IR issues in the inflationary universe

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arXiv:1009.2947, arXiv:1007.0468
PTP122: 779 arXiv:0902.3209
PTP122:1207 arXiv:0904.4415

and with Jaume Garriga (Barcelona univ.)

Phys.Rev.D77:024021 arXiv:0706.0295

Non-Gaussianity

- CMB non-Gaussianity might be measurable!

$$f_{NL}^{local} = 32 \pm 21$$

(Komatsu et al, ApJ supple (2010))

$$f_{NL}^{equil} = 26 \pm 140$$

$$f_{NL}^{orthogonal} = -202 \pm 104$$

⇨ Non-linear dynamics

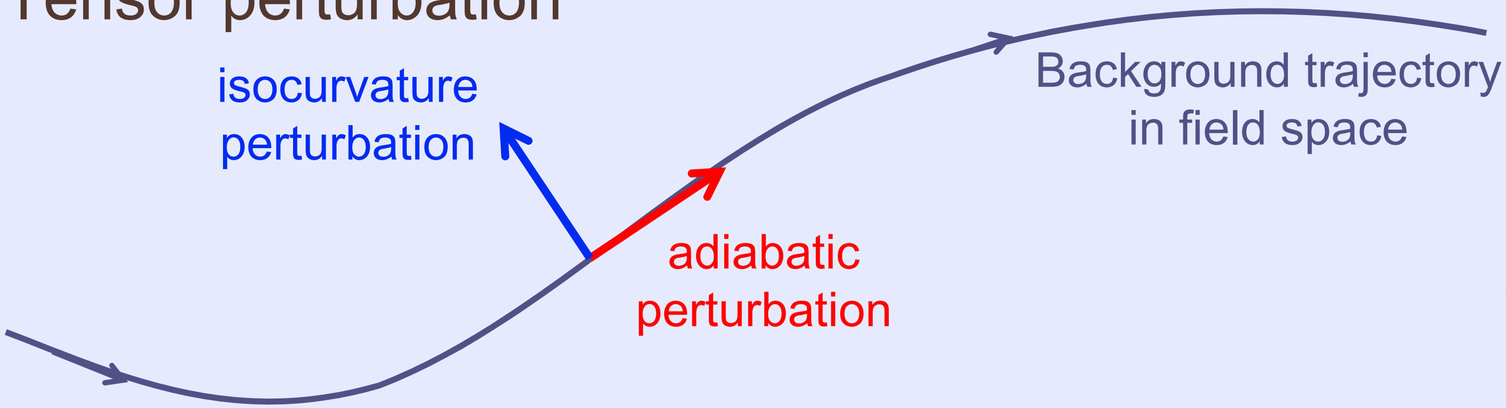
- Free field approximation is not sufficient. But once we take into account interaction, we may feel uneasy to continue to neglect loop corrections,

although it is a completely separate issue whether loop corrections during inflation are under our control or not.

Various IR issues

- IR divergence coming from k-integral
- Secular growth in time $\propto (Ht)^n$

- Adiabatic perturbation,
  - which can be locally absorbed by the choice of time slicing.
- Isocurvature perturbation
  - ≈ field theory on a fixed curved background
- Tensor perturbation



§ Isocurvature perturbation

≈ field theory on a fixed curved background
≈ field theory in de Sitter space

$m^2 > 0$: de Sitter invariant vacuum state with interaction exists. (Marolf and Morrison (1010.5327))

If we choose de Sitter invariant vacuum at the beginning, the state remains unchanged.

So, there is no secular time evolution in this case!

However, if the initial state is different, secular time evolution will happen. (Polyakov)

Question is whether this is just a relaxation process to a true vacuum state or a kind of instability?

For small mass limit, another issue arises.

