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A Stringy Mechanism for A Small Cosmological Constant

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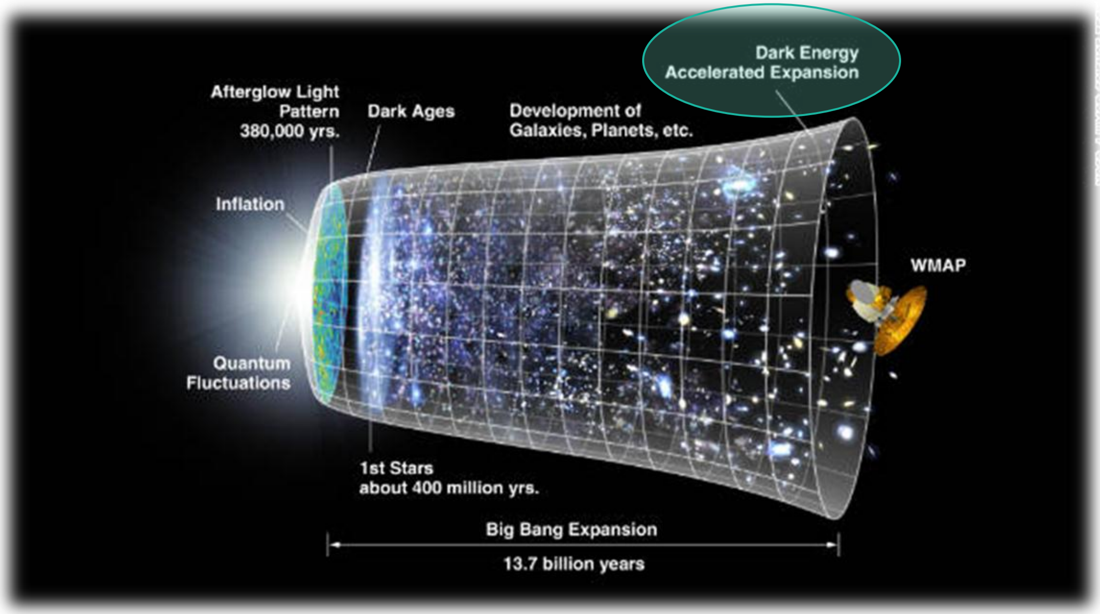
- X. Chen, Shiu, Sumitomo, Tye, arxiv:1112.3338, JHEP 1204 (2012) 026
- Sumitomo, Tye, arXiv:1204.5177
- Sumitomo, Tye, in preparation

Contents

Motivation

Dark Energy

Late time expansion



Awarded Nobel Prize in 2011!

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

Photo: Belinda Pratten, Australian National University

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What can be a source for this?

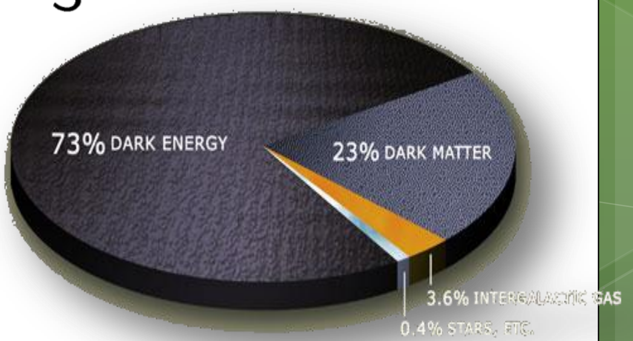
Acceleration

$$\frac{3\ddot{a}}{a} = -4\pi G(3p + \rho)$$

The universe is accelerating if $\rho < -3p$

or pressure-density ratio: $w \equiv \frac{p}{\rho} < -\frac{1}{3}$

Cosmological scale



EOM (Friedmann eq.)

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho}{3}}$$ for flat background

Observationally $\Omega_\Lambda \sim 0.7$ → DE domination

$$\rho_0 = \frac{3H_0^2}{8\pi G}\Omega_\Lambda \sim 10^{-122} M_P^4$$

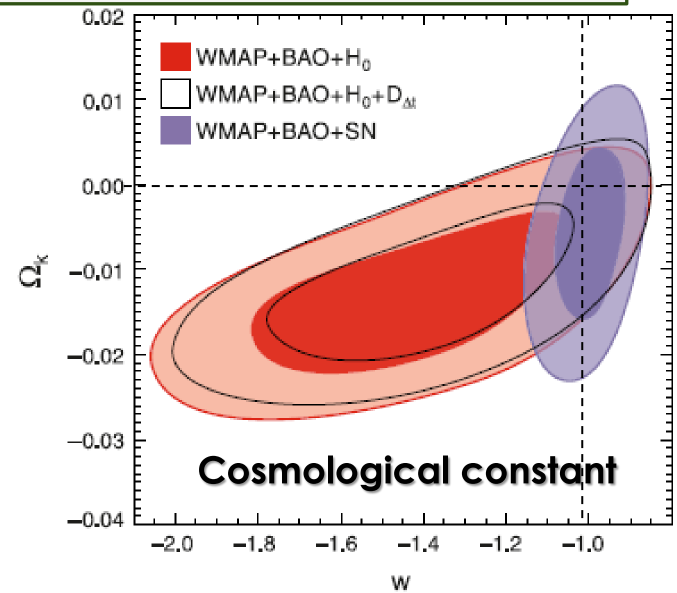
Two possibilities

- For **cosmological constant**

  WMAP+BAO+SN suggests

$$w = -1.10 \pm 0.14 \ (64\% \ \mathrm{CL})$$

for a flat universe $\Omega_k = -\frac{k}{a_0^2 H_0^2} = 0$



- For **time-varying DE**

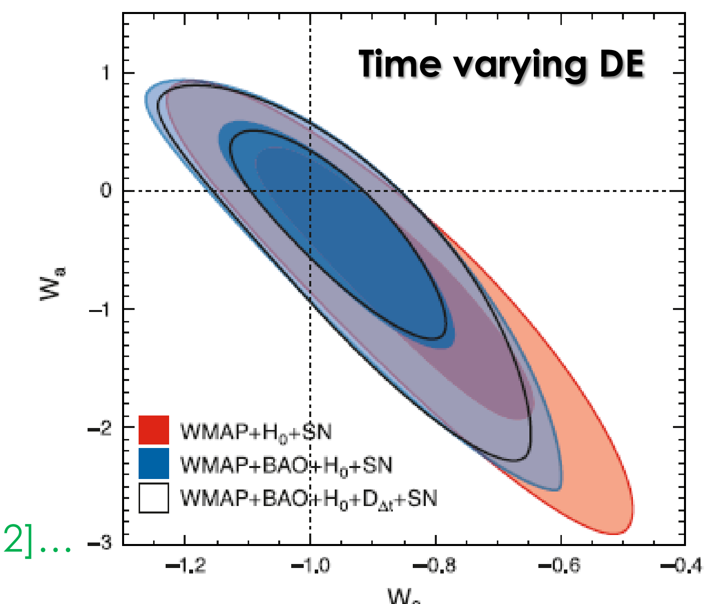
  WMAP+BAO+H0+DΔt+SN suggests

$$w = w_0 + w_a(1 - a(t))$$

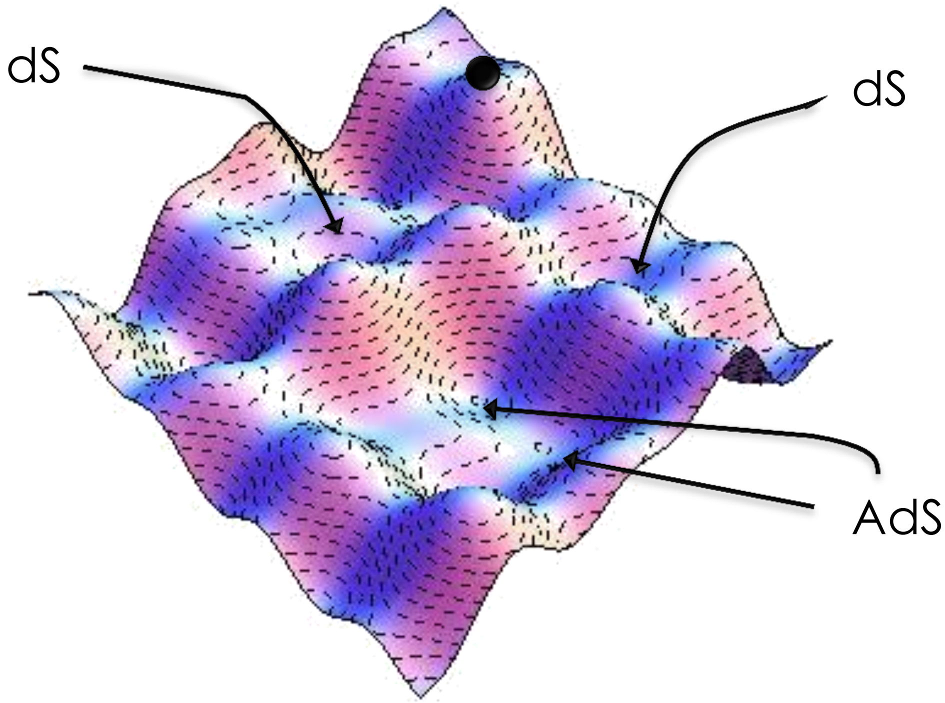
$$w_0 = -0.93 \pm 0.13$$
$$w_a = -0.41^{+0.72}_{-0.71} \ (68\% \ \mathrm{CL})$$

