Probing The Size of Extra Dimension from GW Astronomy

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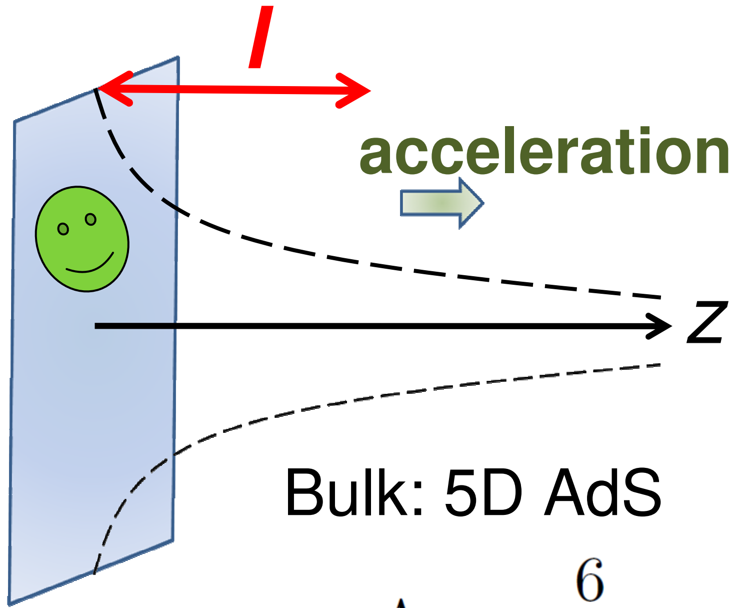
§ 1 INTRODUCTION

1-1 **RS II Braneworld Scenario** Randall & Sundrum (1999)

$$ds^2 = dz^2 + e^{-2|z|/l}(-dt^2 + d\boldsymbol{x}^2)$$

- String theory motivated
- Flat 4D brane with 5D **AdS** bulk
- **Infinite warped extra dimension** with **a characteristic scale *l*** (AdS curvature scale)
- Reproduces **4D G.R.** on the brane



$$V(r) = -G_4 \frac{Mm}{r}\left(1 + \frac{2l^2}{3r^2}\right)$$

$$\Lambda = -\frac{6}{l^2}$$

- **Table top experiment** (Adelberger *et al.* 2007):

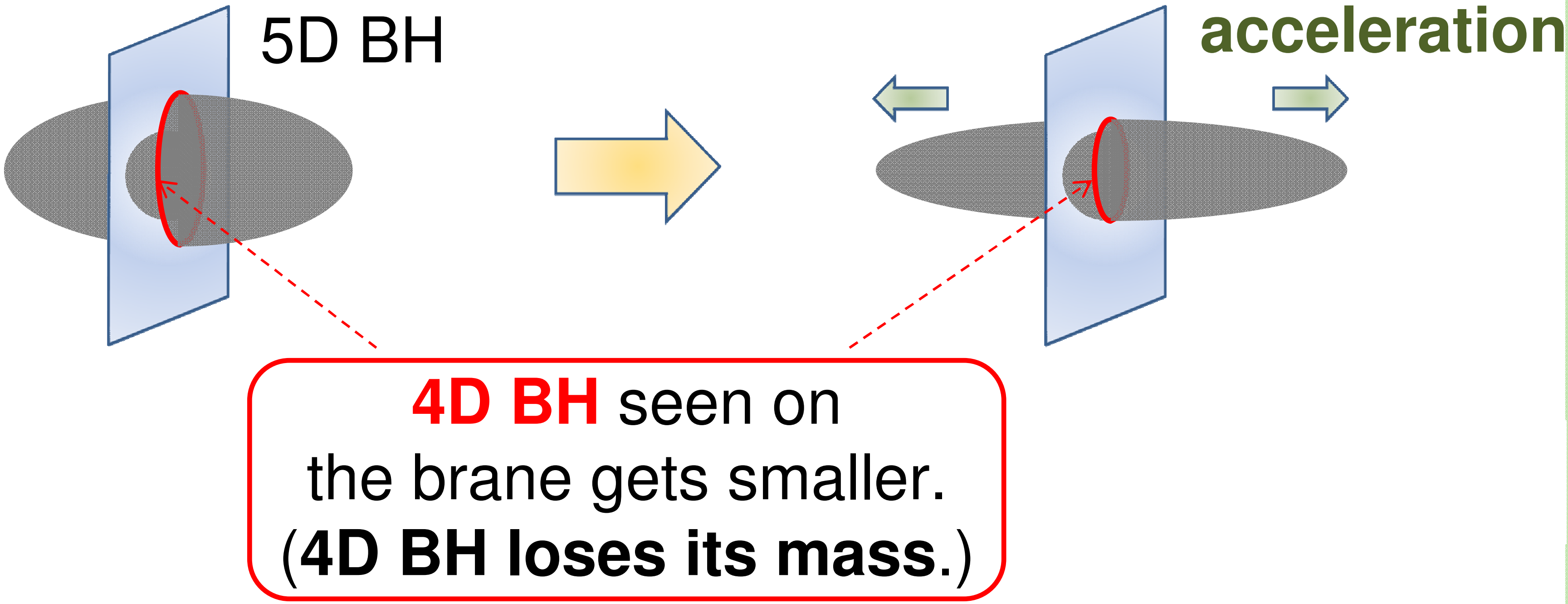
$\Rightarrow$ ***l* < 14 μm**

1-2 Brane-localised BH

- No brane-localised BH has been found.

- Conjecture:
  The brane-localised BH larger than / cannot be static.

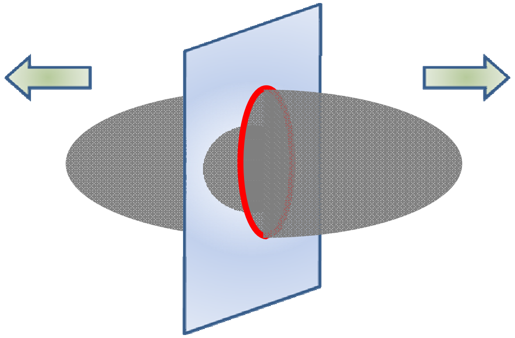
(Emparan *et al*. 2003, Tanaka 2003)



5D

5D deforming BH

(**Classical** mass loss effect on the brane)



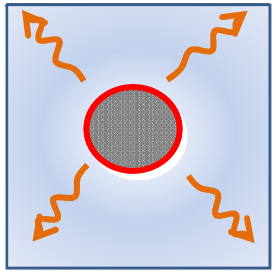
AdS/CFT

Hawking *et al.* (2000)

4D

4D CFT Hawking radiation

(**Quantum** mass loss effect on the brane)



The effect of Hawking radiation is enhanced by

$$\left(\frac{\ell}{l_{\rm pl}}\right)^2 \approx 10^{60}\left(\frac{\ell}{10\mu{\rm m}}\right)^2$$

(4D Planck scale: $l_{\rm pl}=1.6\times 10^{-33}$ cm)

Evaporation time:

$$t_{\rm evap} \approx 10^{68}\left(\frac{M}{5M_\odot}\right)^3 {\rm yr} \text{ (G.R.)} \Rightarrow t_{\rm evap} \approx 10^{8}\left(\frac{M}{5M_\odot}\right)^3\left(\frac{10\mu{\rm m}}{\ell}\right)^2 {\rm yr}$$

Many BHs evaporate within the age of the Universe!

