Matter power spectrum in $f(R)$ gravity with SDSS DR7

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2010.11.9-12 Extra-Dimension Probe by Cosmophysics (ExDiP2010) @ KEK

Outline

1. Introduction of $f(R)$ Gravity

2. Background Universe

3. Density Perturbation

4. Summary

Cosmic acceleration

SNIa

⟶ Accelerating expansion

EoS $w \equiv P/\rho > -1/3$

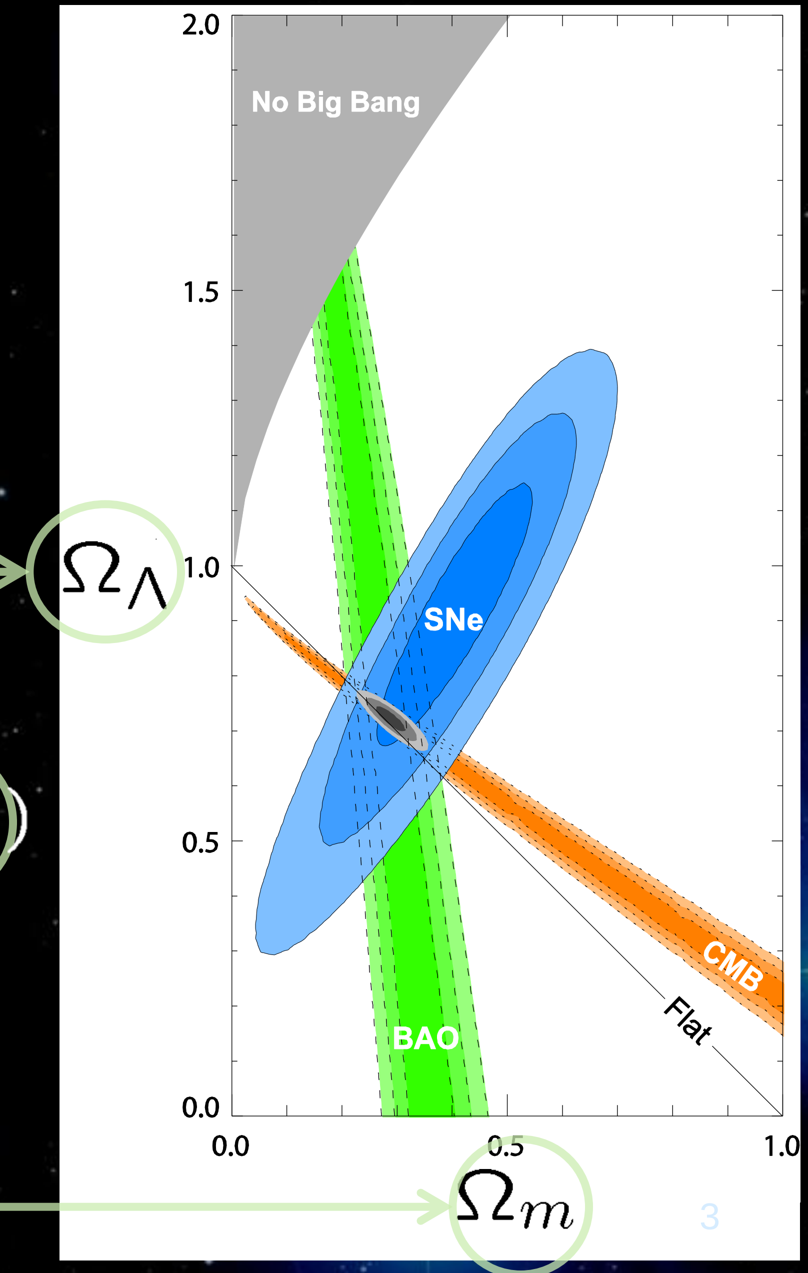
⟶ Decelerating expansion

New component

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(T_{\mu\nu} + T_{\mu\nu}^{\mathrm{DE}})$$

Cosmic acceleration problem:
What is responsible for
accelerating expansion?

Kowalski et al. (2008)



ΛCDM model

Cosmological constant Λ causes accelerating expansion.

- Action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_m$$

- Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$= 8\pi G\left(T_{\mu\nu} + T^{\mathrm{DE}}_{\mu\nu}\right)$$

- Dark energy $T^{\mathrm{DE}}_{\mu\nu} = -\frac{\Lambda g_{\mu\nu}}{8\pi G},\ w_{\mathrm{DE}} \equiv \frac{P_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} = -1$

(New) Cosmological constant problem:
Fine tuning problem $\sim 10^{120}$.

Approaches to cosmic acceleration problem

- ΛCDM model

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R-2\Lambda) + S_m$$

- Modified matter (modify r.h.s. of Einstein eq.)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_\phi + S_m$$

- Modified gravity (modify l.h.s. of Einstein eq.)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m$$

- Inhomogeneous model (modify cosmological principle)

Zero point energy problem (Old CC problem):
We need other mechanism.

$f(R)$ gravity

Function $f(R)$ causes accelerating expansion.

