Quantum Field Theory in de Sitter space

Hiroyuki Kitamoto (Sokendai)
with
Yoshihisa Kitazawa (KEK,Sokendai)

Outline

- Quantum field theory in de Sitter space concerns deep mysteries: inflation in the early universe and dark energy of the present universe
- If a field is massless and minimally coupled to background, propagator in dS space has time dependence
- In an interacting field theory, this time dependence gives time dependence to the energy-momentum tensor of matter field

- So the effective cosmological constant changes with time
- This time dependence has been still investigated in some models. As a model whose result is unknown, we investigate Non-linear sigma model

Propagator in dS space

$$dS_4: \ ds^2 = -dt^2 + e^{2Ht}d\mathbf{x^2}$$
$$= \frac{-d\tau^2 + d\mathbf{x^2}}{H^2\tau^2}$$

$$\tau \equiv -\frac{1}{H}e^{-Ht}$$

H: Hubble constant

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\ g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

massless
minimal coupling

Propagator for a massless and minimal coupled field has IR divergence

$$\langle\varphi(x)\varphi(x')\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{H^2\tau\tau'}{2p}(1-i\frac{1}{p\tau})(1+i\frac{1}{p\tau'})e^{-ip(\tau-\tau')+i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}$$
$$\sim \int \frac{dp}{p} \qquad (p\to 0)$$

In physical momentum $P \equiv p/e^{Ht} = pH|\tau|$,

$$\int \frac{dp}{p} = \int \frac{dP}{P}$$

To avoid IR divergence, we give IR cut-off ε_0

$$\int_{\varepsilon_0 H|\tau|} dP$$

This prescription means that more degrees of freedom go out of the cosmological horizon $P = H$ with cosmic evolution

In contrast, UV cut-off Λ fixes the maximum physical momentum

$$\int^{\Lambda} dP$$

Inside the cosmological horizon $P > H$,
the degrees of freedom are constant

Outside the cosmological horizon $P < H$,
the degrees of freedom increase as time goes on

$$\int dP = \underbrace{\int_{H}^{\Lambda} dP}_{\text{constant}} + \underbrace{\int_{\varepsilon_0 H|\tau|}^{H} dP}_{\text{increase}}$$

Contribution from outside the cosmological horizon gives
growing time dependence to propagator

$$\langle \varphi(x)\varphi(x')\rangle \sim \int_{\varepsilon_0 H|\tau|}^{H} \frac{dP}{P} = \left| \log(\varepsilon_0|\tau|) \right|$$

Effective cosmological constant

Energy-momentum tensor of matter field contributes to the effective cosmological constant

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = 8\pi G$$

$$\Lambda_{eff} \equiv \Lambda - \frac{\kappa}{4}\langle T_\mu{}^\mu\rangle$$

V : potential

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - g_{\mu\nu}V$$

The time dependence of propagator can gives time dependence to the effective cosmological constant

Contribution from a free field

For a free field, propagator has time dependence but energy-momentum tensor does not because of differential operator

$$\langle T_{\mu\nu}\rangle = \langle \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\rangle$$
$$= \ g_{\mu\nu}\frac{3H^4}{32\pi^2} + \underline{(conformal\ anomaly)}$$

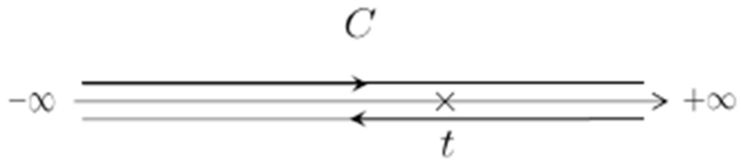
$$= g_{\mu\nu}\frac{29H^4}{15\cdot 16\pi^2}$$

To obtain the time dependence of the energy-momentum tensor, we need to consider interaction

Schwinger-Keldysh formalism

When we consider an interacting field theory in a time dependent background like dS space, we can't prefix the vacuum at $t = \infty$

