Scale-dependence of non-Gaussianity as a probe of the early Universe

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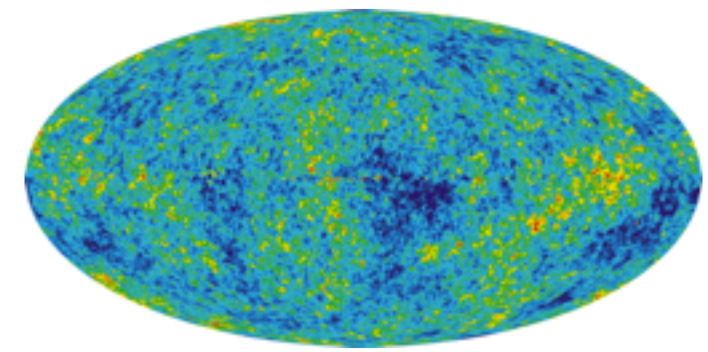
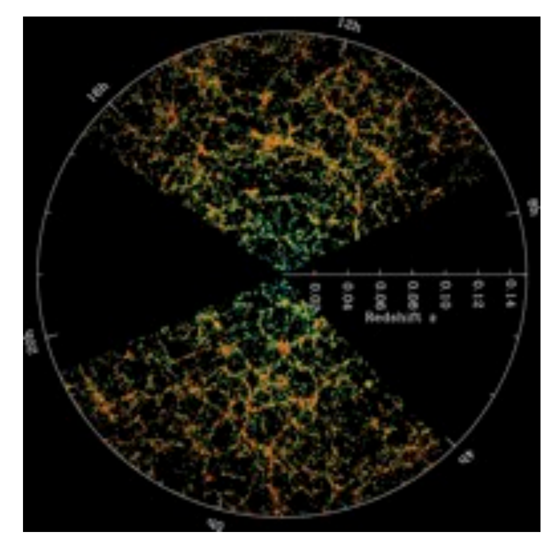
Non-Gaussianity as a probe of the early Universe

- The early Universe can be probed with primordial density fluctuations.



CMB



Large scale structure

Non-Gaussianity as a probe of the early Universe

- Observables I: Power spectrum

$$P_\zeta = A_s \left(\frac{k}{k_{\mathrm{ref}}} \right)^{n_s - 1}$$

- Observables II: Gravitational waves

$$r = \frac{P_T}{P_\zeta} \quad \text{(Tensor-to-scalar ratio)}$$

Non-Gaussianity as a probe of the early Universe

- Observables III: Bispectrum (non-Gaussianity)

Usually parametrized with the parameter $f_{\rm NL}$

$$f_{\rm NL}^{\rm local} = 32 \pm 21 \quad (68\% \text{ CL}) \qquad f_{\rm NL}^{\rm equil} = 26 \pm 140 \quad (68\% \text{ CL})$$

[WMAP7, Komatsu et al, 2010]

(we consider "local-type" in this talk.)

- Standard inflation models → $f_{\rm NL} < \mathcal{O}(1)$
- Curvaton, inhomogeneous reheating, multi-brid inflation, modulated trapping, ... → $f_{\rm NL} = \mathcal{O}(10) \sim \mathcal{O}(100)$ possible

Non-Gaussianity as a probe of the early Universe

- There are many models giving large f_{NL}

  → f_{NL} is NOT enough to differentiate models

- We need something beyond f_{NL}:

  - Using information of trispectrum (4-pt. function)

    [Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

  - Looking at scale-dependence of f_{NL}

    → This talk

Scale-dependence of $f_{\rm NL}$

- Definition: $n_{f_{\rm NL}} \equiv \frac{d \ln |f_{\rm NL}|}{d \ln k}$

$$\rightarrow \quad f_{\rm NL}(k) = f_{\rm NL}(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}} \right)^{n_{f_{\rm NL}}}$$

where

$$k \equiv (k_1 k_2 k_3)^{1/3}$$

In the following, we consider "local type": $\zeta = \zeta_G + \frac{3}{5} f_{\rm NL} \zeta_G^2$

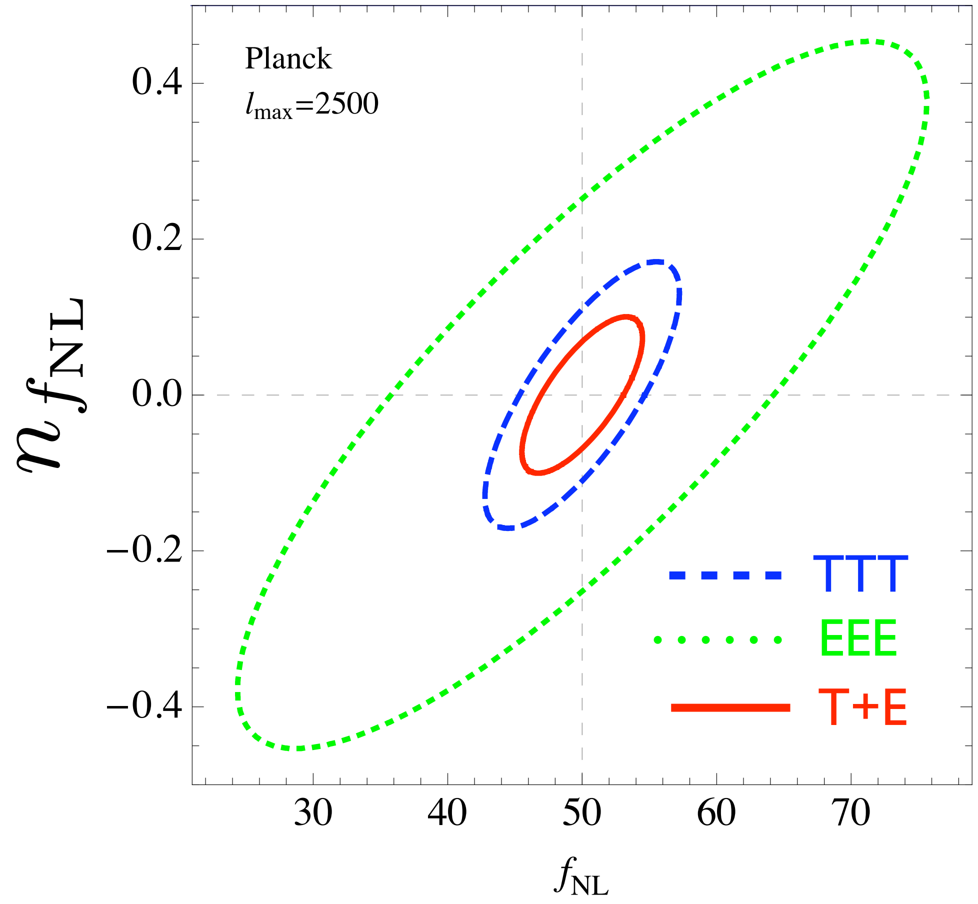
$$B_\zeta(k_1, k_2, k_3) = 2 f_{\rm NL} \Delta_\zeta^2 \left(\frac{1}{k_1^{3-(n_s-1)}} \frac{1}{k_2^{3-(n_s-1)}} + \frac{1}{k_1^{3-(n_s-1)}} \frac{1}{k_3^{3-(n_s-1)}} + \frac{1}{k_2^{3-(n_s-1)}} \frac{1}{k_3^{3-(n_s-1)}} \right)$$

