COSMOLOGY OF A GALILEON FIELD

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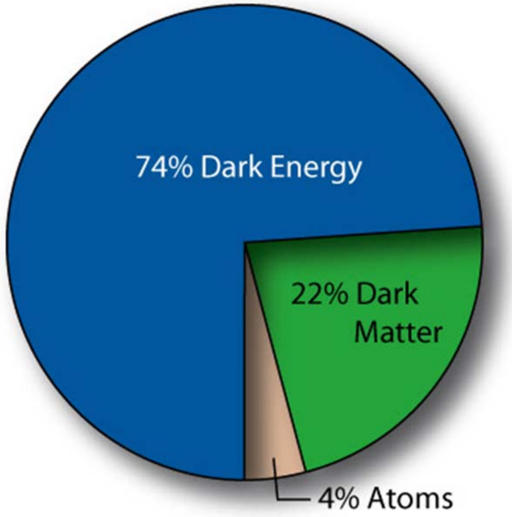
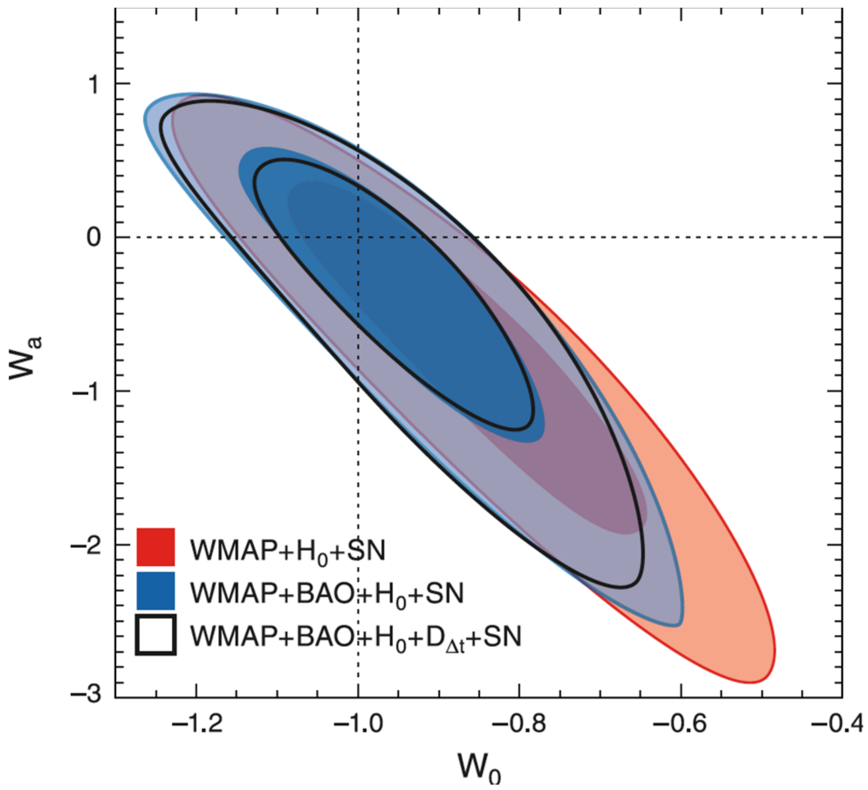
Dark energy problem

The present universe is dominated by dark energy responsible for cosmic acceleration.



Parametrization for the equation of state of dark energy

$$w(a) = w_0 + w_a(1 - a)$$



(Komatsu et al, 2010)

Observational constraints:

$$w_0 = -0.93 \pm 0.13$$

$$w_a = -0.41^{+0.72}_{-0.71}$$

Dark energy problem may imply some modification of gravity on large scales.

Modified gravitational theories

The cosmic acceleration today may originate from the modification of gravity at large scales.

- **Examples of modified gravitational models of dark energy**

1. **f(R) gravity**
2. **Scalar-tensor theories**

Chameleon mechanism can be at work.

3. **DGP braneworld**
4. **Galileon gravity**

Vainshtein mechanism can be at work.

At short-distances these models need to recover the General Relativistic behavior to satisfy solar system constraints.

Recovery of GR behavior on small scales

(i) Chameleon mechanism

Khoury and Weltman, 2004

The effective mass of a scalar field degree of freedom is density-dependent (massive in the region of high density).

Effective potential: $U_{\rm eff} = U(\phi) + \rho_m e^{Q\phi}$

ρ_m : Matter density

Q : Matter coupling with the field ϕ

Applied to f(R) gravity and scalar-tensor theory (with potential)

(ii) Vainshtein mechanism

Vainshtein, 1972

The non-linearity can be important in the high-density region (within a ``Vainshtein radius”).

The scalar-field self interaction such as $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ can allow the possibility to recover the GR behavior even without the field potential.