- Action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m$$

- Field equation $\quad (F = df/dR \quad)$

$$FR_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda)F = 8\pi G T_{\mu\nu}$$

i.e.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G\left(T_{\mu\nu} + T_{\mu\nu}^{\mathrm{DE}}\right)$$

$$T_{\mu\nu}^{\mathrm{DE}} = \frac{1}{8\pi G}\Big[(1-F)R_{\mu\nu} - \frac{1}{2}(R-f)g_{\mu\nu} + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda)F\Big]$$

Viable conditions

- Enough past = ΛCDM model

  $$R \gg H_0^2, \;\; f \simeq R - \mathrm{const.}$$

- No cosmological constant

  $$R \to 0, \;\; f \to 0$$

- Stability of high curvature regime

  $$R \gg H_0^2, \;\; F' > 0$$

- Effective gravitational coefficient is positive.

  $$\forall R < \infty, \;\; F > 0$$

- Reproduce GR

  $$\forall R < \infty, \;\; F < 1$$

- Pass local test

  $$R \simeq H_0^2, \;\; |F - 1| \ll 1$$

de Sitter solution

Trace of the field equation

$$3\Box F - RF + 2f = 8\pi G T$$

The de Sitter solution is obtained by substituting $R = R_1 = \text{const.}$

- de Sitter condition

$$R_1 F_1 = 2 f_1$$

where $f_1 = f(R_1)$ and $F_1 = F(R_1) = f'(R_1)$.

Effective dark energy is

$$8\pi G \rho_{\text{DE},1} = -8\pi G P_{\text{DE},1} = \frac{R_1}{4}$$

$$w_{\text{DE},1} = -1$$

Perturbation around the de Sitter solution

First order perturbation with respect to $\delta R \equiv R - R_1$ gives

$$\delta\ddot{R} + 3H_1\delta\dot{R} + \frac{1}{3}\left(\frac{F_1}{F_1'} - R_1\right)\delta R = 0$$

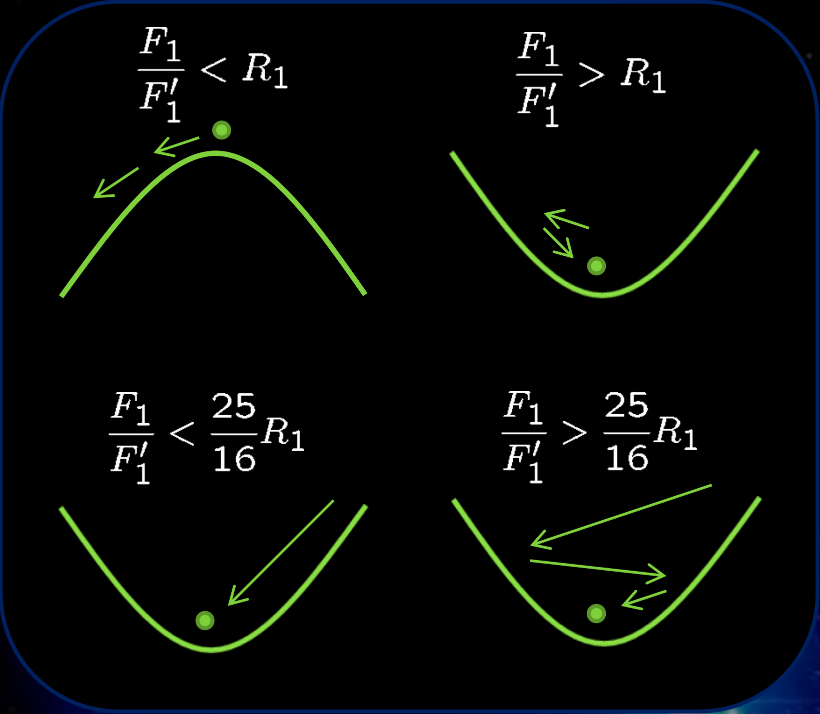
H.M., Starobinsky, Yokoyama (in prep.)

- Stability condition

$$\frac{F_1}{F_1'} > R_1$$

- Oscillation condition

$$\frac{F_1}{F_1'} > \frac{25}{16}R_1$$



Viable $f(R)$ model

- Starobinsky model — Starobinsky (2007)

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-n} - 1 \right]$$

model parameters : n and λ ($R_s \lesssim R_0$)

high curvature limit :

$$f(R) \simeq R + \lambda R_s \left[\left(\frac{R}{R_s} \right)^{-2n} - 1 \right] \simeq R - \lambda R_s$$

2Λ → λR_s

ΛCDM limit :

large n ⟶ $f(R) \simeq R - \lambda R_s$

large λ ⟶ small R_s

⟶ large R/R_s

⟶ $f(R) \simeq R - \lambda R_s$

Summary of section 1

1. Introduction of $f(R)$ Gravity
   - Mimic ΛCDM model without cosmological constant.
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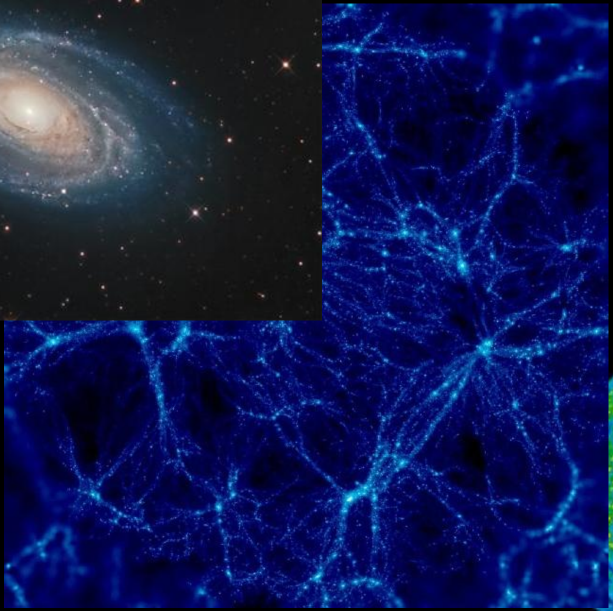
Testing modified gravity