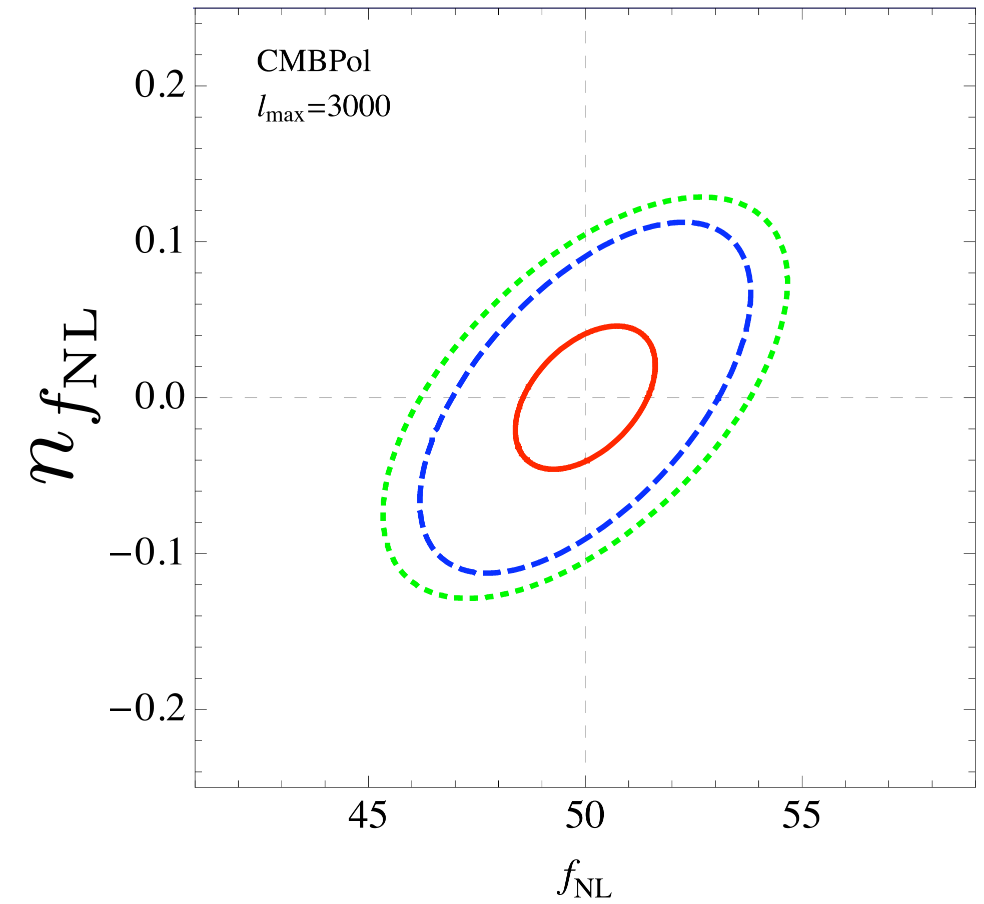
$$\left\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \right\rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Scale-dependence of $f_{\rm NL}$

- Attainable limit:

[Sefusatti et al. 2009]





$$\Delta n_{f_{\rm NL}} = 0.05 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}} \quad \text{(CMBpol)}$$

Worked example for $n_{f_{\rm NL}}$

- Curvaton model

[Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]

Worked example for $n_{f_{\rm NL}}$

- Curvaton model

[Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]

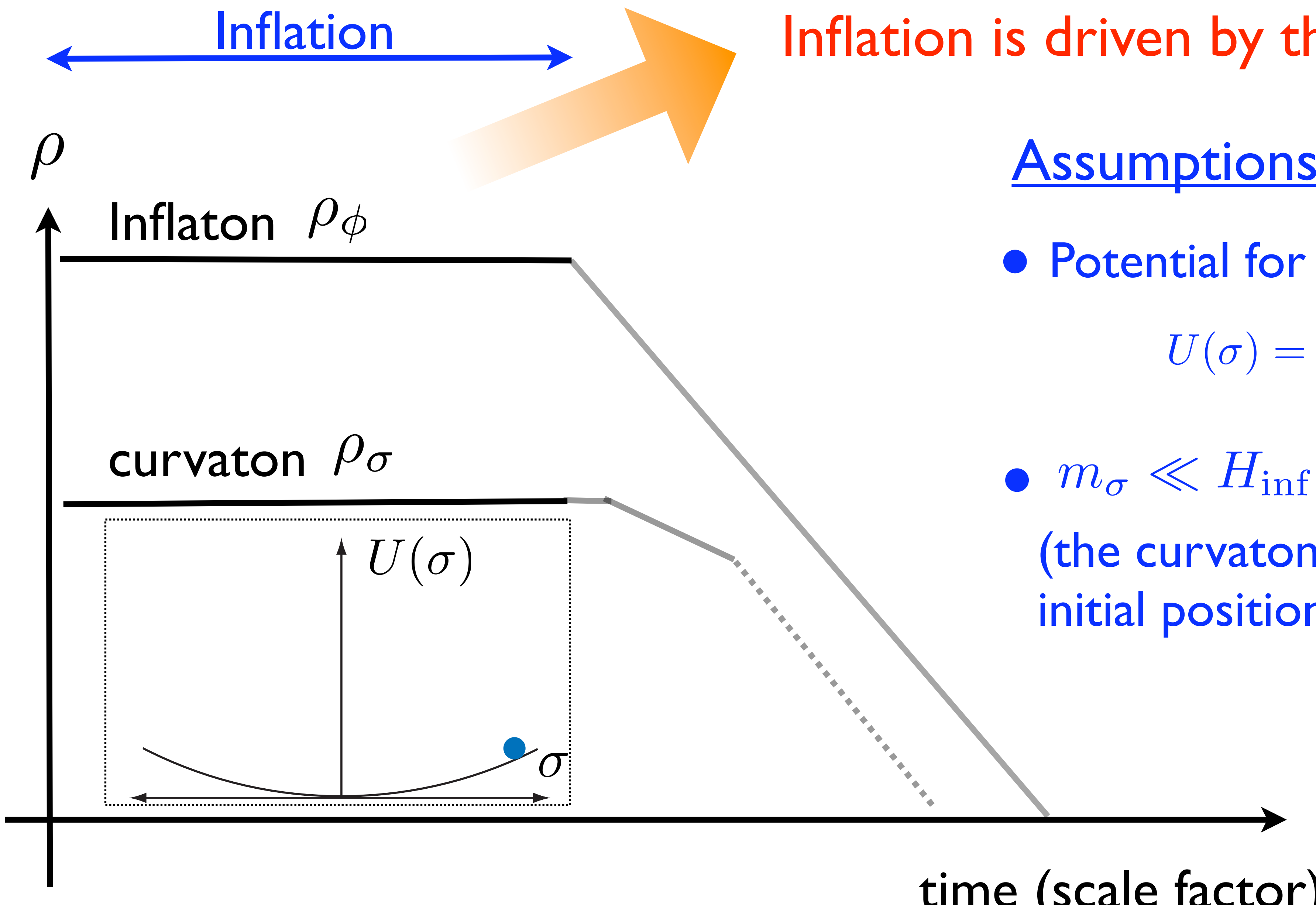
  - Curvaton + Inflaton mixed scenario

[Langlois, Vernizzi, 2004; Moroi, TT, Toyoda, 2005;
Ichikawa, Suyama, TT, Yamaguchi, arXiv:0802.4138]

  - Curvaton with self-interaction

[Enqvist, Nurmi astro-ph/0508573;
Enqvist, TT, arXiv:0807.3069;
Enqvist, Nurmi, Taanila, TT, arXiv:0912:4657]

A brief thermal history of the curvaton scenario



Inflation is driven by the inflaton.