Applied to DGP model and Galileon gravity

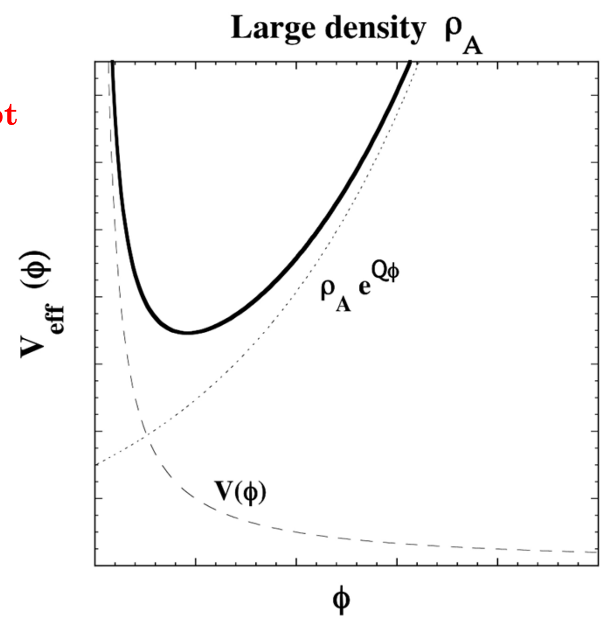
(i) Chameleon mechanism

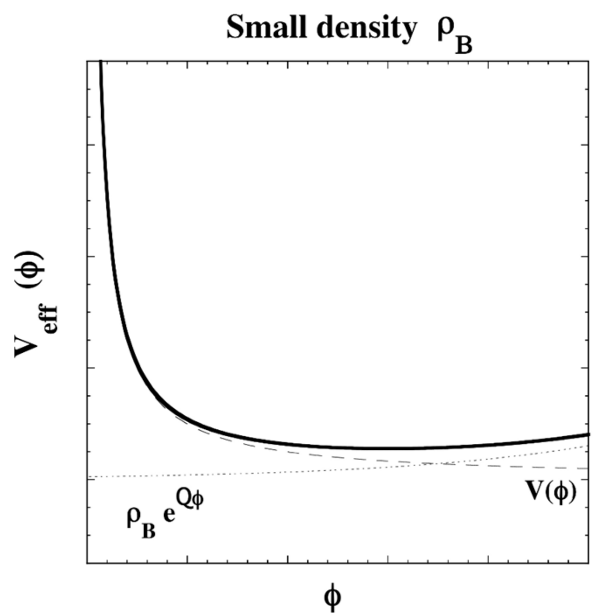
The effective mass of a scalar field degree of freedom is density-dependent.

Effective potential: $U_{\text{eff}} = U(\phi) + \rho_m e^{Q\phi}$

Massive (local region) **Massless (cosmological region)**

The field does not propagate freely in the regions of high density.





Effective coupling: $Q_{\text{eff}} = 3Q\epsilon_{\text{th}}$ **where** $\epsilon_{\text{th}} \simeq \Delta r_c/r_c + 1/(m_A r_c)$

$$Q_{\text{eff}} \ll Q \quad \text{for} \quad \epsilon_{\text{th}} \ll 1$$

(ii) Vainshtein mechanism

How about the case of a massless field?

Scalar-field non-linear self interaction such as

$$\Box\phi(\partial_\mu\phi\partial^\mu\phi)$$

allows the possibility to recover the GR behavior at high energy (without a potential).

The field can be nearly frozen in the regions of high density within a Vainshtein radius.

The Vainshtein mechanism was used for the DGP model.

DGP model: GR can be recovered for

$$r < (r_g r_c^2)^{1/3}$$

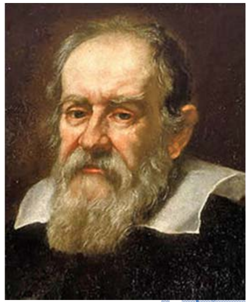
r_g : Schwarzschild radius

$r_c \sim H_0^{-1}$: Crossover scale

The Vainshtein radius is larger than the solar-system scale.

However the DGP model is plagued by a ghost problem as well as the incompatibility with observations.

Galileon gravity



The field self-interaction $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ appearing in the DGP model satisfies the Galilean symmetry in the flat space-time:

$$\partial_\mu\phi \to \partial_\mu\phi + b_\mu$$

What kind of Lagrangians do we get by imposing the Galilean symmetry?

In the flat space-time Nicolis et al. (2008) showed that there are five Lagrangians in total including

$$\mathcal{L}_1 = \phi, \quad \mathcal{L}_2 = (\nabla\phi)^2, \quad \mathcal{L}_3 = (\Box\phi)(\nabla\phi)^2$$

In the curved space-time the Galilean symmetry is in general broken.

One can construct covariant Lagrangians that recover the Galilean symmetry in the limit of the flat space-time.

Covariant Galileons

Deffayet et al. (2009)

There are five covariant Lagrangians that respect the Galilean symmetry in the Minkowski spacetime:

$$\mathcal{L}_1 = M^3\phi\,, \quad \mathcal{L}_2 = (\nabla\phi)^2\,, \quad \mathcal{L}_3 = (\Box\phi)(\nabla\phi)^2/M^3\,,$$

$$\mathcal{L}_4 = (\nabla\phi)^2\left[2(\Box\phi)^2 - 2\phi_{;\mu\nu}\phi^{;\mu\nu} - R(\nabla\phi)^2/2\right]/M^6,$$

$$\mathcal{L}_5 = (\nabla\phi)^2[(\Box\phi)^3 - 3(\Box\phi)\,\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\rho}\phi_{;\rho}{}^{\mu} - 6\phi_{;\mu}\phi^{;\mu\nu}\phi^{;\rho}G_{\nu\rho}]/M^9$$

M: some mass scale

The above Lagrangians are constructed to keep the equations of motion up to the second-order.