Local scale

$\sim 1\mathrm{AU}$:

Solar system test



Intermediate scale

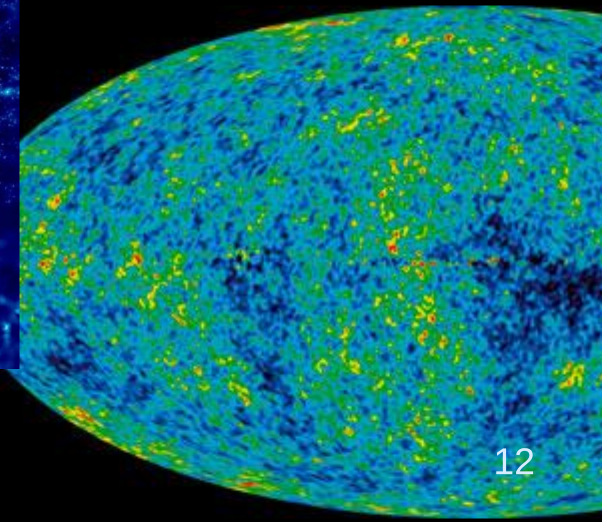
$\sim 100\mathrm{Mpc}$:

Large scale structure

Cosmological scale

$\sim 1\mathrm{Gpc}$:

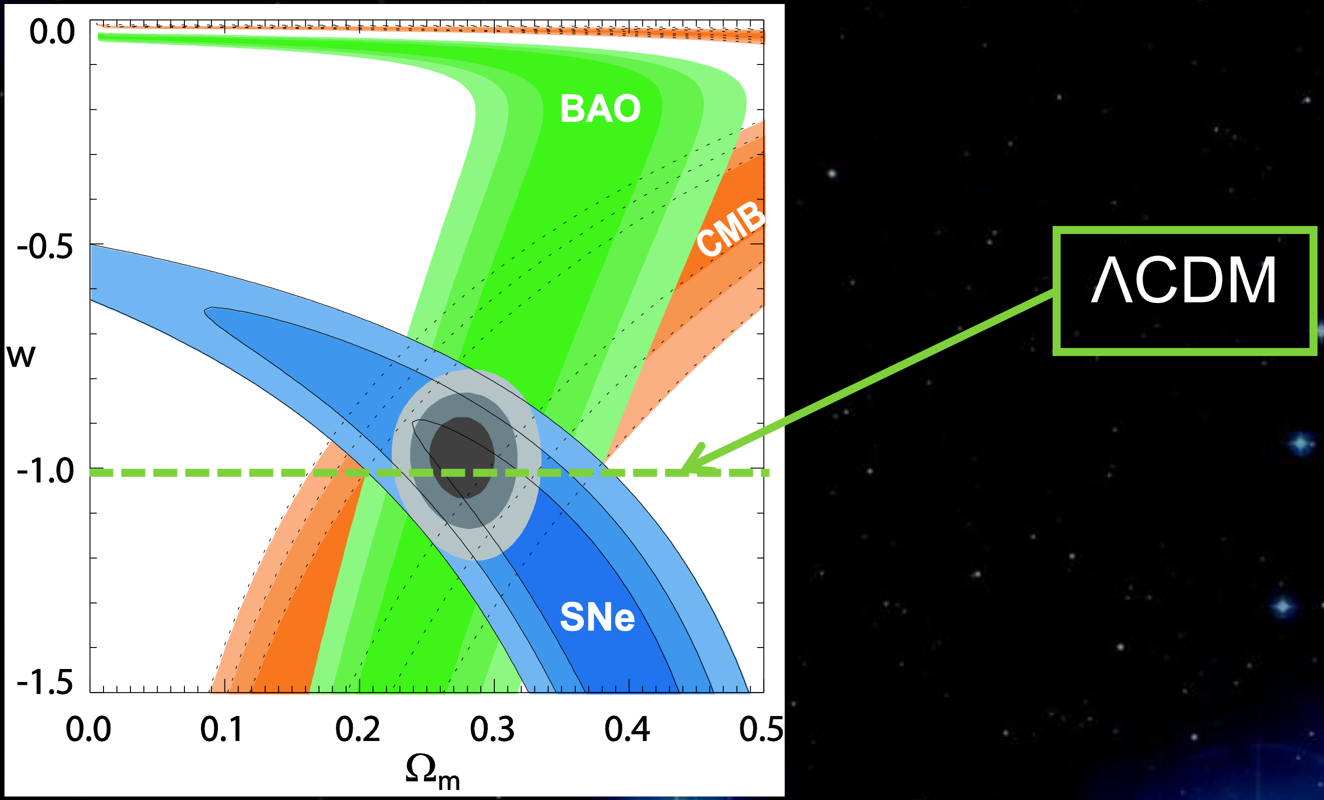
Expansion history



Equation of state parameter $w_{\rm DE}(z)$

If $w_{\rm DE} = {\rm const.}$,

observational constraint is close to $w_{\rm DE} = -1$

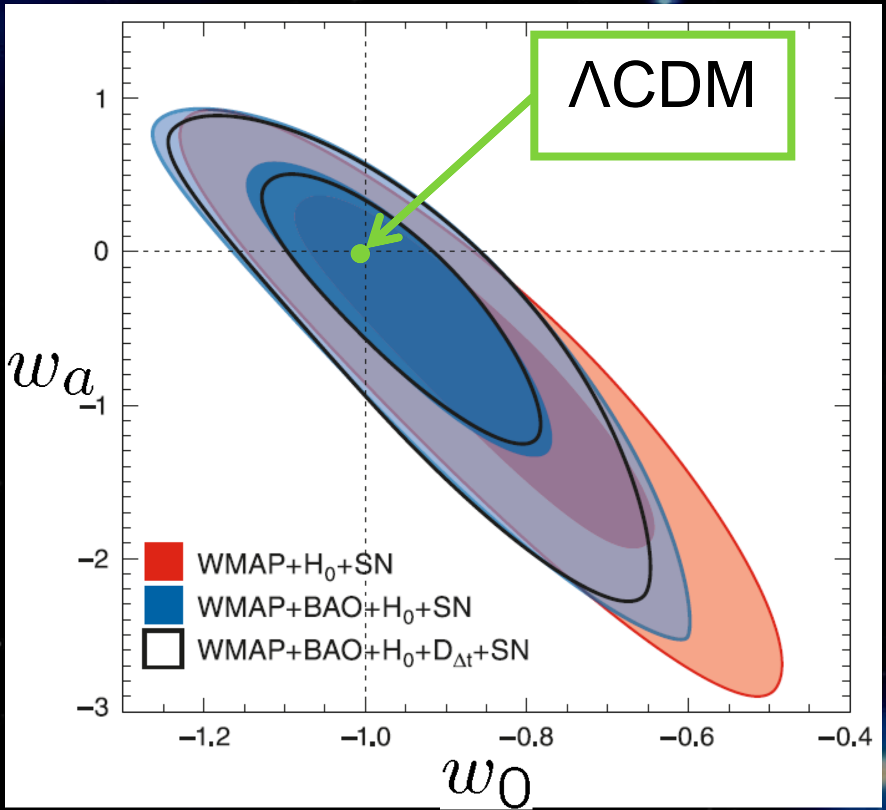


Komatsu et al. (2010)

Time variation of $w_{\mathrm{DE}}(z)$

Time variation of w_{DE} is still allowed.

Crossing the phantom divide $w_{\mathrm{DE}} = -1$.



Komatsu et al. (2010)

- CPL fitting

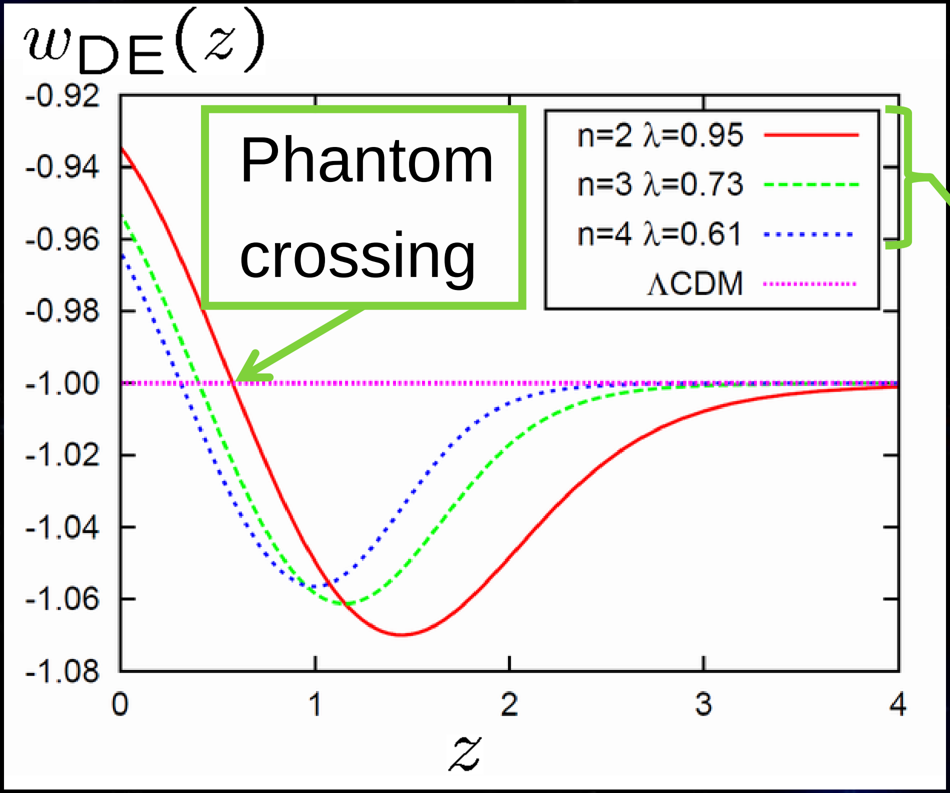
$$w_{\mathrm{DE}} = w_0 + \frac{w_a z}{1+z}$$

Chevallier & Polarski (2001)
Linder (2003)

Phantom crossing in Starobinsky model

Starobinsky model

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-n} - 1 \right]$$



- CPL fitting

w_0	w_a
- 0.92	- 0.23
- 0.94	- 0.22
- 0.96	- 0.21

H.M., Starobinsky, Yokoyama (2010)

Summary of section 2

1. Introduction of $f(R)$ Gravity
   - Mimic ΛCDM model without cosmological constant.
   - Viable conditions restrict the form of $f(R)$.
2. Background Universe
   - Viable $f(R)$ model exhibits phantom crossing.
   - Time dependence of $w_{\rm DE}(z)$ fits the observation.
3. Density Perturbation