e.g. Stringy Quintessence models

[Kiwoon, 99], [Svrcek, 06], [Kaloper, Sorbo, 08], [Panda, YS, Trivedi, 10], [Cicoli, Pedro, Tasinato, 12]…



Landscape



Metastable vacua in moduli space

- Inflation

⇓ rolling down (& tunneling)

- dS vacua

⇓ tunneling

- AdS vacua?

Low energy

We may stay here for a while.

⇒ But how likely with tiny CC?

Stringy Landscape

There are many types of vacua in string theory, as a result of a variety of (Calabi-Yau) compactification.

$$ds_{10}^2 = ds_4^2 + ds_6^2$$

A class of Calabi-Yau gives Swiss-cheese type of volume.

$$\mathcal{V}_6 = \gamma_1(T_1 + \bar{T}_1) - \sum_{i=2} \gamma_i(T_i + \bar{T}_i)\,,$$

E.g. workable models: [Denef, Douglas, Florea, 04]

- $\mathbb{P}^4_{[1,1,1,6,9]}$: $h^{1,1} = 2,\ h^{2,1} = 272$
- $\mathcal{F}_{11}$: $h^{1,1} = 3,\ h^{2,1} = 111$
- $\mathcal{F}_{18}$: $h^{1,1} = 5,\ h^{2,1} = 89$

All can be stabilized *(a la KKLT)*, but in various way.

More recently, for $2 \leq h^{1,1} \leq 4$, 418 manifolds !

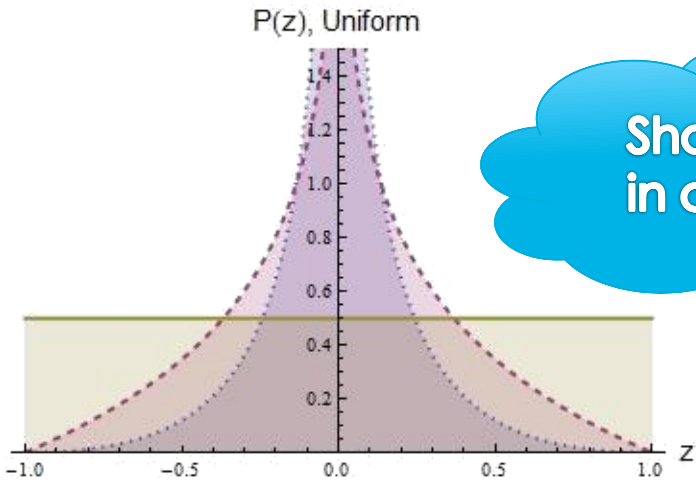
[Gray, He, Jejjala, Jurke, Nelson, Simon, 12]

Any implication of multiple vacua?

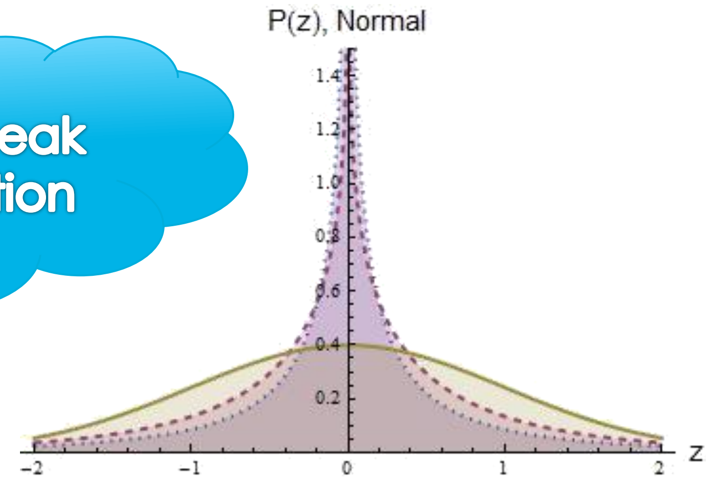
Keys in this talk

Product distribution

Assuming products of random variables: $z = y_1 y_2 y_3 \cdots$



Sharper peak in distribution



Many terms? → Correlation through stabilization

→ $z = y_1 y_2 y_3 \cdots f(y_1, y_2, y_3, \cdots)$ still peaked

We apply this mechanism for cosmological constant (CC)

Before proceeding…

I have to say

we don’t solve cosmological constant problem completely.

But here,

we introduce a tool to make cosmological constant smaller, maybe up to a certain value.

“A Stringy Mechanism for A Small Cosmological Constant”

Moduli stabilization ~random approach~

Gaussian suppression on stability

Various vacua in string landscape

➡ Mass matrix given randomly at extrema

➡ How likely stable minima exist?

Positivity of mass matrix ⟷ Positivity of Hessian $\partial_{\phi_i}\partial_{\phi_j}V\Big|_{\min}$

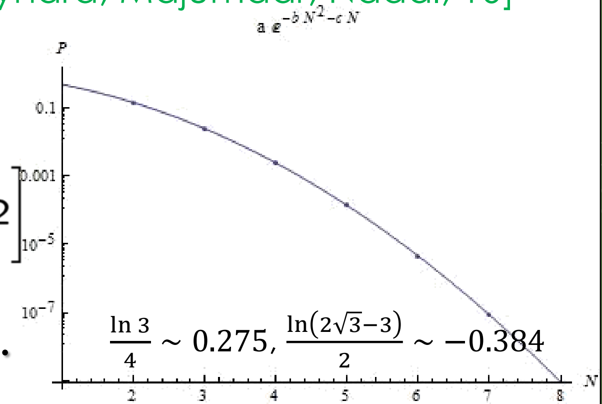
➡ Real/complex symmetric matrix

- Gaussian Orthogonal Emsemble

[Aazami, Easther, 05], [Dean, Majumdar, 08], [Borot, Eynard, Majumdar, Nadal, 10]

$$Z=\int dM_{ij}\, e^{-\frac{1}{2}\mathrm{tr}\, M^2},\, M=M^T$$

$$\mathcal{P}=\exp\left[\underline{-\frac{\ln 3}{4}N^2}+\frac{\ln(2\sqrt{3}-3)}{2}N-\frac{1}{24}\ln N-0.0172\right]$$



$\frac{\ln 3}{4}\sim 0.275,\ \frac{\ln(2\sqrt{3}-3)}{2}\sim -0.384$

Gaussian term dominates even at lower N.

Hierarchical setup

- Assuming hierarchy between diag. and off-diag. comp.

Actual models are likely to have minima at *AdS*.

\+ uplifting term toward *dS* vacua.

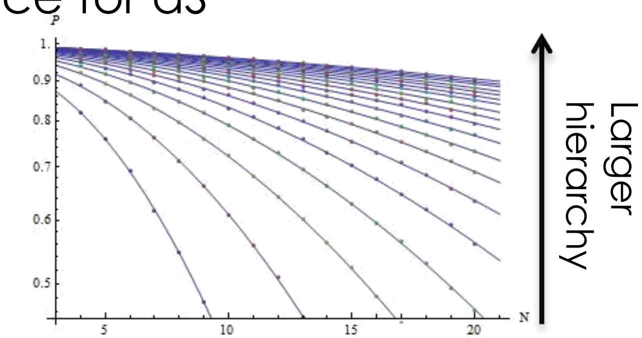
$\text{Hessian} = A + B$ where A: diagonal positive definite with σ_A
B: GOE with σ_B

Still Gaussianly suppressed, but a chance for dS

$$\mathcal{P} = a\, e^{-bN^2 - cN}$$

[X. Chen, Shiu, YS, Tye, 11]

When applying a model in type IIA, quite tiny chance remains.