- **How about the cosmology based on the above Lagrangians ?**
- **The ghost does not appear unlike the DGP model?**

Galileon cosmology

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R+\frac{1}{2}\sum_{i=1}^{5}c_i\mathcal{L}_i\right]+S_m$$

$\mathcal{L}_i \quad (i=1,\cdots,5)$ **: five covariant Galileon Lagrangians**

We set $c_1=0$ to discuss the case in which the late-time cosmic acceleration can be realized by the field kinetic energy.

The equations of motion in the flat FLRW background (in the presence of non-relativistic matter and radiation):

$$3M_{\rm pl}^2H^2=\rho_{\rm DE}+\rho_m+\rho_r$$
$$3M_{\rm pl}^2H^2+2M_{\rm pl}^2\dot{H}=-P_{\rm DE}-\rho_r/3$$

where

$$\rho_{\rm DE}\equiv -c_2\dot{\phi}^2/2+3c_3H\dot{\phi}^3/M^3-45c_4H^2\dot{\phi}^4/(2M^6)+21c_5H^3\dot{\phi}^5/M^9$$
$$P_{\rm DE}\equiv -c_2\dot{\phi}^2/2-c_3\dot{\phi}^2\ddot{\phi}/M^3+3c_4\dot{\phi}^3[8H\ddot{\phi}+(3H^2+2\dot{H})\dot{\phi}]/(2M^6)$$
$$-3c_5H\dot{\phi}^4[5H\ddot{\phi}+2(H^2+\dot{H})\dot{\phi}]/M^9\,.$$

(second-order)

de Sitter solutions in Galileon cosmology

One can have de Sitter solutions with

$$\dot{\phi} = \dot{\phi}_{\rm dS} = {\rm constant}, \qquad H = H_{\rm dS} = {\rm constant}$$

We introduce two dimensionless variables

$$r_1 \equiv \frac{\dot{\phi}_{\rm dS} H_{\rm dS}}{\dot{\phi} H}, \qquad r_2 \equiv \frac{1}{r_1} \left(\frac{\dot{\phi}}{\dot{\phi}_{\rm dS}} \right)^4$$

$r_1 = 1, r_2 = 1$

at the dS point

The dark energy density parameter can be expressed as

$$\Omega_{\rm DE} = \frac{\rho_{\rm DE}}{3M_{\rm pl}^2 H^2} = -\frac{1}{6} c_2 x_{\rm dS}^2 r_1^3 r_2 + c_3 x_{\rm dS}^3 r_1^2 r_2 - \frac{15}{2} c_4 x_{\rm dS}^4 r_1 r_2 + 7 c_5 x_{\rm dS}^5 r_2$$

where $x_{\rm dS} = \dot{\phi}_{\rm dS}/(H_{\rm dS} M_{\rm pl})$

At the dS point one has $\Omega_{\rm DE} = 1$ **.**

This gives one constraint :

$$-\frac{1}{6} c_2 x_{\rm dS}^2 + c_3 x_{\rm dS}^3 - \frac{15}{2} c_4 x_{\rm dS}^4 + 7 c_5 x_{\rm dS}^5 = 1$$

Dynamical equations

The existence of de Sitter solutions requires that

$$c_2 x_{\rm dS}^2 = 6 + 9\alpha - 12\beta\,, \qquad c_3 x_{\rm dS}^3 = 2 + 9\alpha - 9\beta\,.$$

where $$\alpha \equiv c_4 x_{\rm dS}^4\,, \qquad \beta \equiv c_5 x_{\rm dS}^5$$

All the physical quantities can be expressed in terms of

$$\alpha,\ \beta,\ r_1,\ r_2$$

E.g. $$\Omega_{\rm DE} = -\frac{1}{2}(3\alpha - 4\beta + 2) r_1^3 r_2 + (9\alpha - 9\beta + 2) r_1^2 r_2 - \frac{15}{2}\alpha r_1^2 r_2 + 7\beta r_2$$

One can derive autonomous equations for $r_1,\ r_2\ ,\Omega_r$

$$r_1' = \frac{1}{\Delta}(r_1 - 1) r_1 \left[r_1\left(r_1(-3\alpha + 4\beta - 2) + 6\alpha - 5\beta\right) - 5\beta\right]$$
$$\times \left[2\left(\Omega_r + 9\right) + 3r_2\left(r_1^3(-3\alpha + 4\beta - 2) + 2r_1^2(9\alpha - 9\beta + 2) - 15r_1\alpha + 14\beta\right)\right]$$