◎ Tiny *l* can be constrained from **astrophysical observations** !!

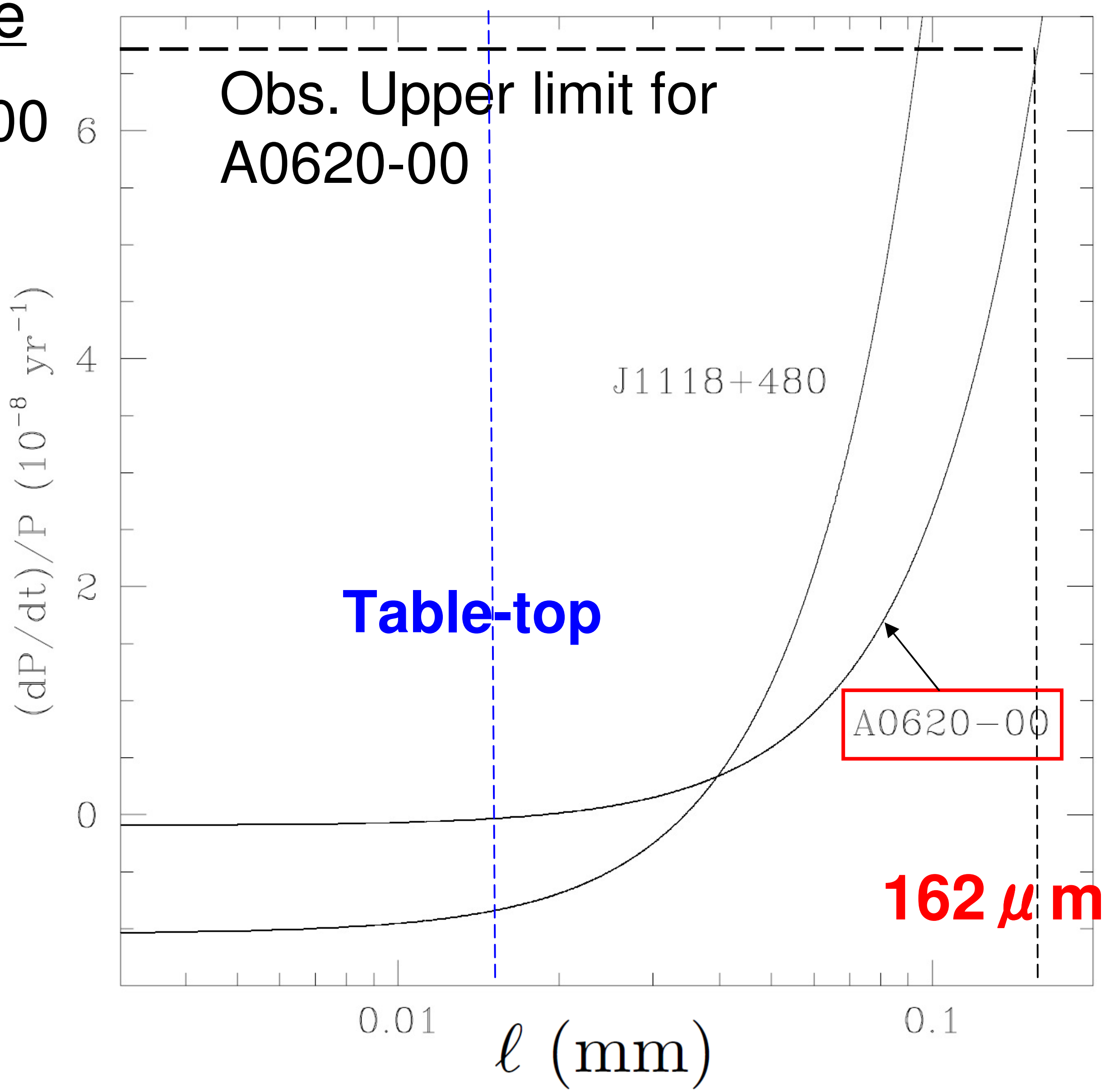
§ 1-3 Astrophysical Constraints

(I) Orbital decay rate

X-ray binary: A0620-00

Johannsen (2009)

$l \leqq 162\,\mu\mathrm{m}$



(II) BH mass & age

$$t_{evap} \approx 10^8 \left(\frac{M}{5M_\odot}\right)^3 \left(\frac{10\mu m}{\ell}\right)^2 \text{yr}$$