- Assuming more randomness in SUGRA at SUSY *AdS*

$$\mathcal{P} = e^{-bN^2}$$

[Bachlechner, Marsh, McAllister, Wrase, 12]

Moduli stabilization ~concrete models~

Type IIB

Sources: $H_3, F_1, F_3, \tilde{F}_5$, dilaton, localized sources

Metric: $ds_{10}^2 = e^{2A} ds_4^2 + e^{-2A} \underline{d\tilde{s}_6^2}$

Calabi-Yau

Then EOM becomes [Giddings, Kachru, Polchinski, 02]

$$\tilde{\nabla}^2(\mathrm{e}^{4A} - \alpha) = \frac{\mathrm{e}^{2A}}{6\,\mathrm{Im}\,\tau}|iG_3 -*_6 G_3|^2 + e^{-6A}|\partial(e^{4A} - \alpha)|^2 + (\text{local sources})$$

LHS=0 when integrating out

positive contributions

$e^{4A} = \alpha,\ iG_3 =*_6 G_3$: imaginary self-dual condition

where α is a function in $\tilde{F}_5$, $G_3 = F_3 - \tau H_3$, $\tau = C_0 + i\, e^{-\phi}$

No-scale structure

Take a scaling: $\tilde{g}_{mn} \to \lambda\, \tilde{g}_{mn}$

$$\begin{cases} e^{4A} = \alpha,\ iG_3 =*_6 G_3 \text{: invariant} \\ \text{The other equations are also unchanged.} \end{cases}$$

No-scale structure

⇒ superpotential $W_0 = \int G_3 \wedge \Omega$ is independent of Kahler

4D effective potential with $K = -3\ln(T + \bar{T})\,, W_0 = \text{const}$

$$V = e^{K/M_P^2}\left(K^{IJ} D_I W_0\, \overline{D_J W_0} - \frac{3}{M_P^2}|W|^2\right) = 0$$

Kahler directions remain flat.

A bonus in type IIB

Hierarchical structure of mass matrix/potential helps to stabilize moduli at positive cosmological constant.

[X. Chen, Shiu, YS, Tye, 12]

Moduli stabilization with positive cosmological constant

- Fluxes → Complex structure & dilaton
- Non-perturbative effect, α'-correction, localized branes

→ Kahler [KKLT, 03], [Balasubramanian, Berglund, Conlon, Quevedo, 05], [Balasubramanian, Berglund, 04]…

$$V = \underbrace{V_{\mathrm{Flux}}}_{\text{Complex}} + \underbrace{V_{\mathrm{NP}} + V_{\alpha'} + \cdots}_{\text{Kahler}}$$

No scale structure → Hierarchy between Kahler and Complex

KKLT

Non-trivial potential for Kahler is generated by NP-corrections.

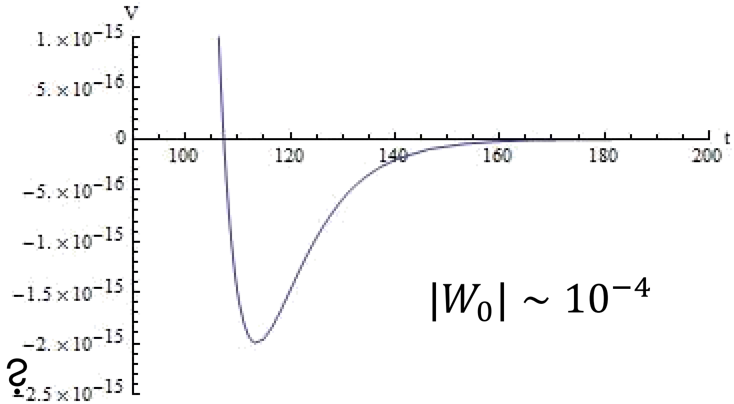
E.g. Gluino condensation on D7-branes

D7-branes wrapping the four cycle: $W_{NP} = A\, e^{-\tilde{a}\, 8\pi^2/g_{D7}} = A\, e^{-a\,T}$

Together with the superpotential from fluxes: $W = W_0 + W_{NP}$

Supersymmetric vacuum
$D_T W = 0$ existes.

But exponentially small W_0 is required.
$|W_0| \sim A\, e^{-a\,T}$, naturally realized?



$|W_0| \sim 10^{-4}$

Large Volume Scenario

[Balasubramanian, Beglund, Conlon, Quevedo, 05]

α'-corrections can break no-scale structure too.

$\mathcal{O}(\alpha'^3)$-correction in type II action [Becker, Becker, Haack, Louis, 02]

$$K = -2\ln\left(\mathcal{V} + \underline{\frac{\xi}{2}\left(-i(\tau+\bar{\tau})\right)^{3/2}}\right) - \ln(-i(\tau+\bar{\tau})) + \cdots$$

scales differently

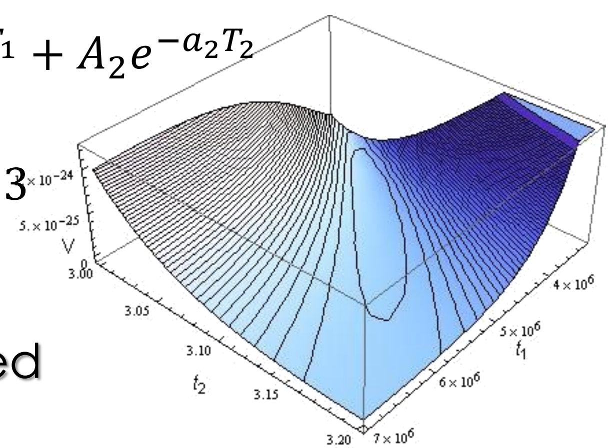
E.g. $\mathbb{P}^4_{[1,1,1,6,9]}$ model (assuming complex sector is stabilized)

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(t_1^{3/2} - t_2^{3/2}\right), \qquad W = W_0 + \cancel{A_1 e^{-a_1 T_1}} + A_2 e^{-a_2 T_2}$$

Solution: $W_0 \sim -20,\ A_1 \sim 1,\ t_1 \sim 10^6,\ t_2 \sim 3$

$V_{\min} \sim -10^{-25}$: *AdS* vacua

$|W_0| \gg |W_{NP}|,\ \mathcal{V} \gg \xi$: naturally realized



Kahler uplifting

[Balasubramanian, Berglund, 04], [Westphal, 06], [Rummel, Westphal, 11], [de Alwis, Givens, 11]

Same setup as that of LVS

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) + \cdots, \qquad \mathcal{V} = \gamma_1(T_1 + \bar{T}_1) - \sum_{i=2} \gamma_i(T_i + \bar{T}_i),$$

$$W = W_0 + \underline{A_1 e^{-a_1 T_1}} + \sum_{i=2} A_i e^{-a_i T_i}$$

Interested in a region where this term plays a roll.

less large volume than LVS, but still $|W_0| \gg |W_{NP}|$, $\mathcal{V} \gg \xi$

E.g. single modulus [Rummel, Westphal, 11]

$$V \sim -\frac{W_0 a_1^3 A_1}{2\,\gamma_1^2}\left(\frac{2C}{9 x_1^{9/2}} - \frac{e^{-x_1}}{x_1^2}\right), \qquad C = \frac{-27\, W_0\, \xi\, a_1^{3/2}}{64\sqrt{2}\gamma_1 A_1}, x_1 = a_1 t_1$$

When $W_0 A_1 < 0$, the $C \propto \xi$ term contributes the uplifting.

