ExDiP2010, KEK, 9 November, 2010

Anisotropic inflation with gauge kinetic function

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The nature of primordial fluctuations

As you know the horizon problem can be resolved by slow roll inflation.

We need to assume the initial homogeneity in order to have inflation.

Once the inflation occurs, the cosmic no-hair conjecture suggests that the exponential expansion erases any classical anisotropy.

It is believed that the origin of the large scale structure of the universe is in remaining quantum fluctuations during de Sitter inflation.

Thus we have the following predictions:

First of all, slow roll condition implies Gaussian statistics $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = P(\mathbf{k}_1,\mathbf{k}_2)$

Moreover, initial homogeneity implies to statistical homogeneity $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$

And, cosmic no-hair suggests statistical isotropy $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(k_1=|\mathbf{k}_1|)$

Finally, deSitter invariance yields scale free spectrum $P(k) \approx const.$

The above predictions are robust at the zeroth order.

Precision tests of inflation

However, precision cosmology forces us to look at fine structures of fluctuations!

Scale free spectrum?

There should be a slight tilt because the expansion is not exactly deSitter.
The deviation from deSitter can be characterized by the slow roll parameter.
Hence, the tilt should be of the order of the slow roll parameter.

Gaussian ?

There is small non-gaussianity of the order of the slow roll parameter.

Statistical isotropy?

Thus, it is legitimate to seek a slight deviation from the statistical isotropy.

In this talk, we argue that the statistical anisotropy is ubiquitous

$$\left\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\right\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$$

In fact, we have the following theoretically well motivated models for that.

Gauge kinetic function in the sky

low energy

Superstring theory → **Supergravity**

- Kahler potential K
- Superpotential W
- Gauge kinetic function f

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial\phi^i \partial\phi^{\bar{j}}}$$

$$D_i W = \frac{\partial W}{\partial \phi^i} + \kappa^2 \frac{\partial K}{\partial \phi^i} W$$

$$S = \int d^4x \left[\sqrt{-g} R + g_{i\bar{j}} \partial^\mu \phi^i \partial_\mu \phi^{\bar{j}} - e^{\kappa^2 K} g^{i\bar{j}} \left(D_i W D_j \overline{W} - 3\kappa^2 |W|^2 \right) - \frac{1}{4} \mathrm{Re}\, f_{ab}(\phi^i) F^{a\mu\nu} F^b_{\mu\nu} + \cdots \right]$$

The cosmological roll of K and W in inflation has been discussed so far.

However, the roll of vector fields in inflationary scenario has been overlooked.

The purpose of this talk is to give a light on the roll of gauge kinetic function in inflation and show that

- **Anisotropic inflation is naturally realized due to gauge kinetic function.**
- **As a consequence, statistical anisotropy is produced in supergravity.**
- **There arises cross correlation between scalar and tensor perturbations.**
- **Thus, cosmological observations can constrain gauge kinetic functions!**

Plan of my talk

1. Inflation with a gauge kinetic function
2. Cosmological perturbation theory
   in a simple Bianchi universe
3. The nature of primordial fluctuations
   in anisotropic inflation
4. Summary

Inflation with a gauge kinetic function

A simple model

Action

gauge kinetic function

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}\left(\partial_\mu\phi\right)^2-V(\phi)-\frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

Scalar　　Vector

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

For homogeneous background, the time component can be eliminated by gauge transformation.