- XTE J1118+480

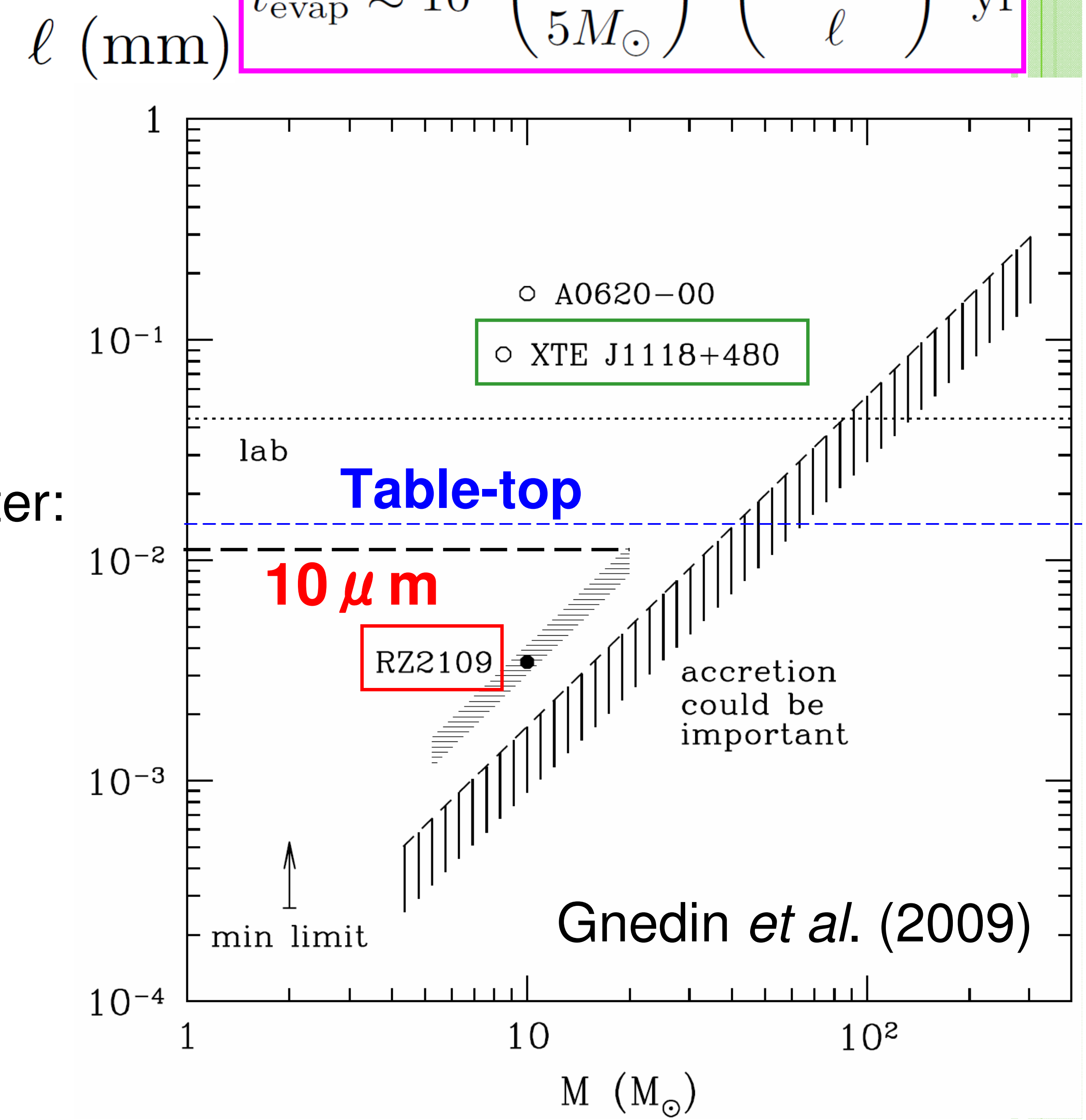
  Psaltis (2007)

  $l \leqq 80\,\mu$m

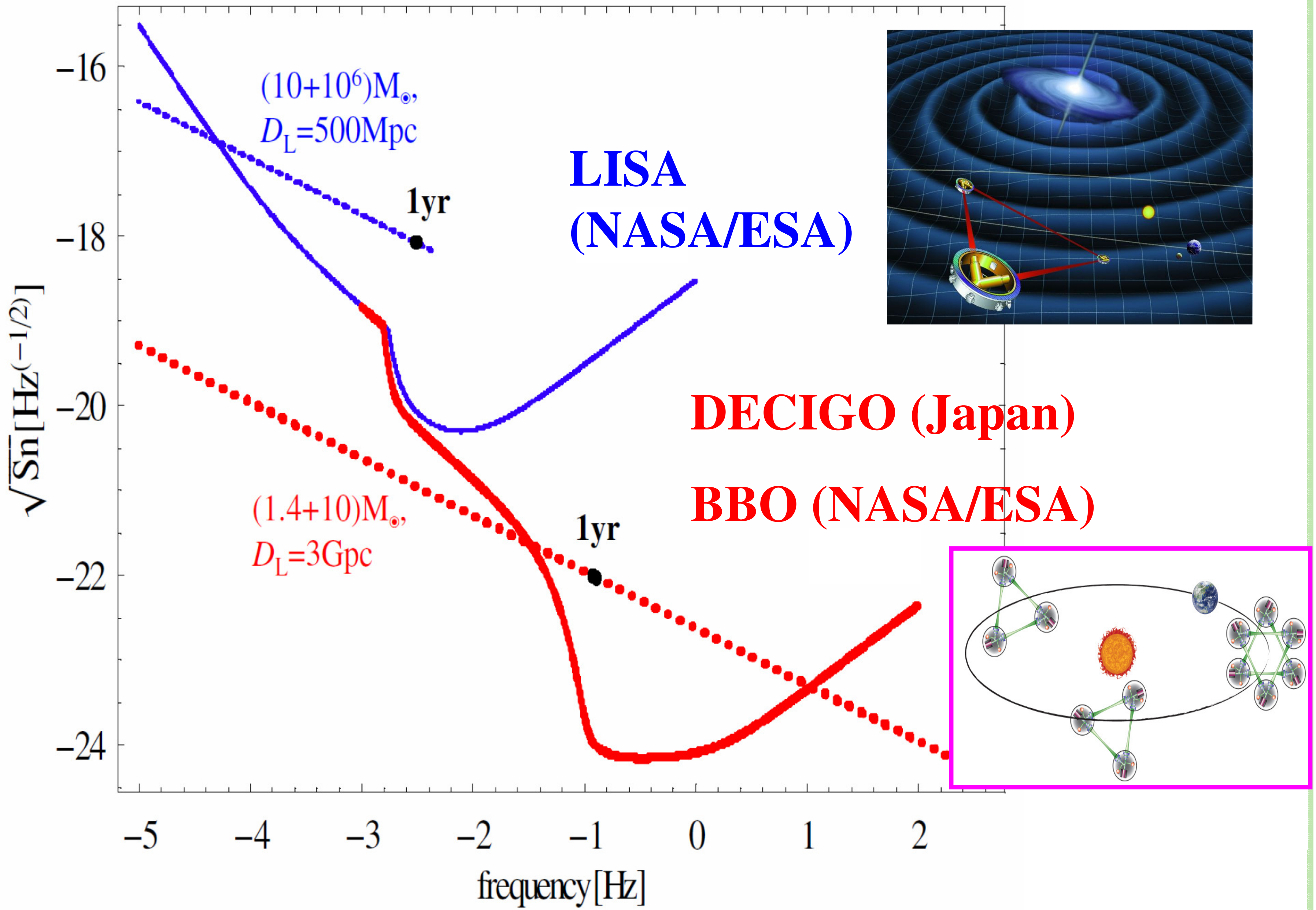
- BH in Globular Cluster: RZ2109

  Gnedin *et al.* (2009)

  $l \leqq 10\,\mu$m

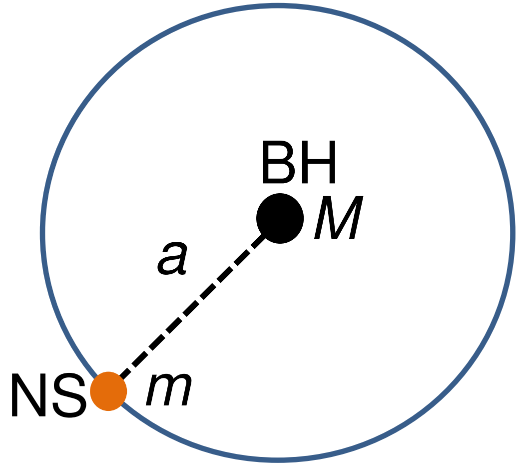


§ 1-4 Space-borne GW Interferometers



§ 2 Constraining ℓ by GW observations with LISA

McWilliams (2010)