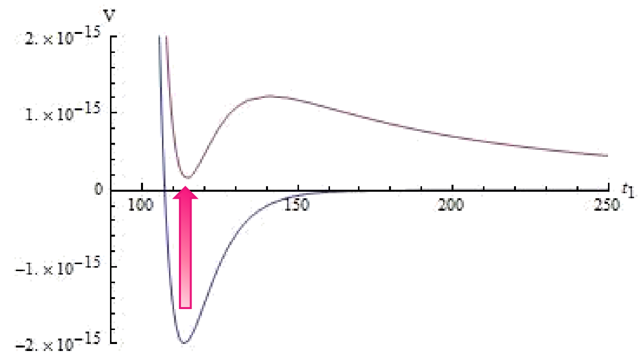
KKLT vs Kahler uplifting

- KKLT

  Add an uplifting potential by hand

$$V = V_{SUGRA} + V_{D3-\overline{D3}}$$

$$V_{D3-\overline{D3}} = 2T_3 \int d^4x \sqrt{-g_4}$$



Backreaction of $\overline{D3}$? ⇒ A singularity exists, but finite action

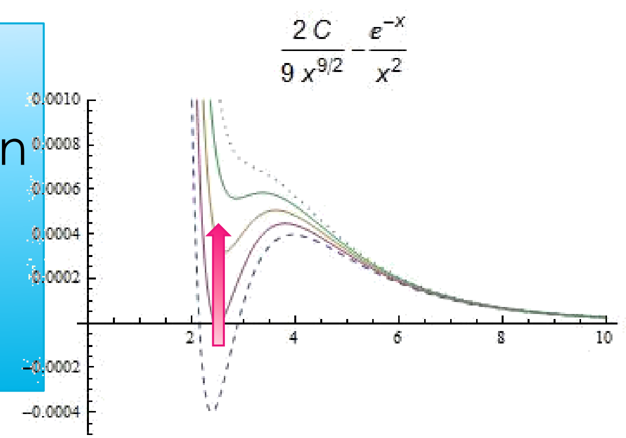
Safe or not? [DeWolfe, Kachru, Mulligan, 08], [McGuirk, Shiu, YS, 09], [Bena, Giecold, Grana, Halmagyi, Massai, 09-12], [Dymarsky, 11],…

- Kahler uplifting

  $V = V_{SUGRA}$ SUGRA + α'-correction

  Owing to $|W_0| \gg |W_{NP}|$

  ⇒ No fine-tuning for W_0



Statistical approach

Further approximation

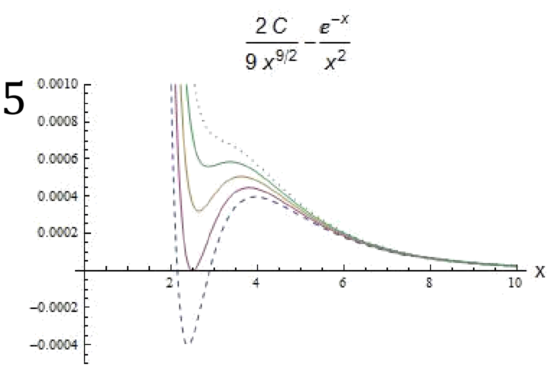
$$\frac{V}{M_P^4} = -\frac{W_0 a_1^3 A_1}{2\,\gamma_1}\left(\frac{C}{9{x_1}^{9/2}} - \frac{e^{-x_1}}{x_1^2}\right), \qquad \mathrm{C} = \frac{-27 W_0 \xi {a_1}^{\frac{3}{2}}}{64\sqrt{2}\gamma_1^2 A_1}, \qquad x_1 = a_1 t_1$$

[Rummel, Westphal, 11]

The stability constraint with positive CC at stationery points:

$$V \geq 0 \longrightarrow 3.65 \leq C < 3.89 \longleftarrow \partial_x^2 V > 0$$

Further focusing on smaller CC region: $C \sim 3.65$



$$\frac{V}{M_P^4} \sim \frac{1}{9}\left(\frac{2}{5}\right)^{\frac{9}{2}} \frac{-W_0 a_1^3 A_1}{\gamma_1^2}(C - 3.65)$$

Neglecting the parameters a_1, γ_1, ξ, the model is simplified to be

$$\Lambda = w_1 w_2 (c - c_0), \qquad c_0 \leq c = \frac{w_1}{w_2} < c_1 \qquad (w_1 = -W_0, w_2 = A_1, c \propto C)$$

Stringy Random Landscape

Starting with the simplified potential: [YS, Tye, 12]

$$\Lambda = w_1 w_2 (c - c_0), \qquad c_0 \leq c = \frac{w_1}{w_2} < c_1$$

Since W_0, A_1 are given model by model (various ways of stabilizing complex moduli), here we impose reasonable randomness on parameters.

➡ $w_1, w_2 \in [0, 1]$, uniform distribution (for simplicity)

Probability distribution function

$$P(\Lambda) = N_0 \int dc \int dw_1 dw_2 \; \delta(w_1 w_2 (c - c_0) - \Lambda) \, \delta\left(\frac{w_1}{w_2} - c\right)$$

N_0: normalization constant

Divergence in product distribution

When $z = w_1 w_2$,

$$P(z) = \int dw_1 dw_2\, \delta(w_1 w_2 - z) = \frac{1}{2} \ln \frac{1}{z}$$

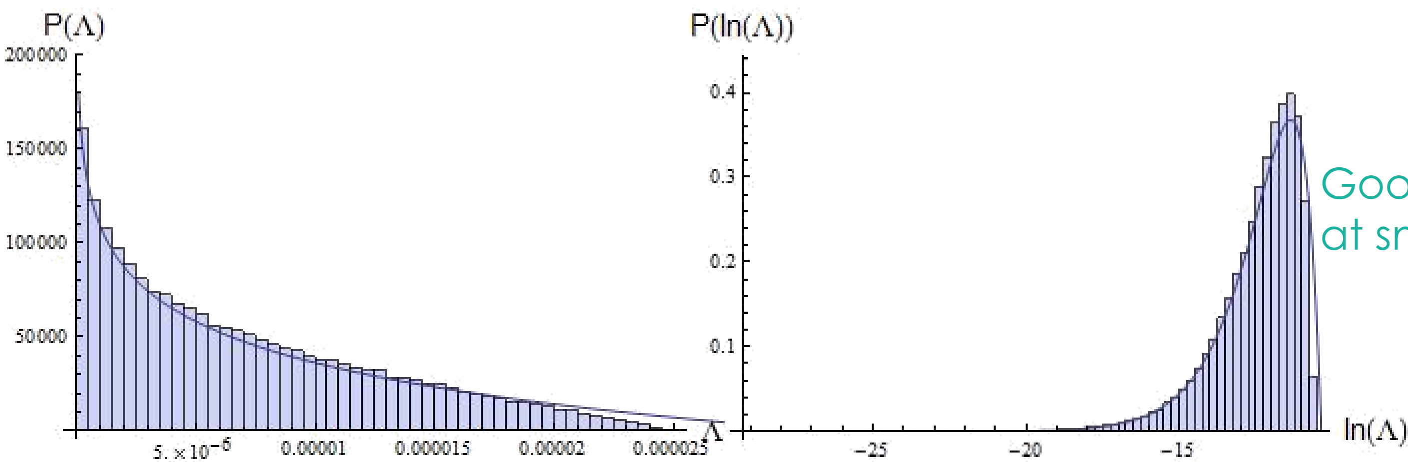
log divergence at $z = 0$

With constraint? $\Lambda = w_1 w_2 (c - c_0)$, $\underbrace{c_0 \le}_{\text{positivity}} c = \frac{w_1}{w_2} \underbrace{< c_1}_{\text{stability}}$

$$\Rightarrow \quad P(\Lambda) = \frac{c_1}{c_1 - c_0} \ln \frac{c_1 - c_0}{c_1 \Lambda}$$

still diverging!!

Comparison to the full-potential (randomizing W_0, A_1 without approx.)



Good agreement at smaller Λ

Zero-ness of parameters

We assumed the parameters W_0, A_1 passing through zero value, but is it true?

- E.g. T^6 model: $W_0 = \left(c_1 + \sum d_i U_i\right) - \left(c_2 + \sum e_i U_i\right) S$

SUSY condition

$$W_0 = 2\,(c_1 + c_2 s) \frac{\prod_k (d_k - e_k s)}{\sum_i (d_i + e_i s) \prod_{j \neq i} (d_j - e_j s)} \qquad s = \mathrm{Re}(S)$$

easy to be zero

- Brane position dependence of A_1 [Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan, 06]

$$A_1 = \hat{A}_1(U_i)\big(f(X_i)\big)^{1/n}, \qquad f(X_i) = \prod X_i{}^{p_i} - \mu^q$$

$f(X_i) = 0$ when D3-brane hits D7-brane (divisor, at μ)

known as *Ganor zero*

More on product distribution

Mellin transform

Product distribution is understood in terms of Mellin integral transformation.