Assumptions:

- Potential for the curvaton is

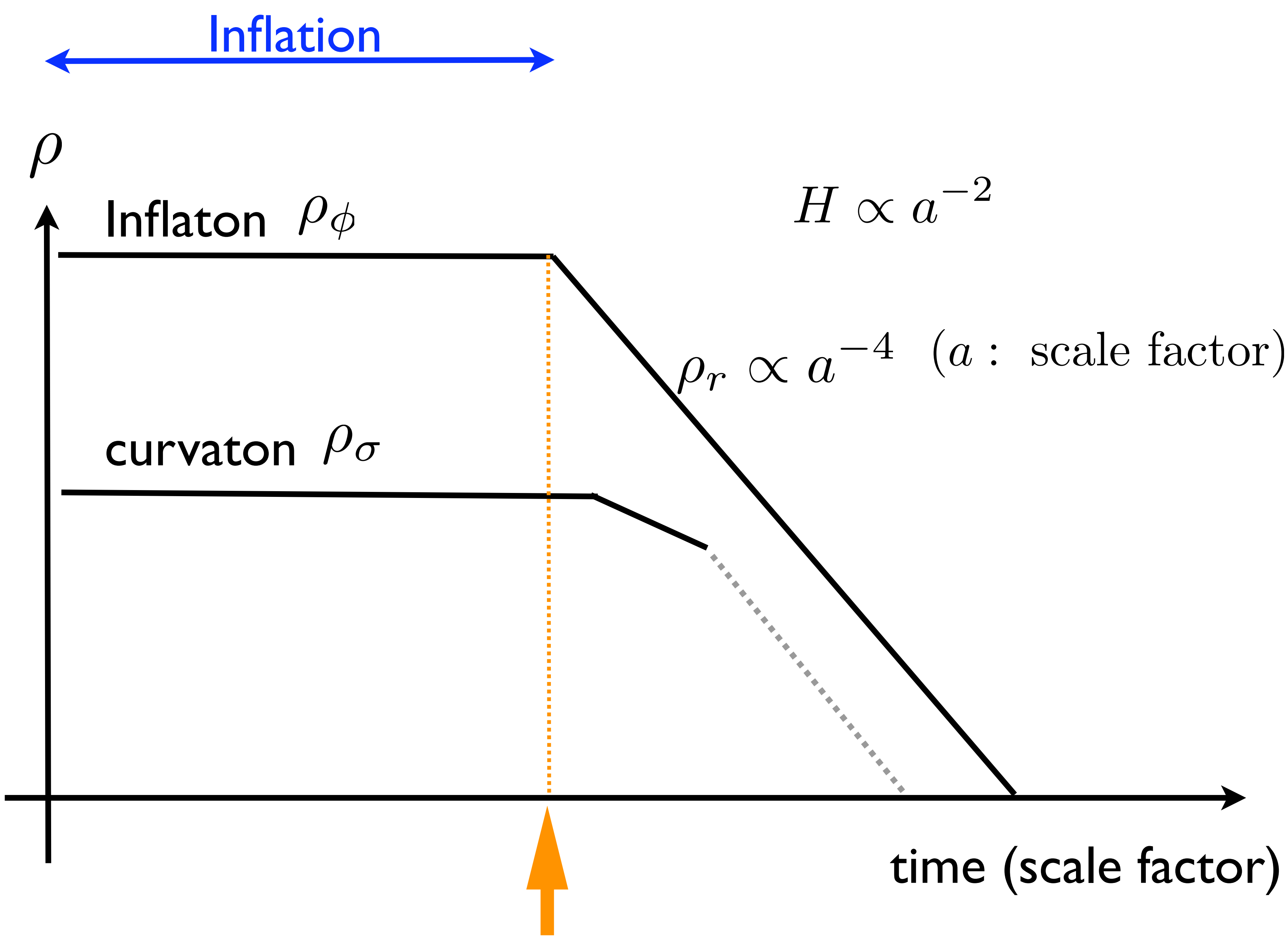
$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2$$

- $m_\sigma \ll H_{\rm inf}$

(the curvaton almost stays at the initial position during inflation.)

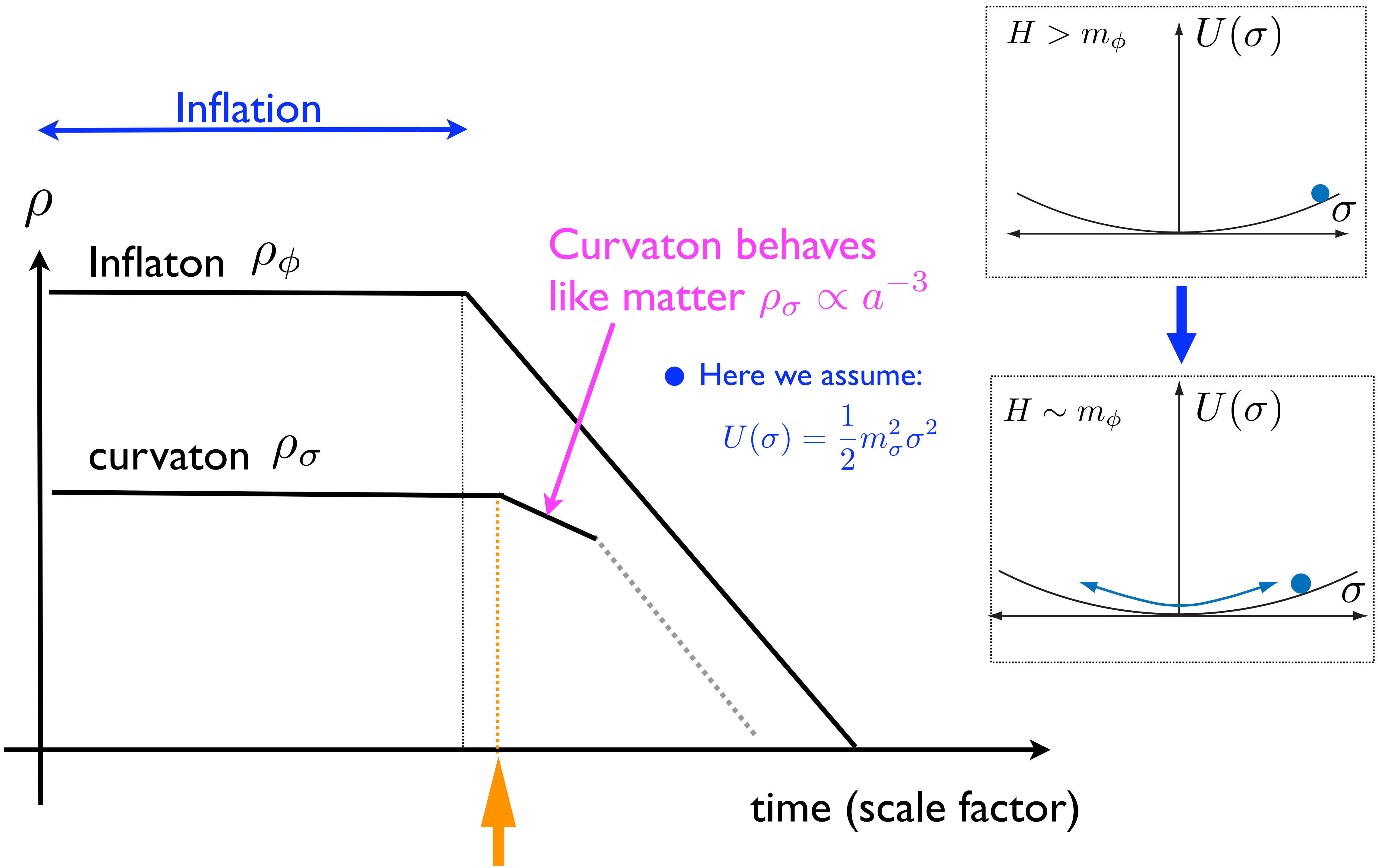
Eq. of motion for a scalar field: $\ddot{\sigma} + 3H\dot{\sigma} + \frac{dU}{d\sigma} = 0$