・Galactic BH/NS binary (spinless) with a quasi-circular orbit

(I) **mass loss effect**:

$$\dot{a}_{\rm H} = -\frac{a\frac{d}{dt}(M+m)}{M+m} = -\frac{\dot{M}}{M+m}a > 0$$

$$\left[\dot{M} = -A\frac{\ell^2}{M^2} = -2.2\times 10^{-7}\left(\frac{5M_\odot}{M}\right)^2\left(\frac{\ell}{44\mu{\rm m}}\right)^2 M_\odot\ {\rm yr}^{-1}\right]$$

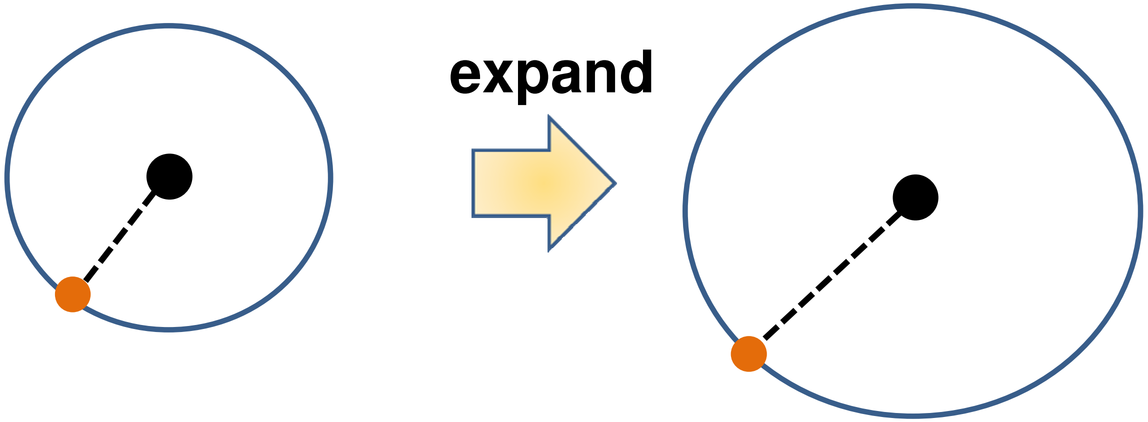
(II) **GW radiation**:

$$\dot{a}_{\rm gw} = -\frac{64G^3Mm(M+m)}{5c^5a^3} < 0$$

$$\left|\frac{\dot{a}_{\rm H}}{\dot{a}_{\rm GW}}\right| \propto a^4$$

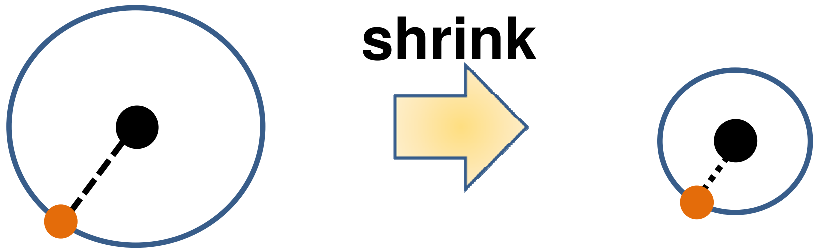
- large $a \Rightarrow \dot{a}_{\mathrm{H}} > \dot{a}_{\mathrm{GW}}$

Hawking effect dominant



- small $a \Rightarrow \dot{a}_{\mathrm{H}} < \dot{a}_{\mathrm{GW}}$

GW effect dominant

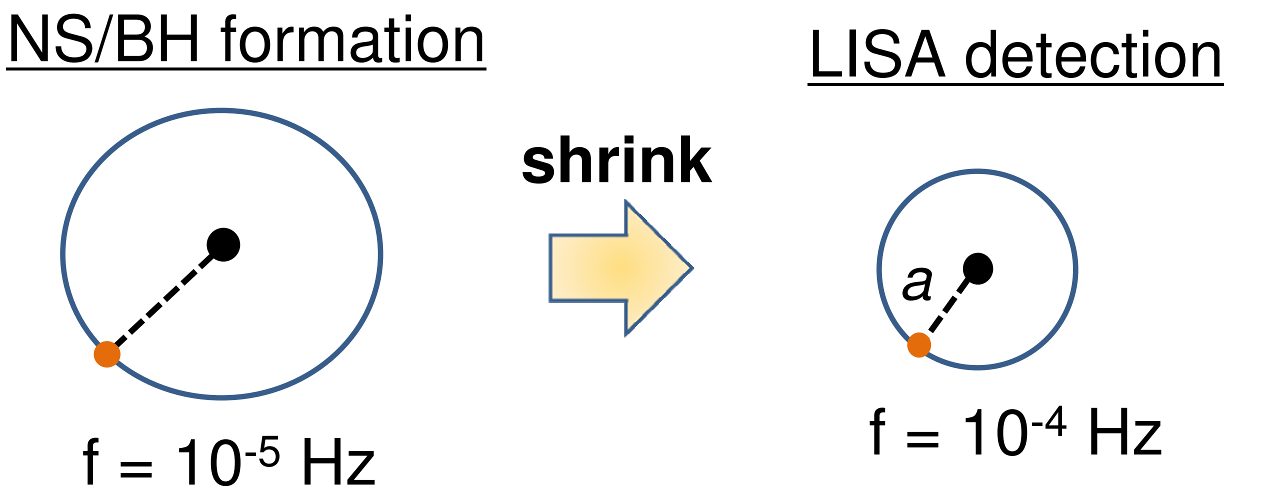


- **The critical separation a_{crit}**: $\dot{a}_{\mathrm{H}} = \dot{a}_{\mathrm{GW}}$

$$a_{\mathrm{crit}} = 6.3 \times 10^{-3} \left(\frac{M}{5M_{\odot}}\right)^{3/4} \left(\frac{m}{2M_{\odot}}\right)^{1/4} \sqrt{\left(\frac{M+m}{7M_{\odot}}\right)\left(\frac{44\mu\mathrm{m}}{\ell}\right)}\ \mathrm{AU}$$

・<u>Assumption</u>: LISA detects a **monochromatic** binary signal from **a (2+5)$M_\odot$ NS/BH binary @ f=10^{-4}Hz**.

