Axion and dark energy

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1 Introduction

- **Axion is a field introduced to solve strong CP problem**
  The QCD Lagrangian includes a CP-violating term

$$\mathcal{L} = \theta \frac{g^2}{32\pi^2} G^{\mu\nu a} \tilde{G}^a_{\mu\nu}.$$

Limits on the neutron electric dipole moment imply

$$\theta \leq 10^{-10}$$

but this is unnatural fine tuning.

Spontaneously broken global Peccei-Quinn $U(1)_{PQ}$ symmetry gives a solution to this problem, but this symmetry is broken by an anomaly

$$\mathcal{L} = \left(\theta - \frac{\phi_A}{f_A}\right) \frac{g^2}{32\pi^2} G^{\mu\nu a} \tilde{G}_{\mu\nu}. \quad \phi_A: \text{ axion,} \quad f_A: \text{ decay constant.}$$

Axion potential is induced and the axion gets VEV to cancel this term.

$$\textbf{Axion mass: } m_A \sim \frac{f_\pi m_\pi}{2 f_A}.$$

For the QCD, f_A was taken as $f_A \sim v_{weak}$, but this is now completely ruled out by experiment.

Axions with $f_A \gg v_{weak}$ are viable:
Typically there are two kinds.

- **(Kim-Shifman-Vainstein-Zakharov)**
  New heavy quark coupled to the axion, leaving ordinary quarks without tree-level axion couplings. Heavy quark is electrically neutral.
- **(Dine-Fischler-Srednicki-Zhitnitsky)**
  Two Higgs doublets and ordinary quarks and leptons carry PQ charge.

In all of them, one electroweak singlet scalar gets VEV and breaks PQ symmetry.

String theory may give many pseudo-scalar particles coupled to the topological charge, upon compactification to four dimensions.

If they really exists, depending on their masses, various physical phenomena are expected to occur, which might be observed in the future astrophysical experiments. $\Rightarrow$ Axiverse

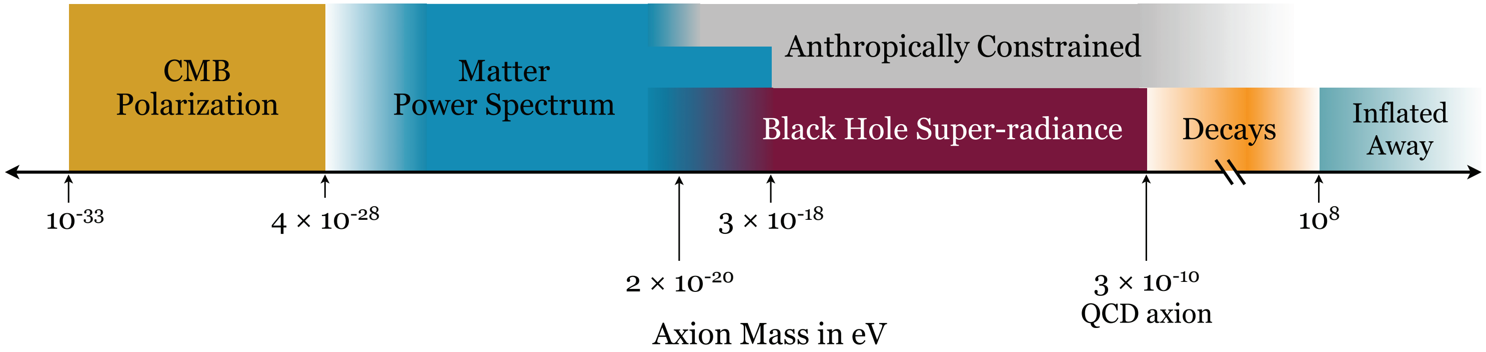


Figure 1: **Map of the Axiverse: (From A. Arvanitaki et al. Phys. Rev. D 81 (2010) 123530** [**arXiv:0905.4720** [**hep-th**]]**.** The signatures of axions as a function of their mass, assuming $f_a \approx M_{GUT}$ and $H_{inf} \sim 10^8$ eV.

Current experimental bound is:

$$10^{12}\ \mathbf{GeV} > f_A > 10^9\ \mathbf{GeV}$$

On the other hand, string consideration gives typically $f_A \geq 10^{16}$ GeV. (Svrcek and Witten [hep-th/0605206])

This is true both for

Model-independent axion: from $B_{\mu\nu}$.