Let the direction of the vector be x - axis

$$A_\mu=(0,\ v(t),\ \ 0\ \ ,\ \ 0\ \ )\qquad\qquad \phi=\phi(t)$$

Then, the metric should be Bianchi Type-I

Anisotropy

$$ds^2=-dt^2+e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2+e^{2\sigma(t)}\left(dy^2+dz^2\right)\right]$$

Scale Factor　　Plane Symmetry

The action reduces to

$$S=\int d^4x\ e^{3\alpha}\left[\frac{3}{\kappa^2}(-\dot{\alpha}^2+\dot{\sigma}^2)+\frac{1}{2}\dot{\phi}^2-V(\phi)+\frac{1}{2}f^2(\phi)e^{-2\alpha+4\sigma}\dot{v}^2\right]$$

$$\dot{v}=f^{-2}(\phi)e^{-\alpha-4\sigma}E$$

const. of integration

Basic equations

Hamiltonian Constraint $\bullet = \partial_t$

$$\dot{\alpha}^2 = \dot{\sigma}^2 + \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{E^2}{2} f^{-2}(\phi) e^{-4\alpha-4\sigma}\right]$$

Scale factor

$$\ddot{\alpha} = -3\dot{\alpha}^2 + \kappa^2 V(\phi) + \frac{\kappa^2 E^2}{6} f^{-2}(\phi) e^{-4\alpha-4\sigma}$$

Anisotropy

$$\ddot{\sigma} = -3\dot{\alpha}\dot{\sigma} + \frac{\kappa^2 E^2}{3} f^{-2}(\phi) e^{-4\alpha-4\sigma}$$

Scalar field $' = \partial_\phi$

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - V_\phi(\phi) + E^2 f^{-3}(\phi) f_\phi(\phi) e^{-4\alpha-4\sigma}$$

Isotropic Power-law Inflation

Let us start with a natural choice for potential and gauge kinetic functions.

Exponential functional $V = V_0 e^{\lambda \frac{\phi}{M_p}}$ $f = f_0 e^{\rho \frac{\phi}{M_p}}$

In this case, we know there exists power-law inflation

Power-law ansatz $\alpha = x \log t$ $\sigma = y \log t$ $\frac{\phi}{M_p} = z \log t + \phi_0$

$$\Longrightarrow \quad x = \frac{2}{\lambda^2} \qquad y = 0 \qquad z = -\frac{2}{\lambda} \qquad \frac{V_0}{M_p^2} e^{\lambda \phi_0} = \frac{2(6-\lambda^2)}{\lambda^4}$$

This solution represents an isotropic power-law inflation

$$ds^2 = -dt^2 + t^{4/\lambda^2}\left(dx^2 + dy^2 + dz^2\right)$$

In order to have a sufficient inflation, we need $\lambda \ll 1$

Is this a unique exact solution?

Anisotropic Power-law inflation

With the same ansatz, we obtained the following new solution

Anisotropic inflation

$$ds^2 = -dt^2 + t^{2x}\left[t^{-4y}dx^2 + t^{2y}\left(dy^2 + dz^2\right)\right]$$

$$x = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)} \qquad y = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)} \qquad z = -\frac{2}{\lambda}$$

consistency check

$$\frac{E^2 f_0^{-2}}{M_p^2} e^{-2\rho\phi_0} = \frac{\left(\lambda^2 + 2\rho\lambda - 4\right)(-\lambda^2 + 4\rho\lambda + 12\rho^2 + 8)}{2\lambda^2(\lambda + 2\rho)^2} \Longrightarrow \lambda^2 + 2\rho\lambda - 4 > 0$$

Apparently, the expansion is anisotropic and its degree of anisotropy is given by

$$\frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{1}{3} I \varepsilon$$

$$I = \frac{\lambda^2 + 2\rho\lambda - 4}{\lambda^2 + 2\rho\lambda} \qquad \varepsilon = -\frac{\dot{H}}{H^2} = \frac{6\lambda(\lambda + 2\rho)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}$$

$$0 \le I < 1$$

slow roll parameter

Phase space analysis

To see which one is dynamically selected, we move on to

Autonomous system $X = \frac{\dot{\sigma}}{\dot{\alpha}}$ $Y = \frac{1}{M_p}\frac{\dot{\phi}}{\dot{\alpha}}$ $Z = \frac{f(\phi)}{M_p} e^{-\alpha+2\sigma} \frac{\dot{v}}{\dot{\alpha}}$