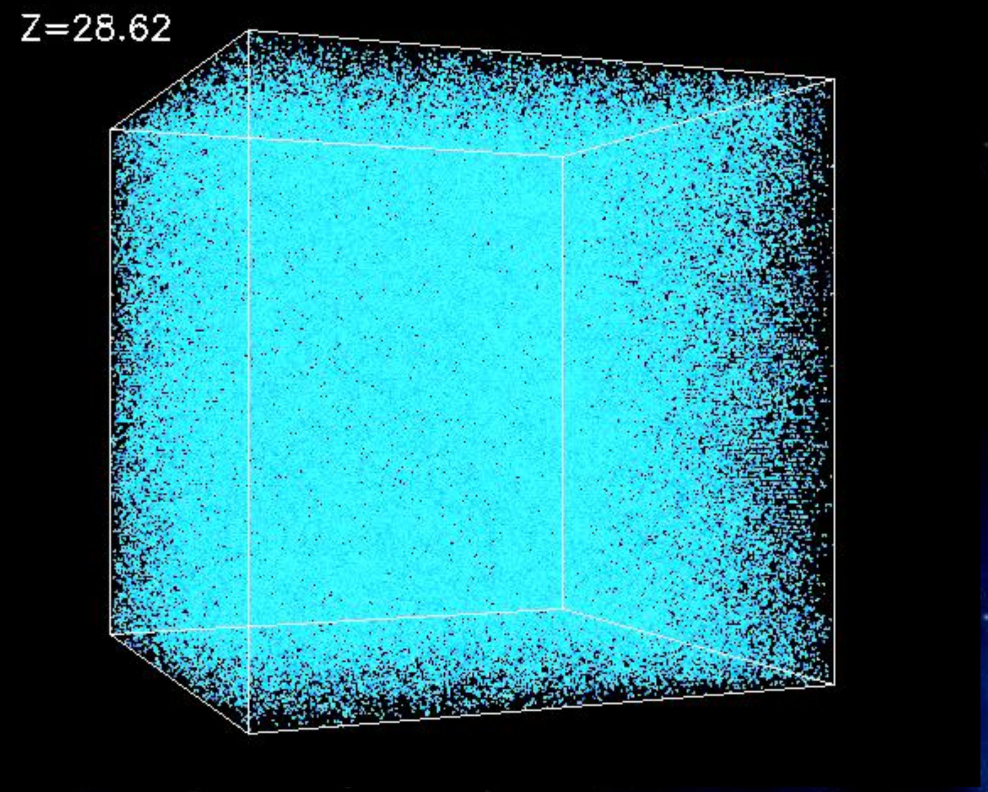
4. Summary

Density fluctuations

Large scale structure is constructed from density fluctuation δ.

$$\delta = \frac{\delta\rho}{\rho}$$

We can study its properties in f(R) gravity by using linear perturbation.



The Center for Cosmological Physics

The evolution equation for density fluctuations

- ΛCDM model

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - 4\pi G\rho\delta_{\mathbf{k}} = 0$$

- f(R) gravity Tsujikawa (2007)

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - 4\pi G_{\mathrm{eff}}(k)\rho\delta_{\mathbf{k}} = 0$$