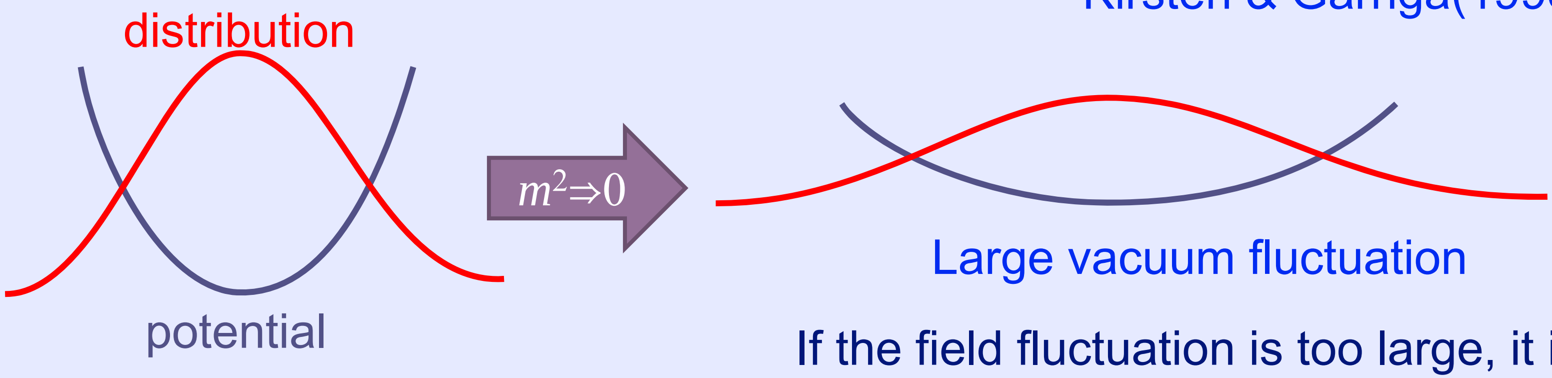
ϕ : a minimally coupled scalar field with a small mass ($m^2 \ll H^2$) in dS.

$$\left\langle \phi^2 \right\rangle^{reg} \approx \int_0^{aH} d^3k \, \frac{H^2}{k^3} \left(\frac{k}{aH} \right)^{\frac{2m^2}{3H^2}} \approx \frac{H^4}{m^2}$$

summing up only long wavelength modes beyond the Horizon scale

De Sitter inv. vac. state does not exist in the massless limit.

Allen & Folacci(1987)
Kirsten & Garriga(1993)



If the field fluctuation is too large, it is easy to imagine that a naïve perturbative analysis will break down once interaction is introduced.

Stochastic interpretation

- Let's consider local average of ϕ: (Starobinsky & Yokoyama (1994))

$$\bar{\phi} = \int_0^{aH} d^3k\, \phi_k e^{i\mathbf{kx}}$$

More and more short wavelength modes participate in $\bar{\phi}$ as time goes on.

Equation of motion for $\bar{\phi}$:

$$\frac{d\bar{\phi}}{dN} = -\frac{V'(\bar{\phi})}{3H^2} + \frac{f}{H}$$

in slow roll approximation

Newly participating modes act as random fluctuation

$$\langle \phi_k \phi_{-k} \rangle \approx H^2 / k^3$$

$$\Rightarrow \langle f(N) f(N') \rangle \approx H^4 \delta(N - N')$$

In the case of massless $\lambda\phi^4$: $\langle \bar{\phi}^2 \rangle \rightarrow \dfrac{H^2}{\sqrt{\lambda}}$

Namely, in the end, thermal equilibrium is realized : $V \approx T^4$

Wave function of the universe ~parallel universes

- Distant universe is quite different from ours.

- Each small region in the above picture
  gives one representation of many parallel universes.
- However: wave function of the universe
  = "a <u>superposition</u> of all the possible parallel universes"
  to be so to keep translational invariance of the wave fn. of the universe
- Now, "simple expectation values are really observables for us?"

“Are simple expectation values
really observables for us?”

No!

