Axions as Quintessence in String Theory

Sandip Trivedi, TIFR, Mumbai

November 2010, KEK, Japan.

``Axions as Quintessence in String Theory''

S. Panda, Y. Sumitomo and S. P. T.

To appear.

Warning: Work In Progress

Results are tentative, subject to some change.

Hopefully not major ones.

Outline

1.Introduction

2. Basic Model

3. String Embedding and a Twist

4. Discussion and Conclusions

Introduction

Dark Energy: A Central Mystery

Cosmological Constant: Leading Explanation.

Observationally: Agrees with Data so far.

Theoretically: Well motivated, can arise in string theory.
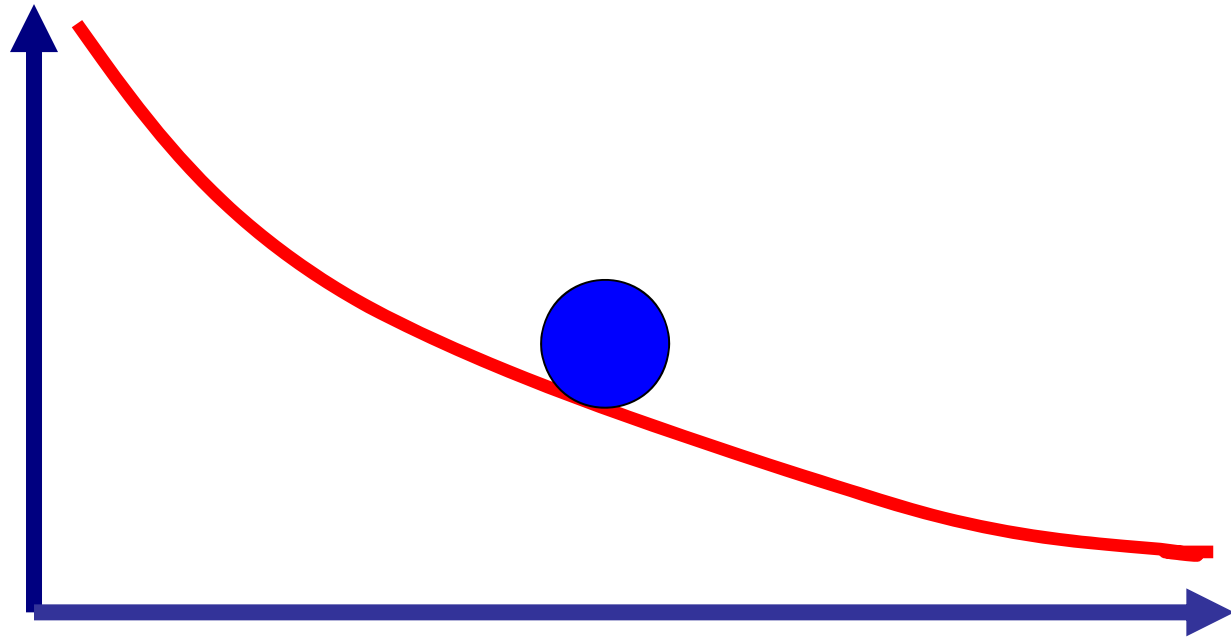
Introduction

However :

Data Allows For Other Possibilities.

Quintessence: An Alternative

Scalar Field with potential which evolves slowly thereby relaxing the vacuum energy,

$$p = \omega \rho$$

ω : Time dependent

Quintessence

- Allowed by Data

- Useful to Develop Idea further as a foil to the cosmological constant.

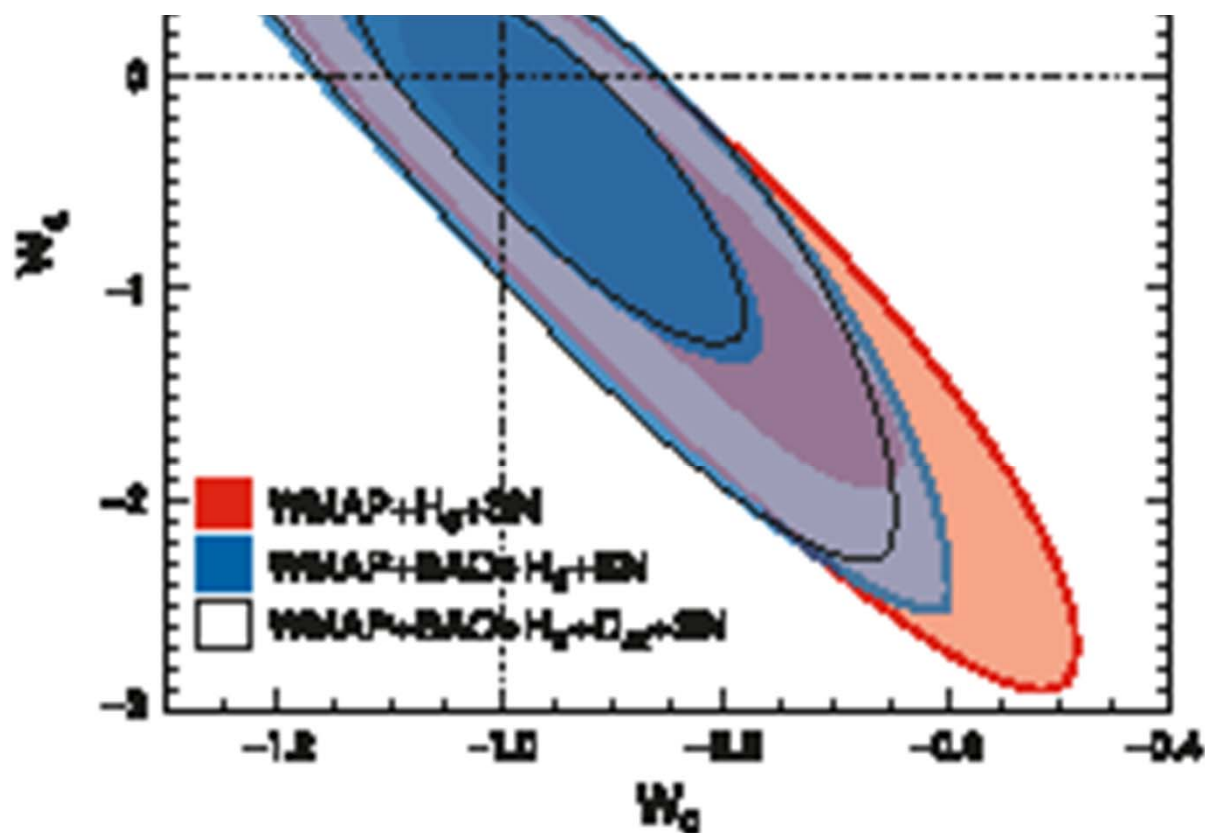- Theoretically: Can Quintessence arise in string theory?

FIG. 13.— Joint two-dimensional marginalized constraint on the linear evolution model of dark energy equation of state, $w(a) = w_0 + w_a(1 - a)$. The contours show the 68% and 95% CL from $WMAP+H_0+$SN (red), $WMAP+BAO+H_0+$SN (blue), and $WMAP+BAO+H_0+D_{\Delta t}+$SN (black), for a flat universe.

careful about the treatment of perturbations in dark energy when w crosses -1. We use the "parametrized post-Friedmann" (PPF) approach, implemented in the CAMB code following Fang et al. (2008).