There is a fixed point at $r_1 = 1$ **, which corresponds to a tracker solution. Along the tracker we have**

$$\Omega_{\rm DE} = r_2$$

Tracker solution in Galileon cosmology

There is a tracker solution characterized by

$$r_1 = 1, \quad \text{i.e.} \quad \dot{\phi} \propto H^{-1}$$

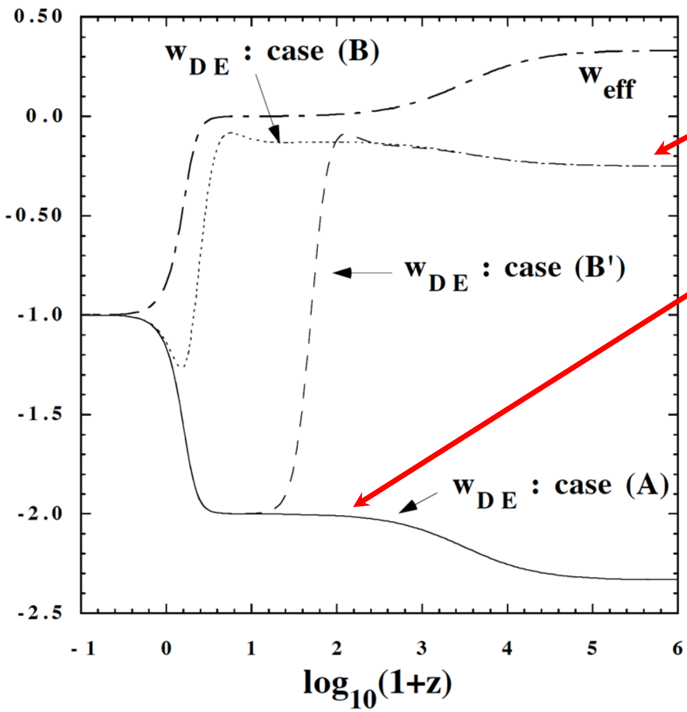
De Felice and S.T.,
PRL (2010)

Along this tracker we have

$$w_{\rm DE} \equiv \frac{P_{\rm DE}}{\rho_{\rm DE}} = -\frac{\Omega_r + 6}{3(r_2 + 1)}$$

This evolve as -7/3 (radiation), -2 (matter), and -1 (dS).

($r_2 \ll 1$ during radiation and matter eras)



Initial conditions with $r_1 \ll 1$

Tracker solution
$(r_1 = 1)$

The approach to the tracker occurs at a later epoch for smaller initial r_1.

Early epoch before reaching the tracker

Let us consider the regime: $r_1 \ll 1,\ r_2 \ll 1$

$$\left.\begin{aligned} r_1' &\simeq (9+\Omega_r)r_1/8 \\ r_2' &\simeq (3+11\Omega_r)r_1/8 \end{aligned}\right\} \Rightarrow \begin{aligned} r_1 &\propto a^{9/8} \\ r_2 &\propto a^{3/8} \end{aligned}$$

in the matter era

The equation of state of dark energy is

$$w_{\rm DE} \simeq -(1+\Omega_r)/8 \Rightarrow \begin{cases} w_{\rm DE} \simeq -1/4 & \textbf{(radiation era)} \\ w_{\rm DE} \simeq -1/8 & \textbf{(matter era)} \end{cases}$$

This correspond to the regime in which the term L_5 provides the dominant contribution:

$$\Omega_{\rm DE} \simeq 7\beta r_2$$

For the initial conditions with $r_2 > 0$ **the no-ghost condition gives**

$$\beta > 0$$

The scalar propagation speed squared is

$$c_S^2 \simeq (1+\Omega_r)/40 > 0$$

$\Rightarrow$ **The Laplacian instability can be avoided.**

Conditions for the avoidance of ghosts and instabilities

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \frac{1}{2}\sum_{i=1}^{5} c_i\mathcal{L}_i\right] + S_m$$

S_m : $\underbrace{\text{radiation}}_{\text{Velocity potential } v_1}$, $\underbrace{\text{matter}}_{\text{Velocity Potential } v_2}$

For the perturbed metric

$$\mathrm{d}s^2 = -(1+2\Psi)\mathrm{d}t^2 + a^2(t)(1+2\Phi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

We can expand the above action at second-order:

$$S^{(2)} = \frac{1}{2}\int d^4x\, a^3\left[\dot{\vec{\mathcal{Q}}}^t\mathbf{K}\dot{\vec{\mathcal{Q}}} - \frac{1}{a^2}\nabla\vec{\mathcal{Q}}^t\mathbf{G}\nabla\vec{\mathcal{Q}} - \vec{\mathcal{Q}}^t\mathbf{B}\dot{\vec{\mathcal{Q}}} - \vec{\mathcal{Q}}^t\mathbf{M}\vec{\mathcal{Q}}\right]$$

where $\vec{\mathcal{Q}} = (v_1, v_2, \Phi)$

(i) No-ghost conditions

The eigenvalues of 3×3 matrix K need to be positive.

One of three conditions is important (two of them are satisfied).