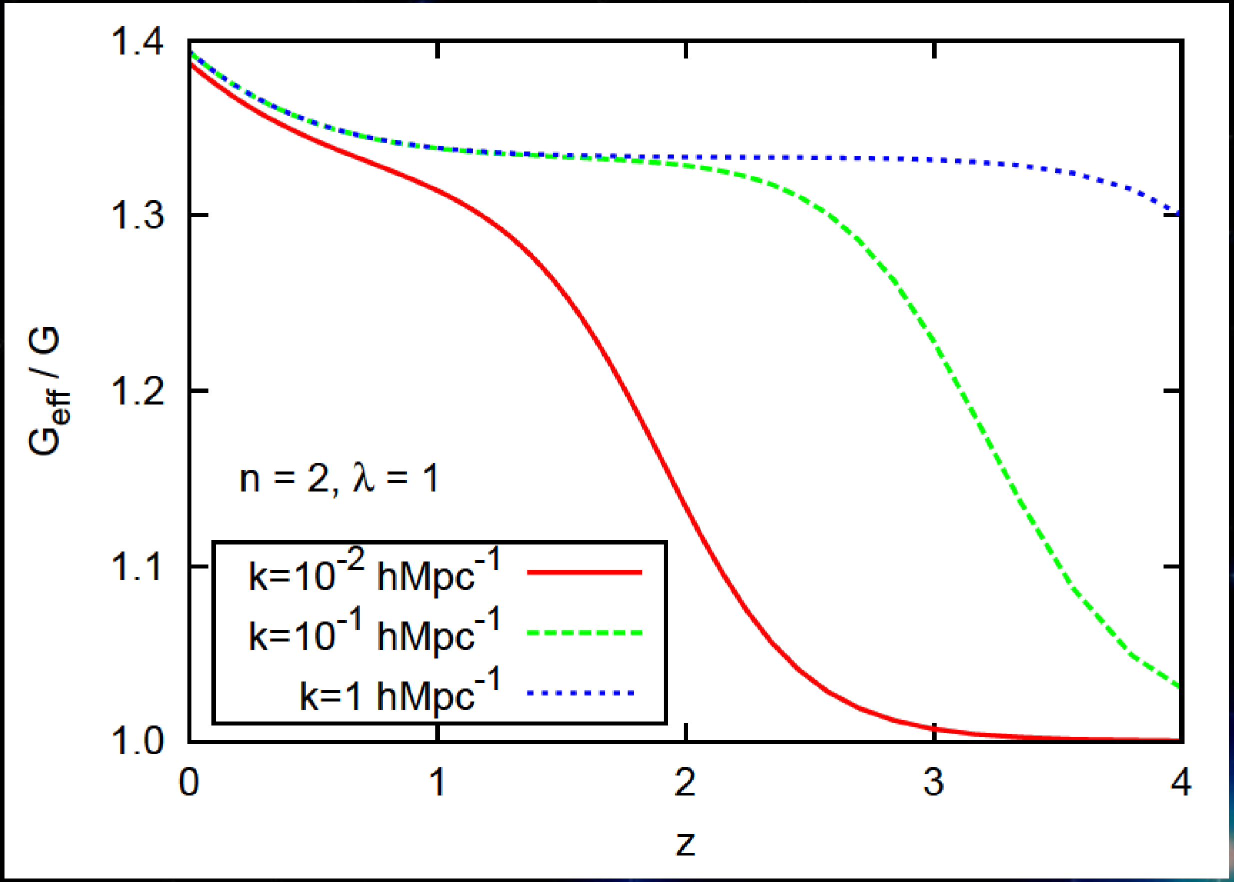
Effective gravitational coefficient

$$G_{\mathrm{eff}}(k) = \frac{G}{F}\left[1 + \frac{1}{3}\frac{\frac{k^2}{a^2}\frac{3F'}{F}}{1 + \frac{k^2}{a^2}\frac{3F'}{F}}\right]$$

$$\simeq \begin{cases} G/F & (3k^2F'/a^2F \ll 1) \\ 4G/3F & (3k^2F'/a^2F \gg 1) \end{cases}$$

Time evolution of effective gravitational coefficient in $f(R)$ gravity

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - 4\pi G_{\text{eff}}(k,z)\rho\delta_{\mathbf{k}} = 0$$