In Figure 13, we show the 7-year con-
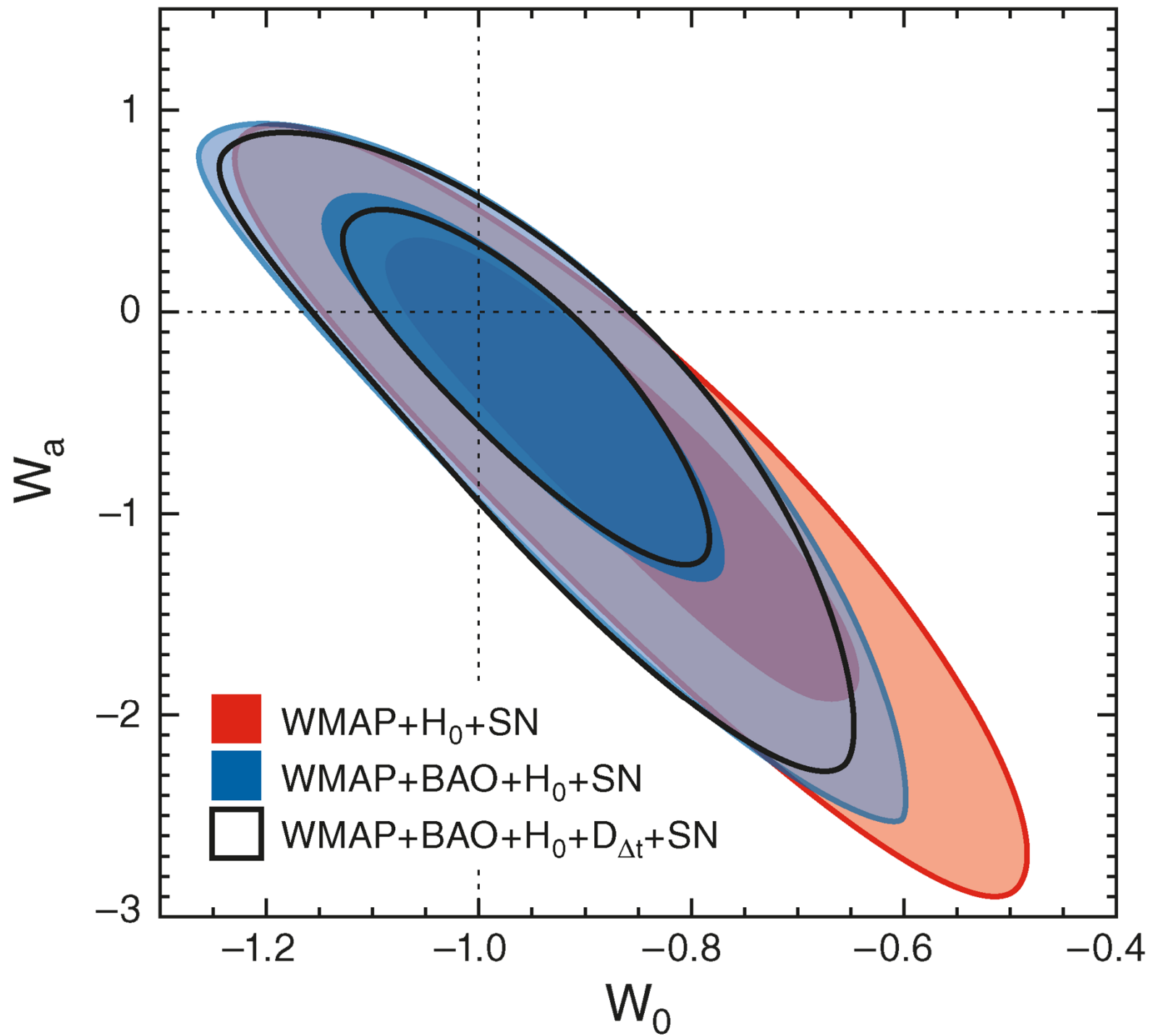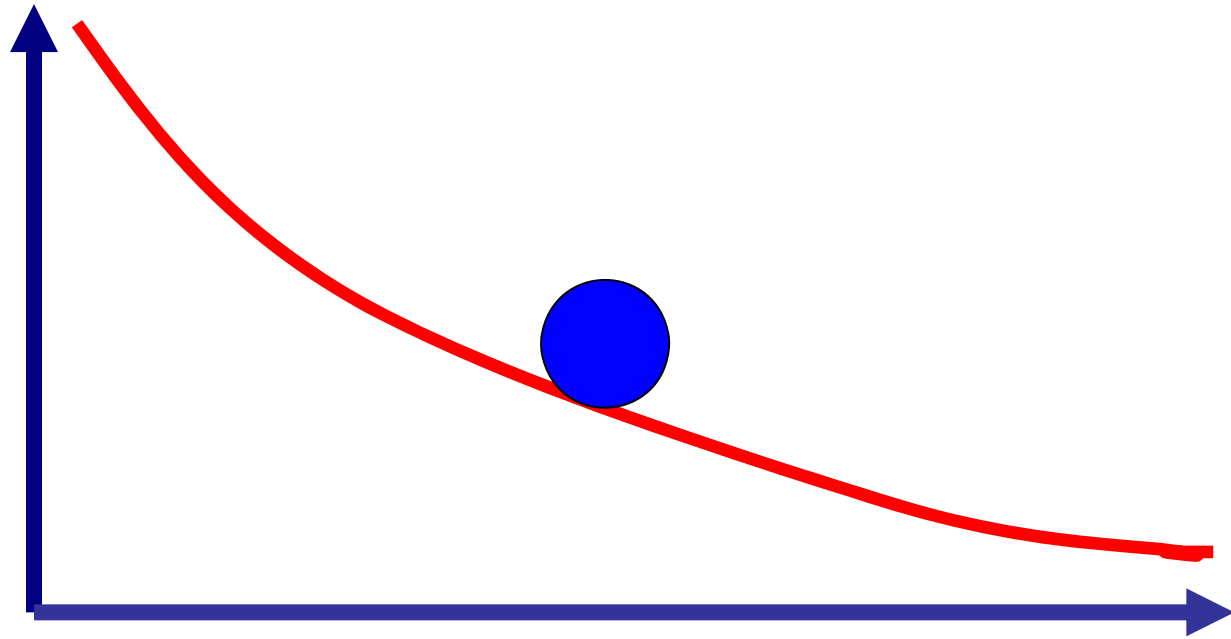
Quintessence:

Idea Analogous To Inflation.

Important Differences:

Energy Scale Much Lower etc.

Reviews: Batra and Peebles, RMP ,75,'03.
E. V. Lindner 0704.2604

Quintessence

In general terms two major challenges:

1) Minimum should be at (nearly) zero : Cosmological constant problem.

2) Need to ensure that the potential is ``shallow'' enough on cosmological scales, even though susy breaking scale is at least ~ 1-10 Tev.

Quintessence: Challenges

- Slow Roll conditions include

$$m^2 \sim V'' \leq H^2$$

$$H \sim 10^{-33} ev$$

- Susy breaking: $\qquad m_{SB} \sim 1 Tev$

$$\frac{M_{SB}}{H} \sim 10^{45}$$

- These two scales are very far apart! This makes it challenging.

Challenges:

- In Inflation the slow-roll conditions impose a condition on dim. 6 operators.

- Here the constraint is on operators upto dim. 8. Much more severe!

- This is why string theory is important.

- Actually will turn out to be even more severe.

Quintessence and String Theory

- At first sight quintessence seems like a natural possibility in string theory since getting such a ``runaway'' type of potential is quite natural for moduli.

- However the challenges discussed above have proven hard to overcome.

Quintessence And String Theory

Two possibilities:

1) A non-compact modulus, e.g. dilaton or overall volume runs away to infinity. Typically this leads to time varying coupling constants.

Quite ambitious.

Quintessence and String Theory

2) A scalar field runs to the origin.

- Best motivated for a situation with an approximate shift symmetry, e.g., for an axion.
- Sort of a goldstone boson.