(ii) Avoidance of Laplacian instability

The dispersion relation is $\det(c_s^2\,\mathbf{K} - \mathbf{G}) = 0$ **. We require** $c_s^2 > 0$

(i) No-ghost conditions

Scalar mode: $Q_S \equiv -s/(1+\mu_3)^2 > 0$

where $s \equiv 6(1+\mu_1)(\mu_1+\mu_2+\mu_1\mu_2-2\mu_3-\mu_3^2)$

$$\mu_1 \equiv 3\alpha r_1 r_2/2 - 3\beta r_2$$
$$\mu_2 \equiv (3\alpha-4\beta+2)r_1^3 r_2/2 - 2(9\alpha-9\beta+2)r_1^2 r_2 + 45\alpha r_1 r_2/2 - 28\beta r_2$$
$$\mu_3 \equiv -(9\alpha-9\beta+2)r_1^2 r_2/2 + 15\alpha r_1 r_2/2 - 21\beta r_2/2$$

Tensor mode: $Q_T \equiv 3\alpha r_1 r_2/4 - 3\beta r_2/2 + 1/2 > 0$

(ii) Avoidance of Laplacian instability

Scalar mode:
$$c_s^2 = \{(1+\mu_1)^2[2\mu_3' - (1+\mu_3)(5+3w_{\rm eff}) + 3\Omega_m + 4\Omega_r] - 4\mu_1'(1+\mu_1)(1+\mu_2) + 2(1+\mu_3)^2(1+\mu_4)\}/s > 0$$

where $\mu_4 \equiv -\alpha r_1 r_2/2 + 3\beta r_2(3+3w_{\rm eff} - 3r_1'/r_1 - r_2'/r_2)/2$

$$w_{\rm eff} = -1 - 2H'/3H$$

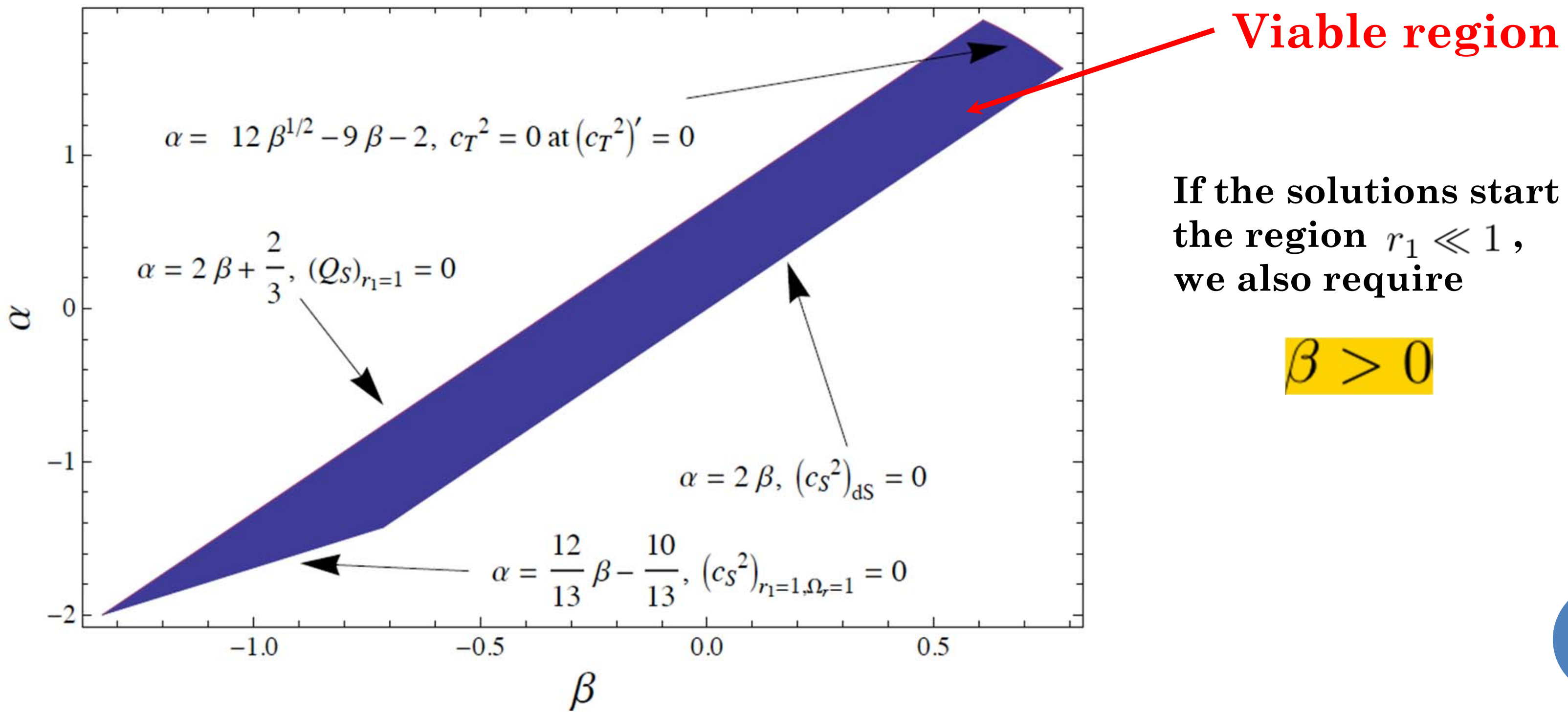
Tensor mode:
$$c_T^2 = \frac{2r_1(2-\alpha r_1 r_2) - 3\beta(r_2 r_1' + r_1 r_2')}{2r_1(2+3\alpha r_1 r_2 - 6\beta r_2)} > 0$$

Viable parameter space

The conditions for the avoidance of ghosts and instabilities of scalar and tensor perturbations restrict the viable parameter space in terms of

$$\alpha = c_4 x_{\rm dS}^4, \quad \beta = c_5 x_{\rm dS}^5$$

For the tracker solution and the de Sitter solutions, one can analytically find the viable region.