So we can't use the Feynman-Dyson perturbation theory (in-out formalism), we need to use the Schwinger-Keldysh formalism (in-in formalism)



In this formalism, we can deal with an interacting field theory which has time dependence e.g. $\lambda\varphi^4$ *theory*

kinetic: $\langle \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\rangle = g_{\mu\nu}\frac{3H^4}{32\pi^2} - \frac{1}{H^2\tau^2}\delta_\mu{}^0\delta_\nu{}^0\frac{\lambda H^4}{2^6\cdot 3\pi^4}\Big|\log(\varepsilon_0|\tau|)\Big|$

potential: $\langle -g_{\mu\nu}V\rangle = -g_{\mu\nu}\frac{\lambda H^4}{2^7\pi^4}\Big|\log(\varepsilon_0|\tau|)\Big|^2$

Compared to potential term, kinetic term is subdominant
and required for conservation law $D_\mu\langle T^\mu{}_\nu\rangle = 0$

1-loop effect increases the cosmological constant

$$\Lambda_{eff} = \Lambda + \kappa\Big(- \frac{3H^4}{32\pi^2} + \frac{\lambda H^4}{2^7\pi^4}\Big|\log(\varepsilon_0|\tau|)\Big|^2\Big)$$

V. K. Onemli, R. P. Woodard ’02

Stochastic approach

Perturbation theory breaks down when $\lambda \left| \log(\varepsilon_0 |\tau|) \right|^2 \sim 1$,
and so we need non-perturbative method

If we extract the leading log contribution,
field equation becomes the Langevin equation

A. A. Starobinsky,
J. Yokoyama '94

$$3H \frac{\partial}{\partial t}\varphi(x) = 3H f(x) - \frac{\partial V(\varphi(x))}{\partial \varphi} \qquad \langle f(x) f(x') \rangle = \frac{H^3}{4\pi^2}\delta(t - t')$$

By using the Fokker-Planck equation, we can estimate
non-perturbative IR effect

$$\frac{\partial}{\partial t}\rho(t, \varphi) = \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial \varphi^2}\rho(t, \varphi) + \frac{1}{3H}\frac{\partial}{\partial \varphi}\left[V(\varphi)\rho(t, \varphi)\right]$$

$\rho(t, \varphi)$: probability density

At $t \to \infty,\ \rho(t,\varphi) \to \rho_\infty(\varphi)$, the result is as follows

$$\langle V \rangle = \langle \frac{\lambda}{4!}\varphi^4 \rangle = \frac{\lambda}{4!}\int_{-\infty}^{\infty} d\varphi\ \rho_\infty(\varphi)\varphi^4 = \frac{3H^4}{32\pi^2}$$

$$\begin{aligned} \langle T_{\mu\nu} \rangle &= \langle \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - g_{\mu\nu}V \rangle \\ &= \ g_{\mu\nu}\frac{3H^4}{32\pi^2} - g_{\mu\nu}\frac{3H^4}{32\pi^2} = 0 \end{aligned}$$

In φ^4 theory, non-perturbative IR effect doesn't contribute to the energy-momentum tensor

Other examples

It depends on model whether energy-momentum tensor increases the cosmological constant or decrease

Following models have been still investigated

$Yukawa\ \bar{\psi}\psi\varphi :$ 1-loop effect increases the cosmological constant

S. P. Miao, R. P. Woodard '06

$Scalar\ QED :$ 1-loop effect decreases the cosmological constant

T. Prokopec, N. C. Tsamis,
R. P. Woodard '08

Non-linear sigma model

As a model whose result is unknown, we investigate Non-linear sigma model

$$S = \frac{1}{2g^2}\int \sqrt{-g}d^4x \left[G_{ij}(-g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j) - R_{iajb}(\xi^a\xi^b)(-g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j) + (-g^{\mu\nu}D_\mu\xi^a D_\nu\xi^a) - \frac{1}{3}R_{cadb}(\xi^a\xi^b)(-g^{\mu\nu}D_\mu\xi^c D_\nu\xi^d) + \cdots\right]$$