- Approximate shift symmetry helps keep mass small.
- Time dependence avoided in coupling constants, G_N, α_{em}
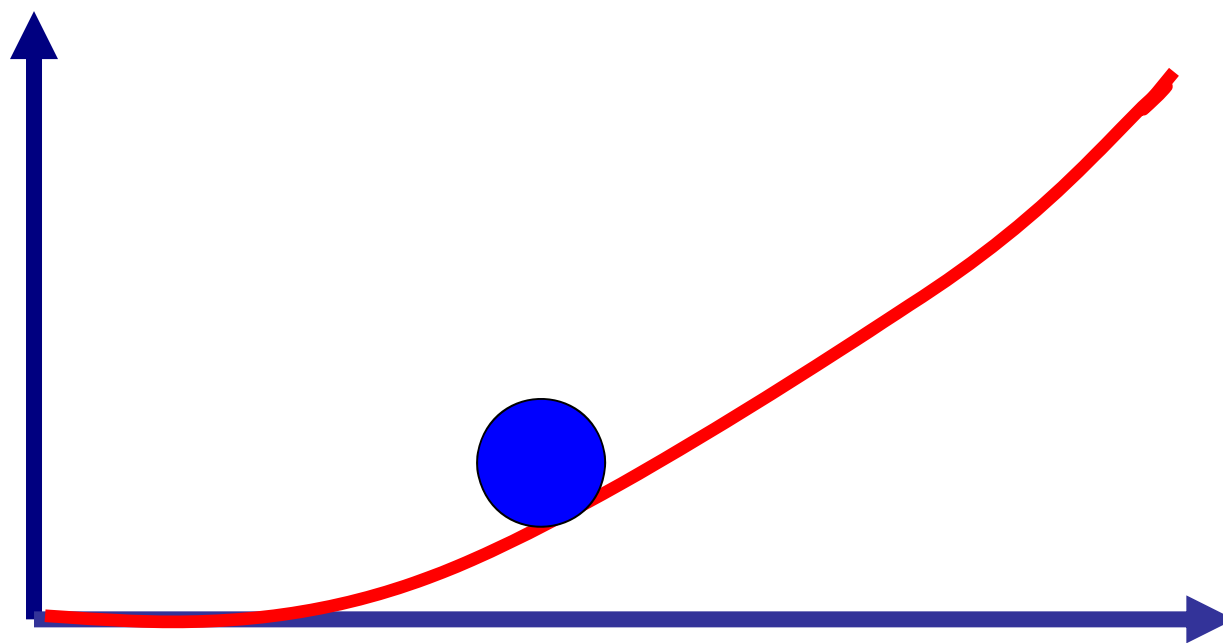
Quintessence in String Theory

- So far constructing such a model in string theory has not proved easy.

- Typically the shift symmetry is broken by non-perturbative effects and the resulting potential is periodic.

$$L_{axion} = \frac{1}{2} f_a^2 (\partial a)^2 - V(a)$$

$$V(a) = M^4 \cos(a)$$

Slow roll conditions:

$$\frac{V''}{V} M_{Pl}^2 \leq 1, \left(\frac{V'}{V}\right)^2 M_{Pl}^2 \leq 1$$

Lead to $\quad f_a > M_{Pl}$

- More correctly, with

$$M^4 \sim e^{-S_{inst}}$$

- Condition really is $\quad f_a \leq M_{Pl}/S_{inst}$

- Now $\quad V \sim M^4 \sim (10^{-3})^4$

- This gives, $\quad S_{inst} \sim 200 - 300$

Svercek, Svercek and Witten

New Feature of our Model:

- Potential is Not Periodic.
Instead Linear.

$$V = \mu^4 a$$

- Now slow roll conditions give

$$\phi = f_a a > M_{Pl}$$

- This is the First Important Condition.

Conditions on The Model

The second important condition is that total energy in quintessence is of required value:

$$V \sim (10^{-3} ev)^4$$

$$\mu^4 a \sim (10^{-3} ev)^4$$

(Rest of Talk use notation: $\Lambda^4 = (10^{-3} ev)^4$

- Total Excursion of the Quintessence Field:

$$\Delta\phi \sim \dot{\phi}H^{-1} \sim \frac{M_{Pl}^2}{\phi} \sim O(M_{Pl})$$

We will need to ensure that the potential meets required conditions over this whole range.

The Model is based on

McAllister, Silverstein, Westphal,
arXiv: 0808.0706

The Model In String Theory

- We will work in IIB string theory.

- Axion will arise from a two-form integrated over a two-cycle.

$$a = \int_{\Sigma_2} C_2$$

- For simplicity we only keep dependence on overall volume modulus. And set $g_s = 1$

Then easy to see that

$$V_I = L^6 (\alpha')^3$$

Internal volume

Dimensionless

String scale

$$M_{Pl}^2 \sim L^6 / \alpha'$$

The axion decay constant in the model is

$$f_a \sim \frac{M_{Pl}}{L^2}$$

Slow Roll conditions give:

$$f_a a > M_{Pl}$$

$$a > L^2$$

More About the Model

The axion shift symmetry

$$a \rightarrow a + c$$

Is unbroken to all orders in quantum loop perturbation theory (in absence of boundaries).

More About the Model

- To break shift symmetry:

- We will introduce a 5-brane wrapping the same 2-cycle.

- To keep the scale of breaking small the 5-brane lives in a highly warped ``throat'' region of the internal compactification.

Warping
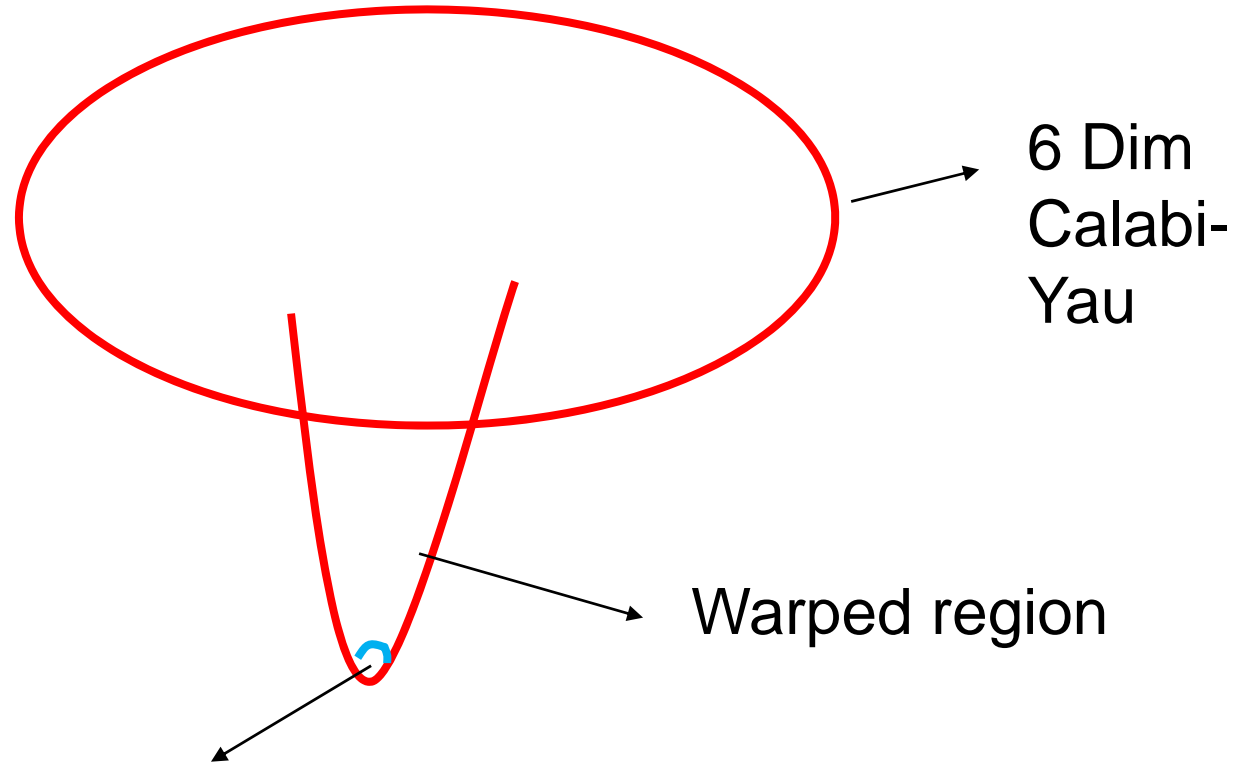
Essential idea:

Metric of warped compactification

$$ds^2 = e^{2A(y)}dx_\mu dx^\mu + e^{-2A(y)}g_{ab}dy^a dy^b$$

Warp factor

Warping arises due to flux.

More on The Model

- We will work in IIB string theory.