For $z = x_1 x_2$, with distributions $P_1(x_1),\ P_2(x_2)$,

$$P(z) = \int\int dx_1 dx_2\, P_1(x_1)\, P_2(x_2)\ \delta(x_1 x_2 - z)$$

When multiplying z^{s-1} and integrating over z,

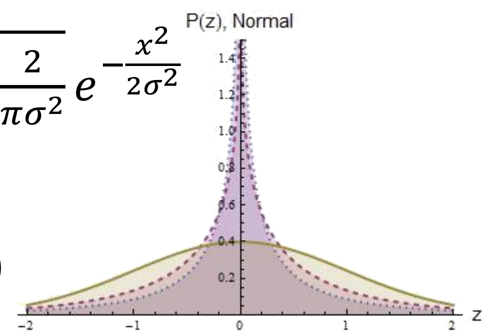
$$M\{P(z)|s\} = M\{P_1(x_1)|s\} \cdot M\{P_2(x_2)|s\},$$

$M\{f(w)|s\} \equiv \int dw\, w^{s-1} f(w)$: Mellin integral transform

E.g. Normal (Gaussian) distribution: $P(x) = \sqrt{\frac{2}{\pi\sigma^2}}\, e^{-\frac{x^2}{2\sigma^2}}$

$P(z)$ is written as Meijer G-function.

Again, log-diverging behavior toward $z = 0$



Expectation value

How we can check the likelihood of small CC?

⇨ Expectation value is a good measure to see tendency.

$$\langle z\rangle = \int dz\, z\, P(z) = M\{P(z)|2\}$$

For $z = x_1 \cdots x_n$, all uniform

$$\langle z\rangle = M\{P(z)|2\} = M\{P_1(x_1)|2\}\cdots M\{P_n(x_n)|2\} = \prod\langle x_i\rangle$$

$= e^{-n\ln 2}$: exponentially suppressed

What happens in case of $z = x_1^p$?

The factorization of Mellin transform doesn't apply.

$$P(z) = \frac{z^{-1+\frac{1}{n}}}{n} P_1\left(z^{1/n}\right) \xrightarrow{\text{uniform}} \langle z\rangle = \frac{1}{n+1}$$

More divergent, but not suppressed so much.

Independent random variables are important.

Ratio distribution

What happens when random variables show up in denominator?

- For $z = \frac{x_1 \cdots x_m}{y_1 \cdots y_n}$, all obey uniform distribution

$$P(z) = \int dx_1 \cdots dx_m dy_1 \cdots dy_n \ \delta\left(\frac{x_1 \cdots x_m}{y_1 \cdots y_n} - z\right)$$

$$\xrightarrow{z=0} \frac{(-1)^{m-1}}{(m-1)!\,(n-1)!\,2^n} (\ln z)^{m-1}$$

- For $z = \frac{x_1^m}{y_1^n}$, all uniform

$$P(z) = \begin{cases} \frac{1}{m+n} z^{-1+1/m} & \text{for } 0 \le z \le 1 \\ \frac{1}{m+n} z^{-1-1/n} & \text{for } 1 \le z \end{cases}$$

Therefore divergence is same as that in the numerator.

Sum distribution

Sum distribution smooths out the divergence and moves the peak.

E.g. $z = x_1^{n_1} + x_2^{n_2} + \cdots + x_p^{n_p}$

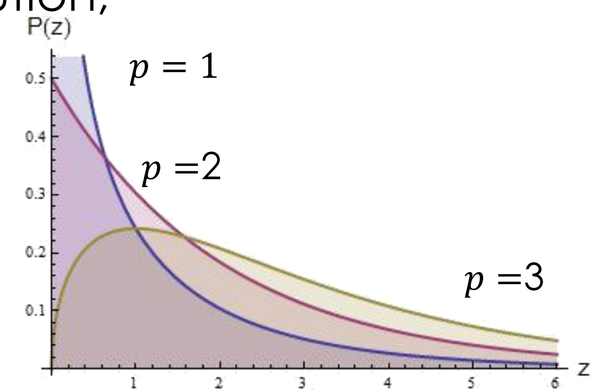
- Each has divergent peak: $P\left(w_i = x_i^{n_i}\right) \propto w_i^{-1+\frac{1}{n_i}}$
- Independent of each other, no correlations.

But uncorrelated summation gives $P(z) \propto z^{-1+\sum\frac{1}{n_i}}$.

When all $n_i = 2$, and $x_i \in$ normal distribution,

$$P(z) = \frac{e^{-p/2} z^{-1+p/2}}{2^{p/2}\Gamma(p/2)}$$

known as *Chi-squared distribution*

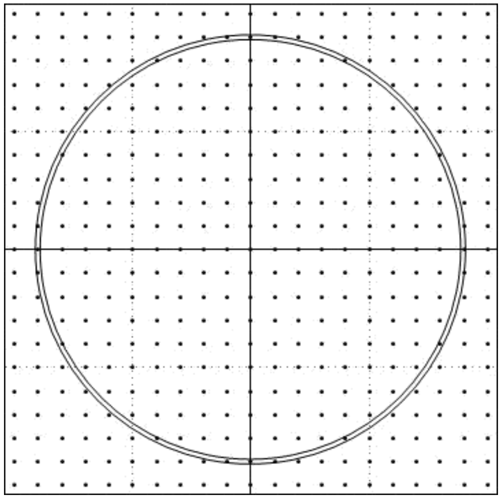


Bousso-Polchinski

4-form quantization

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{M_P^2}R - \Lambda_{\mathrm{bare}} - \frac{Z}{2\times 4!}F_4^2\right)$$

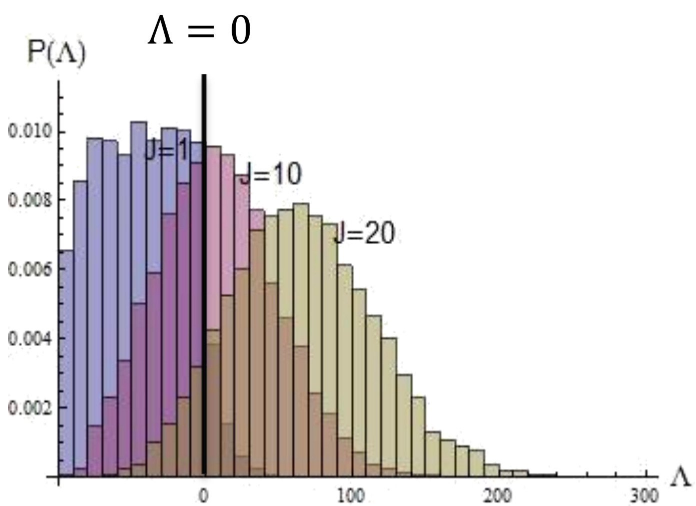
$$\Rightarrow \quad \Lambda = \Lambda_{\mathrm{bare}} + \frac{1}{2}\sum^{J} n_i^2\, q_i^2$$



Assume randomness in Bousso-Polchinski;

n_i: random integer, $0 \le q_i \le 1$: uniform,

$-100 \le \Lambda_{\mathrm{bare}} \le 0$: uniform



But… **Moduli fields couple each term**

$$\Lambda \sim -W_0 A_1\left(\frac{C}{9{x_1}^{9/2}} - \frac{e^{-x_1}}{x_1^2}\right)$$

correlation generated via stabilization

Multi-moduli analyses

Multi-moduli stabilization

[Sumitomo, Tye, in preparation]

Again, we work in the region: $|W_0| \gg |W_{NP}|$, $\mathcal{V} \gg \xi$.

Assuming stabilization of complex structure moduli and dilaton at higher energy scale,

$$\frac{V}{M_P^4} = -\frac{A_1 W_0 a_1^3}{2\,\gamma_1}\left(\frac{2C}{9\tilde{\mathcal{V}}^3} - \frac{x_1 e^{-x_1}}{\tilde{\mathcal{V}}^2} - \sum_{i=2} \frac{B_i x_i e^{-x_i}}{\tilde{\mathcal{V}}^2}\right),$$

$$\tilde{\mathcal{V}} = x_1^{3/2} - \sum_{i=2} \delta_i x_i^{3/2}, \quad x_i = a_i t_i,\ C = \frac{-27 W_0 \xi a_1^{3/2}}{64\sqrt{2}\gamma_1 A_1},\ B_i = \frac{A_i}{A_1},\ \delta_i = \frac{\gamma_i a_i^{3/2}}{\gamma_1 a_1^{3/2}}$$

- Now we have $N_K \times N_K$ mass matrix.