Identifying the dominant component of IR fluctuation

If we subtract the local average

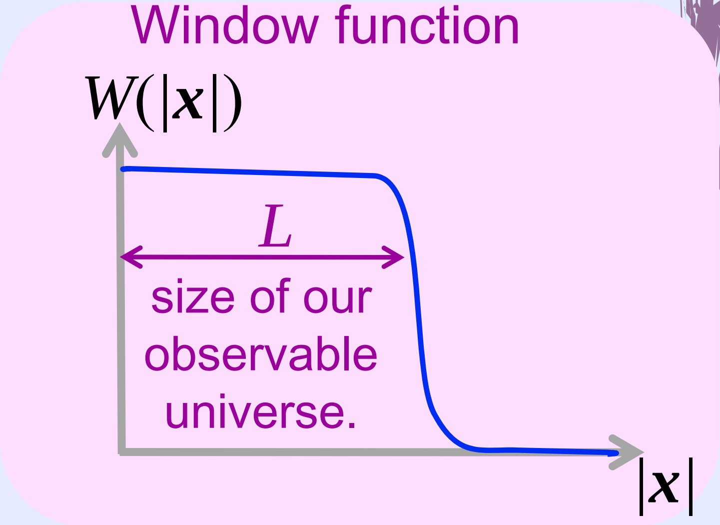
$$\delta\phi := \phi(x) - \bar{\phi}$$

with $\bar{\phi} = \int d^3x W(\boldsymbol{x})\phi(\boldsymbol{x})$

then,

$$\delta\phi = \int d^3k \left(e^{i\boldsymbol{kx}} - W_{\boldsymbol{k}}\right)\tilde{\phi}_{\boldsymbol{k}}$$

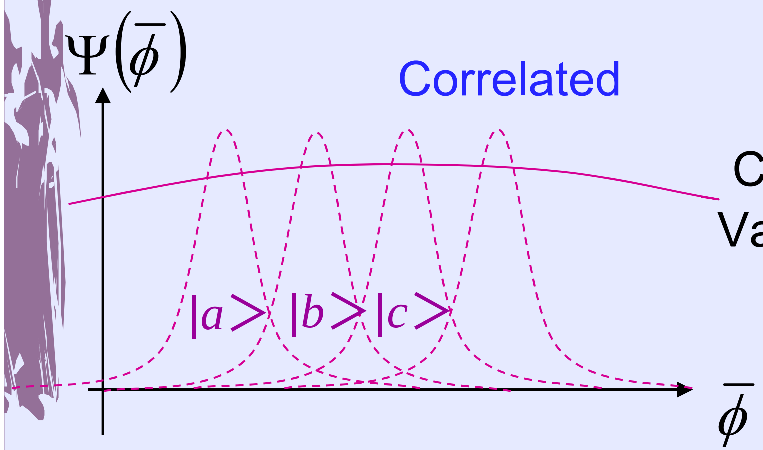
$O(k)$ unless $|\boldsymbol{x}| \gg L$



Dominant IR fluctuation is concentrated on $\bar{\phi}$

Decoherence of the wave function of the universe

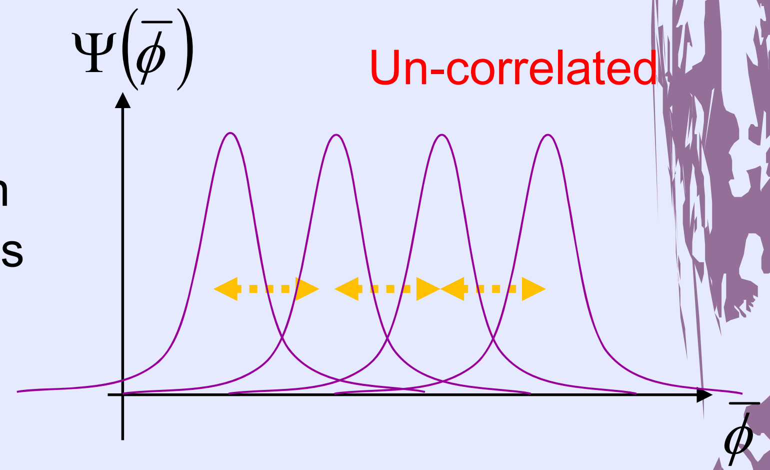
Before



Cosmic expansion
Various interactions

Decoherence

After



Superposition of wave packets

$$\rho = |\Psi(\bar{\zeta})\rangle\langle\Psi(\bar{\zeta})|$$

$$= (|a\rangle + |b\rangle + |c\rangle + \cdots)(\langle a| + \langle b| + \langle c| + \cdots)$$

Coarse graining
Unseen d.o.f.