If the solutions start from the region $r_1 \ll 1$, we also require

$$\beta > 0$$

Observational constraints on Galileon cosmology

One can place observational constraints on the Galileon model at the background level by using the combined datasets of

Supernovae Ia, CMB shift parameters, and BAO

We also include the cosmic curvature K.

For the tracker the Hubble parameter is known in terms of the redshift z, as

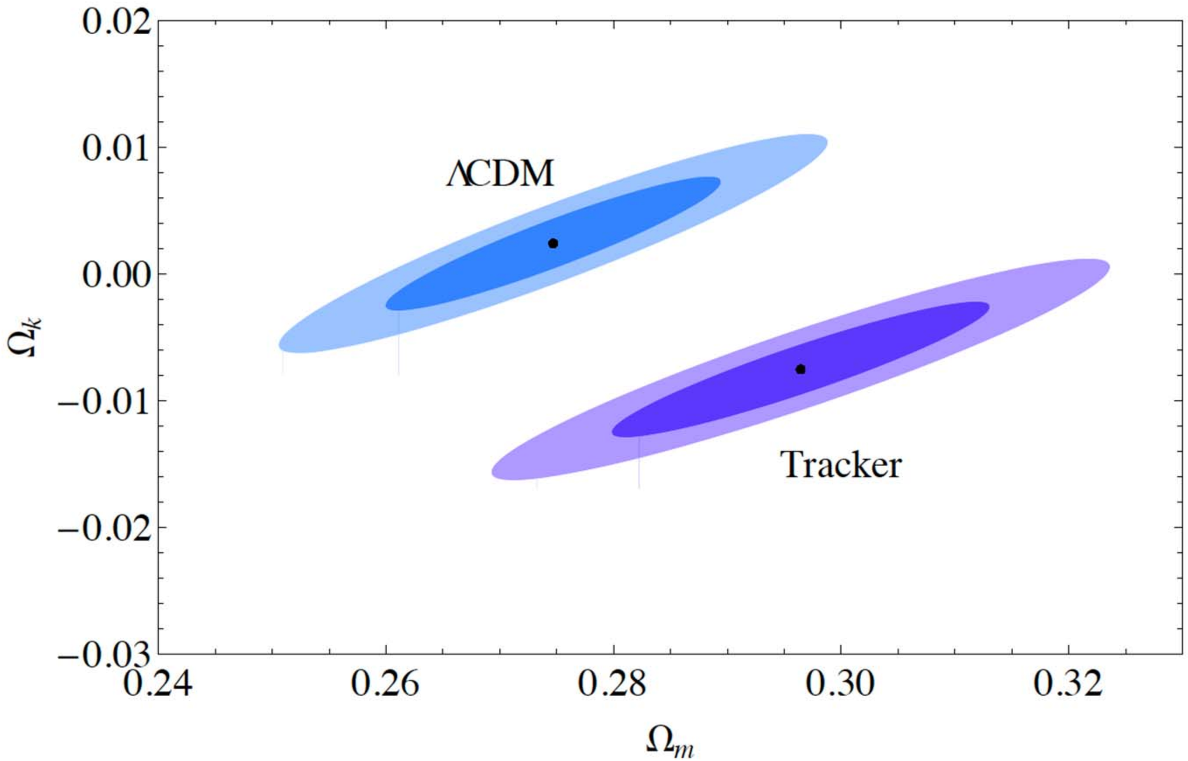
$$\left(\frac{H(z)}{H_0}\right)^2 = \frac{1}{2}\Omega_K^{(0)}(1+z)^2 + \frac{1}{2}\Omega_m^{(0)}(1+z)^3 + \frac{1}{2}\Omega_r^{(0)}(1+z)^4 + \sqrt{1-\Omega_m^{(0)}-\Omega_r^{(0)}-\Omega_K^{(0)}+\frac{(1+z)^4}{4}\left[\Omega_K^{(0)}+\Omega_m^{(0)}(1+z)+\Omega_r^{(0)}(1+z)^2\right]^2}$$

where $\Omega_K^{(0)} = -K/(a_0 H_0)^2$

For general solutions that approach the tracker at late times we need to solve the background equations numerically.

Observational constraints on the tracker solution

Nesseris, De Felice, S.T. (2010)



The CMB and BAO data favor smaller $\Omega_m^{(0)}$, but the SN Ia data prefer larger $\Omega_m^{(0)}$.

