Axionic Instabilities of Black Holes

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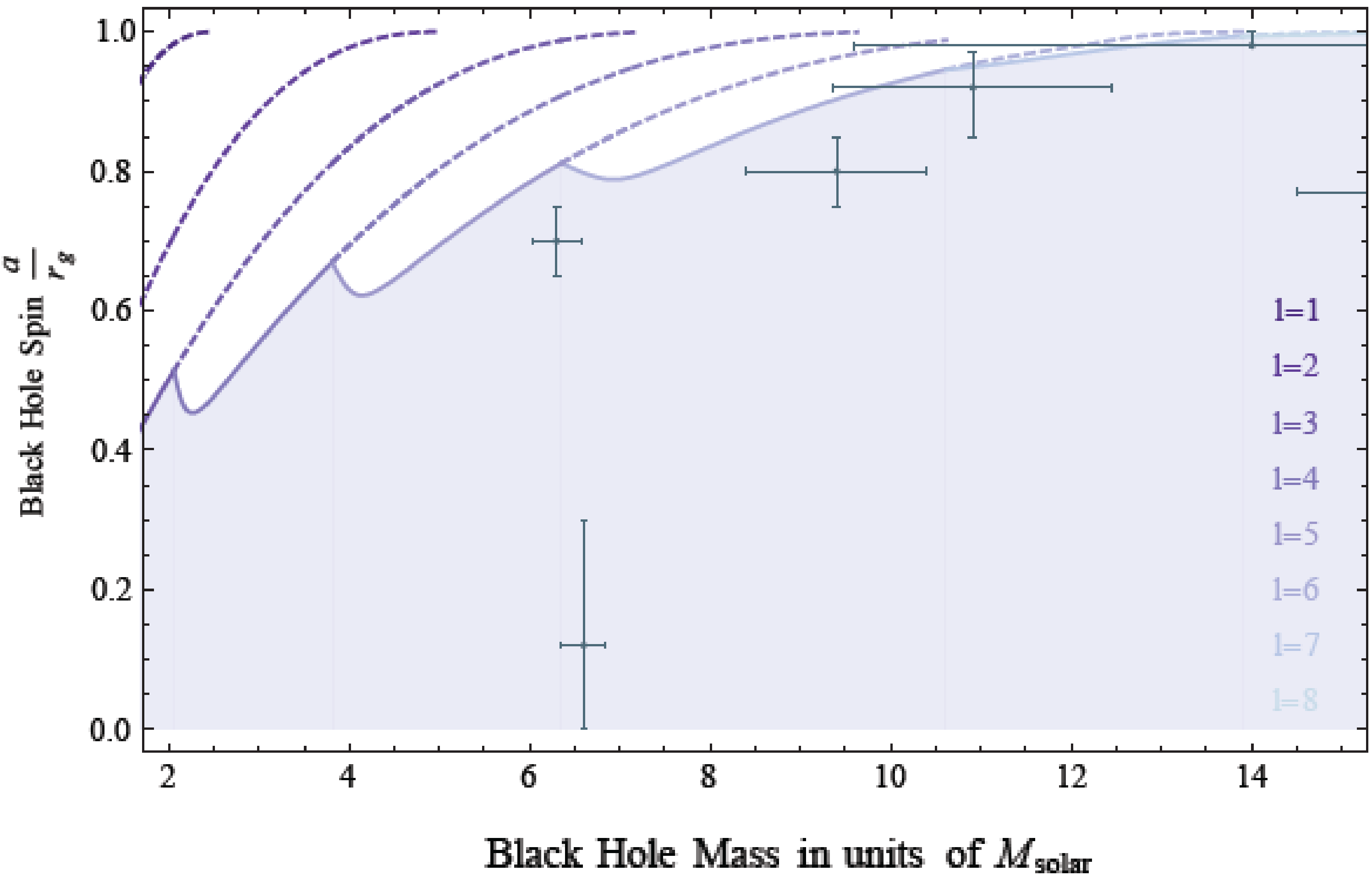
$$d = 4$$

Extra-dimension probe by Cosmophysics

Spin measurements by thermal X-ray spectra of black hole X-ray binaries

McClintock et al 0911.5408



Near future advances in black hole observations

- X-ray astronomy, magnetohydrodynamical simulations, etc.