Sorry, but this process is too complicated.

Statistical ensemble

$$\rho = |a\rangle\langle a| + |b\rangle\langle b| + |c\rangle\langle c| + \cdots$$

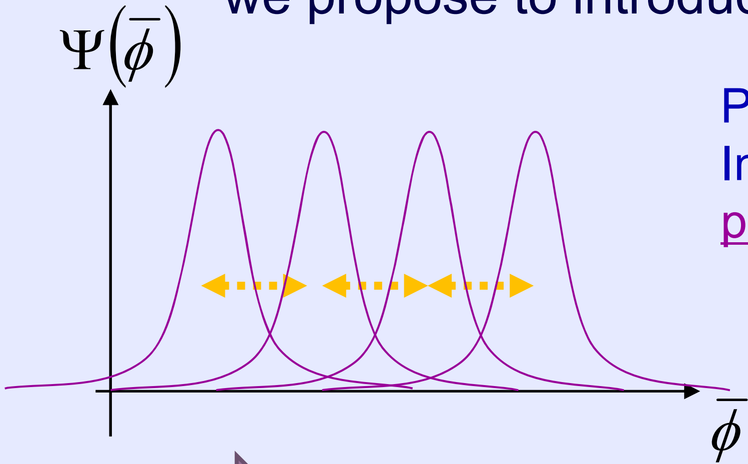
Our classical observation picks up one of the decohered wave packets.

Substitute of picking up one decohered history

(Urakawa & Tanaka PTP122:1207)

- Discussing quantum decoherence is annoying.
  - Which d.o.f. to coarse-grain?
  - What is the criterion of classicality?
- To avoid subtle issues about decoherence,

  we propose to introduce a "projection operator".



Picking up one history is difficult.
Instead, we throw away the other histories presumably uncorrelated with ours.

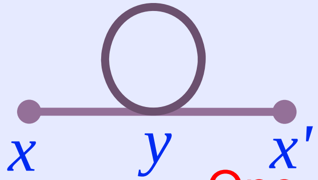
⇒ over-estimate of fluctuations

⇒ We compute

$$\langle PO \rangle / \langle P \rangle \quad \text{with} \quad P = \exp\left(-\frac{\bar{\phi}^2}{2\sigma^2} \right)$$

IR finiteness

Projection acts only on the external lines.
How the contribution from the IR modes at $k \approx k_{\min}$ is suppressed?



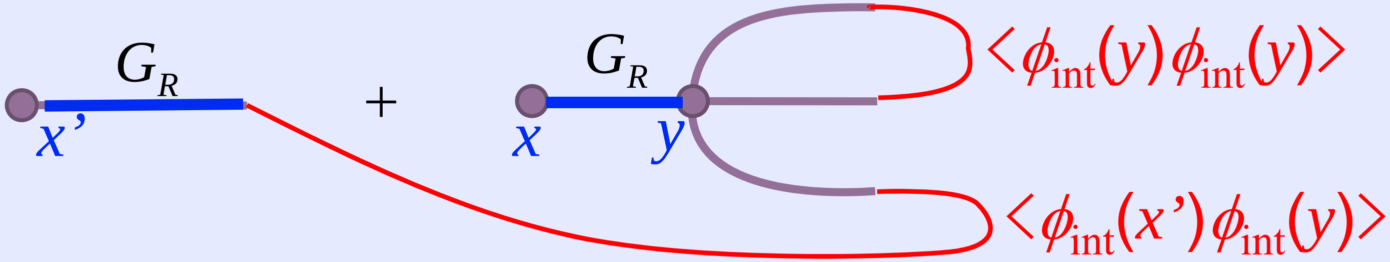
$$\approx \int d^4y\, G(x,y)\, G(x',y)\, G(y,y)$$

One of Green fns. is retarded, G_R.