  All upper-left sub-determinants are positive (*Sylvester's criteria*).

  ⇨ N_K extremal equations + N_K stability constraints

- Stability at positive CC requires $B_i > 0$.

  ⇨ Uplifting is controlled by the first term.

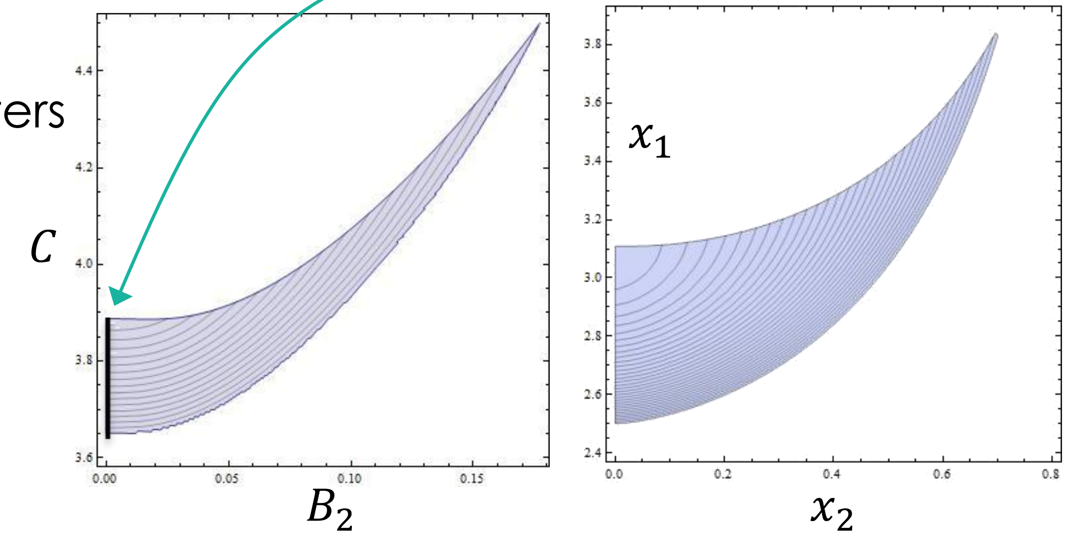
Two moduli

Stability constraint at $N_K = 1$

We can find stability points when parameters are in the region.

$$3.65 \lesssim C \lesssim 4.50$$

$$0 \leq B_2 \lesssim 0.177$$



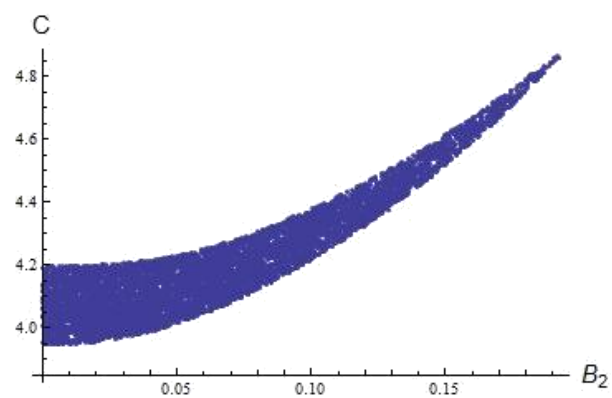
Let's compare with full-potential analysis.

(therefore without $|W_0| \gg |W_{NP}|$, $\mathcal{V} \gg \xi$)

The parameter region is shifted slightly:

$$3.95 \lesssim C \lesssim 4.87, \qquad 0 \leq B_2 \lesssim 0.193$$

But not changed so much.



Three moduli

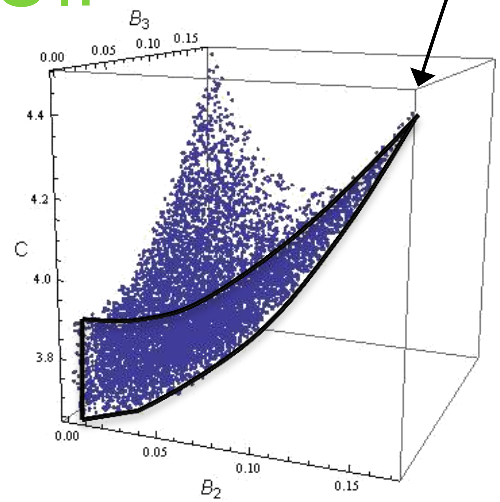
Stable points exist if

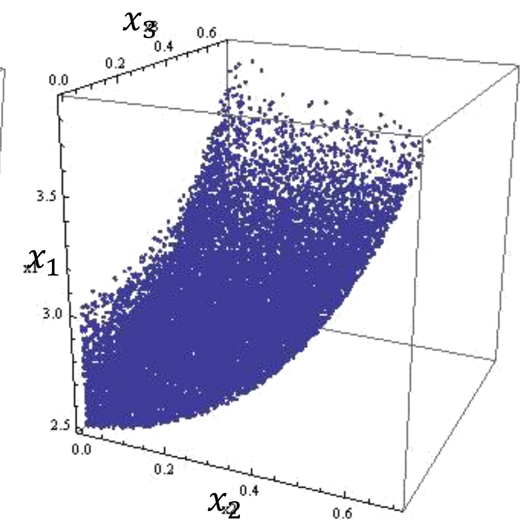
$$3.65 \lesssim C \lesssim 4.50$$

$$0 \leq B_{2,3} \lesssim 0.177$$

But parameter region is further constrained.

Stability constraint at $N_K = 2$

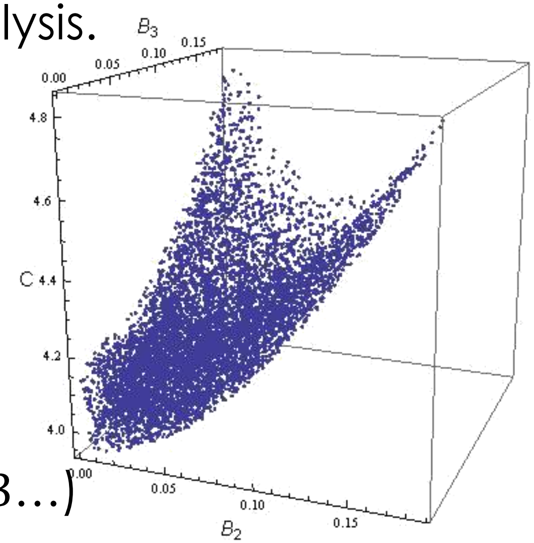




Again, let's compare with full-potential analysis.

(without $|W_0| \gg |W_{NP}|$, $\mathcal{V} \gg \xi$)

$$3.95 \lesssim C \lesssim 4.87, \qquad 0 \leq B_{2,3} \lesssim 0.193$$



Though the resultant parameters are slightly shifted, the essential feature wouldn't be changed.

(not easy to use full-potential beyond $N_K = 3$…)

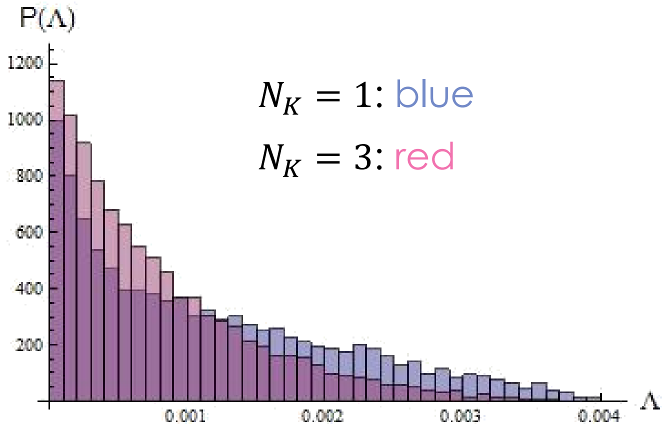
Multi-Kahler statistics

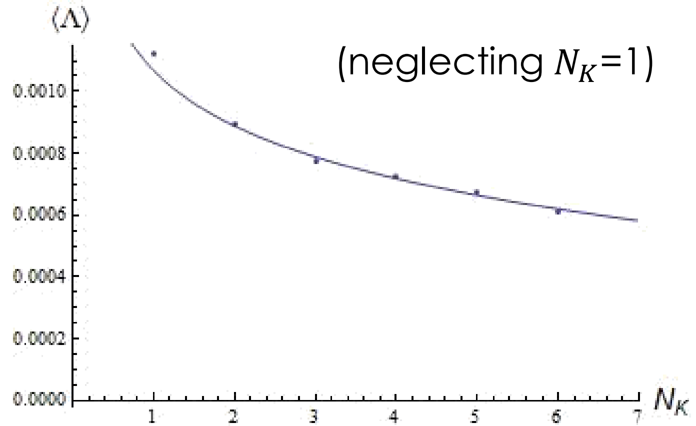
Still complicated system

$$\frac{V}{M_P^4} = -\frac{A_1 W_0 a_1^3}{2\,\gamma_1}\left(\frac{2C}{9\tilde{\mathcal{V}}^3} - \frac{x_1 e^{-x_1}}{\tilde{\mathcal{V}}^2} - \sum_{i=2} \frac{B_i x_i e^{-x_i}}{\tilde{\mathcal{V}}^2}\right)$$