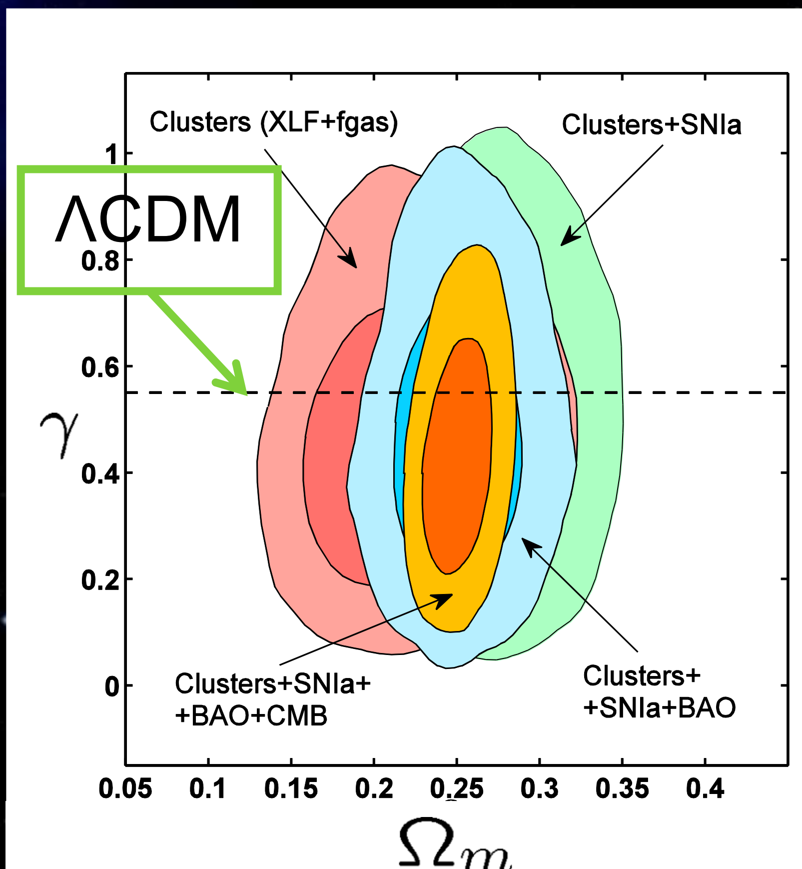


Growth index $\gamma(z)$

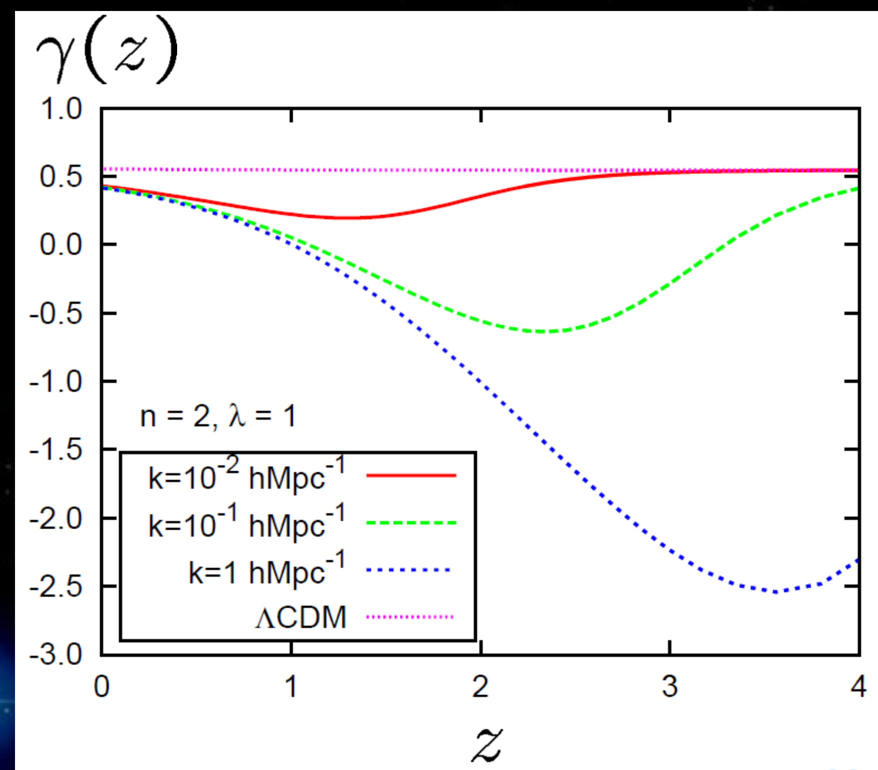
$$\beta(z) = \frac{d \log \delta}{d \log a} = \Omega_m(z)^{\gamma(z)}$$

growth index $\gamma(z)$ (ΛCDM: almost constant $\gamma(z) \simeq 6/11$)

$f(R)$ gravity: $\gamma(z)$
time and scale dependent



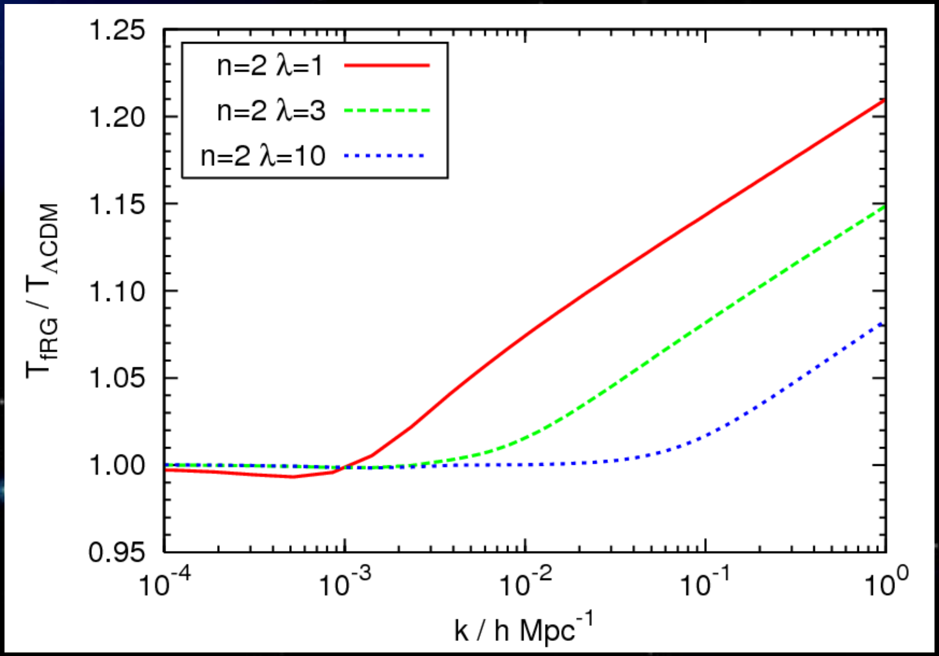
Rapetti et al. (2009)