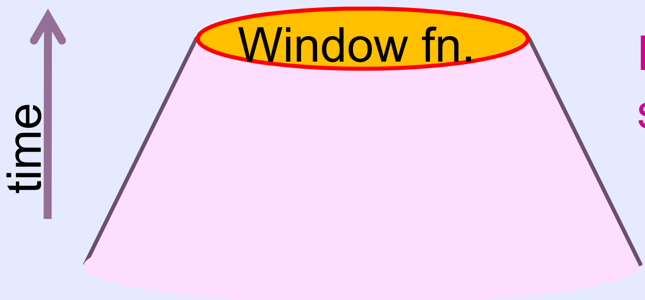
$G(y,y) \leftarrow \langle \phi_{\rm int}(y)\phi_{\rm int}(y)\rangle$

∵ Expansion in terms of interaction-picture fields:

$$\phi(x) = \phi_{\rm int}(x) + \int d^4y\, \lambda_{4-{\rm point}}(y)\, G_R(x,y)\, \phi_{\rm int}(y)^3 + \cdots$$



Integration over the vertex y is restricted to the region within the past light-cone.

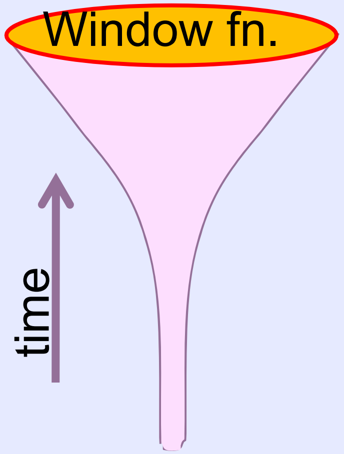


For each η_y, IR fluctuation of $\phi_{\rm int}(y)$ is suppressed since $\bar{\phi}$ is restricted.

OK to any order of loop expansion!

IR finiteness

~ secular growth in time



Past light cone during inflation
shrinks down to horizon size.

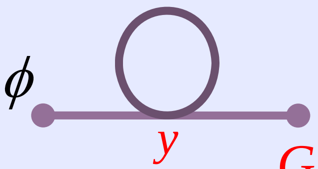
$$ds^2_{de\ Sitter} \approx \frac{1}{(-H\eta)^2}\left(-d\eta^2 + dr^2 + \cdots\right)$$

$-\eta \approx \Delta\eta = \Delta r$:past light cone

$$R_{light\ cone} = \frac{\Delta r}{-H\eta} \to \frac{1}{H}$$

However, for $\eta_y \to -\infty$, the suppression due to constraint on $\bar{\phi}$ gets weaker.

□□□□□ Then, $G(y,y) \approx \langle \phi_{\rm int}(y)\phi_{\rm int}(y)\rangle$ becomes large.



$$\phi \approx \int d^4x\, G_R(x,y) G(x',y) G(y,y)$$

$G_R(x,y) \to$ constant for $\eta_y \to -\infty$

η_y-integral looks divergent, but
homogeneous part of ϕ is constrained by the projection.

$\partial_x G_R(x,y) \to 0$ faster than $G_R(x,y)$ for $\eta_y \to -\infty$

looks OK, at least, at one-loop level !

§ About Graviton loop

2-loop order computation of $\langle h_{\mu\nu} \rangle$ in pure gravity with cosmological constant.

$$H_{eff}(t) = H\left\{1 - \kappa^4 H^4 (Ht)^2\right\}$$

(Tsamis & Woodard (1996,1997))

Screening of Λ?

There are several issues:

1) Initial vacuum is dS inv. *free* vacuum, so it might be just a relaxation process toward the true interacting dS inv. vac.

   Graviton is frequently analogous to a massless minimally coupled scalar field, but dS inv. vacuum exists for gravition. (Alen & Turyn (1987))

2) The expansion rate of the universe is not gauge invariant when there is no marker to specify the hypersurface. (Unruh (1998))

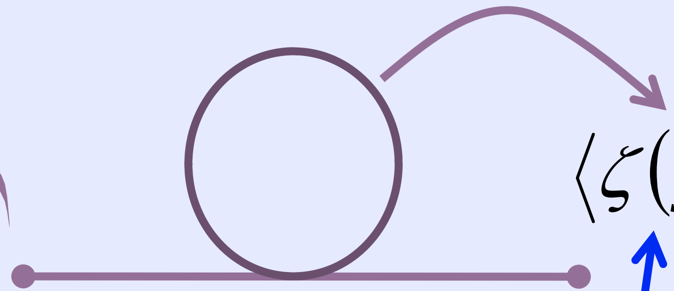
3) If we evaluate the scalar curvature R instead of H, R is really constant. (Garriga & Tanaka (2008))

Graviton in the long wavelength limit is locally gauge, isn't it?