A brief thermal history of the curvaton scenario



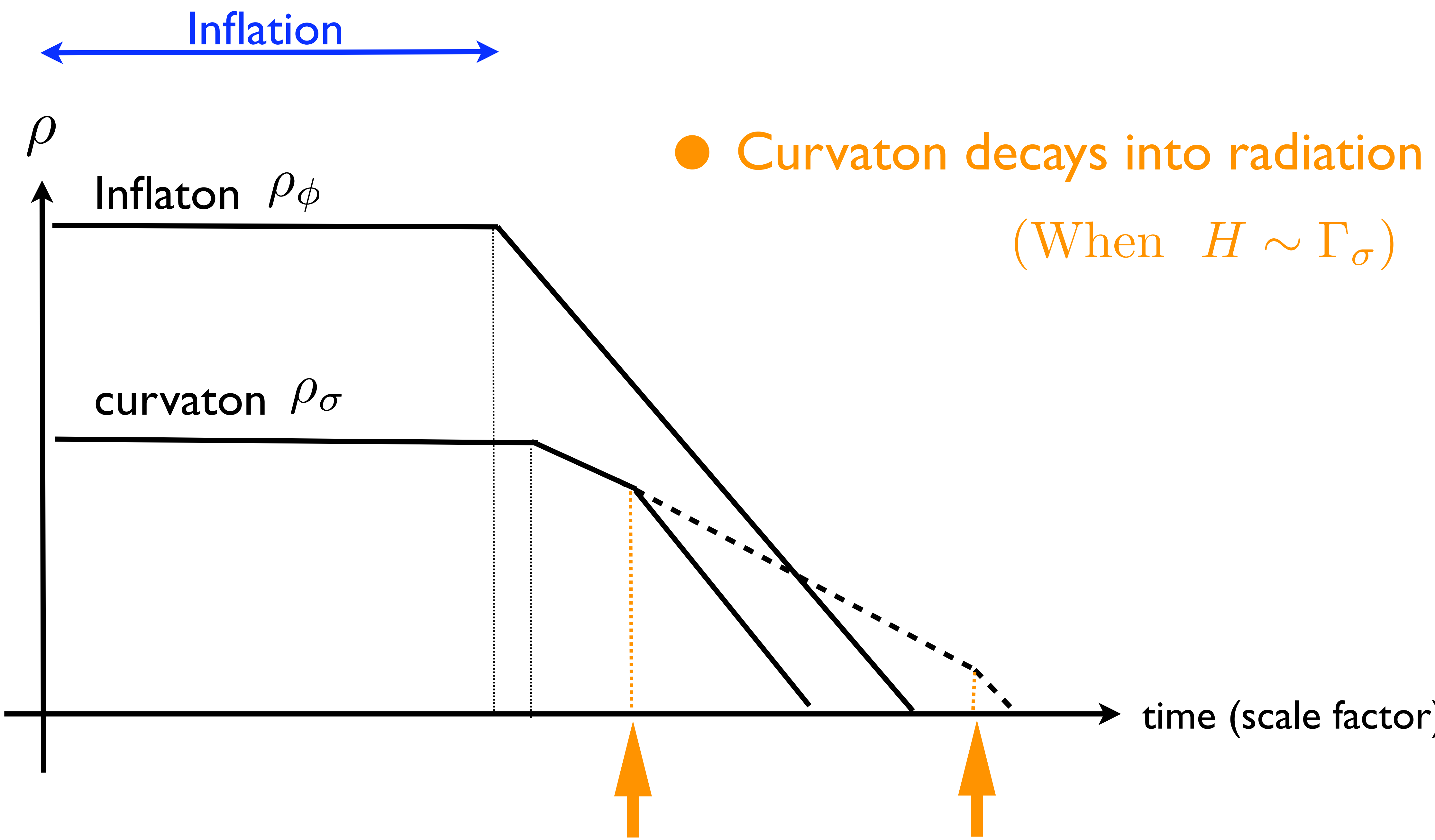
Inflaton decays into radiation

A brief thermal history of the curvaton scenario



Curvaton begins to oscillate (When $H \sim m_\sigma$)

A brief thermal history of the curvaton scenario



- Curvaton decays into radiation

  $(\text{When} \quad H \sim \Gamma_\sigma)$

- Curvaton could decay before/after it dominates the Universe

Mixed curvaton-inflaton model

Mixed curvaton-inflaton model

Even in the curvaton model, fluctuations of the inflaton can exist and contribute to the curvature fluc.

- the curvature perturbation:

$$\zeta = \underbrace{N_\phi \delta\phi_* + \frac{1}{2} N_{\phi\phi} \delta\phi_*^2}_{\text{Inflaton} \ \zeta_{\rm inf}} + \underbrace{N_\sigma \delta\sigma_* + \frac{1}{2} N_{\sigma\sigma} \delta\sigma_*^2}_{\text{Curvaton} \ \zeta_{\rm cur}}$$

- relative size

$$\frac{\zeta_{\rm cur}}{\zeta_{\rm inf}} \sim f_{\rm dec} \sqrt{\epsilon} \frac{M_{\rm pl}}{\sigma_*}$$

where:

$$f_{\rm dec} = \left. \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma} \right|_{\rm dec} \sim \left. \frac{\rho_\sigma}{\rho_{\rm total}} \right|_{\rm dec}$$

$$\epsilon = \frac{1}{2} M_{\rm pl}^2 \left(\frac{V_\phi}{V} \right)^2$$

Mixed curvaton-inflaton model

- Spectral index, tensor mode and nonG scale-dep.

  - $n_s - 1 = -2\epsilon + 2\eta_{\sigma\sigma} - \dfrac{4\epsilon - 2\eta}{1+R}$

  - $r = \dfrac{16\epsilon}{1+R}$

  - $n_{f_{\rm NL}} = \dfrac{4}{1+R}\left(2\epsilon + \eta_{\sigma\sigma} - \eta\right)$

where: $R \equiv \dfrac{P_{\rm cur}}{P_{\rm inf}}$

$$\eta_{\sigma\sigma} = M_{\rm pl}^2 \frac{U_{\sigma\sigma}}{3H_*^2}$$

$$\frac{6}{5} f_{NL} = \frac{N_\sigma^2 N_{\sigma\sigma} + N_\phi^2 N_{\phi\phi}}{(N_\sigma^2 + N_\phi^2)^2}$$

$$= \frac{R^2 f_{\rm NL}^{(\rm cur)} + f_{\rm NL}^{(\rm inf)}}{(R+1)^2}$$

- Small-field inflation case $(\epsilon \ll \eta)$

  --- $n_{f_{\rm NL}} \simeq -2(n_s - 1) \sim 0.08$

  --- $r \ll \mathcal{O}(1)$

(might be)
detectable even with Planck!

[Byrnes et al. 2009]

Mixed curvaton-inflaton model

- Spectral index, tensor mode and nonG scale-dep.