- We just randomize W_0, A_i obeying uniform distribution, while keeping other parameters fixed.
- Solve for t_i (or x_i)

$$-15 \le W_0 \le 0,\; 0 \le A_i \le 1$$





More moduli bring shaper peak.
(though mild suppression)

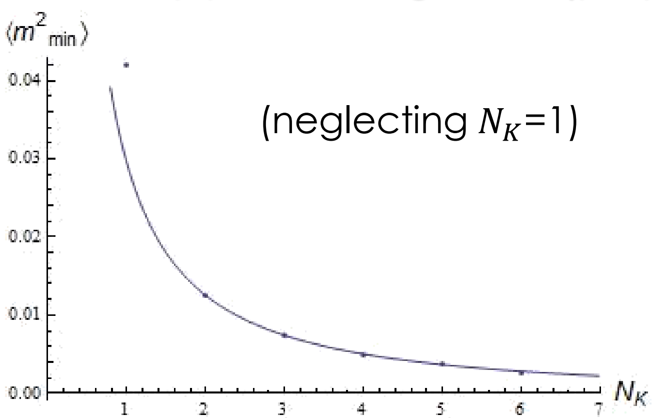
$$\langle\Lambda\rangle \sim 1.1 \times 10^{-3} N_K^{0.23} e^{-0.027\, N_K} M_P^4$$

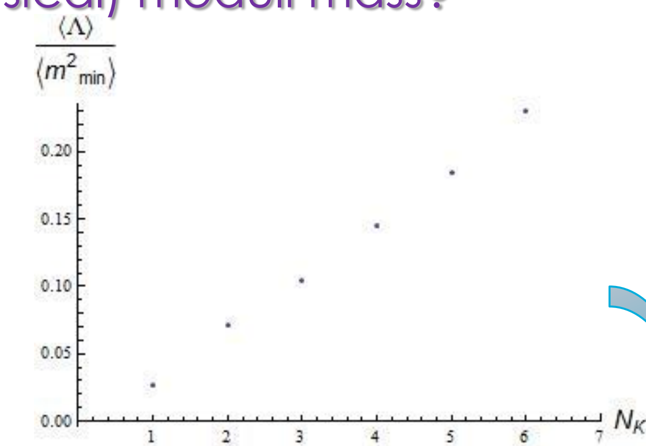
Cosmological moduli problem

Reheating for BBN: $T_r \geq \mathcal{O}(10)$ MeV $\qquad T_r \sim \sqrt{M_P \Gamma_\phi},\ \Gamma_\phi \sim \frac{m_\phi^3}{M_P}$

$\Rightarrow$ $m_\phi \geq \mathcal{O}(10)$ TeV $\sim 10^{-15}\, M_P$

What happens in lightest (physical) moduli mass?





$\langle m^2_{\rm min} \rangle = 0.031\, N_K^{1.0} e^{-0.10\, N_K} M_P^2$: also suppressed

Suppression of mass is relatively faster than Λ.

$\Rightarrow$ $\langle m^2_{\rm min} \rangle \sim 10^{-30} M_P^2$ is likely met earlier than $\langle \Lambda \rangle \sim 10^{-122} M_P^4$

More peaked parameters

So far we assumed uniform distribution for W_0, A_i. But realistic models have a number of complex moduli and others.

⇒ Different distributions for W_0, A_i

Consider the effect of multiple independent parameters.

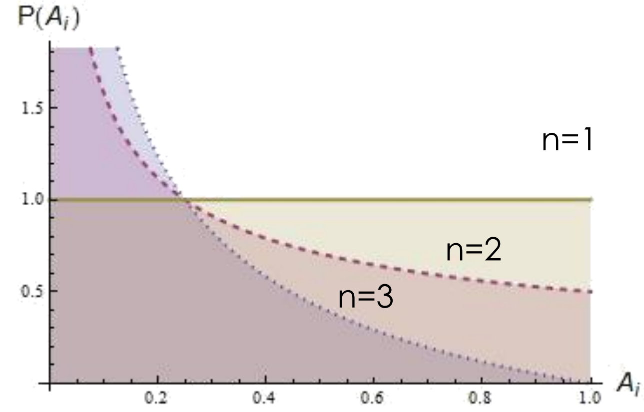
$$W_0 = -w_1 w_2 \cdots w_n, \qquad A_i = y_1^{(i)} y_2^{(i)} \cdots y_n^{(i)}$$

$0 \le w_i \le 15^{\frac{1}{n}},\ 0 \le y_j^{(i)} \le 1$, all obey uniform distribution.

Now,

$$P(W_0) = \frac{1}{15(n-1)!}\left(\ln \frac{15}{|W_0|}\right)^{n-1},$$

$$P(A_i) = \frac{1}{(n-1)!}\left(\ln \frac{1}{A_i}\right)^{n-1}$$



See how CC is affected by "n"

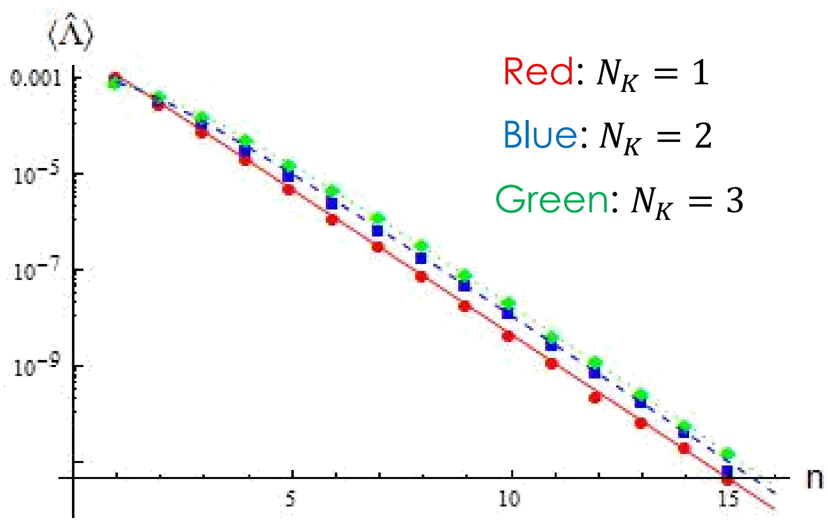
Cosmological constant

We cannot simply consider effect of the coefficient.

$$\frac{V}{M_P^4} = -\frac{A_1 W_0 a_1^3}{2\,\gamma_1}\left(\underline{\frac{2C}{9\tilde{\mathcal{V}}^3} - \frac{x_1 e^{-x_1}}{\tilde{\mathcal{V}}^2} - \sum_{i=2} \frac{B_i x_i e^{-x_i}}{\tilde{\mathcal{V}}^2}}\right)$$

Dynamics also affects.