・Typical frequency @ binary formation: f=10^{-5}Hz



$$a < a_{\rm crit} \Rightarrow \ell \leq 22 \left(\frac{M}{5M_\odot}\right)^{\frac{3}{2}} \left(\frac{m}{2M_\odot}\right)^{\frac{1}{2}} \left(\frac{M+m}{7M_\odot}\right)^{\frac{1}{3}} \left(\frac{f}{10^{-4}{\rm Hz}}\right)^{\frac{4}{3}} \mu{\rm m}$$

Smaller mass, **lower *f*** puts stronger constraint.

-<u>Problem:</u>

・Some binaries may form with **f > 10^{-4}Hz**. (Belczynski *et al*. 2002)

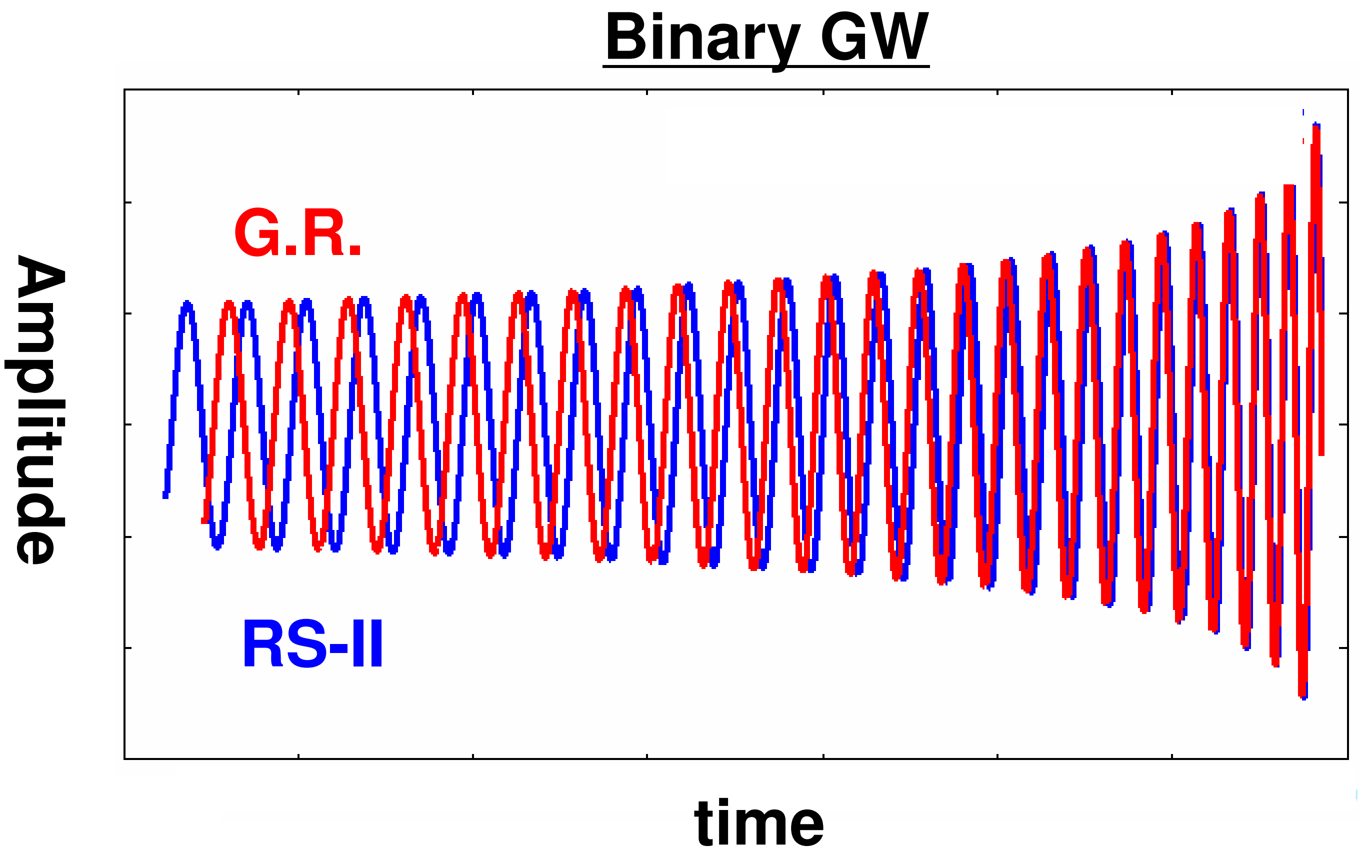
⇒ There remains some uncertainty for claiming $a < a_{\rm crit}$.

⇒ We need to detect **chirp signals**!!

§ 3 Constraining l with DECIGO/BBO

3-1 Modified GW waveform

・We derive **a leading correction** to the gravitational waveform **phase** due to the mass loss effect.



Waveform (Fourier component) $\tilde{h}(f) \propto \exp(i\Psi(f))$

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi\mathcal{M}_0 f)^{-5/3}$$
$$\left(1 - AC_L L x_0^{-4} + (\text{higher PN terms})\right)$$

-4PN correction

$$C_L = \frac{25}{11968} \frac{(3 - 26\eta_0 + 34\eta_0^2) + (-3 + 20\eta_0)\sqrt{1 - 4\eta_0}}{2\eta_0^4}$$