  - $$n_s - 1 = -2\epsilon + 2\eta_{\sigma\sigma} - \frac{4\epsilon - 2\eta}{1+R}$$ where: $$R \equiv \frac{P_{\rm cur}}{P_{\rm inf}}$$

  - $$r = \frac{16\epsilon}{1+R}$$ $$\eta_{\sigma\sigma} = M_{\rm pl}^2 \frac{U_{\sigma\sigma}}{3H_*^2}$$

  - $$n_{f_{\rm NL}} = \frac{4}{1+R}(2\epsilon + \eta_{\sigma\sigma} - \eta)$$

- Large-field inflation case (e.g. chaotic inflation) $(\epsilon \sim \eta)$

  sizable r *(>O(0.01))* and large *fNL (>O(10))* can both possible

  (Most models with large *fNL* give very small r)

Self-interacting curvaton model

Self-interacting curvaton model

- In some curvaton models, the curvaton potential can deviate from a (purely) quadratic form.

For example,

- When an MSSM flat direction is the curvaton, its potential can be given as:

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{\lambda^2 \sigma^{2(n-1)}}{2^{n-1} M^{2(n-3)}}$$

→ The form of the potential can deviate from the quadratic one.

- Interesting prediction for the scale-dependence of non-Gaussianity.

$n_{f_{\rm NL}}$ in the self-interacting curvaton

In the following, we assume: $V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^n$

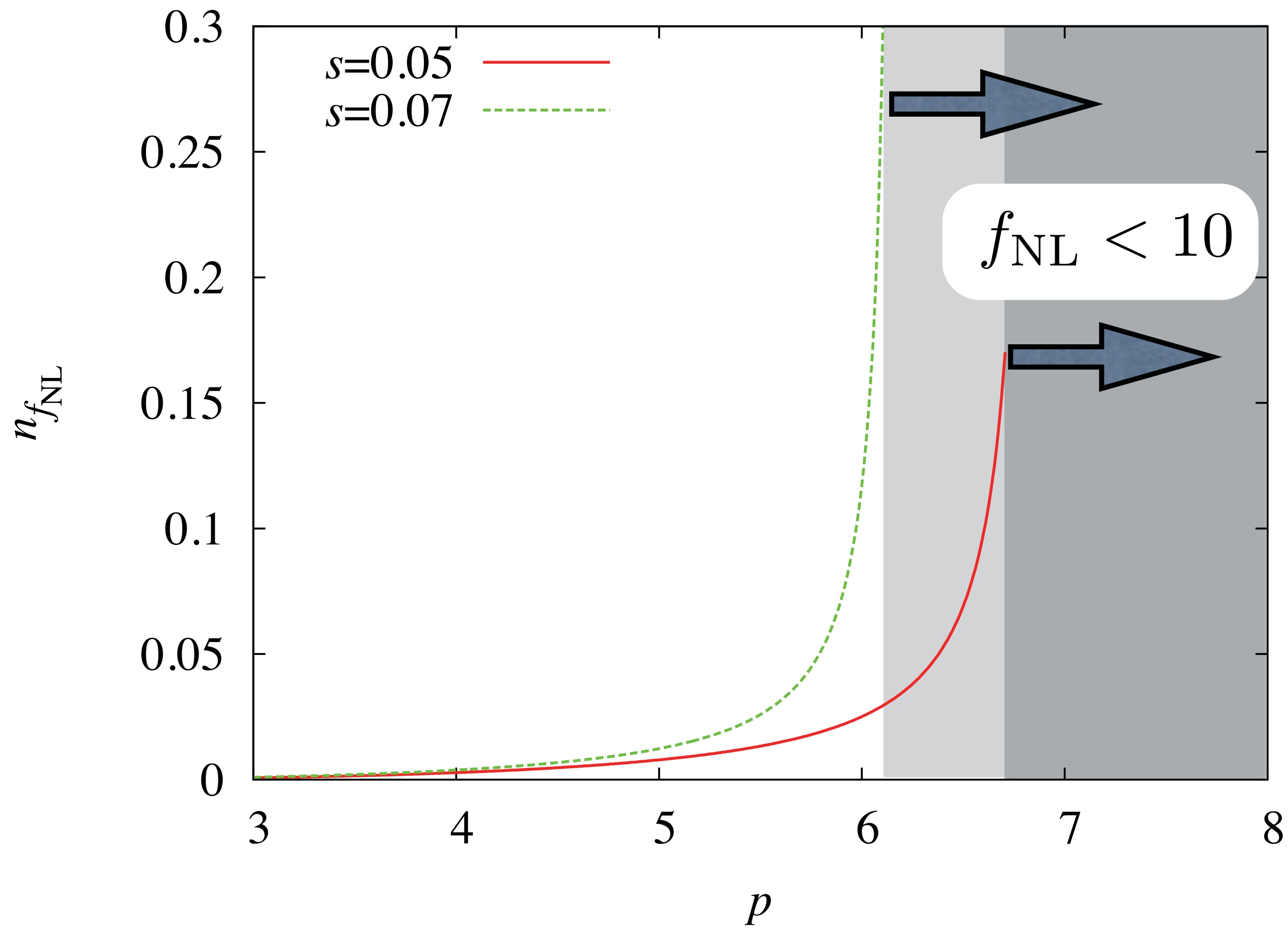
■ Scale-dependence of $f_{\rm NL}$

$$n_{f_{\rm NL}} = \frac{V'''(\sigma_*)}{3H_*^2} \left(\frac{\sigma_{\rm osc}\sigma'_{\rm osc}}{(\sigma'_{\rm osc})^2 + \sigma_{\rm osc}\sigma''_{\rm osc}} \right)$$

- When the potential is purely quadratic, no scale-dependence

- Non-zero $n_{f_{\rm NL}}$ may indicate a self-interacting curvaton.

$n_{f_{\rm NL}}$ in the self-interacting curvaton

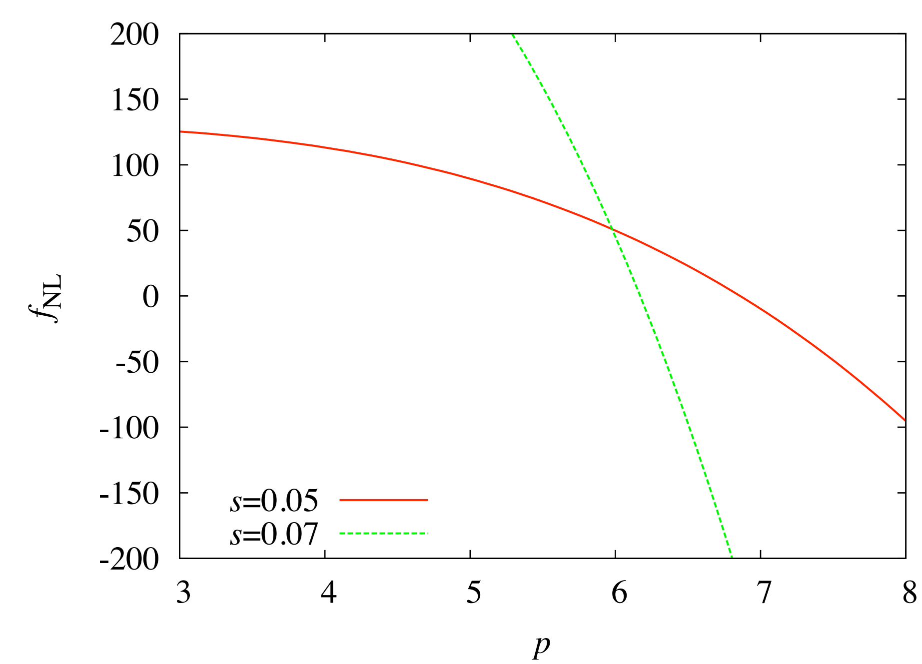


where

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^p$$

$$s \equiv 2\lambda \left(\frac{\sigma_*}{m_\sigma} \right)^{p-2}$$

[Byrnes, Enqvist, TT 2010]