But, proving that there is no IR effect from graviton is not trivial.

§ IR divergence in single field inflation

Details will be explained by Y.U. next.



Factor coming from this loop:

$$\langle \zeta(y)\zeta(y)\rangle \approx \int d^3k\ \underline{P(k)} \approx \log\left(aH / k_{\min}\right)$$

$\propto 1/k^3$

scale invariant spectrum
- no typical mass scale

curvature perturbation in co-moving gauge.

$$\begin{cases} h_{ij} = e^{2N+2\zeta}\left(\delta_{ij} + h_{ij}\right) \\ \delta\phi = 0 \end{cases}$$

Transverse traceless

Special property of single field inflation

Yuko Urakawa and T.T., PTP122: 779 arXiv:0902.3209

- In conventional cosmological perturbation theory, gauge is not completely fixed.

Time slicing can be uniquely specified: $\delta\phi = 0$ OK!

but spatial coordinates are not.

$$h_j^{\ j} = 0 = h_{i\ ,j}^{\ j}$$

Residual gauge d.o.f.

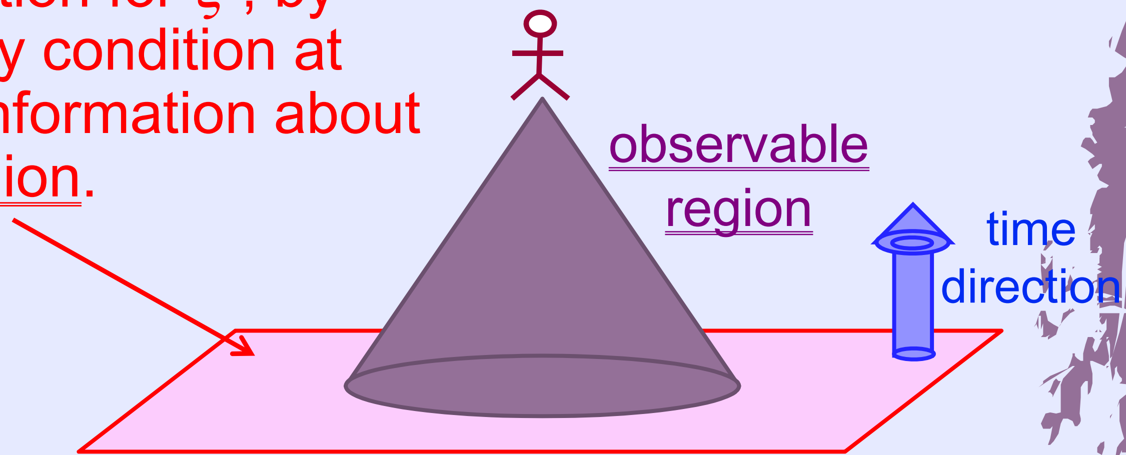
$$\delta_g h_{ij} = \xi_{i,j} + \xi_{j,i}$$

Elliptic-type differential equation for ξ^i.

$$\Delta \xi^i = \cdots$$

Not unique locally!

- To solve the equation for ξ^i, by imposing boundary condition at infinity, we need information about un-observable region.



Basic idea of the proof of IR finiteness in single field inflation

- The local spatial average of ζ can be set to 0 identically by an appropriate gauge choice.