$$\eta \equiv \frac{mM}{(m+M)^2} \qquad \dot{M} = -A\frac{\ell^2}{M^2}$$

$$\mathcal{M} \equiv (m+M)\eta^{3/5} \qquad x \equiv v^2 = (\pi(m+M)f)^{2/3}$$

- Subscript 0 represents the value at the time of coalescence

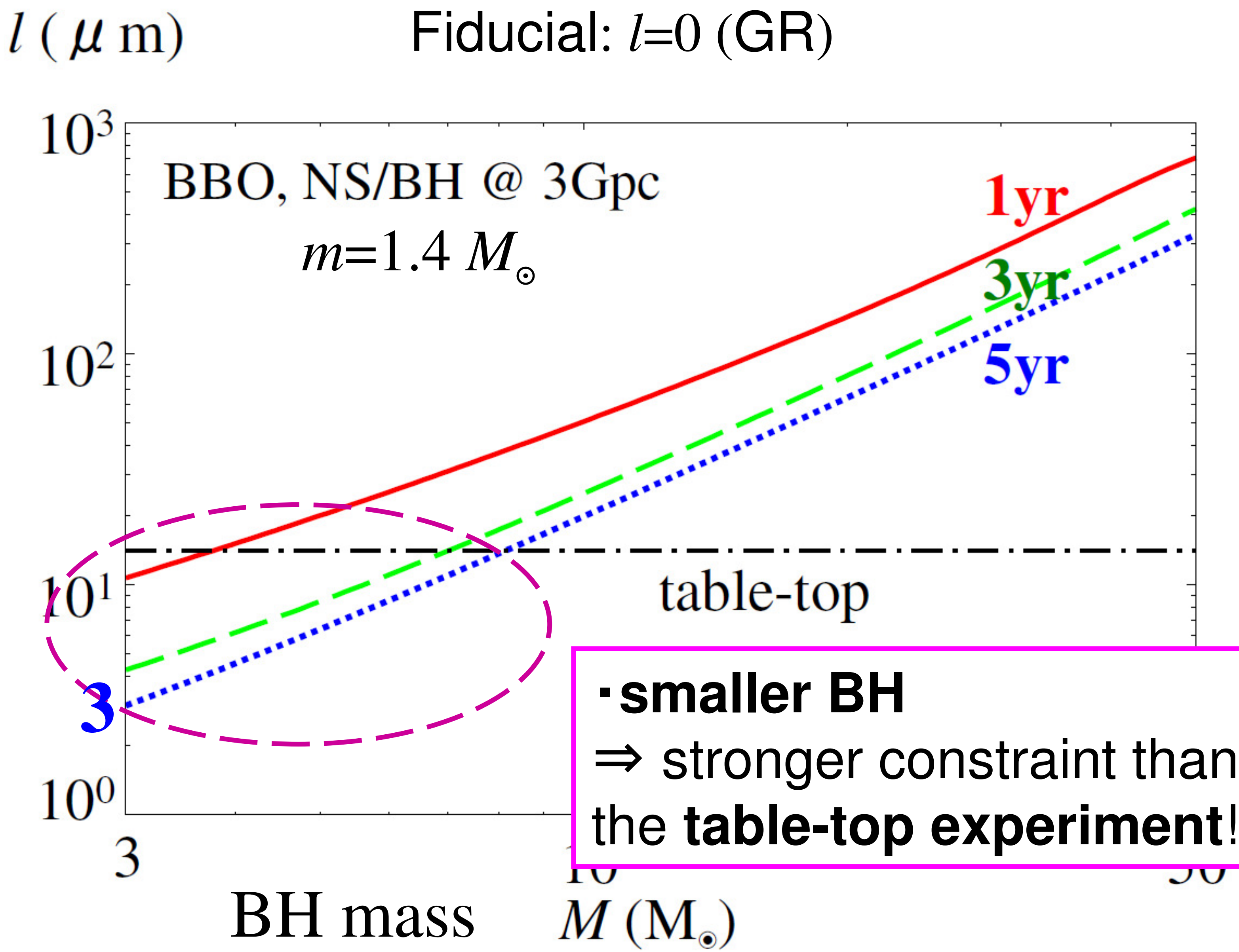
$$L \equiv \frac{\ell^2}{(m+M)^2}$$

- Binary parameters: $\theta^a = (\mathcal{M}, \eta, t_c, \phi_c, D_L, L)$

§ 4 Fisher Analysis Results

4-1 Constraints from a single binary signal

Fiducial: l=0 (GR)



- **smaller BH**

⇒ stronger constraint than the **table-top experiment**!

4-2 Measuring *l* from 10^5 binary signals (statistical analysis)

- **Detection rate** of BH/NS for DECIGO/BBO: 10^5 /yr (GR)

  ≦10^5 /yr (RS-II)

Detection rate (GR) : $$\int_{M_{0,\min}}^{M_{0,\max}} \int_{0}^{z_{\max}} n_{\ell=0}(z, M_0) dz dM_0 = 10^5 \mathrm{yr}^{-1}$$

- Δ *l* can be **improved** by employing this **statistical advantage**

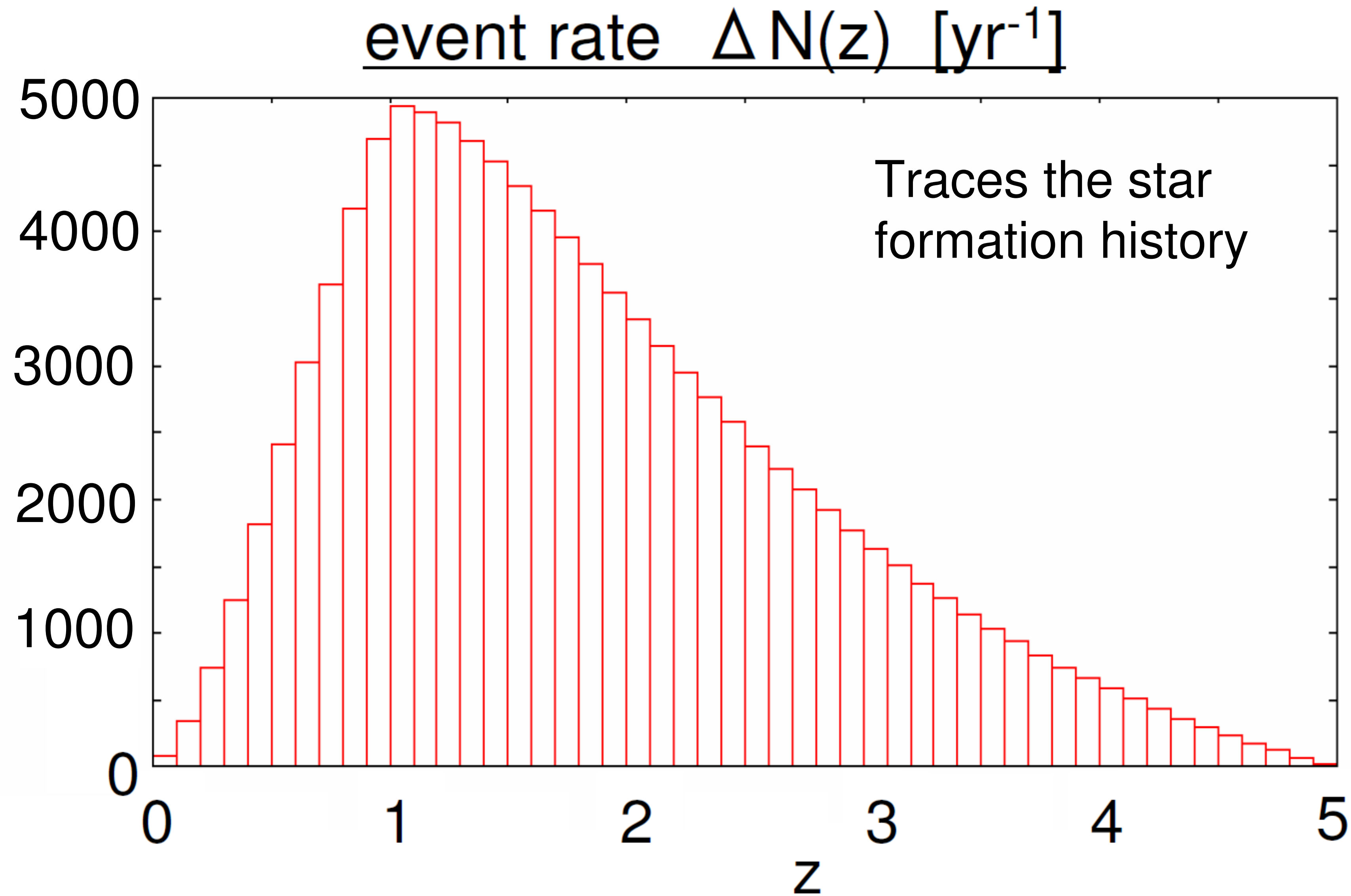
σ_L^2 : total variance of *L*

$$\sigma_L^{-2} = T_{\mathrm{obs}} \int_{M_{0,\min}}^{M_{0,\max}} \int_{0}^{z_{\max}} \underline{n_\ell(z, M_0)} \sigma_L(z, M_0)^{-2} dz dM_0$$