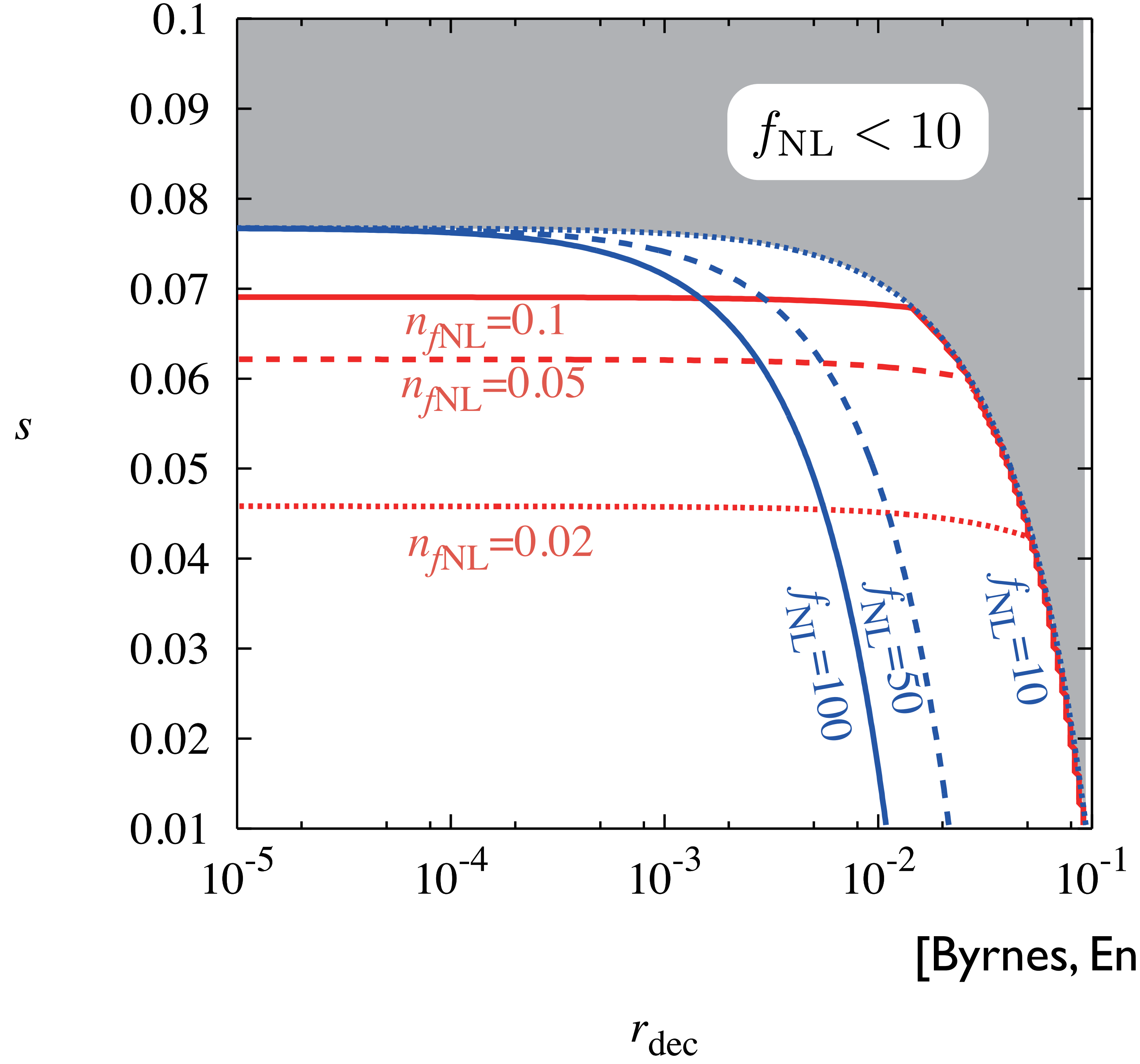


- Detectable region (CMBpol):

$$\Delta n_{f_{\rm NL}} = 0.05 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}}$$

$n_{f_{\rm NL}}$ in the self-interacting curvaton

(case with n=6)

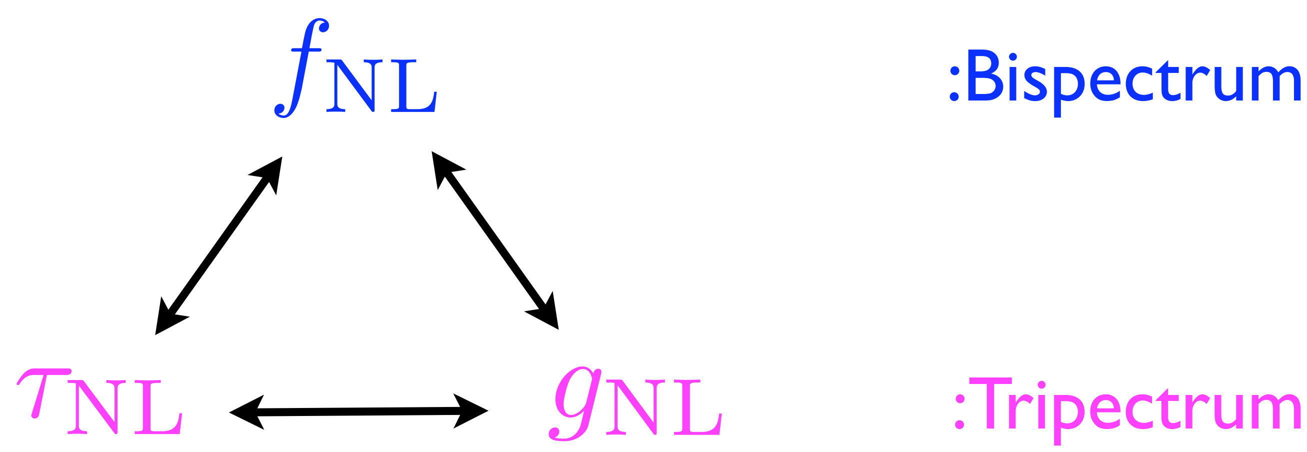


[Byrnes, Enqvist, TT 2010]