Model-dependent axion: B-form contracted with some harmonic 2-form in the compactified dimensions.

Another cosmological effects related to QCD sector.

The recent cosmological observations: $\Rightarrow$ the early inflationary epoch and the accelerated expansion of the present universe.

Observation: the universe has an unknown form of energy density, named the dark energy, about 75% of the total energy density.

The simplest possibility: cosmological constant (origin ?)

Vacuum energy: easily incorporated in the quantum field theory.

Standard model of particle physics: Higgs field $\Rightarrow$ electroweak phase transition $\Rightarrow$ changes the value of the vacuum energy.

The vacuum fluctuations in quantum field theory naturally induce such a vacuum energy, but the problem is how to control the size of it.

The contribution of quantum fluctuations in known fields up to 300 GeV $\Rightarrow$ a vacuum energy density of order (300 GeV)^4. Vastly larger than the observed dark energy density $(3 \times 10^{-3}$ eV$)^4$ by a factor of order 10^{56}.
Assuming the tree-level contribution is zero, it is a great challenge how to understand the origin of this tiny energy density.

Very interesting suggestion on the origin of a cosmological constant:

The cosmological constant from the ghost fields in QCD.

The ghosts are required to exist for the resolution of the U(1) problem.

The ghosts are decoupled from the physical states and make no contribution in the flat Minkowski space.

Once they are in the curved space or time-dependent background, the cancellation of their contribution to the vacuum energy is off-set, leaving a small energy density $\rho \sim H\Lambda_{QCD}^3$, ($H$: the Hubble parameter, Λ_{QCD}: the QCD mass scale 100 MeV).

$\Rightarrow$ With $H \sim 10^{-33}$ eV, this gives the right magnitude $(3 \times 10^{-3}$ eV$)^4$. This coincidence is remarkable $\Rightarrow$ we are on the right track.
However it was argued with a four-dimensional analogue of two-dimensional toy models replacing the vector gauge field by two scalar fields.

However the QCD ghost must be intrinsically vector field in order for the U(1) problem to be consistently resolved.

2 Decoupling of the vector ghost in the Minkowski space

The low-energy effective Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_{NS}^2 S^2 + \frac{1}{2F_S^2(m_S^2 - m_{NS}^2)}(\partial_\mu K^\mu)^2 - \frac{1}{F_S} S \partial_\mu K^\mu,$$

S: **a flavor-singlet pseudoscalar field with the decay constant** F_S,
K^μ: **an axial vector "field"**

$$K^\mu = 2N_f \frac{g^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma} A_\nu^a \left(\partial_\lambda A_\sigma^a + \frac{1}{3} g f^{abc} A_\lambda^b A_\sigma^c\right), (N_f : \textbf{the number of flavors})$$

Invariant under the gauge transformation

$$K^\mu \to K^\mu + \epsilon^{\mu\nu\lambda\sigma}\partial_\nu \Lambda_{\lambda\sigma}, (\Lambda_{\lambda\sigma} : \textbf{an arbitrary antisymmetric tensor})$$

Reflects the color gauge invariance of the underlying QCD.

$$[Q_B, K^\mu] = 2iN_f \frac{g^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}\partial_\nu (C^a \partial_\lambda A_\sigma^a),$$

Q_B: **the BRST charge,** C^a: **the Fadeev-Popov ghost.**

Quantization: Add gauge fixing

$$\frac{1}{4F_S^2(m_S^2 - m_{NS}^2)\alpha}(\partial_\mu K_\nu - \partial_\nu K_\mu)^2, \quad (\alpha : \textbf{ a gauge parameter}).$$

Choose $\alpha = 1 \Rightarrow$ Feynman rules:

$$K_\mu\textbf{-propagator: } \frac{i\eta_{\mu\nu}}{k^2}F_S^2(m_S^2 - m_{NS}^2), \qquad S\textbf{-propagator: } \frac{i}{k^2 - m_{NS}^2},$$

$$S - K_\mu\textbf{-mixing: } \frac{1}{F_S}k_\mu.$$

A scalar field S with mass m_{NS} and a massless vector?

No: the mass of the scalar S gets shifted to m_S due to the mixing of the scalar and vector modes.