$$\begin{cases} \frac{dX}{d\alpha} = \frac{1}{3}Z^2(X+1) + X\left\{3\left(X^2-1\right)+\frac{1}{2}Y^2\right\} \\ \frac{dY}{d\alpha} = (Y+\lambda)\left\{3\left(X^2-1\right)+\frac{1}{2}Y^2\right\} + \frac{1}{3}YZ^2 + \left(\rho+\frac{\lambda}{2}\right)Z^2 \\ \frac{dZ}{d\alpha} = Z\left[3\left(X^2-1\right)+\frac{1}{2}Y^2+\frac{1}{2}Y^2-\rho Y+1-2X+\frac{1}{3}Z^2\right] \end{cases}$$

Isotropic fixed point $(X,Y,Z) = (0,-\lambda,0)$

Anisotropic fixed point

$$(X,Y,Z) = \frac{2}{A}\left(\lambda^2+2\rho\lambda-4, -6(\lambda+2\rho), \frac{3\sqrt{2}}{2}\sqrt{\left(\lambda^2+2\rho\lambda-4\right)\left(A-2\lambda^2-4\rho\lambda\right)}\right)$$

This exists only for $\lambda^2+2\rho\lambda-4>0$

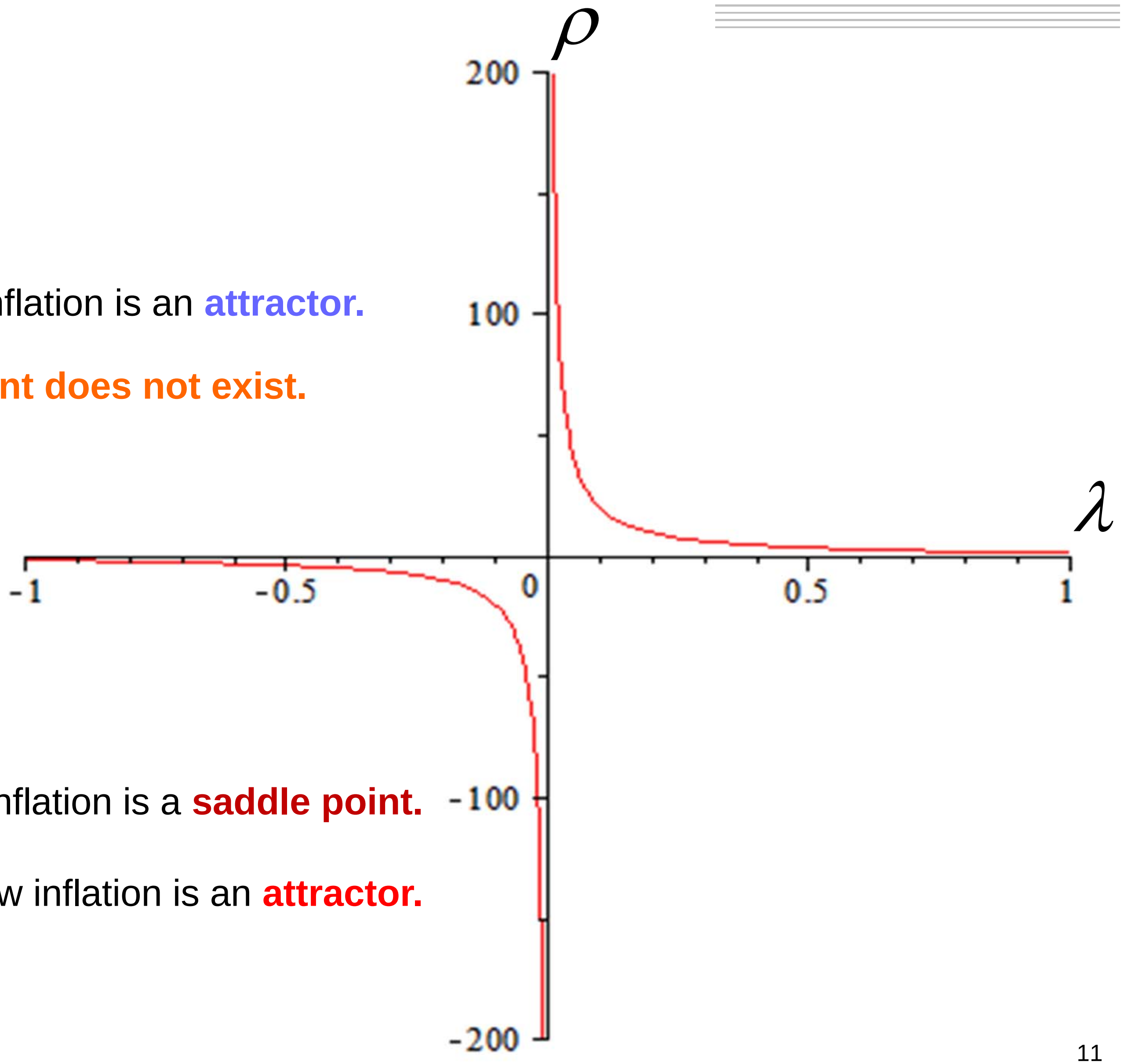
$$A = \lambda^2+8\rho\lambda+12\rho^2+8$$