- Flux: F_5
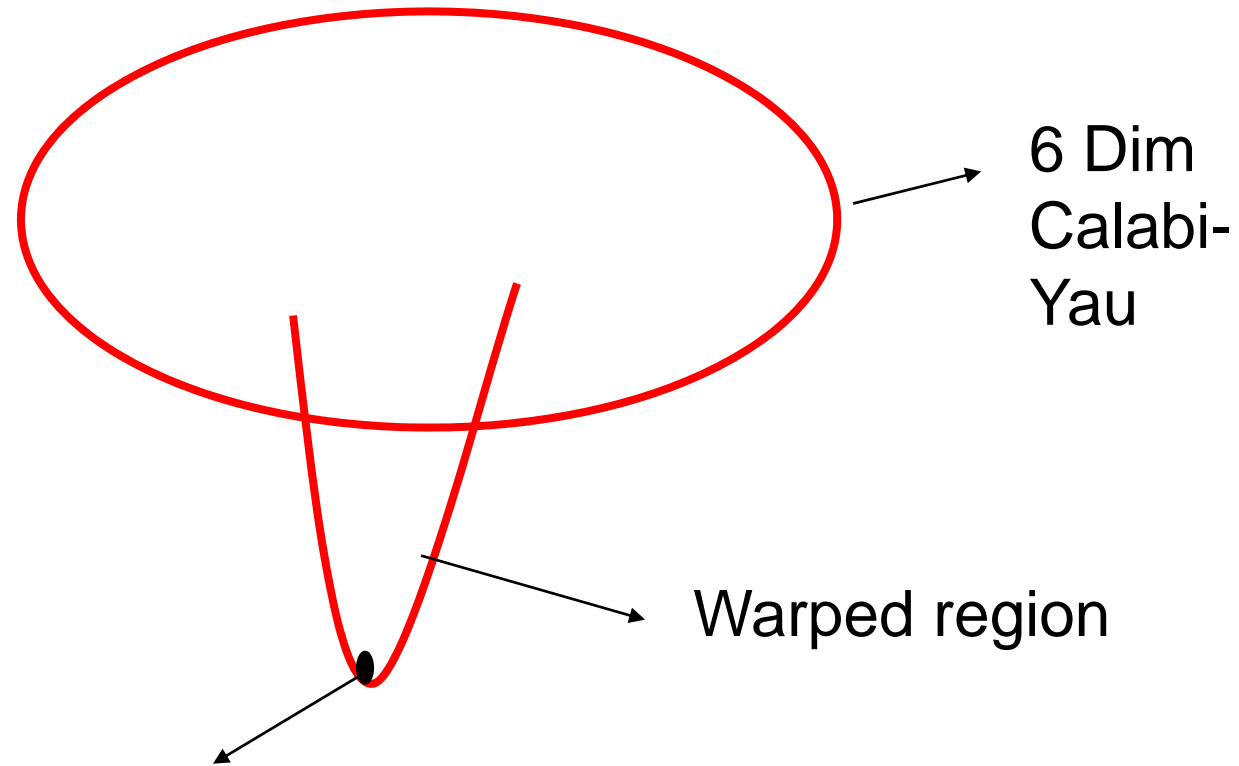- Sourced by D3 branes.

- Other Sources for flux as well.

6 Dim Calabi-Yau

Warped region

Redshift at bottom $e^{A_0} << 1$

Mass of string at bottom $m \sim M_s e^{A_0}$

More on Warping

- Modest ratio of fluxes can give rise to exponentially small value for e^{A_0}

- In this way we can generate a scale of order $\Lambda \sim 10^{-3} ev$

- By inputting some modestly small numbers.

6 Dim Calabi-Yau

Warped region

- A D3 brane at the bottom
- Point like in the internal directions.

- Result in potential: $\delta V = e^{4A_0} T_3$

More on the Model

• In our model the D3 brane charge will be linearly proportional to axion a.

•In this way we will get a potential which is linear and by adjusting e^{A_0} of order Λ

- The resulting potential is

$$V_0 = a T_3 e^{4A_0}$$

- If the Throat is supported by N units of flux and

$$a \ll N$$

- The warp factor e^{A_0} is to leading order independent of axion. By varying the flux can be varied e^{A_0} independently of a
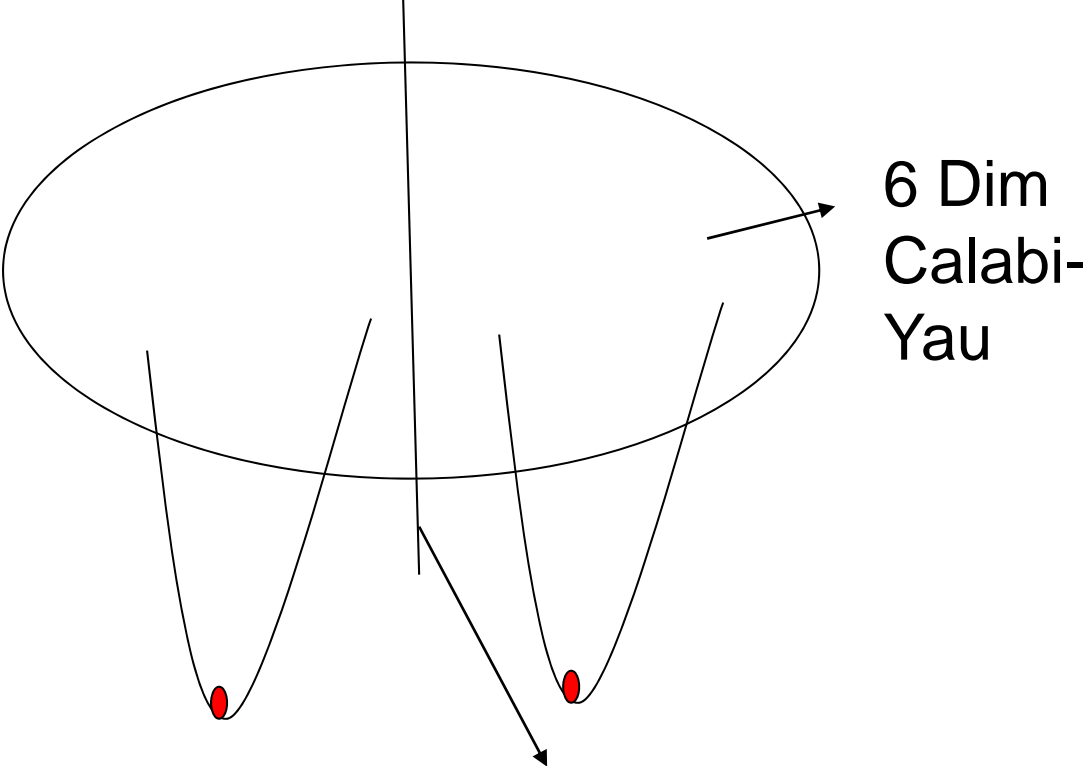
More About the Model

- To break shift symmetry:

- Take a NS 5 brane wrapping a 2-cycle at the bottom of the ``throat''.

- This will lead to induced D3 charge and tension that depends linearly on the axion.

6 Dim Calabi-Yau

Z^4
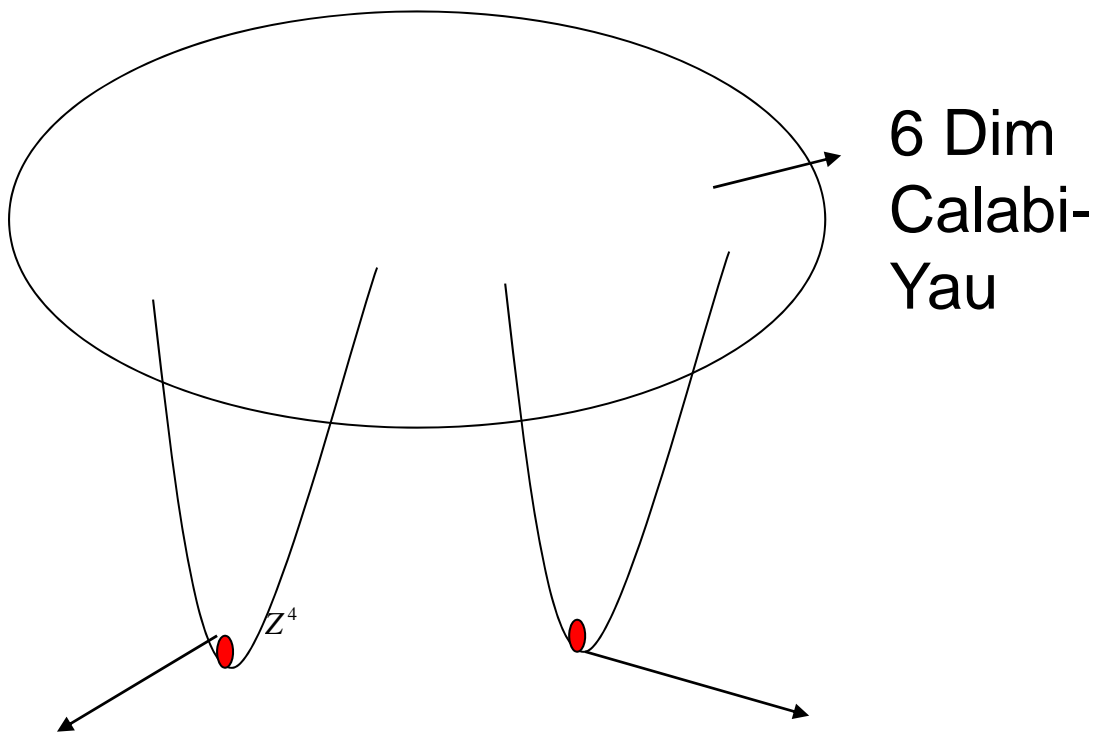
NS 5-brane wrapping 2-cycle

Anti NS 5-brane wrapping 2-cycle

6 Dim Calabi-Yau

Plane of Z_2 symmetry

Induced D3 brane charge

On brane world volume is coupling:

$$\int C_2 \wedge C_4$$

This will give rise to induced D3 charge

$$N_{induced} = a$$

And potential $V_0 = c_1 M_s^4 e^{4A_0} a$

Meeting The Slow-Roll Conditions

The axion decay constant in the model is

$$f_a \sim \frac{M_{Pl}}{L^2}$$

Slow Roll conditions are met if:

$$f_a a > M_{Pl}$$

$$a > L^2$$

Meeting The Slow-Roll Conditions

e.g., For $\qquad$ $L \sim O(1)$

Slow roll conditions can be met for

$$a >\sim O(1)$$

Second condition is on the potential

$$V = \mu^4 a = M_S^4 e^{4A_0} a \sim M_{Pl} e^{4A_0} \sim \Lambda^4$$

This can be met by choosing fluxes so that e^{A_0} takes the required value.

This can be achieved for reasonably modest values of the flux.

At first glance then the model looks encouraging, we have met the two required conditions.

However we are not done yet!

Main Worry

What about moduli stabilisation and susy breaking?

These effects should not generate an unacceptably large mass or potential for the axion.

Incorporating Moduli Stabilisation

- The concrete model of moduli stabilisation and susy breaking that we use is the KKLT model.

- The volume modulus is stabilised by non-perturbative effects in this model

Kachru, Kallosh, Linde, S.P.T. PRD68,(2003) 046005

The KKLT Construction

Has several ingredients:

1) An internal manifold: Calabi-Yau Manifold.

2) Fluxes

3) Non-perturbative effects

4) Anti-Branes.

Moduli Stabilisation and the Axion

- The superpotential for Kahler modulus in KKLT:

