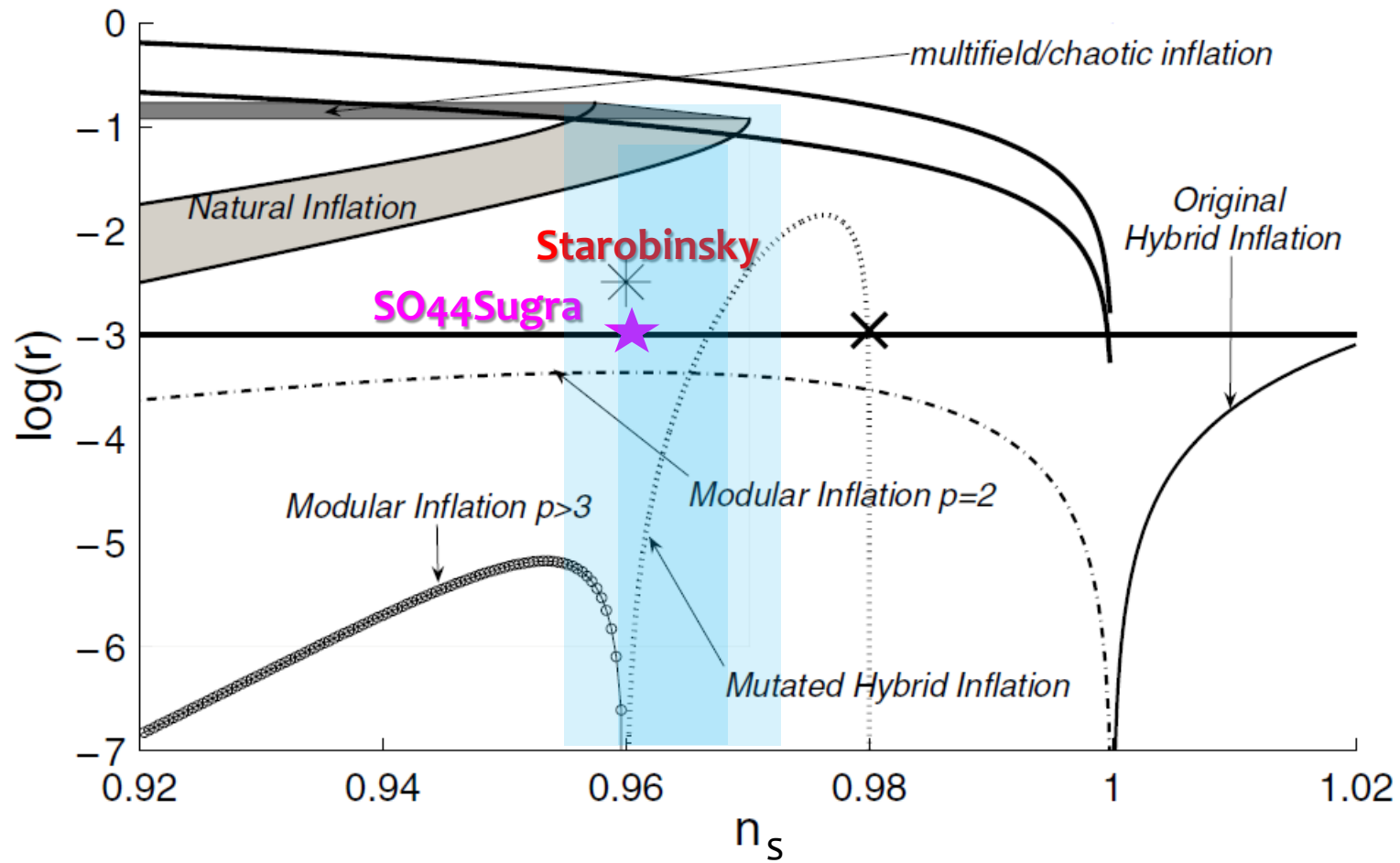
- Tensor-scalar ratio:

Inflation in our theory is the intermediate scale type and similar to that in the Starobinsky model:

$$0.96 \times 10^{-3} \lesssim r = 16\epsilon \lesssim 1.37 \times 10^{-3}$$

- Scalar perturbation amplitudes:

$$\mathcal{P}_\zeta = \frac{1}{2\epsilon} \left( \frac{H}{2\pi m_{\text{pl}}} \right)^2 \simeq 2 \times 10^{-9} \Rightarrow g \simeq 4 \times 10^{-6}$$



# »» Conclusion

# Summary

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- Cosmology of a multi-component inflaton system corresponding to the  $SO(3) \times SO(3)$  –invariant sector in the  $SO(4,4)$  gauging.

When the deformation parameter  $s$  is very close to the critical value,

- a multi-dimensional version of the Starobinsky-type inflation is realised;
  - $n_s$  is miraculously consistent with the Planck result;
  - $r \sim 10^{-3}$ ;
  - No chromonatural-type inflation or anisotropic inflation occurs.
- Next task is to find a string uplifting of this theory.