Linear stability analysis

$$\lambda^2 + 2\rho\lambda - 4 < 0$$

Isotropic power-law inflation is an attractor.

Anisotropic fixed point does not exist.



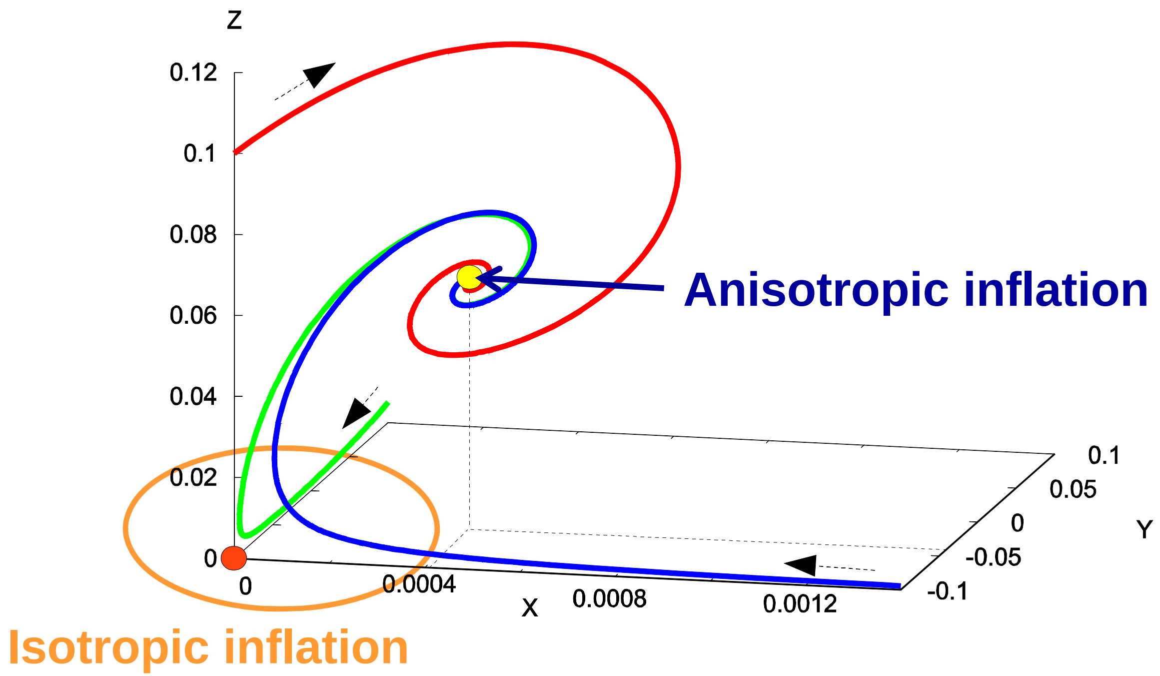
$$\lambda^2 + 2\rho\lambda - 4 > 0$$

Isotropic power-law inflation is a saddle point.