$$W = W_0 + Ae^{-cS}$$

- If we work with the RR two-form.
- And W arises due to gaugino condensation.
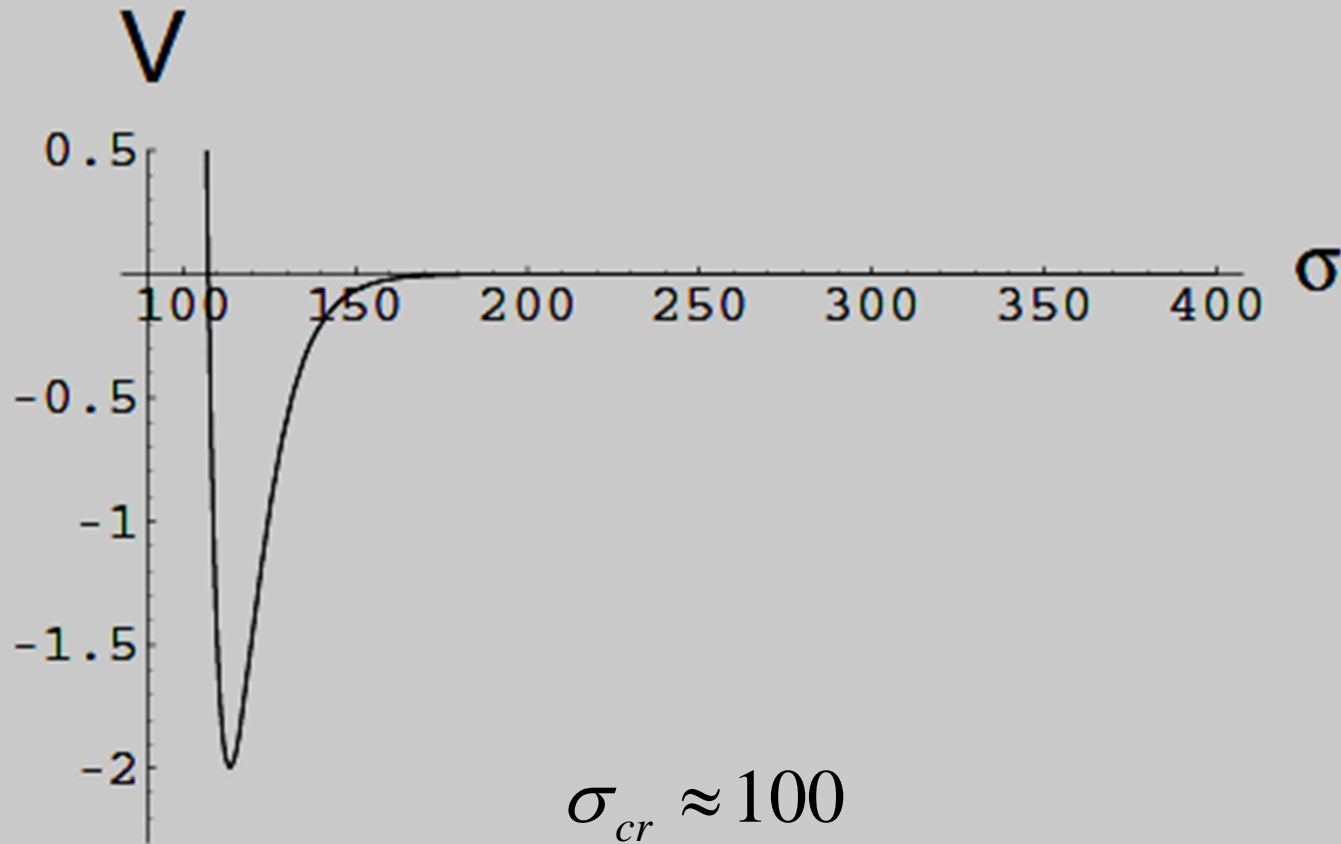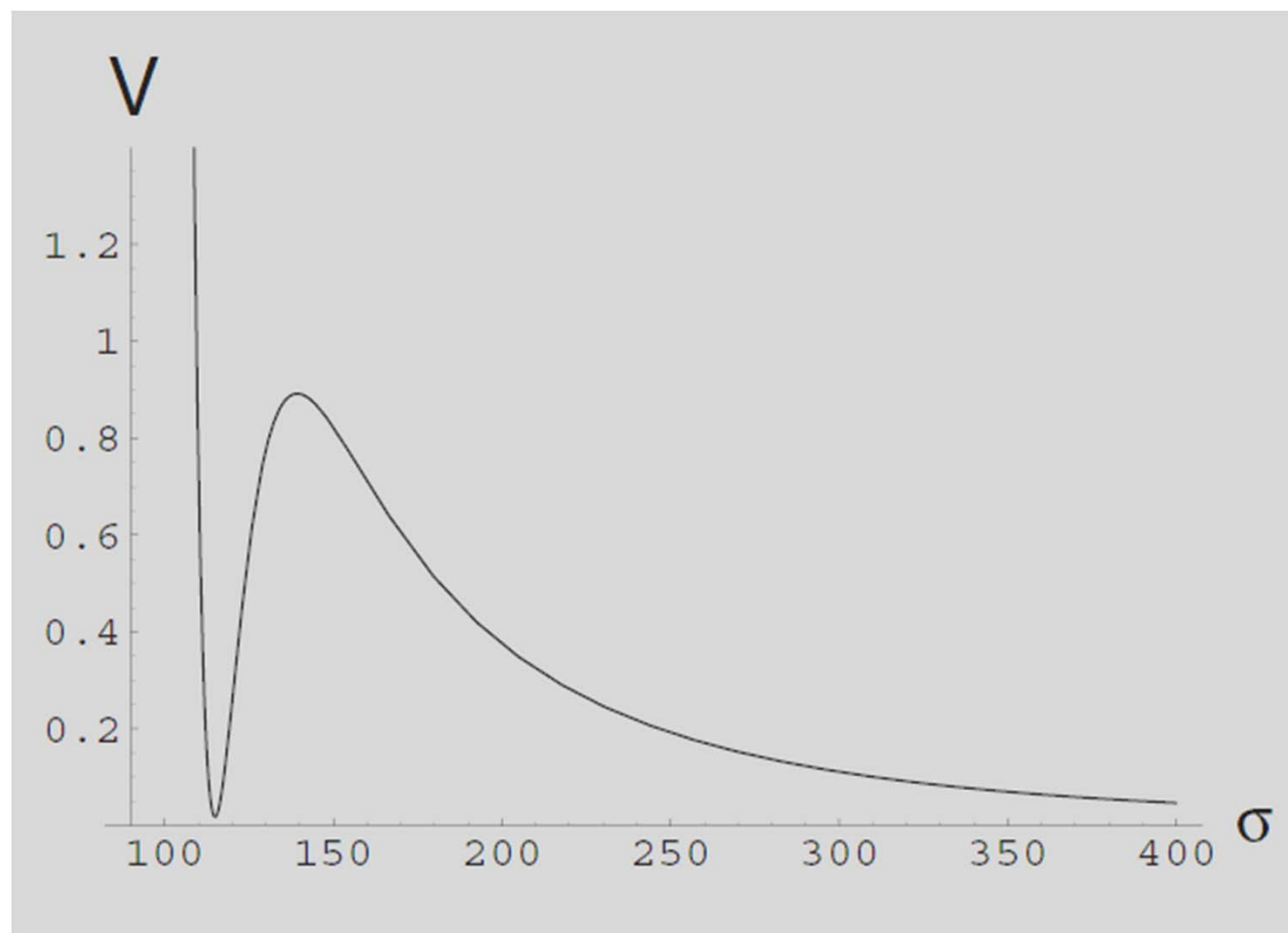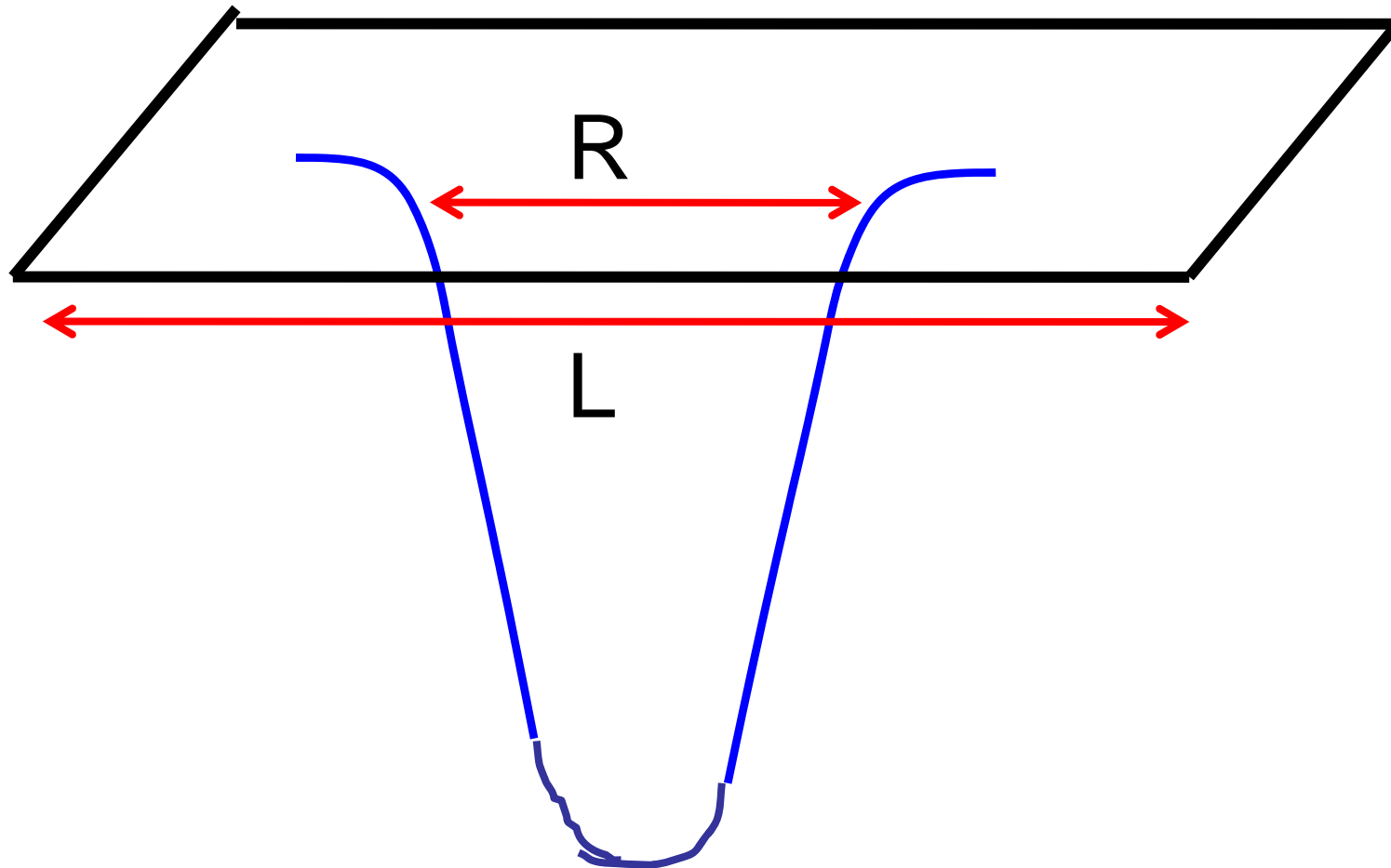- The resulting contribution to axion potential is small

FIG. 1: Potential (multiplied by 10^{15}) for the case of exponential superpotential with $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$. There is an AdS minimum.

$\sigma_{cr} \gg 1$ if W_0 <<1. With 10^(100) vacua this leaves many possibilities.

Additional Important Contribution

This arises because the axion changes the extent of warping and thus the volume.

R

L

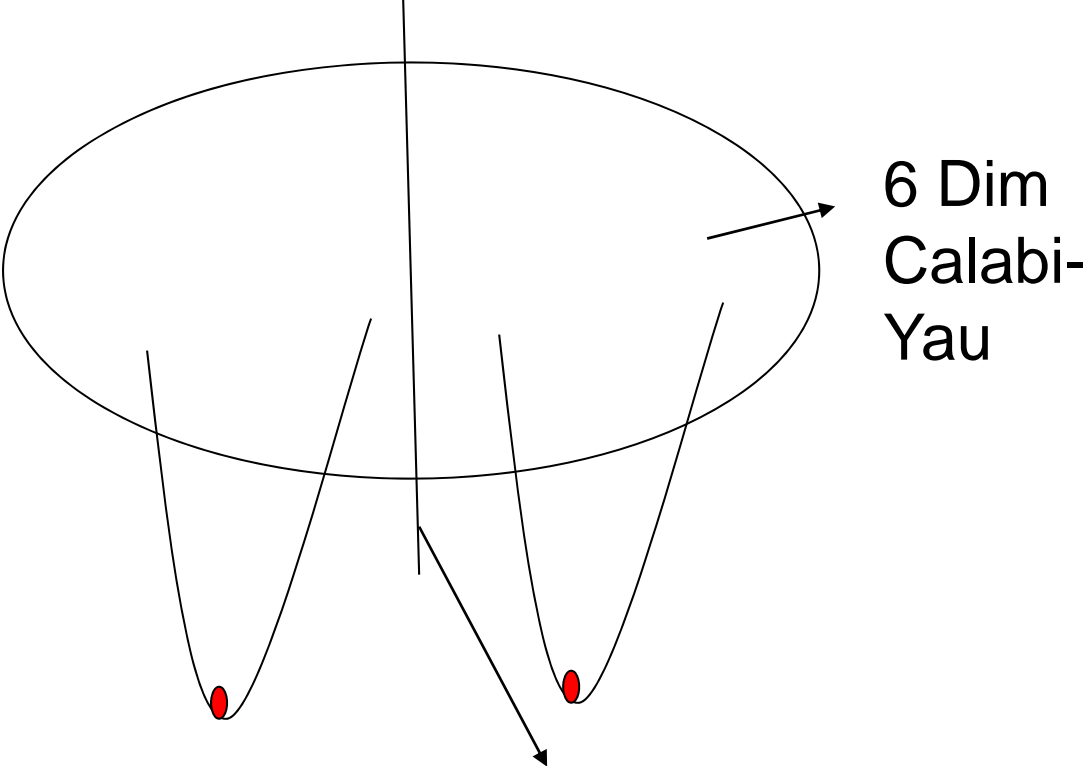As R changes so does the total volume.