Using both Bispectrum and trispectum

Using bispectrum and trispectum

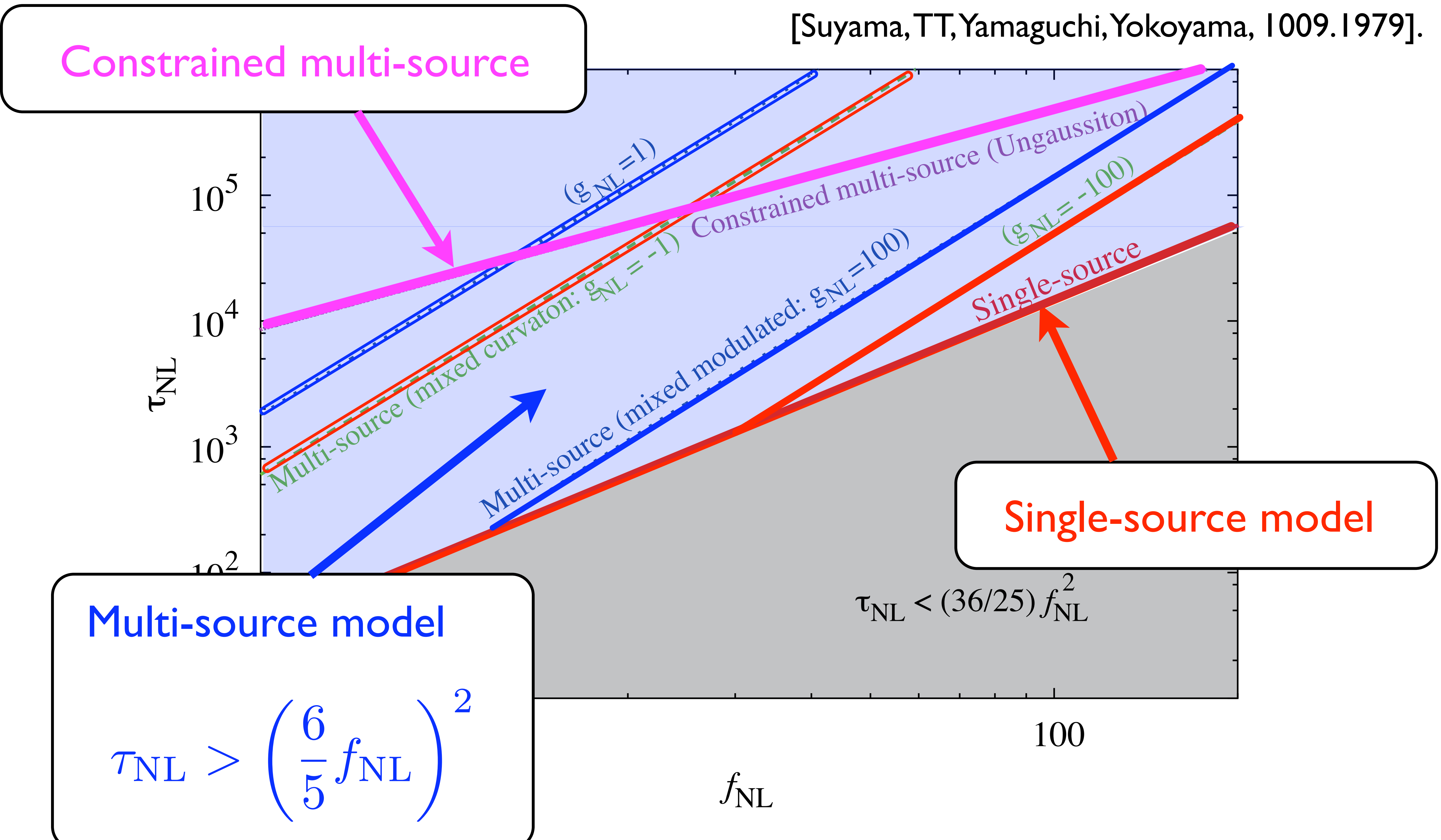
- There are some relation between the non-linearity parameters in most models:



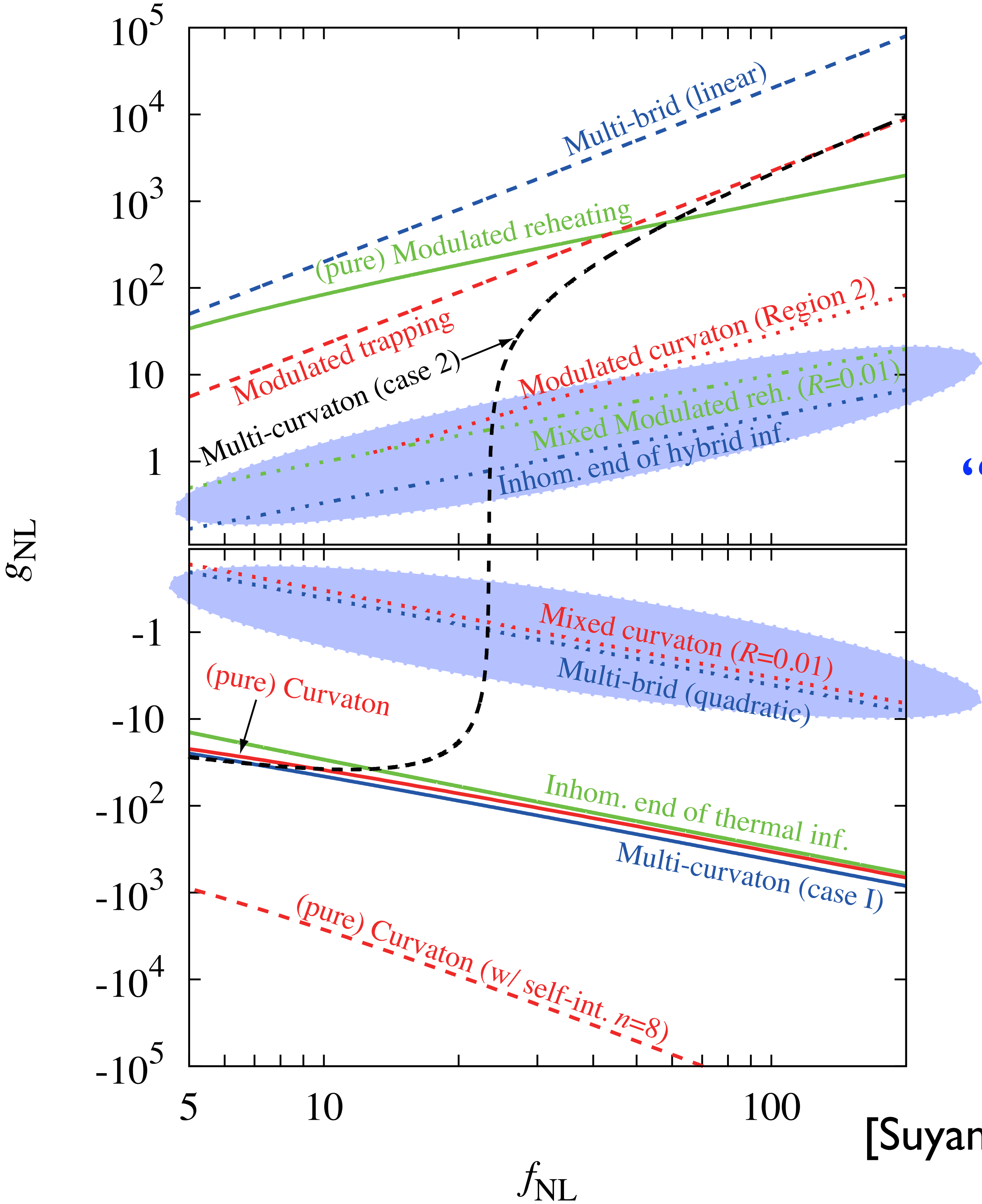
By using "consistency relation" between these parameters, we can divide the models into some categories.

f_{NL} - τ_{NL} diagram

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].