Alternatively, in the general gauge, the two-point functions

$$\begin{aligned} T\langle K_\mu(x)K_\nu(y)\rangle_M &= \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}i\left\{\frac{k^2 - m_{NS}^2}{k^2 - m_S^2}\frac{k_\mu k_\nu}{(k^2)^2}\right. \\ &\quad \left. + \frac{\alpha}{k^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\right\}F_S^2(m_S^2 - m_{NS}^2), \\ T\langle S(x)S(y)\rangle_M &= \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}\frac{i}{k^2 - m_S^2}, \\ T\langle S(x)K_\mu(y)\rangle_M &= \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}\frac{F_S(m_S^2 - m_{NS}^2)}{k^2(k^2 - m_S^2)}, \end{aligned}$$

T: the time-order, the subscript M: the expectation value by the Minkowski vacuum.

The two-point function of $S \Rightarrow$ it has the shifted mass m_S instead of the original m_{NS}.

This is one of the consequences of the massless mode K_μ and gives a resolution of the $U(1)$ problem.

What happens to the massless mode in the system?

Field equations:

$$\Box K_\mu - \partial_\mu\{(1-\alpha)\partial^\nu K_\nu + F_S(m_S^2 - m_{NS}^2)S\} = 0,$$
$$\Box S + m_{NS}^2 S + \frac{1}{F_S}\partial^\mu K_\mu = 0.$$

$$\Rightarrow \Box(\partial_\mu K_\nu - \partial_\nu K_\mu) = 0.$$

The subsidiary condition on the physical states:

$$(\partial_\mu K_\nu - \partial_\nu K_\mu)^{(+)}|\mathbf{phys}\rangle = 0.$$

The mode expansion of the vector field in terms of the canonically normalized field $K'_\mu(x) \equiv F_S\sqrt{m_S^2 - m_{NS}^2}K(x)$:

$$K'_\mu(x) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2k_0}} e_\mu^{(\lambda)}[e^{-ik\cdot(x-y)}a(\boldsymbol{k},\lambda) + e^{ik\cdot(x-y)}a^\dagger(\boldsymbol{k},\lambda)],$$

$e_\mu^{(1)} = (0, \boldsymbol{e}^{(1)})$, $e_\mu^{(2)} = (0, \boldsymbol{e}^{(2)})$: **the transverse modes**
$e_\mu^{(3)} = (0, \boldsymbol{e}^{(3)})$ **the longitudinal mode, and** $e_\mu^{(0)} = (1, \boldsymbol{0})$ **the time component with**

$$\boldsymbol{e}^{(3)} = \frac{\boldsymbol{k}}{|\boldsymbol{k}|}, \quad \boldsymbol{e}^{(1)} \cdot \boldsymbol{e}^{(2)} = \boldsymbol{e}^{(1)} \cdot \boldsymbol{e}^{(3)} = \boldsymbol{e}^{(2)} \cdot \boldsymbol{e}^{(3)} = 0.$$

Canonical quantization $\Rightarrow [a(\boldsymbol{k}, \lambda), a^\dagger(\boldsymbol{k}', \lambda')] = \eta_{\lambda\lambda'} \delta^3(\boldsymbol{k} - \boldsymbol{k}')$.

The transverse modes have the opposite sign to the usual gauge fields.

The two transverse components $a(\boldsymbol{k}, 1), a(\boldsymbol{k}, 2)$ **and the combination** $\frac{1}{\sqrt{2}}[a(\boldsymbol{k}, 3) - a(\boldsymbol{k}, 0)]$ **should annihilate the physical state.** $\Rightarrow$ **Forbids the states generated by the two transverse components and by** $\frac{1}{\sqrt{2}}[a^\dagger(\boldsymbol{k}, 3) + a^\dagger(\boldsymbol{k}, 0)]$**.**

The remaining component $\frac{1}{\sqrt{2}}[a^\dagger(\boldsymbol{k}, 3) - a^\dagger(\boldsymbol{k}, 0)]$ **can only produce zero norm states, so that all the component of** K_μ **are completely decoupled from the physical state.**

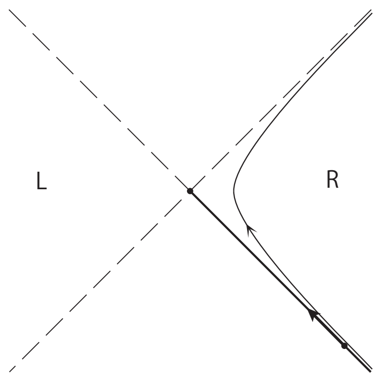
What effects this massless mode may produce if our space is not just the Minkowski but curved space or time-dependent?