Additional Important Contribution

- The warping effects change the volume.

- Since the volume modulus has been stabilised this gives to an extra cost in energy which depends on the warping effects.

- That is a term in potential dependent on the axion.

Let U_{mod} be the potential for the volume modulus.

Then $$U_{mod} \sim M^4_{susybreaking}$$

The extra cost in energy is then related to the fractional change in internal volume caused by warping

6 Dim Calabi-Yau

Plane of Z_2 symmetry

Additional Contribution

$$V_1 \sim U_{mod} e^{2A_0} \frac{\sqrt{N}}{L^6} a$$

This formula is an estimate.
Valid when $a \ll N$

Current technology does not allow a precise calculation. Thus not possible to go beyond this limit.

Shiu, Torroba, Underwood, Douglas, 0803.3068

Other related references:

DeWolfe, Kachu, Mulligan, 0801.1520

Bena, Grana, Halmagyi, 0912.3519

Additional Contribution

- Thus full potential for axion is

$$V = V_0 + V_1$$
$$V_0 = c_1 M_s^4 e^{4A_0} a$$

- It is Linear.

•Our earlier analysis therefore still goes through.

•Adjusting axion to meet $a > L^2$ the slow roll conditions are met.

•And then e^{A_0} can be adjusted so that the total potential V is of required value Λ^4

Meeting The Conditions in The Model

- Thus as a First pass we have a viable model of quintessence.

- A few different ingredients, these occur quite generically.

- Next step would be a full string embedding. A particular Calabi-Yau, with required warped throats, non-perturbative effects etc.

Shortcomings:

- Not a complete model for Particle Physics: Have not incorporated the Standard Model.

- Not a complete model for cosmology:
- Inflation, Moduli problem etc.

Relative Importance of the Two Contributions

- For $L \sim O(1), M_s \sim M_{Pl}$

- With $U_{mod} \sim M_{SB}^4 \sim (1 Tev)^4$

- One finds both contributions comparable.

- The warped down string scale is $M_s e^{A_0} \sim 10^{-3} ev$

For bigger values of L, U_{mod}

And one gets:

$$V_1 > V_0$$

$$V_0 \sim \Lambda^4 \left(\frac{(1Tev)^8}{U_{mod}^2}\right)\left(\frac{1}{L^2}\right)$$

And warped down string scale:

$$M_{WD} \sim 10^{-3} ev \left(\frac{(1Tev)^2}{\sqrt{U_{mod}}}\right) L^4$$

- In KKLT: $$U_{mod} \sim M_{SB}^4$$

- For
$$U_{mod} \sim (1Tev)^4$$

$$V_0^{1/4} \sim 10^{-3}ev$$

$$M_{WS} \sim 10^{-3}ev$$

- For
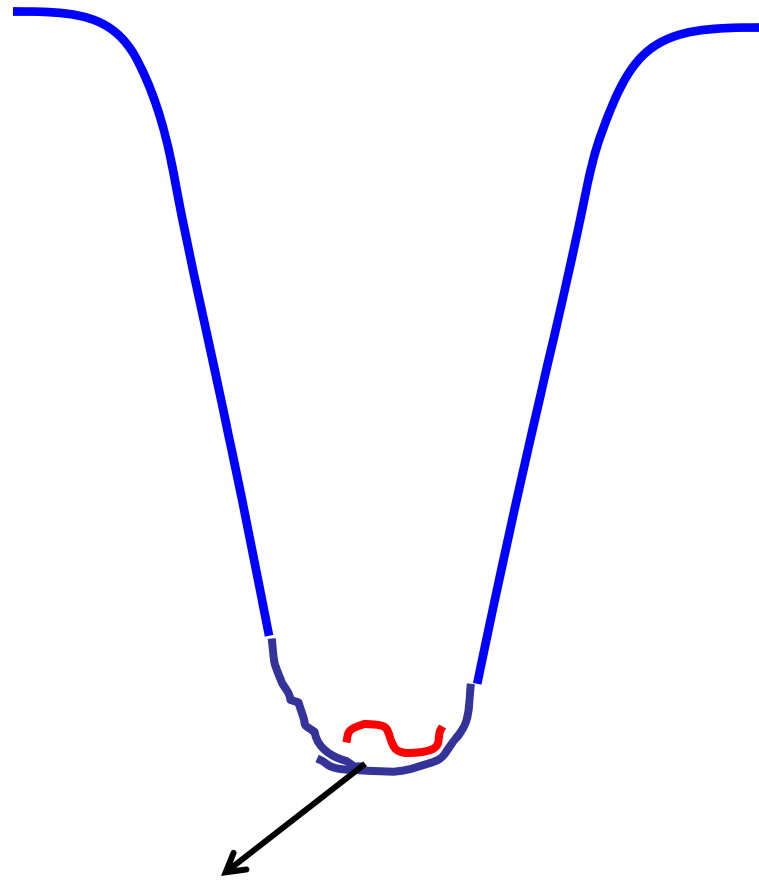$$U_{mod} \sim (10^{10}Gev)^4$$

$$V_0^{1/4} \sim 10^{-10}ev$$

$$M_{WS} \sim 10^{-17}ev$$

Light Fields: General Feature of Model

Thus: The Model requires a very highly warped throat region and as a result many light fields.

Light states lie at the
bottom of the throat

Highly Warped region is like Randall Sundrum II

For $\qquad N \sim O(1), L \sim O(1)$

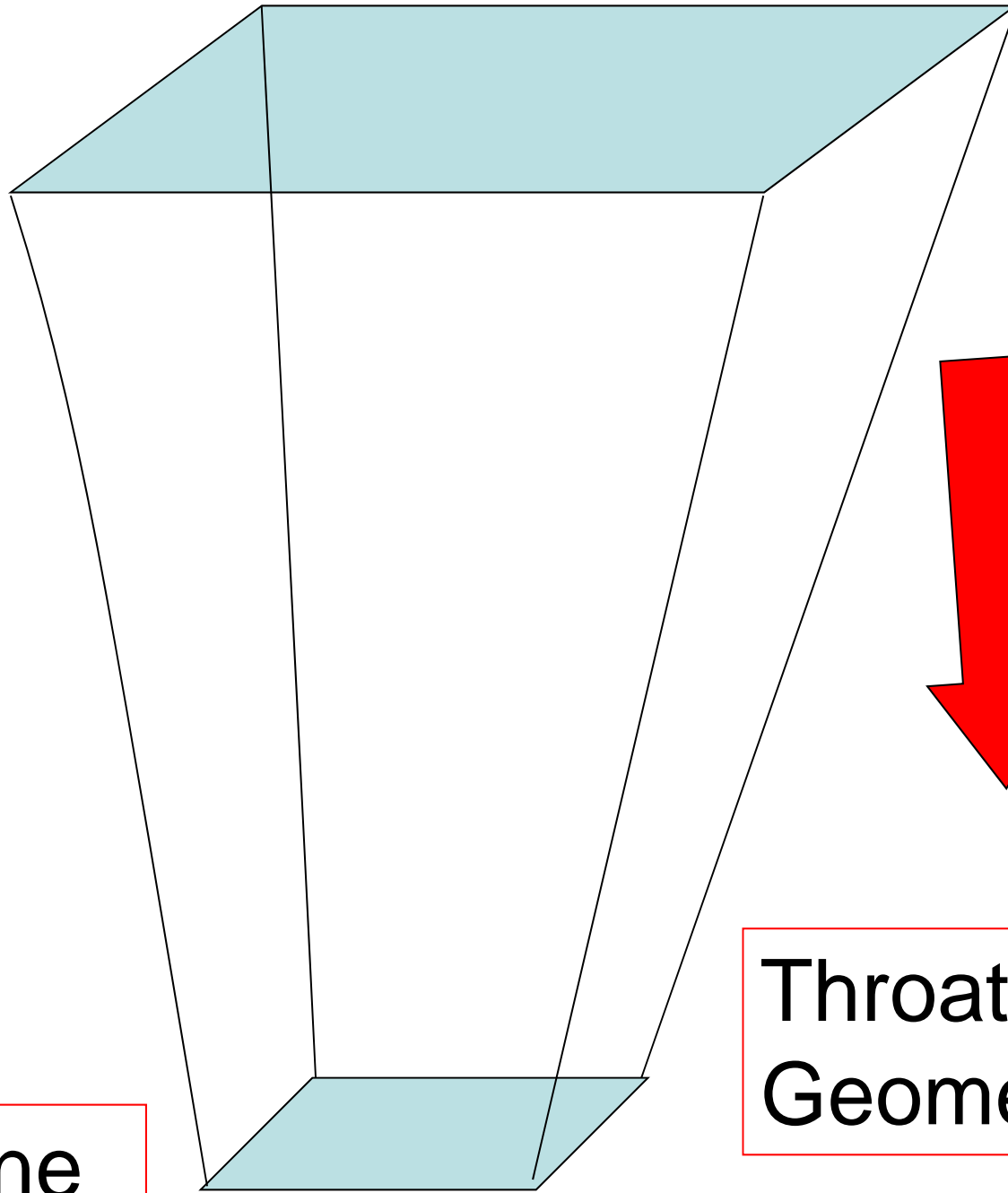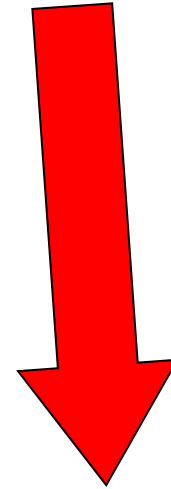in a process with characteristic momentum p, effects of light states suppressed by