It is difficult for the tracker solution to be compatible with all datasets. The difference of χ^2 from the LCDM is

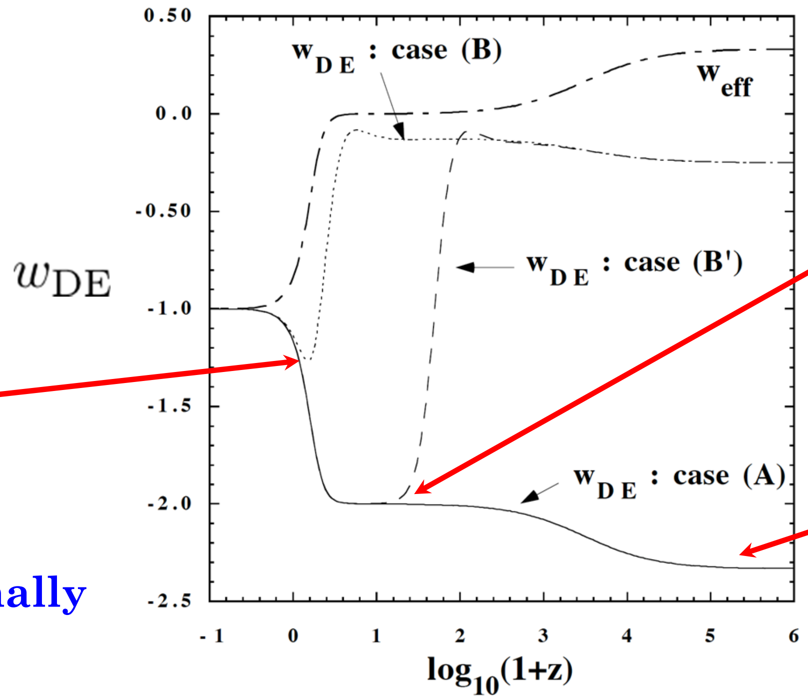
$$\delta\chi^2 \sim 22$$

General solutions

The general solutions with a late-time tracking can be compatible with combined data analysis.

Late-time tracking

Observationally favored



Early-time tracking

Tracker

The combined data analysis based on SN Ia +CMB +BAO gives

$$\alpha = 1.401 \pm 0.159, \quad \beta = 0.425 \pm 0.064$$

and

$$\Omega_m^{(0)} = 0.287 \pm 0.014, \quad \Omega_K^{(0)} = -0.003 \pm 0.005$$

Matter density perturbations

The modified evolution of matter density perturbations can allow us to distinguish the Galileon model from the LCDM.

Consider the perturbed metric

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$$

and the gauge-invariant matter perturbation

$$\delta_m \equiv \delta\rho/\rho + 3Hv$$

The full equations for perturbations are very complicated, but they are simplified under a quasi-static approximation on sub-horizon scales (i.e. picking up the terms including $\delta_m, \ k^2/a^2$).

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi \underline{G_{\rm eff}} \rho_m \delta_m \simeq 0$$

Modified growth of matter perturbations

The effective gravitational coupling is different from the bare gravitational constant G.

Modified evolution of perturbations for sub-horizon modes

Since the observations favor the late-time tracker, consider the regime

$$r_1 \ll 1, \quad r_2 \ll 1$$

The effective gravitational coupling is approximately given by

$$G_{\rm eff} \simeq (1 + \underbrace{255\beta r_2/8}_{>0 \text{ to avoid ghosts}})G > G$$

The growth rate of matter perturbations is enhanced.

We define an effective gravitational potential and an anisotropic parameter

$$\Phi_{\rm eff} = (\Psi - \Phi)/2, \quad \eta = -\Phi/\Psi$$

We obtain the Poisson equation in the following form

$$\Phi_{\rm eff} \simeq -\frac{3}{2}\underbrace{\frac{G_{\rm eff}}{G}}_{>1}\underbrace{\frac{1+\eta}{2}}_{>1}\Omega_m \delta_m \left(\frac{aH}{k}\right)^2$$

where

$$\eta \simeq 1 + 129\beta r_2/8 > 1$$

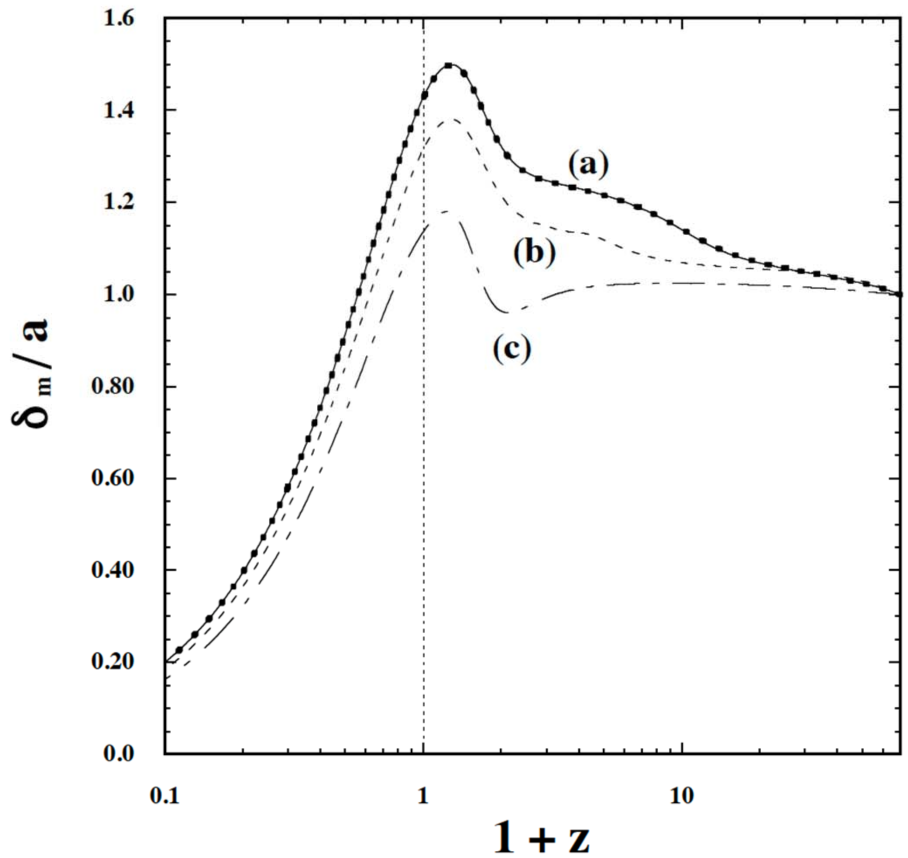
The effective gravitational potential is also enhanced.