McClintock et al 0911.5408

- Gravitational radiations from inspiraling binaries

e.g. Extreme-Mass-Ratio-Inspirals (EMRIs)

Gair et al CQG21, S1595 (2004)

a few $\times 10^2$ events for 3 year observations by LISA

High precision measurements of mass & spin parameters

w/ accuracy $10^{-3} \sim 10^{-5}$

\+ probes of near horizon geometries

Precision black hole physics

Precision black hole physics

- Astrophysics
- Probing fundamental physics

  Beyond the Standard Model physics

  Black holes in modified gravity theories

  -- Extra-dimensions / higgs phases ... etc

- Exploring the String Axiverse:

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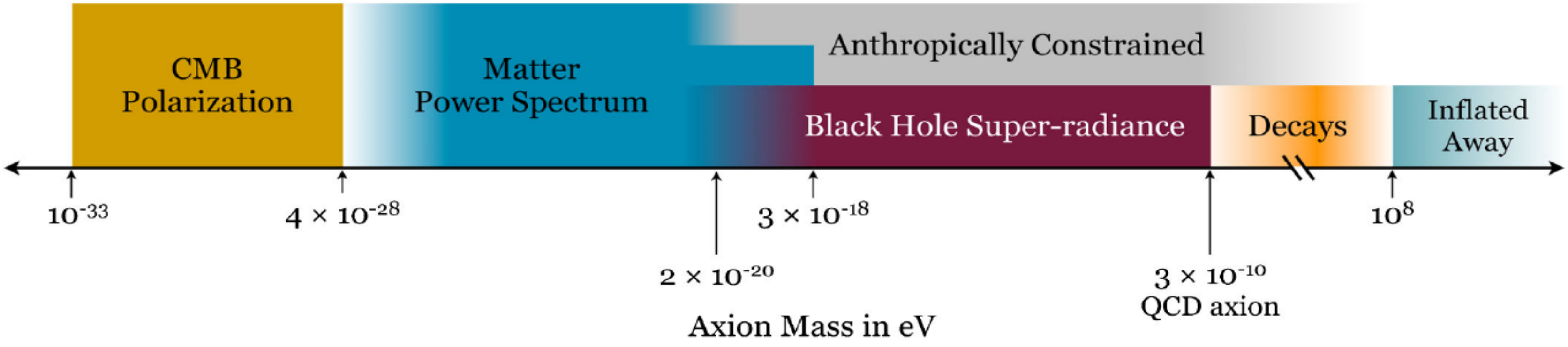
String Axiverse

- Plenitude of Axions



- Probing Axiverse by precision black hole observations

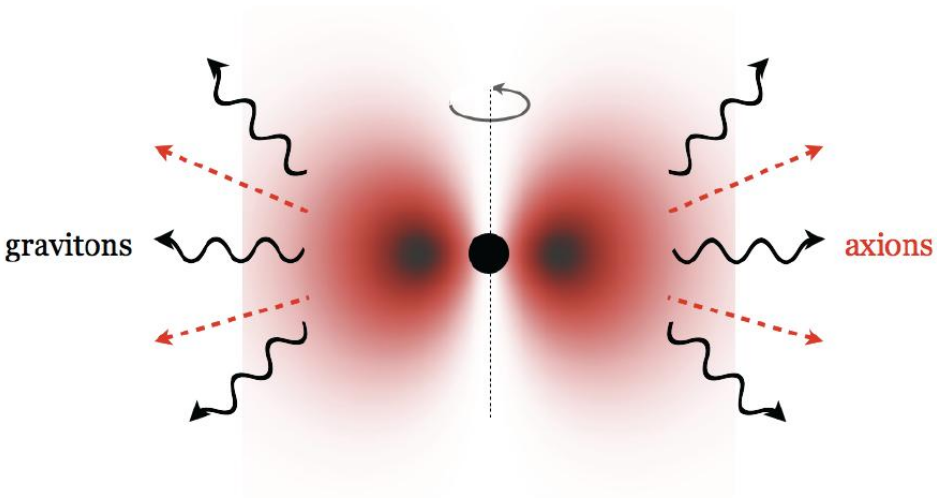
- Instability of Axion around a rotating black

- Compton wavelength of order $M_\odot \sim 10^{12} M_\odot$ BH size

$$\mu = 10^{-9} \sim 10^{-21}\ \text{eV}$$

A black hole surrounded by an axionic cloud

- Looks like a huge quantum mechanical system
  similar to a hydrogen atom



with each level occupied by bosons rather than electrons

- Axion cloud may behave like an Bose-Einstein Condensate (BEC)

Purpose

- Discuss possible observational consequences of the presence of axions around astrophysical black holes