Anisotropic power-law inflation is an attractor.

The whole picture

$$\lambda^2 + 2\rho\lambda - 4 > 0$$



After a transient isotropic inflationary phase,
the universe enter into an anisotropic inflationary phase.

General Potential

Consider the slow roll phase $\varepsilon \ll 1$

$$\dot{\alpha}^2 = \frac{\kappa^2}{3}\left[V(\phi) + \frac{E^2}{2} f^{-2}(\phi) e^{-4\alpha-4\sigma}\right]$$

In order for the vector contribution to increase, we need the condition $\frac{f_\phi}{\kappa f}\frac{V_\phi}{\kappa V} > 2$

Once the vector contributes the dynamics of the inflaton field, the ratio does not increase any more

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - V_\phi(\phi) + E^2 f^{-3}(\phi) f_\phi(\phi) e^{-4\alpha-4\sigma}$$

opposite to the potential force

The vector energy density saturates at $E^2 f^{-2}(\phi) e^{-4\alpha-4\sigma} = V_\phi \frac{f}{f_\phi}$

Inflation continues if $\frac{E^2 f^{-2}(\phi) e^{-4\alpha-4\sigma}}{2V} = \frac{1}{2}\frac{V_\phi}{V}\frac{f}{f_\phi} < 1$ at this saturating point

Because of this vector contribution, we have anisotropy of the order of

$$\frac{\Sigma}{H} \approx \frac{E^2 f^{-2} e^{-4\alpha-4\sigma}}{V} \approx \frac{V_\phi}{V}\frac{f}{f_\phi} < \frac{1}{\kappa^2}\left(\frac{V_\phi}{V}\right)^2 \approx \varepsilon$$

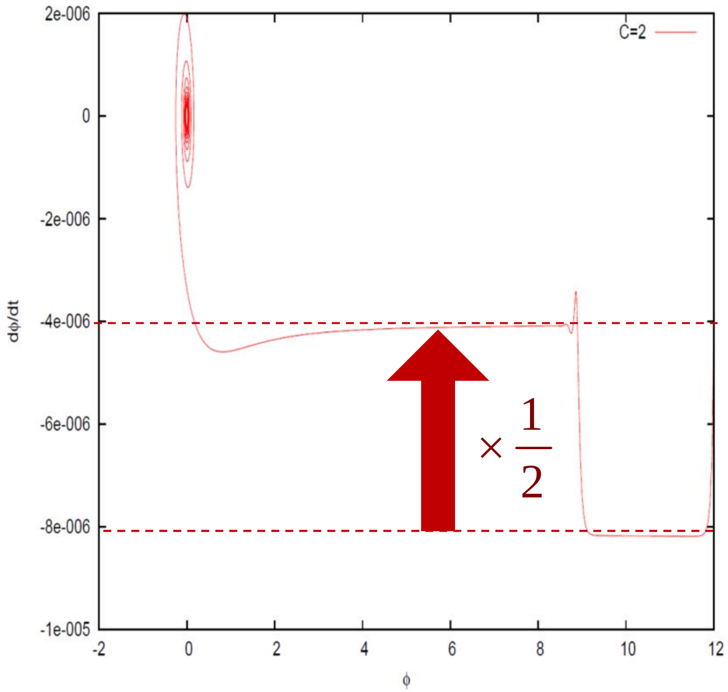
Example：chaotic inflation $V = \frac{1}{2} m^2 \phi^2$