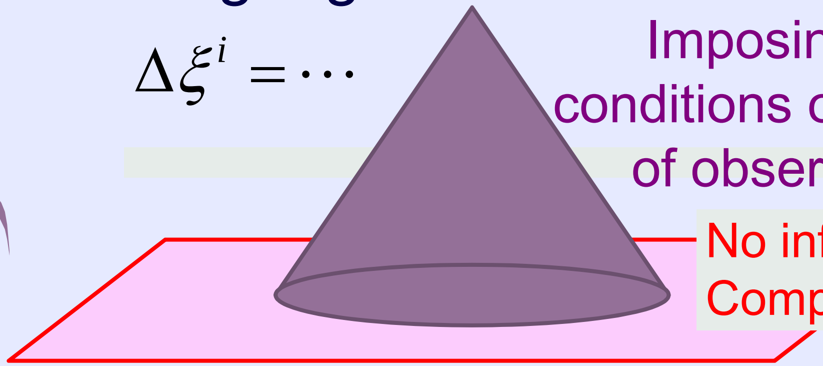
- Even if we choose such a local gauge, the evolution equation for ζ formally does not change and, it is hyperbolic. So the interaction vertices are localized inside the past light cone.

- Therefore, IR effect is completely suppressed as long as we compute ζ in this local gauge.

Complete gauge fixing vs.
Genuine gauge-invariant quantities

- Local gauge conditions.

$\Delta \xi^i = \cdots$



Imposing boundary conditions on the boundary of observable region ⇒ But unsatisfactory? The results depend on the choice of boundary conditions

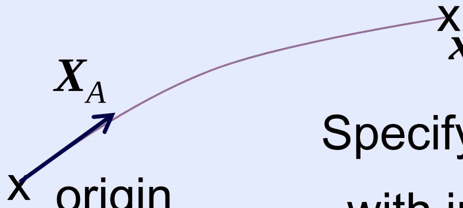
No influence from outside
Complete gauge fixing ☺

- Genuine gauge-invariant quantities.

Correlation functions for 3-d scalar curvature on ϕ=constant slice.

$\langle R(\boldsymbol{x}_1)\, R(\boldsymbol{x}_2)\rangle$ Coordinates do not have gauge invariant meaning.

(Giddings & Sloth 1005.1056)
(Byrnes et al. 1005.33307)



$\boldsymbol{x}(\boldsymbol{X}_A, \lambda=1) = \boldsymbol{X}_A + \delta \boldsymbol{x}_A$

Specify the position by solving geodesic eq $D^2 x^i / d\lambda^2 = 0$

with initial condition $Dx^i / d\lambda \big|_{\lambda=0} = X^i$

$^gR(\boldsymbol{X}_A) := R(\boldsymbol{x}(\boldsymbol{X}_A, \lambda=1)) = R(\boldsymbol{X}_A) + \delta \boldsymbol{x}_A \nabla R(\boldsymbol{X}_A) + \ldots$

$\langle {}^gR(\boldsymbol{X}_1)\, {}^gR(\boldsymbol{X}_2)\rangle$ should be genuine gauge invariant.

Translation invariance of the vacuum state takes care of the ambiguity in the choice of the origin.

Summary of what we found

1) To avoid IR divergence, the initial quantum state must be "scale invariant/Bunch Davies" in the slow roll limit.

"Wave function must be homogeneous in the residual gauge direction"

2) To the second order of slow roll, a generalized condition of "scale invariance" to avoid IR divergence was obtained, and found to be consistent with the EOM and normalization.

3) Giddings and Sloth's computation assumed adiabatic vacuum and they found no IR divergence. This means that our generalized condition of scale invariance should be compatible with the adiabatic vacuum choice.

Summary

- Isocurvature mode
  - Potentially large IR fluctuation of isocurvature mode is physical.
    - Stochastic inflation -
  - But what we really measure is not a simple expectation value.
  - We need to develop an efficient method to compute "conditional probability" in field theory.
- Tensor perturbation
  - There seem to be no IR cumulative effect of tensor modes.
  - But rigorous proof is lacking.
- Adiabatic mode
  - Adiabatic perturbation in the long wavelength limit is locally gauge.
  - In the local gauge, in which the local average of perturbation is set to 0, there is no large IR effect.
  - But, computation in the local gauge is not easy to perform.
  - Even if we compute in global gauge, true gauge-invariant observables should be free from large IR effects.
  - However, possible quantum state is restricted since the residual gauge is not completely fixed.