f_{NL} - g_{NL} diagram

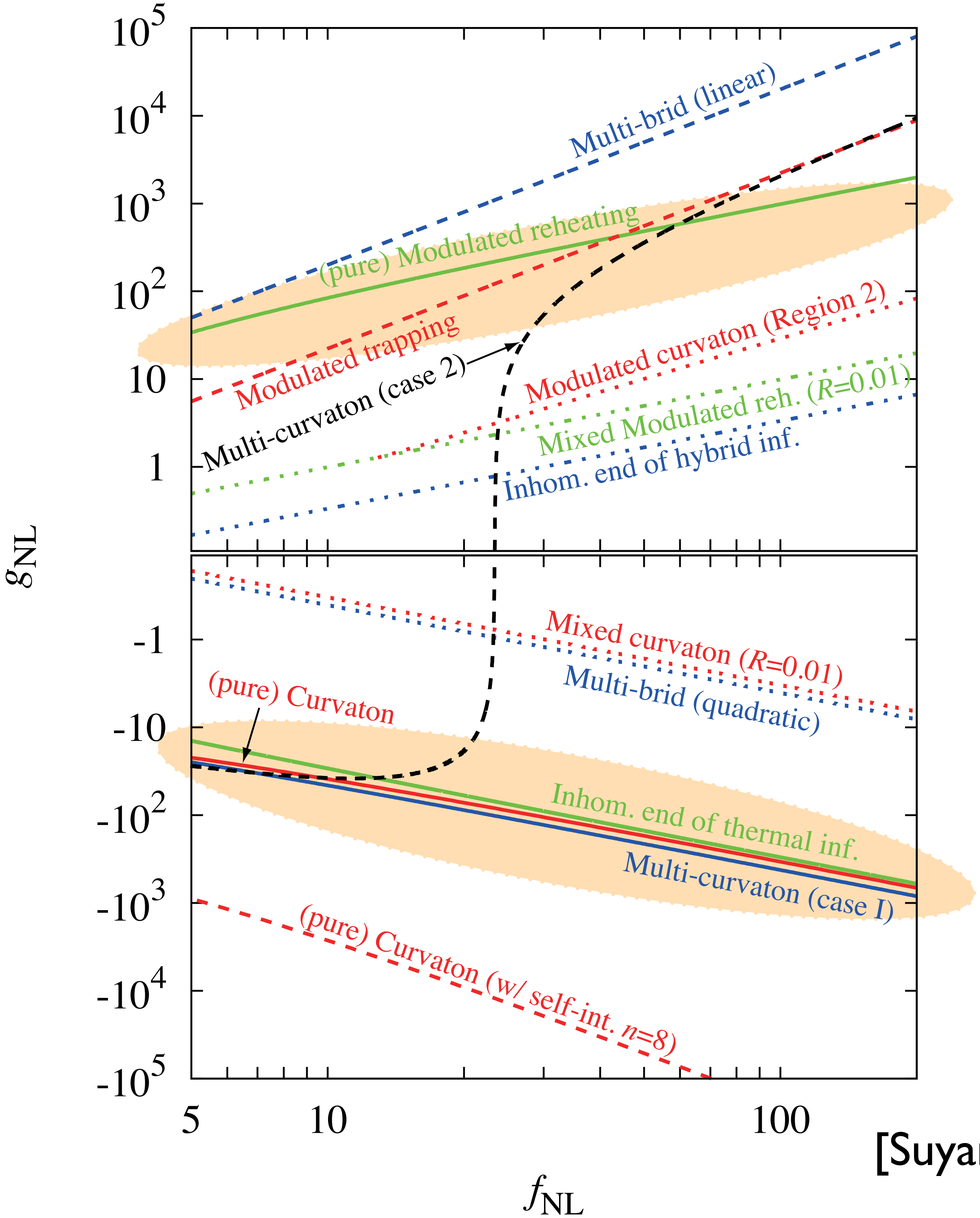


“Suppressed” g_{NL} Type

$$g_{NL} \sim (\text{suppression factor}) \times f_{NL}$$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

$f_{\rm NL}$ - $g_{\rm NL}$ diagram

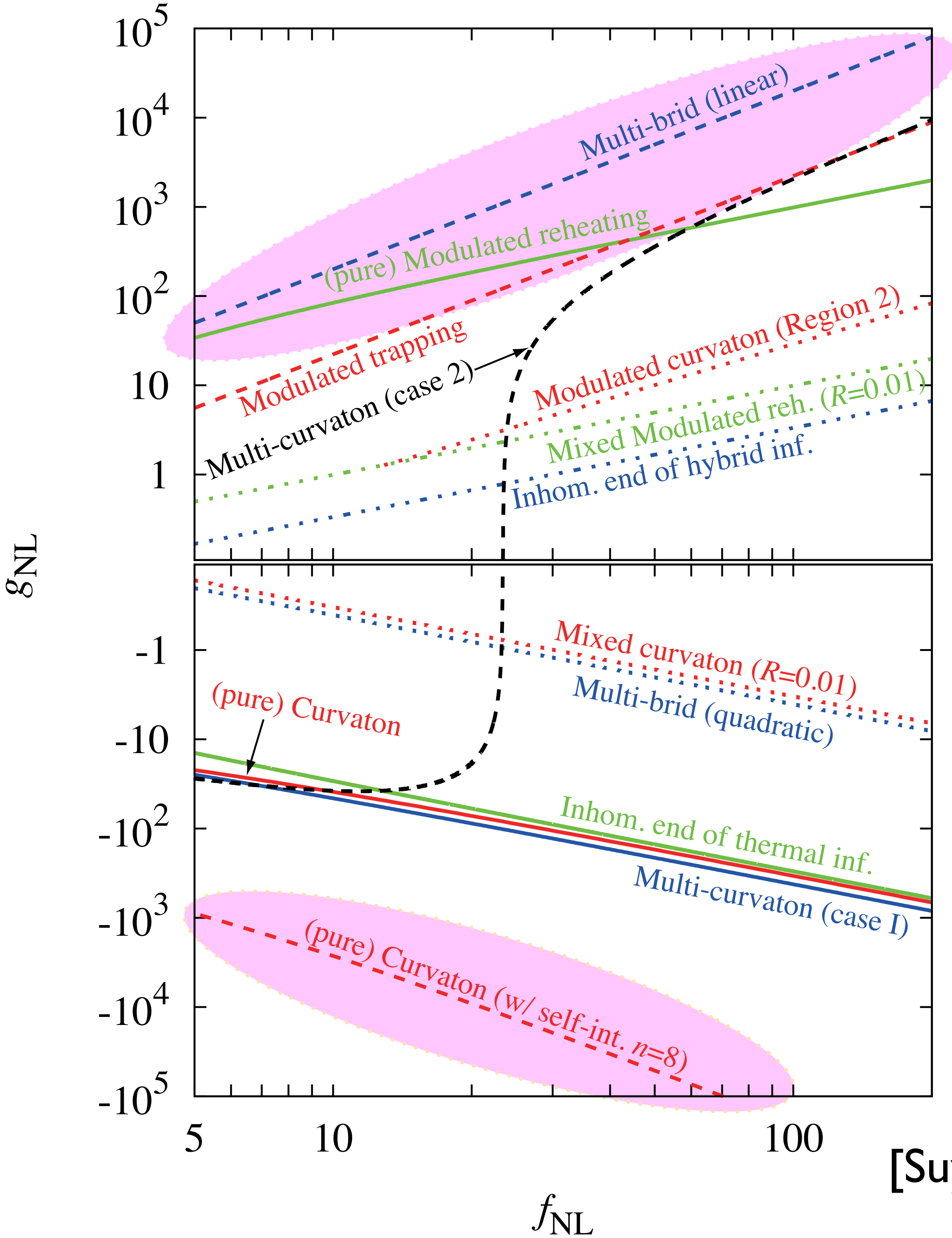


"Linear" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}$$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

$f_{\rm NL}$ - $g_{\rm NL}$ diagram



"Enhanced" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}^n$$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

Summary

- Information on $f_{\rm NL}$ is NOT enough to differentiate models of primordial fluctuations.

- Scale-dependence of non-Gaussianity ($n_{f_{\rm NL}}$) can be useful to discriminate models of large non-G.

- Some models (e.g.. mixed, self-interacting curvaton) predict large $n_{f_{\rm NL}}$ which can be testable with future obs.

- Scale-dependence of non-G. might be worth investigating more (e.g., in other models)