φ^i : background field ξ^i : quantum fluctuation

$g_{\mu\nu}$: dS_4

G_{ij} : S_{N-1} $\Longleftarrow$ O(N) symmetry $i = 1 \sim N-1$

$$R_{ikjl} = 1 \cdot (G_{ij}G_{kl} - G_{il}G_{jk})$$

Effective coupling

$$\frac{1}{2g^2}G_{ij}(-g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j) - \frac{1}{2g^2}R_{iajb}\langle\xi^a(x)\xi^b(x)\rangle(-g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j)$$

$$= \frac{1}{2g^2}\left\{1-(N-2)\langle\xi(x)\xi(x)\rangle\right\} G_{ij}(-g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j)$$

$$\Longrightarrow \qquad g_{eff}^2 = g^2 + (N-2)g^2\langle\xi(x)\xi(x)\rangle$$

As we know, Non-linear sigma model in 2-dimentional Minkowski space is asymptotic free

$$g_{\mu\nu} : M_2 \qquad \langle\xi(x)\xi(x)\rangle = g^2\int\frac{dp}{2\pi}\frac{1}{2p} = g^2\frac{1}{4\pi}\log\frac{\Lambda}{\mu}$$

4-dimentional dS space' case is similar

$$g_{\mu\nu} : dS_4 \qquad \langle\xi(x)\xi(x)\rangle \sim g^2\int\frac{d^3p}{(2\pi)^3}\frac{H^2}{2p^3} = g^2\frac{H^2}{4\pi^2}\log\frac{\Lambda}{\varepsilon_0|\tau|}$$

The coupling constant increases as time goes on

Effective cosmological constant

1-loop effect increases the cosmological constant

$$\langle T_{\mu\nu}\rangle = \langle(\delta_\mu{}^\rho\delta_\nu{}^\sigma - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma})(\partial_\rho\xi^a\partial_\sigma\xi^a - \frac{1}{3}R_{acbd}\partial_\rho\xi^a\partial_\sigma\xi^b\xi^c\xi^d)\rangle$$
$$= g_{\mu\nu}(N-1)\frac{3H^4}{32\pi^2} - g_{\mu\nu}(N-1)(N-2)\frac{g^2H^6}{2^5\cdot 3\pi^4}\left|\log(\varepsilon_0|\tau|)\right|$$

$$\Lambda_{eff} = \Lambda + \kappa\left(-(N-1)\frac{3H^4}{32\pi^2} + (N-1)(N-2)\frac{3g^2H^6}{2^8\pi^4}\left|\log(\varepsilon_0|\tau|)\right|\right)$$

This sign is decided by the sign of the Ricci scalar

If $G_{ij} : S_{N-1} \to H_{N-1}$,

At 1-loop effect, the coupling constant and the cosmological constant decrease as time goes on

$$g_{eff}^2 = g^2 - (N-2) g^4 \frac{H^2}{4\pi^2} \left| \log(\varepsilon_0 |\tau|) \right|$$

$$\Lambda_{eff} = \Lambda + \kappa \left(-(N-1) \frac{3H^4}{32\pi^2} - (N-1)(N-2) \frac{3g^2 H^6}{2^8 \pi^4} \left| \log(\varepsilon_0 |\tau|) \right| \right)$$

Summary

- Increase in degrees of freedom outside the cosmological horizon gives time dependence to the effective cosmological constant
- In non-linear sigma model, 1-loop effect increases the coupling constant and the cosmological constant on $G_{ij} : S_{N-1}$ and decreases the coupling constant and the cosmological constant on $G_{ij} : H_{N-1}$

Future work

- If there are differential interaction, the stochastic approach is not available. We need an alternative method to estimate non-perturbative effect