$\underline{n_\ell(z, M_0)}$ depends on

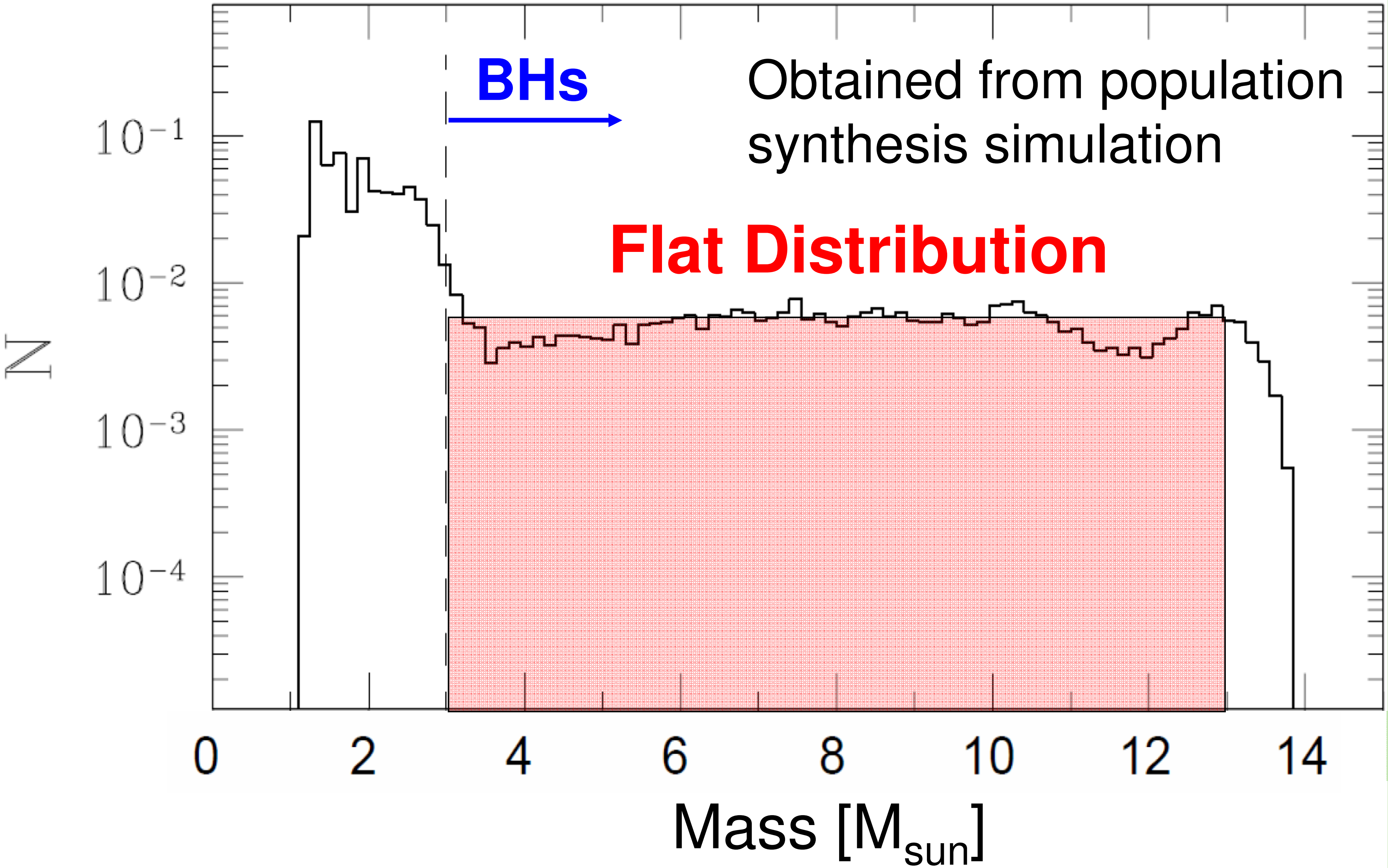
(I) Redshift distribution of sources
(II) BH mass distribution
(III) Reduction factor due to evaporation