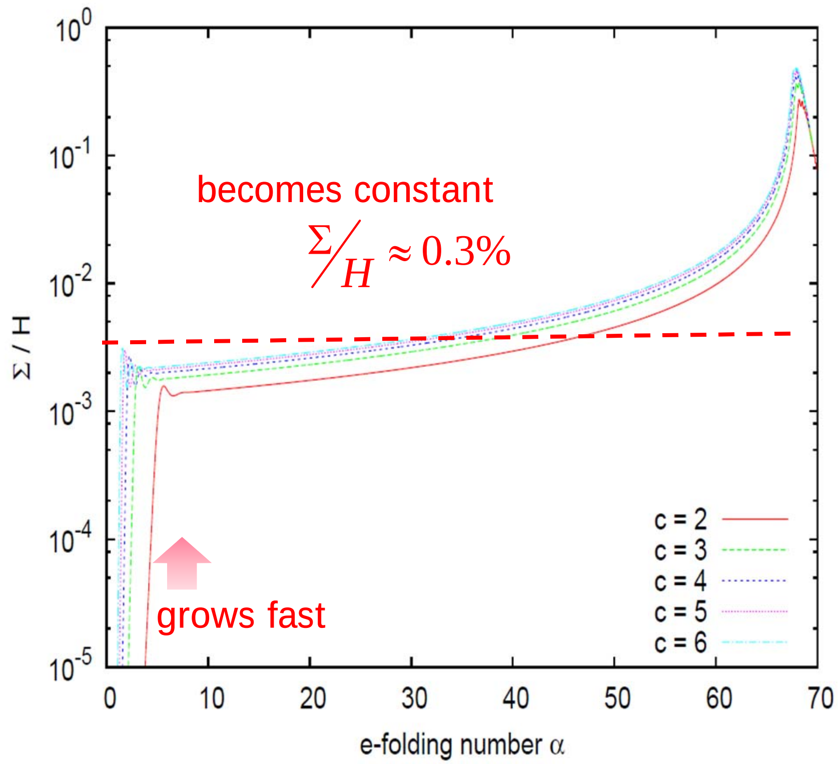
A simple choice is $f(\phi) = e^{2c\kappa^2 \int \frac{V}{V_\phi} d\phi}$ $c > 1$

We find that the degree of anisotropy is written by the slow-roll parameter.

$$\frac{\Sigma}{H} = \frac{1}{3} I \varepsilon_H$$

: A universal relation $I = \frac{c-1}{c}$





COSMOLOGICAL PERTURBATION THEORY IN A SIMPLE BIANCHI UNIVERSE

Flat slicing gauge in anisotropic universe

In our case, we have only 2-dimensional rotational symmetry

$$ds^2 = a^2(\eta)\left[-d\eta^2 + dx^2\right] + b^2(\eta)\left[dy^2 + dz^2\right]$$

Vector type perturbations can be characterized in a special frame as follows

$$\vec{k}_{2D} = (k_y, 0) \qquad\qquad V^i{}_{,i} = 0 \Rightarrow V_2 = 0$$

Scalar type perturbations are V_2

There is no tensor type perturbations in 2-d.

vector perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & b^2\beta_3 \\ * & 0 & 0 & b^2\Gamma \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix} \qquad\qquad \delta A_\mu = \left(0, 0, 0, D\right)$$

scalar perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} -2a^2\Phi & a\beta_1 & a\beta_2 & 0 \\ * & 2a^2 G & 0 & 0 \\ * & * & 2b^2 G & 0 \\ * & * & * & -2b^2 G \end{pmatrix} \qquad \delta A_\mu = \left(\delta A_0, 0, J, 0\right) \qquad \delta\phi$$

The blue variables are physical.