The result:



Red: $N_K = 1$

Blue: $N_K = 2$

Green: $N_K = 3$

$$\langle\Lambda\rangle_{N_K=1} = 4.7 \times 10^{-3}\, n^{0.080} e^{-1.40\, n}$$

$$\langle\Lambda\rangle_{N_K=2} = 3.7 \times 10^{-3}\, n^{0.97} e^{-1.49\, n}$$

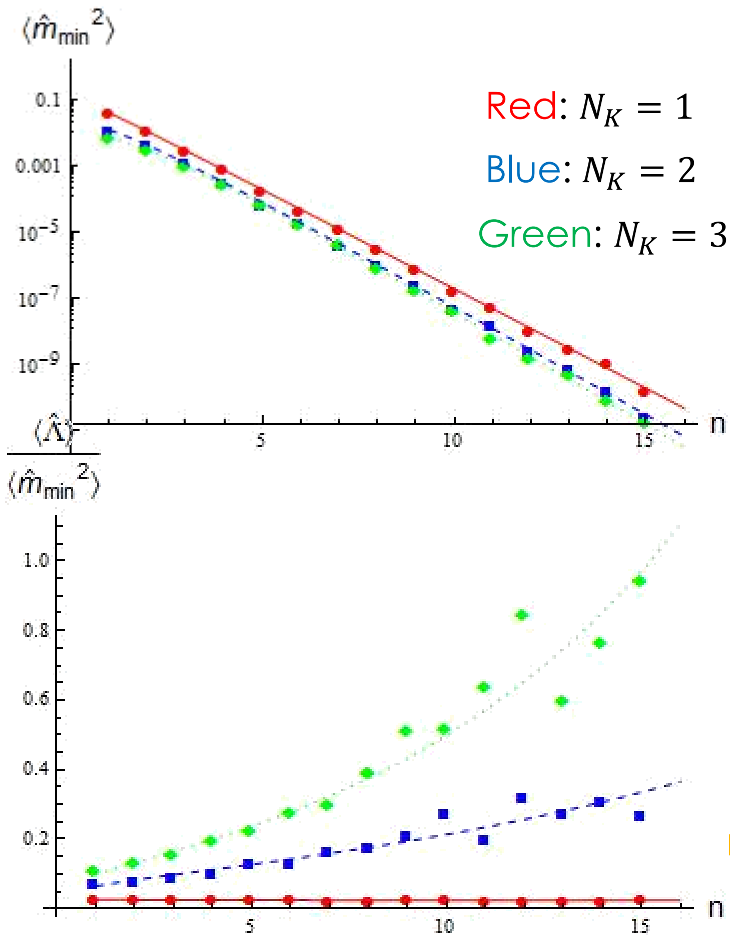
$$\langle\Lambda\rangle_{N_K=3} = 3.4 \times 10^{-3}\, n^{1.5} e^{-1.55\, n}$$

More than the effect of the coefficient!

$$\langle A_1 W_0 \rangle \sim 15\, e^{-1.39\, n}$$

Moduli mass

We worry about the cosmological moduli problem.



$$\langle \mathrm{m}_{\min}^2 \rangle_{N_K=1} = 0.18\, n^{0.14} e^{-1.40\, n}$$

$$\langle \mathrm{m}_{\min}^2 \rangle_{N_K=2} = 0.061\, n^{0.73} e^{-1.56\, n}$$

$$\langle \mathrm{m}_{\min}^2 \rangle_{N_K=3} = 0.039\, n^{1.2} e^{-1.66\, n}$$

Compare with CC

$$\langle \Lambda \rangle \propto e^{-1.40\, n},\ e^{-1.49\, n},\ e^{-1.55\, n}$$

Suppression in mass is getting larger as increasing N_K.

also suggests

Estimation

Using the estimated functions, we get

$N_K(=h^{1,1})$	1	2	3
$\langle\Lambda\rangle \sim 10^{-122}M_P^4$	$n \sim 197$	$n \sim 188$	$n \sim 182$
$\langle m^2\rangle \sim 10^{-30}M_P^2$	$n \sim 48$	$n \sim 44$	$n \sim 42$

n: number of product in W_0, A_i

Rather considerable number, e.g.

- $\mathbb{P}^4_{[1,1,1,6,9]}$: $h^{1,1} = 2,\ h^{2,1} = 272$
- $\mathcal{F}_{11}$: $h^{1,1} = 3,\ h^{2,1} = 111$

and the other moduli (e.g. brane position, open string) come in a complicated way, like

- $A_1 = \hat{A}_1(U_i)\big(f(X_i)\big)^{1/n},\ f(X_i) = \prod {X_i}^{p_i} - \mu^q$

While, without help of product distribution in W_0, A_i

$N_K \sim 10100$ for $\langle\Lambda\rangle \sim 10^{-122}M_P^4$, $N_K \sim 1350$ for $\langle m^2\rangle \sim 10^{-30}M_P^2$

Distribution in W_0

Consider the simplest model:

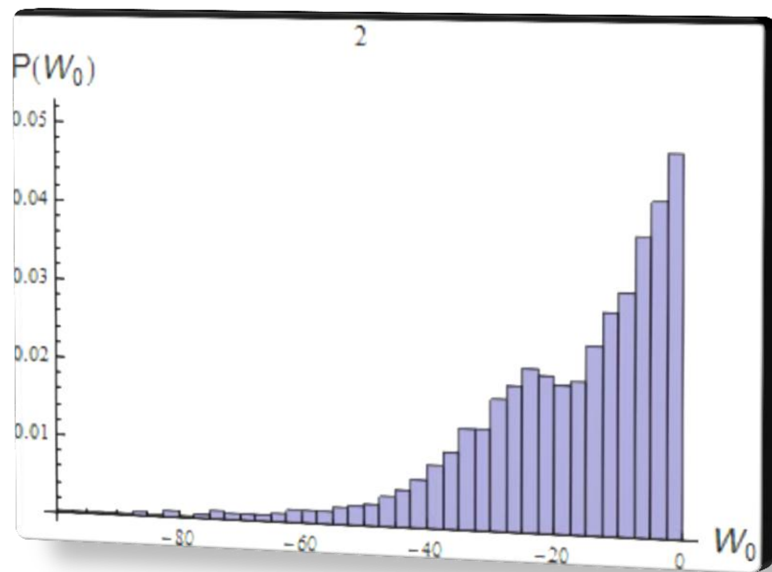
$$W_0 = -\left(c_1 + \sum_{i=1}^{h^{2,1}} e_i\, U_i\right) - \left(c_2 + \sum_{i=1}^{h^{2,1}} f_i\, U_i\right) S$$

SUSY stabilization

$$D_{U_i} W_0 = 0, \qquad D_S W_0 = 0$$

with Re $U_i > 0$, Re $S = g_s^{-1} > 1$

When N_C ↑, the dist. gets more sharply peaked!



Mass matrix

Physical mass matrix is a linear combination of $\partial_{x_i}\partial_{x_j}V|_{\min}$.

Assuming uniformly distributed $-15 \leq W_0 \leq 0,\ 0 \leq A_i \leq 1$,

$$\left\langle \left| \partial_{x_i}\partial_{x_j}V \right|_{\min} \right| \right\rangle \sim 10^{-3} \times \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & \cdots & & x_{N_K} \end{array} \\ \begin{pmatrix} 7 & 4 & \cdots & \cdots & 4 \\ 4 & 60 & 1 & \cdots & 1 \\ \vdots & 1 & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 4 & 1 & \cdots & 1 & 60 \end{pmatrix} \end{array}$$

some hierarchical structures

Though off-diagonal comp. are relatively suppressed, eigenvalue repulsion gets more serious when increasing N_K.

e.g. 2×2 matrix: $\begin{pmatrix} a & b \\ b & c \end{pmatrix} \Longrightarrow \lambda_\pm = \frac{1}{2}\left(a + c \pm \sqrt{(a-c)^2 + \underline{4b^2}}\right)$

The lowest mass eigenvalue is generically suppressed more than CC.

Summary & Discussion

Summary & Discussion

- Stringy Random Landscape

We may expect that stringy motivated models have the following properties:

- Product of parameters
- Correlation of each term by dynamics

⇒ Both works for smaller CC.

- A number of Kahler moduli

Correlation makes CC smaller. But the effect is modest.

- A number of complex moduli and other moduli

Those are likely to produce more peakiness in parameters

⇒ Interesting to see detailed effect in concrete models

Summary & Discussion

- A potential problem

Lightest moduli mass is suppressed simultaneously.

→ cosmological moduli problem
before reaching $\Lambda \sim 10^{-122} M_P^4$.

Other than "product" and "correlation" effect,
"eigenvalue repulsion" also makes the value smaller.

This is presumably a generic problem
when taking statistical approach without fine-tuning.

→ Once finding a way out, the stringy mechanism naturally explain why CC is so small.

Thermal inflation, coupling suppression to SM,
or some other corrections may help?