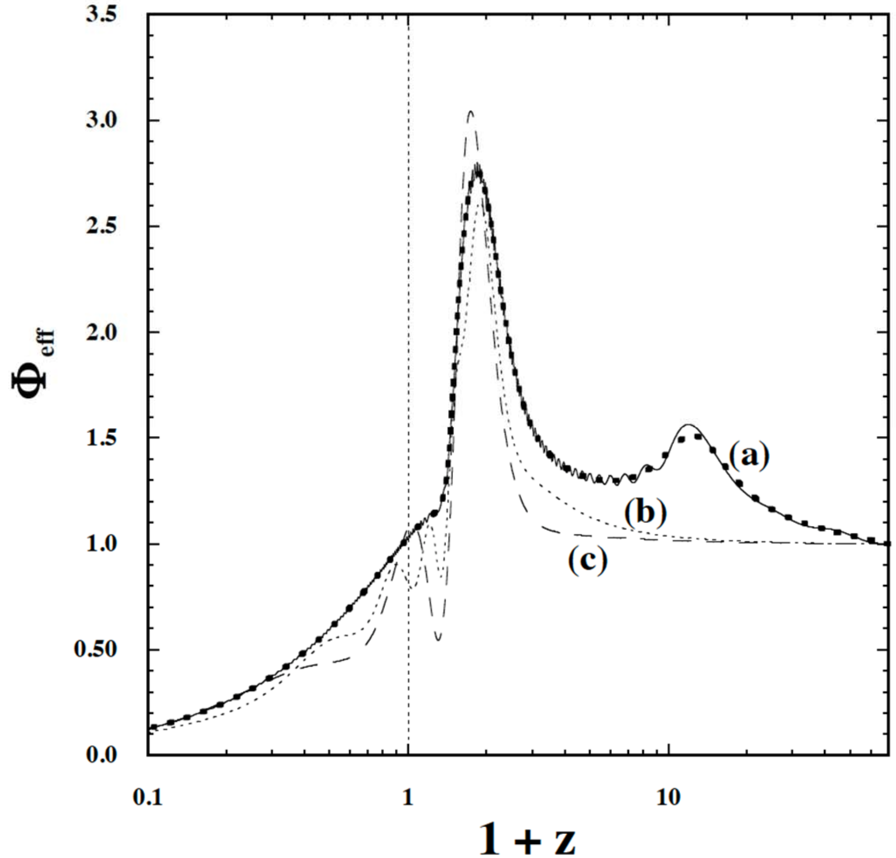
Full numerical results

De Felice, Kase, S.T., to appear.

Consider the case in which the solutions reach the tracker at late times.

$$\text{(a) } k = 300a_0H_0, \quad \text{(b) } k = 30a_0H_0, \quad \text{(c) } k = 5a_0H_0$$





The quasi-static approximation (the dotted line) agrees with full numerical results for the modes deep inside the Hubble radius.

Observational signatures

We define the growth index γ , as

$$\dot{\delta}_m/(H\delta_m) = (\Omega_m)^\gamma$$

In the LCDM, $\gamma \simeq 0.55$

- For the modes relevant to large-scale structure, we find that γ varies in time with the present value

$$\gamma_0 = 0.3 \sim 0.4$$

This property can be distinguished from the LCDM.

- Even for large-scale modes relevant to the ISW effect in CMB, $\Phi_{\rm eff}$ shows a temporal growth from the end of the matter era to the accelerated epoch.

It will be of interest to carry out the full CMB likehood analysis including perturbations.

Conclusions and outlook

- The Galileon model allows the late-time cosmic acceleration preceded by radiation and matter eras.
- There are viable parameter spaces in which the appearance of ghosts and Laplacian instabilities can be avoided.
- The solutions with different initial conditions converge to a common trajectory (tracker).
- From the combined data analysis of SN Ia, CMB, and BAO, the late-time tracking behavior is favored.
- The modified growth of perturbations affects the large-scale structure, the ISW effect in CMB, and weak lensing.