Structure of couplings

The main features of the action can be understood by looking at the following term

Key term $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} f^2(\phi) F_{\mu\nu} F_{\alpha\beta}$$

Notice the following relations

Background quantity $$\frac{f^2 v'^2}{a^2} \approx I\varepsilon_H \qquad \frac{f_\phi}{f} \approx \frac{\kappa^2 V}{V_\phi} \approx \frac{1}{\sqrt{\varepsilon_H}} \qquad I = \frac{c-1}{c}$$

Now, we take variations

vector-tensor $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi) F_{\mu\nu}}_{f^2 v'} F_{\alpha\beta} \qquad f v' \approx \sqrt{I\varepsilon_H}$$

vector-scalar $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi)}_{f f_\phi \delta\phi} \underbrace{F_{\mu\nu}}_{v'} F_{\alpha\beta} \qquad f_\phi v' \approx \frac{f_\phi}{f} f v' \approx \sqrt{I}$$

scalar-tensor $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi)}_{f f_\phi \delta\phi} \underbrace{F_{\mu\nu} F_{\alpha\beta}}_{v'^2} \qquad f_\phi v'^2 \approx I\sqrt{\varepsilon_H}$$

Reduced Quadratic Action: Slow roll Approximation

$$-\frac{\dot{H}}{H^2}=\varepsilon_H \qquad \frac{\Sigma}{H}=\frac{1}{3}I\varepsilon_H \qquad \sin\theta=\frac{k_y a}{kb} \qquad I=\frac{c-1}{c}$$

$$S^{\text{vector}}=S_{free}\left(\Gamma,D\right)+\int d\eta d^3k\left[\frac{\sqrt{6I\varepsilon_H}}{2}(-\eta)^{-1}\sin\theta\left(\Gamma' D^*+\Gamma'^* D\right)-\frac{\sqrt{6I\varepsilon_H}}{2}(-\eta)^{-2}\sin\theta\left(\Gamma D^*+\Gamma^* D\right)\right]$$

vector-tensor

$$S^{\text{scalaor}}=S_{free}\left(G,J,\delta\phi\right)$$

$$+\int d\eta d^3k\left[-3I\sqrt{\varepsilon_H}\,(-\eta)^{-2}\sin^2\theta\left(G\delta\phi^*+G^*\delta\phi\right)\right.$$

scalar-tensor

$$+\sqrt{6I}\,(-\eta)^{-1}\sin\theta\left(\delta\phi'^*J+\delta\phi'^*J\right)-\sqrt{6I}\,(-\eta)^{-2}\sin\theta\left(\delta\phi^*J+\delta\phi J^*\right)$$

vector-scalar

$$\left.-\frac{\sqrt{6I\varepsilon_H}}{2}(-\eta)^{-1}\sin\theta\left(G'^*J+G'J^*\right)+\frac{\sqrt{6I\varepsilon_H}}{2}(-\eta)^{-2}\sin\theta\left(G^*J+GJ^*\right)\right]$$

The nature of primordial fluctuations in anisotropic inflation

Statistical anisotropy for a test field

In slow-roll phase $(H \equiv \dot{\alpha}, \Sigma \equiv \dot{\sigma}$ are almost const.) , we have the metric:

$$ds^2 = -dt^2 + e^{2Ht}\left[e^{-4\Sigma t}dx^2 + e^{-2\Sigma t}\left(dy^2 + dz^2\right)\right]$$

Since the expansion is anisotropic, we expect statistically anisotropic fluctuations.

Ackerman et al. (2007) analyzed a test field on this background and found

the power spectrum:

$$P_\psi(\mathbf{k}) = P_0(k)\left[1 + g(\hat{\mathbf{k}}\cdot\mathbf{n})^2\right]$$

Deviation from isotropic part depends on $\hat{\mathbf{k}}$

Isotropic part

may be detectable if

$$g > 0.025 \times \left(\frac{400}{\ell_{\max}}\right)^{1.27} \approx 0.3\% \quad \text{with } \ell_{\max} = 2000$$ (Planck)

Groeneboomn & Eriksen (2008)

Since we have a concrete anisotropic inflation model, we can go beyond the test field analysis.