$$(p/M_{Pl})^2$$

Very small.

Planck Brane

IR Brane

Throat Like Geometry

- Includes Corrections to Newton's Law, Energy loss in gravitational radiation, self-interactions of gravitons themselves.

- Thus Model not ruled out by particle physics considerations.

Tracker Behaviour:
In radiation era

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mu^4}{f_a} = 0$$

Has a solution

$$\phi = \phi_0 - \frac{\mu^4}{5 f_a} t^2$$
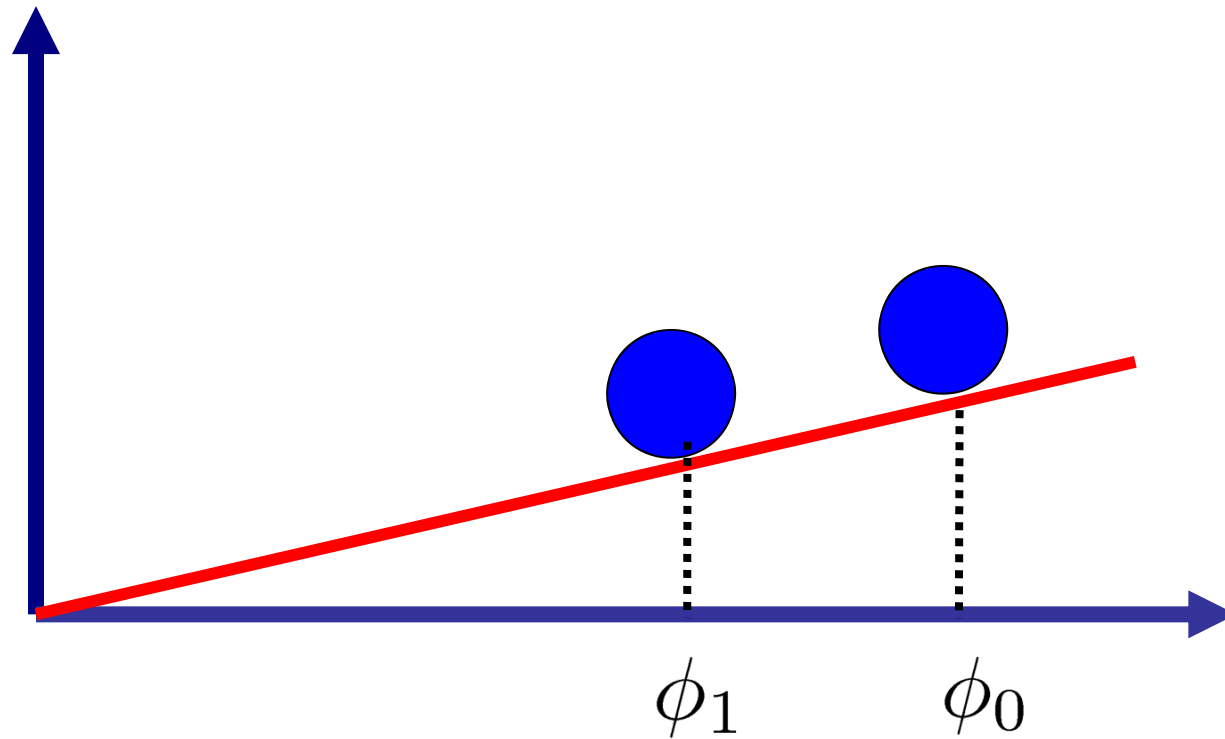
Small perturbations go as:

Not quite a tracker.

$$\delta\phi = d_1 + d_2 t^{-1/2}$$

Tracker Behaviour

- Extra integration constant d_1 tied to choice of initial condition ϕ_0

- In turn tied to the cosmological constant.

- Also no solution to the coincidence problem. The initial conditions have to be just right.

- Not satisfactory.

- Maybe Anthropics?

$$\delta\phi = \phi_0 - \phi_1 \sim \phi_0 \sim M_{Pl}$$

One important omission:

- Have Not Addressed how to realise the standard model in this kind of a set up.

- Becomes particularly important if we want to search for observational signals of the model.

Coupling:

$$\int d^4x \sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \zeta aF\tilde{F}\right]$$

Allowed because axion psuedoscalar.

Causes direction of polarisation for linearly polarised light to rotate.

Harari, Sikivie; Carrol, Field, Jackiw

For the CMB this leads to a mixing between the E and B modes.

Proportional to $\Delta\alpha = \zeta a$

Rotation of the Polarisation of Light

Observationally:

$$\Delta\alpha = -1.1 \deg \pm 1.3 \deg \pm 1.5 \deg$$

Central value $\Delta\alpha \sim 10^{-2} radians$

Komatsu et. Al. WMAP 7-year results

In This Model:

- This coupling, almost certainly, cannot arise in a susy preserving way.

- However it could arise if we do not preserve susy.

Suppose Electromagnetism is the U(1) gauge field associated with a D5-brane. Then there is a coupling:

$$L_{EM} = \frac{(2\pi\alpha')^2 T_5}{L^2} \int d^4x \sqrt{-g} \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{L^2}{2} a F \tilde{F}\right]$$

$$\zeta = \frac{L^2}{2}$$

Implications for the Model

$$\epsilon \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{M_{Pl}^2}{2\phi^2}$$

$$\omega_{axion} = -1 + \frac{\epsilon^2}{9}$$

$$\Delta\alpha \sim \sqrt{\epsilon}$$

Comparing with observation

$$\epsilon \leq 10^{-2}$$

Thus

$$\omega_{axion} \simeq -1 + O(10^{-4})$$

Conclusion:

If coupling to electromagnetism arises due to a 5-brane, best hope of detecting signal is rotation of polarisation.

Conclusions

- We have constructed a Model of Quintessence in String Theory.

- It meets the non-trivial constraints imposed due to the relatively high scale of susy breaking.

Conclusions

- The model has many very light particles with mass $10^{-3}ev$ or lighter
- Which are coupled very weakly, akin to Randall-Sundrum II.

- There could potentially be an interesting signal from the rotation E to B mode of the CMB due to varying axion.

Conclusions

- This is a first pass.

- The model needs to be developed further. A more complete string embedding incorporating the standard model.

Conclusions

- The cosmological constant problem still present, and related to choice of initial conditions.

- Perhaps solved by anthropics.