3 Vector ghost in Rindler space

Minkowski space:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \equiv d\bar{u}d\bar{v} - dx^2 - dy^2,$$

$$\bar{u} = t - z, \quad \bar{v} = t + z.$$



local Rindler horizon

Transformation

$$t = \frac{1}{a}e^{a\xi}\sinh a\eta, \quad z = \frac{1}{a}e^{a\xi}\cos a\eta, \quad (-\infty < \eta, \xi < \infty, a > 0),$$

$$\Rightarrow ds^2 = e^{a(v-u)}dudv - dx^2 - dy^2 = e^{2a\xi}(d\eta^2 - d\xi^2) - dx^2 - dy^2,$$
$$\bar{u} = -\frac{1}{a}e^{a(\xi-\eta)} \equiv -\frac{1}{a}e^{-au}, \quad \bar{v} = \frac{1}{a}e^{a(\xi+\eta)} \equiv \frac{1}{a}e^{av},$$

The Rindler coordinates η and ξ describes only the quadrant part $z > |t|$ called R.

The opposite quadrant part L: $z < -|t|$ is described by changing the signs.

The rest of our Minkowski space are described by analytic continuation of these coordinates.

The wave function: solutions for the scalar wave equation

$$\phi^{;\alpha}{}_{;\alpha} = 0.$$

Explicitly this becomes in our coordinate system

$$[e^{-2a\xi}(\partial_\eta^2 - \partial_\xi^2) - \partial_x^2 - \partial_y^2]\,\phi = 0.$$

Wave function which asymptotes, in the R region,

$${}^{R}u(\boldsymbol{k}) \simeq \begin{cases} e^{-ik_0 u} e^{-i(k_1 x + k_2 y)} & \textbf{in R} \\ \mathbf{0} & \textbf{in L} \end{cases}$$

along the surface $v = -\infty, \bar{v} = 0$, the past horizon of the Rindler coordinate.

Wave function in the L region:

$${}^{L}u(\boldsymbol{k}) \simeq \begin{cases} \mathbf{0} & \textbf{in R} \\ e^{ik_0 v} e^{-i(k_1 x + k_2 y)} & \textbf{in L} \end{cases}$$

The positive-frequency Minkowski modes: analytic and bounded in the lower half complex $\bar{u}$ plane on $\bar{v} = 0$.

$$\frac{1}{[2\sinh(\pi k_0/(2a))]^{1/2}}(e^{\pi k_0/(2a)}\ {}^{R}u(\boldsymbol{k}) + e^{-\pi k_0/(2a)}\ {}^{L}u(-\boldsymbol{k})^*),$$

$$\frac{1}{[2\sinh(\pi k_0/(2a))]^{1/2}}\ (e^{-\pi k_0/(2a)}\ {}^{R}u(-\boldsymbol{k})^* + e^{\pi k_0/(2a)}\ {}^{L}u(\boldsymbol{k})),$$

where ${}^L u(-\boldsymbol{k})$ and ${}^R u(-\boldsymbol{k})$ denote the wave function with minus momenta, have precisely this property, so we can use these modes to express our Minkowski space field:

$$
\begin{aligned}
K'_\mu(x) \;=\; & \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2k_0}} \frac{e_\mu^{(\lambda)}}{[2\sinh(\pi k_0/(2a))]^{1/2}} [(e^{\pi k_0/(2a)}\ {}^R u(\boldsymbol{k}) + e^{-\pi k_0/(2a)}\ {}^L u(-\boldsymbol{k})^*) a^{(1)}(\boldsymbol{k},\lambda) \\
& + (e^{-\pi k_0/(2a)}\ {}^R u(-\boldsymbol{k})^* + e^{\pi k_0/(2a)}\ {}^L u(\boldsymbol{k})) a^{(2)}(\boldsymbol{k},\lambda) + \textbf{h.c.}],
\end{aligned}
$$

Minkowski vacuum:

$$
a^{(i)}(\boldsymbol{k},1)|0_M\rangle = a^{(i)}(\boldsymbol{k},2)|0_M\rangle = [a^{(i)}(\boldsymbol{k},3) - a^{(i)}(\boldsymbol{k},0)]|0_M\rangle = 0, \quad (i=1,2).
$$