Perturbative estimates of statistical anisotropy

In the isotropic limit, we have

Mode functions $\delta\phi = u(\eta)a_k + u(\eta)^* a_k^\dagger$ $\quad u(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right)$

Interaction Hamiltonian $H_I = \int d^3k \left[-\sqrt{\frac{6I}{1-I}} (-\eta)^{-1} \sin\theta \left(\delta\phi^\dagger J + \delta\phi J^\dagger \right) + \cdots \right]$

Assuming that I is small, we can calculate corrections to the power spectrum

$$\frac{\delta\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle}{\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle} = \frac{i^2}{\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle} \int_{\eta_{in}}^{\eta} d\eta_2 \int_{\eta_{in}}^{\eta_2} d\eta_1 \langle 0|\left[H_I(\eta_1), \left[H_I(\eta_2), \delta\phi_k(\eta)\delta\phi_p(\eta) \right]\right]|0\rangle$$

$$\approx \frac{24I}{1-I} \sin^2\theta N^2(k)$$

Predictions of anisotropic inflation

Thus, we found the following nature of primodordial fluctuations in anisotropic inflation.

statistical anisotropy in curvature perturbations

$$P_s(\mathbf{k}) = P_s(k)\left[1 + g_s \sin^2\theta\right] \qquad g_s = 24 I N^2(k)$$

statistical anisotropy in primordial GWs

$$P_t(\mathbf{k}) = P_t(k)\left[1 + g_t \sin^2\theta\right] \qquad g_t = 6 I \varepsilon_H N^2(k)$$

cross correlation between curvature perturbations and primordial GWs

$$\frac{\langle \zeta G \rangle}{\langle \zeta\zeta \rangle} = -24 I \varepsilon_H N^2(k)$$

TB correlation in CMB

small linear polarization in primordial GWs

CMB spectrum

Angular power spectrum of X and Y reads

$$C_{\ell\ell'}^{XY} \propto \int d\Omega_k P_{XY}\left(\mathbf{k}\right) {}_{-s}Y_{\ell m}^{*}\left(\hat{\mathbf{k}}\right) {}_{-s'}Y_{\ell' m'}\left(\hat{\mathbf{k}}\right)$$

For isotropic spectrum, $P\left(\mathbf{k}\right) = P\left(k\right)$ **, we have** $C_{\ell\ell'}^{XY} \propto \delta_{\ell\ell'}$

For anisotropic spectrum, there are off-diagonal components.

For example,

$$C_{\ell\ell'}^{TB} \propto \int d\Omega_k P_{TB}\left(\mathbf{k}\right) Y_{\ell m}^{*}\left(\hat{\mathbf{k}}\right) {}_{-2}Y_{\ell' m'}\left(\hat{\mathbf{k}}\right)$$

$$\propto \delta_{\ell,\ell'\pm 1}$$

The off-diagonal part of the angular power spectrum tells us if the gauge kinetic function plays a role in inflation.

Observational implication

The current observational constraint is given by

WMAP constraint Pullen & Kamionkowski (2007) $g_s = 24 I N^2(k) < 0.3$

Now, suppose we detected $g_s = 24 I N^2(k) = 0.2$ $\varepsilon_H = 0.01$

Then we could expect

- **statistical anisotropy in GWs** $g_t = 10^{-3}$
- **cross correlation between curvature perturbations and GWs** $\frac{\langle \zeta G \rangle}{\langle \zeta \zeta \rangle} = -2 \times 10^{-3}$

Cf . current constraint $TB / TE < 10^{-2}$

If these predictions are proved, it must be an evidence of anisotropic inflation!

Summary

We have shown that anisotropic inflation can be realized once we take into account a gauge kinetic function.

We have given the predictions:

- ✓**the statistical anisotropy in scalar and tensor fluctuations**
- ✓**the cross correlation between scalar and tensor**
- ✓**the linear polarization of tensor fluctuations**

Off-diagonal angular power spectrum can be used to prove or disprove our scenario.

Our analysis gives a first cosmological constraint on gauge kinetic functions.

As a by-product, we found a counter example to the cosmic no-hair conjecture.

As a future issue, it is interesting to construct a model from string theory.