(I) Redshift Distribution



Schneider *et al*. 2001

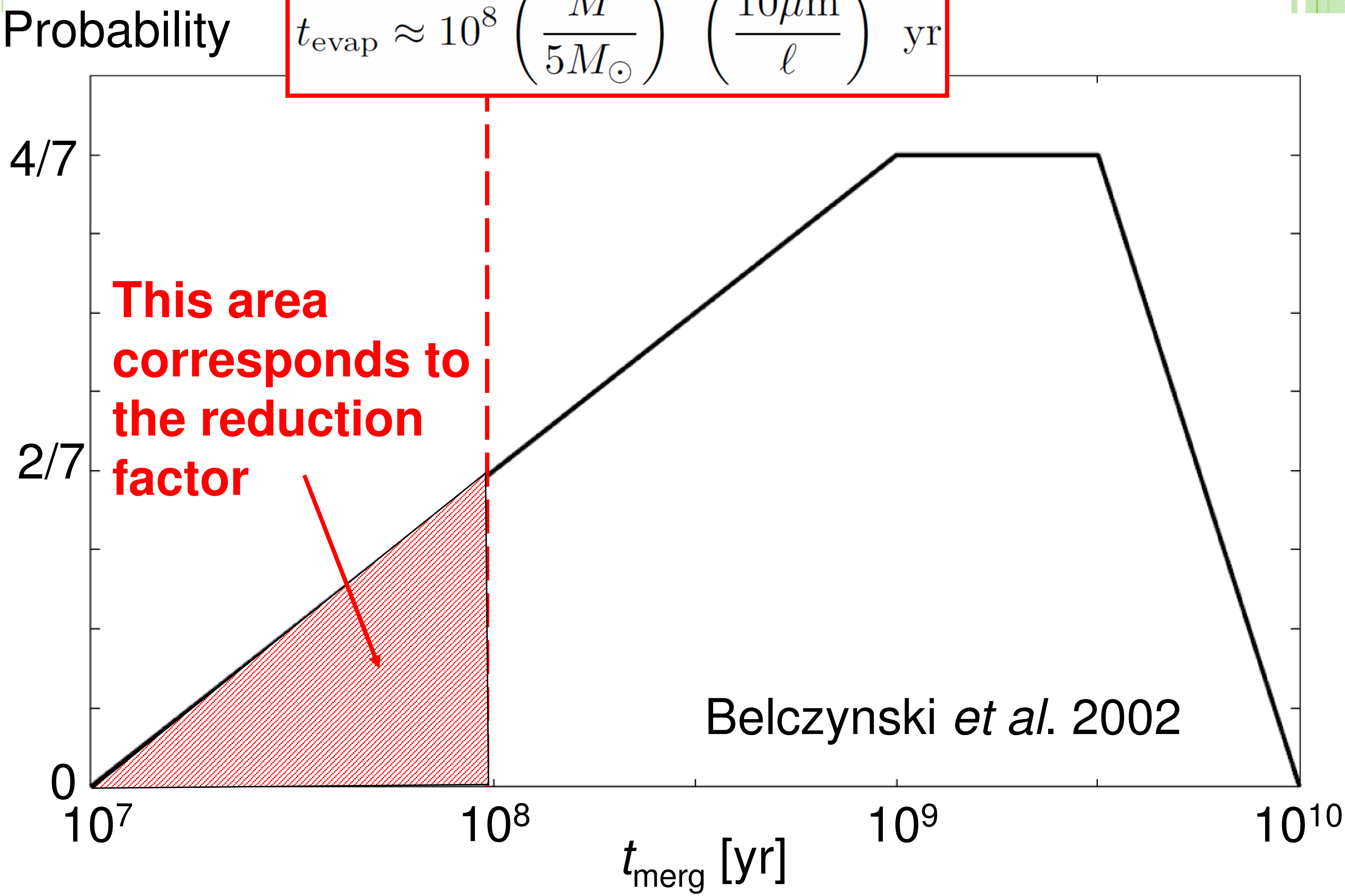
(II) BH Mass Distribution

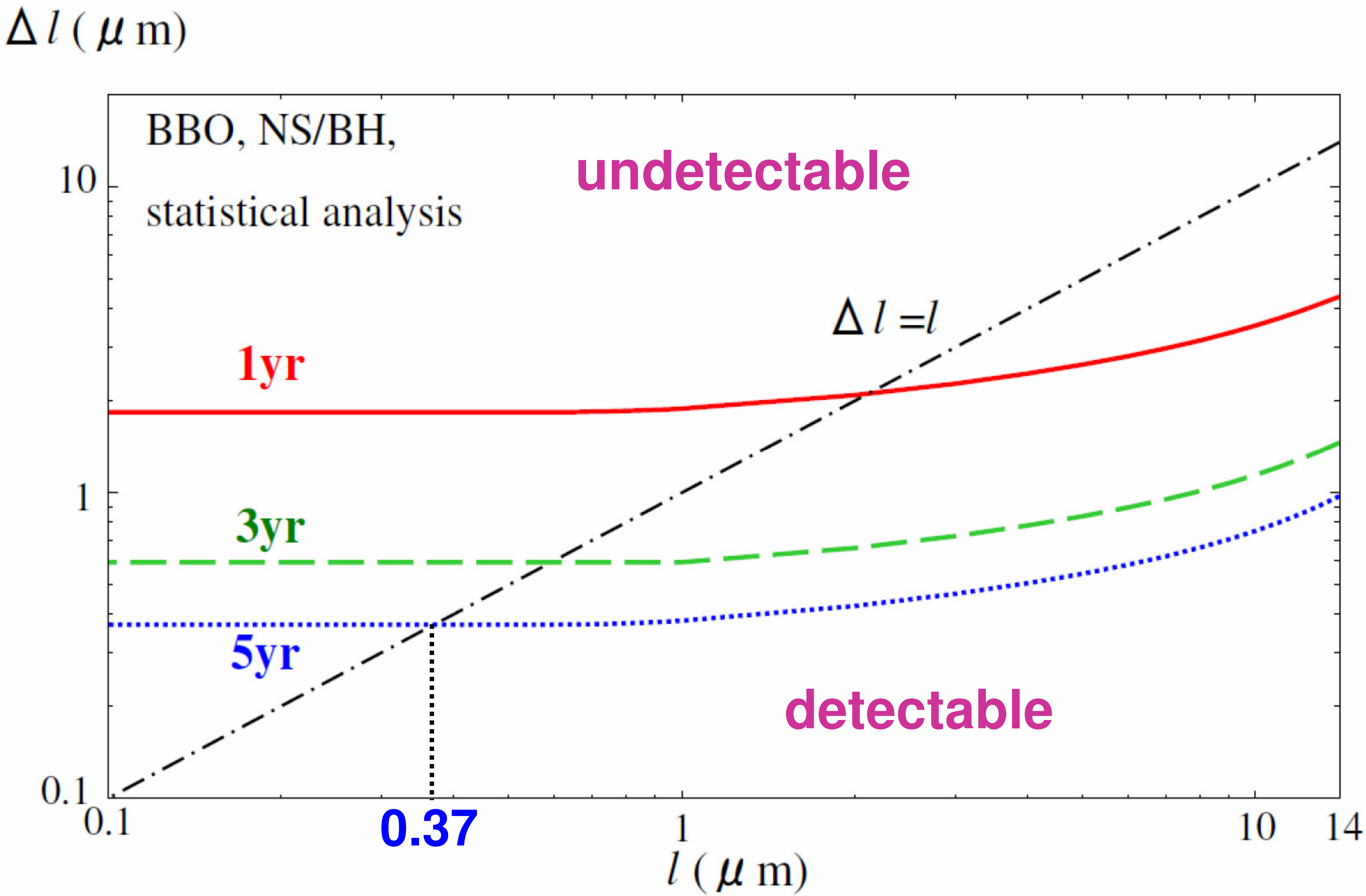


Belczynski *et al*. 2002

(III) Reduction factor due to BH evaporation

$$t_{\rm evap} \approx 10^8 \left(\frac{M}{5M_\odot}\right)^3 \left(\frac{10\mu{\rm m}}{\ell}\right)^2 {\rm yr}$$





- 40 times stronger than the table-top experiment!

§ 5 Conclusions

・We estimated **the constraint on *l*** by detecting **GW signals** from **NS/BH binaries** with **DECIGO/BBO**.

・We derived **the -4PN correction term** to the binary waveform phase.

・Performing Fisher analysis, we found that DECIGO/BBO might be able to put more than **10 times stronger constraint than the table-top experiment**.

・**DECIGO/BBO are very powerful in probing theories of gravity**. (c.f. **K.Y. & Tanaka (2009)** with **Brans-Dicke theory**)