The field in the Rindler space

$$
\begin{aligned}
K'_\mu(x) \;=\; & \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2k_0}} e_\mu^{(\lambda)} [{}^L u(\boldsymbol{k})\ b^{(1)}(\boldsymbol{k},\lambda) + {}^L u(\boldsymbol{k})^*\ b^{(1)\dagger}(\boldsymbol{k},\lambda) \\
& + {}^R u(\boldsymbol{k})\ b^{(2)}(\boldsymbol{k},\lambda) + {}^R u(\boldsymbol{k})^*\ b^{(2)\dagger}(\boldsymbol{k},\lambda)],
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow b^{(1)}(\boldsymbol{k},\lambda) \;&=\; \frac{1}{\sqrt{2\sinh\frac{\pi k_0}{a}}} \Big[e^{\pi k_0/(2a)} a^{(2)}(\boldsymbol{k},\lambda) + e^{-\pi k_0/(2a)} a^{(1)\dagger}(-\boldsymbol{k},\lambda) \Big], \\
b^{(2)}(\boldsymbol{k},\lambda) \;&=\; \frac{1}{\sqrt{2\sinh\frac{\pi k_0}{a}}} \Big[e^{\pi k_0/(2a)} a^{(1)}(\boldsymbol{k},\lambda) + e^{-\pi k_0/(2a)} a^{(2)\dagger}(-\boldsymbol{k},\lambda) \Big].
\end{aligned}
$$

The resulting energy for each mode

$$\langle 0_M| \int d^3\boldsymbol{k}' \sum_{\lambda,\lambda'=0}^{3} k_0 \eta_{\lambda\lambda'} b^{(1)\dagger}(\boldsymbol{k},\lambda) b^{(1)}(\boldsymbol{k}',\lambda')|0_M\rangle$$
$$= \langle 0_M| \int d^3\boldsymbol{k}' \sum_{\lambda,\lambda'=0}^{3} k_0 \eta_{\lambda\lambda'} b^{(2)\dagger}(\boldsymbol{k},\lambda) b^{(2)}(\boldsymbol{k}',\lambda')|0_M\rangle = \frac{4k_0}{e^{2\pi k_0/a}-1}.$$

The vector field has four degrees of freedom and all of them are decoupled in the flat Minkowski space.

However, they all contribute to the vacuum energy in the Rindler space.

The contribution of high frequency modes are suppressed by the factor $e^{-2\pi k_0/a}$ and the main contribution comes from $k_0 \sim a$.

Cosmological context: $a \sim H$ and hence $k_0 \sim H$, giving the vacuum energy proportional to the Hubble parameter.

strongly interacting confining QCD: this effect occurs only in the time direction and their wave function in other space directions is expected to have the size of QCD energy scale.

⇒ The ghost gives the vacuum energy density $H\Lambda_{QCD}^3$ of the right magnitude $\sim (3 \times 10^{-3}$ eV$)^4$.

The vacuum energy arises due to the mismatch between the vacuum energies computed in slowly expanding universe and Minkowski space.

4 Discussions and conclusions

- **The decoupling mechanism of the QCD ghost in the flat Minkowski space clarified.**
- **The contribution of the QCD ghost to the vacuum energy density in the Rindler space as a typical example of time-dependent spacetime e3valuated.**
- **Found it gives the vacuum energy proportional to the Hubble parameter.**

Extremely interesting feature.

1. **does not assume new degrees of freedom only to produce nonzero cosmological constant. Rather it is induced by the already existing field just because the universe is expanding.**

2. gives the amazing result of the cosmological constant of right magnitude without artificial fine tuning. Note that the vacuum energy is not just a constant but depends on the Hubble parameter.

The unphysical modes or polarization of QED photon may also contribute to the dark energy of the similar amount?

QED is weakly interacting theory unlike QCD and also does not have nontrivial topological structure, and hence there is no restriction on the wave function as in QCD.

The contribution to the energy density is very small of order H^4 by dimensional reason and need not be considered.

Physical evidence?:

The same ghost may also generate magnetic field in an expanding universe. It would be interesting to check if this mechanism makes sense with the vector ghost.

Another interesting question is to try to find if there is any other effects to check the proposed mechanism.

Another important question is to see if there is any connection with string axion.

We hope to see if our results have experimental evidence!!

Thank you!