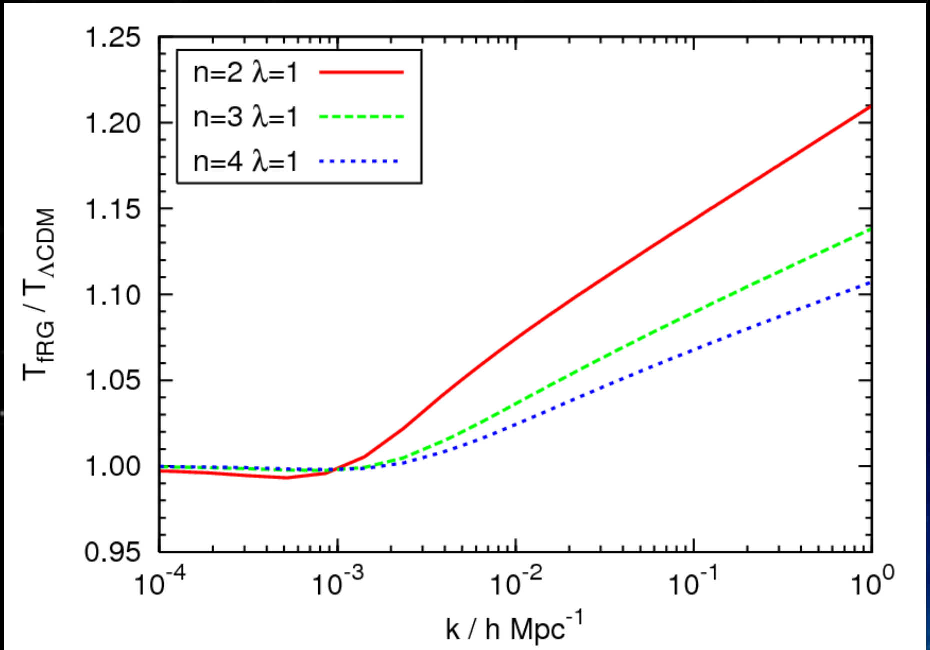


Transfer function in $f(R)$ gravity

Scale and time dependence for $\gamma(z)$

= Additional transfer function





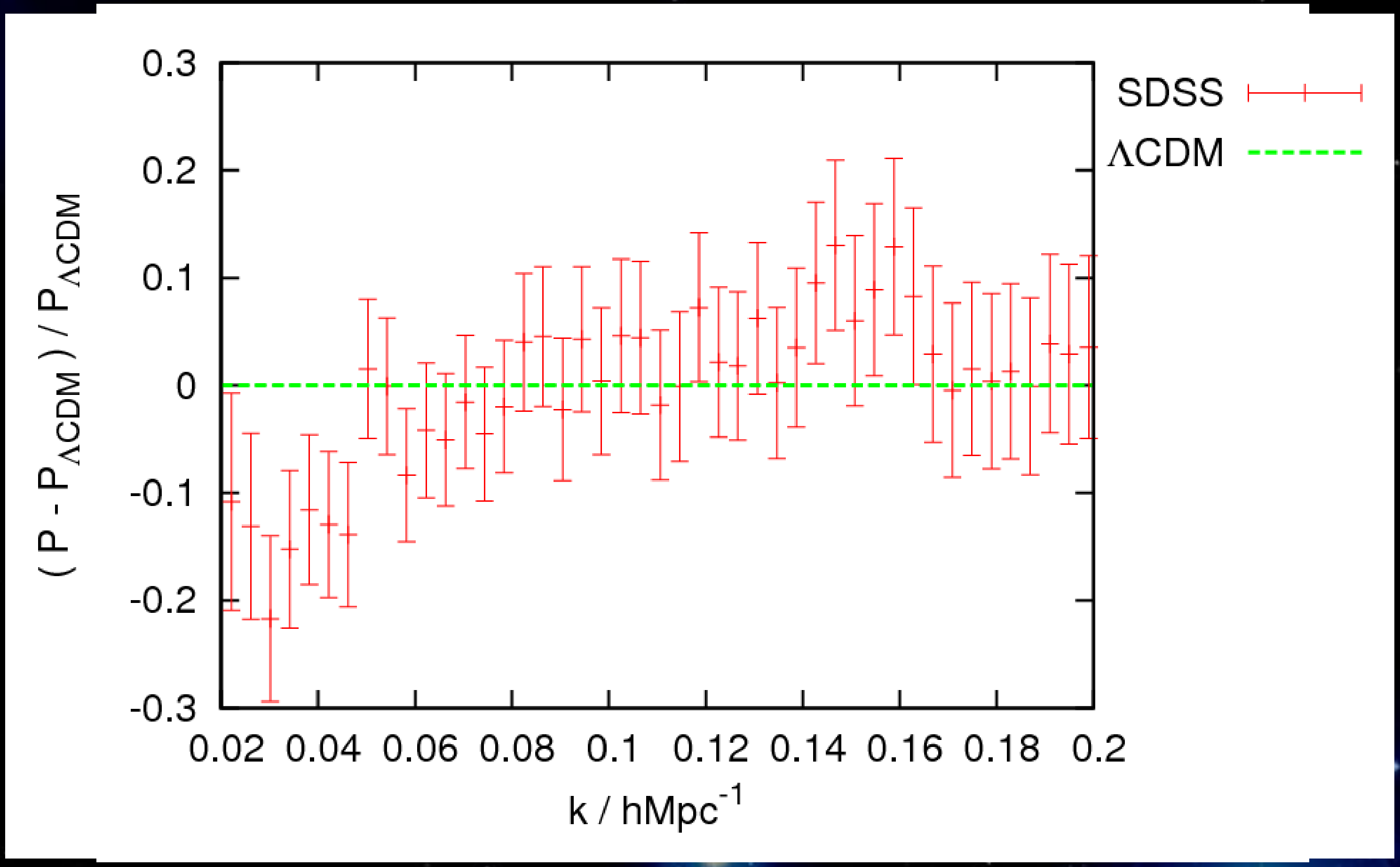
It gives scale dependent enhancement in power spectrum.

--- compare it observational data from SDSS DR7.

ΛCDM model

Reid et al. (2009)
Komatsu et al. (2010)

- Cosmological parameters: WMAP7 data



Massive neutrinos

- Free streaming scale

$$k_{\mathsf{fs}}(z) \simeq \frac{0.35}{(1+z)^{1/2}} \left(\frac{m_\nu}{1\ \mathrm{eV}}\right) \left(\frac{\Omega_m}{0.27}\right)^{1/2} h\ \mathrm{Mpc}^{-1}$$

Massive neutrinos break the structure below k_{fs} .

- Bounds for total neutrino mass

0.058 eV < $\sum m_\nu$ < 0.3 – 0.6 eV

< 0.58 eV (WMAP7 only)

Summary

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   - Viable $f(R)$ model exhibits phantom crossing.
   - Time dependence of $w_{\rm DE}(z)$ fits the observation.
3. Density Perturbation
   - $f(R)$ gravity enhances the structure formation.
   - $\gamma(z)$ ($G_{\rm eff}$) depends on time and scale.
   - Massive neutrinos are allowed without scale dependent bias in $f(R)$ gravity.