(i) – review briefly what have so far been proposed

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(ii) – attempt to clarify prospects and problems in these ideas

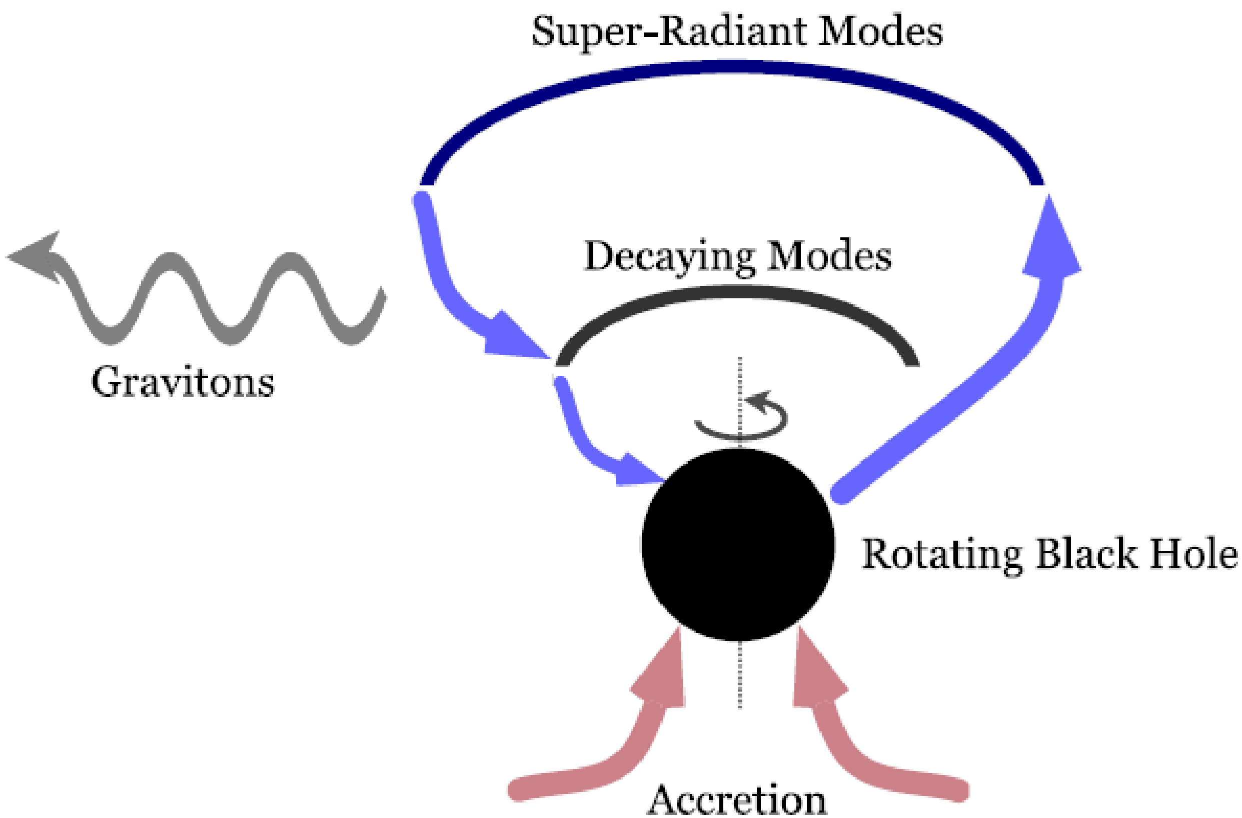
Main target

- Superradiant instability

  A consequence of the presence of an Axion cloud around rotating black holes

- Gravity waves from superradiant axion clouds

  A direct possibility to detect the presence of an Axion cloud by high precision observations



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Outline

- Axionic superradiant instability
- Gravity waves from Axion cloud

  Possible processes (linear analysis)

  Level transitions

  Annihilations
- Discussions:

  Analogy w/ quantum mechanical system

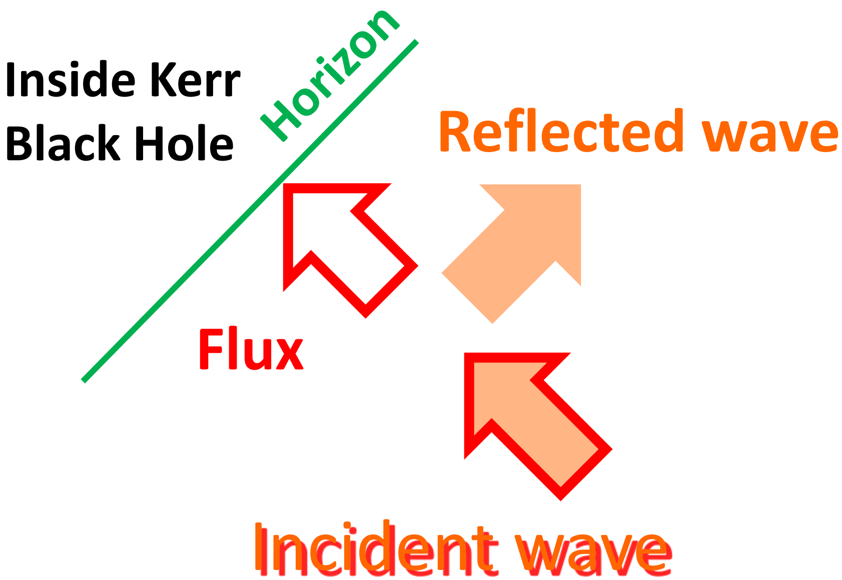
  (self-interaction)

  w/ classical fluid mechanics

  (self-gravity)

SUPERRADIANT INSTABILITY

Wave scattering by a black hole



Conervation law:

$$E_I = E_R + \mathit{Flux}$$

If $\mathit{Flux} < 0$ ➡ $E_R > E_I$

Reflected wave gets amplified

Zel'dovich 72 Starobinsky73

e.g. Scalar field of modes : $\phi = e^{-i\omega t + im\varphi} f(r, \theta)$

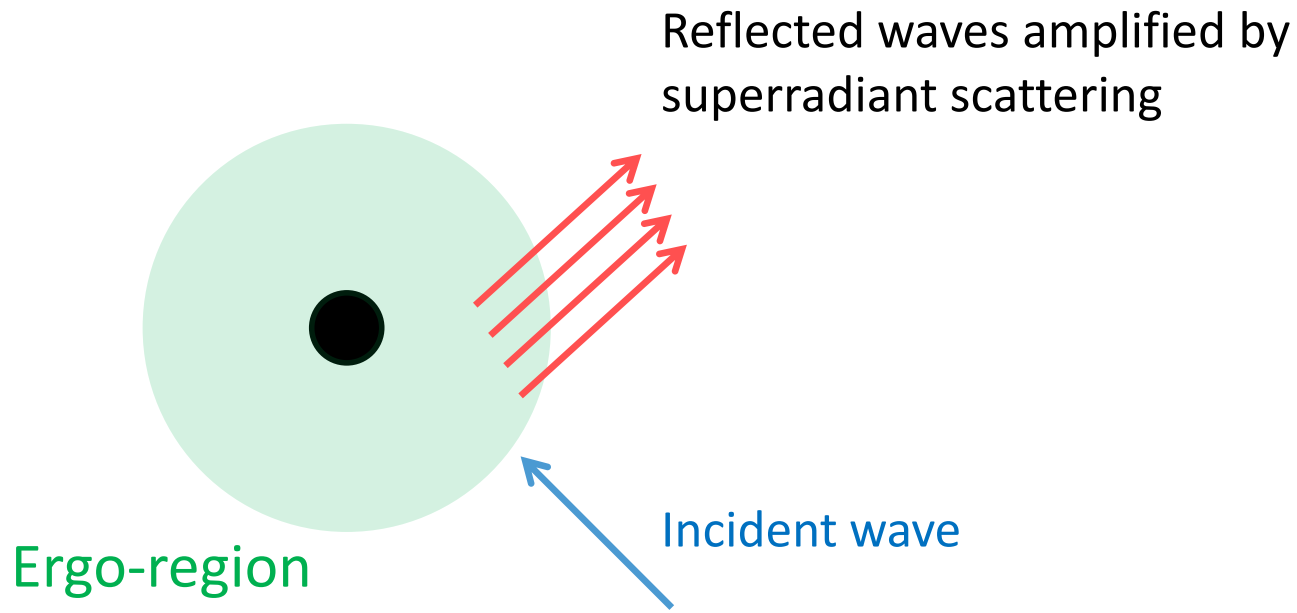
$$\mathit{Flux} = \omega(\omega - m\Omega_H) \int_H |f|^2$$

Ω_H: Horizon angular velocity

$\mathit{Flux} < 0$ for $0 < \omega < m\Omega_H$ Superradiant modes

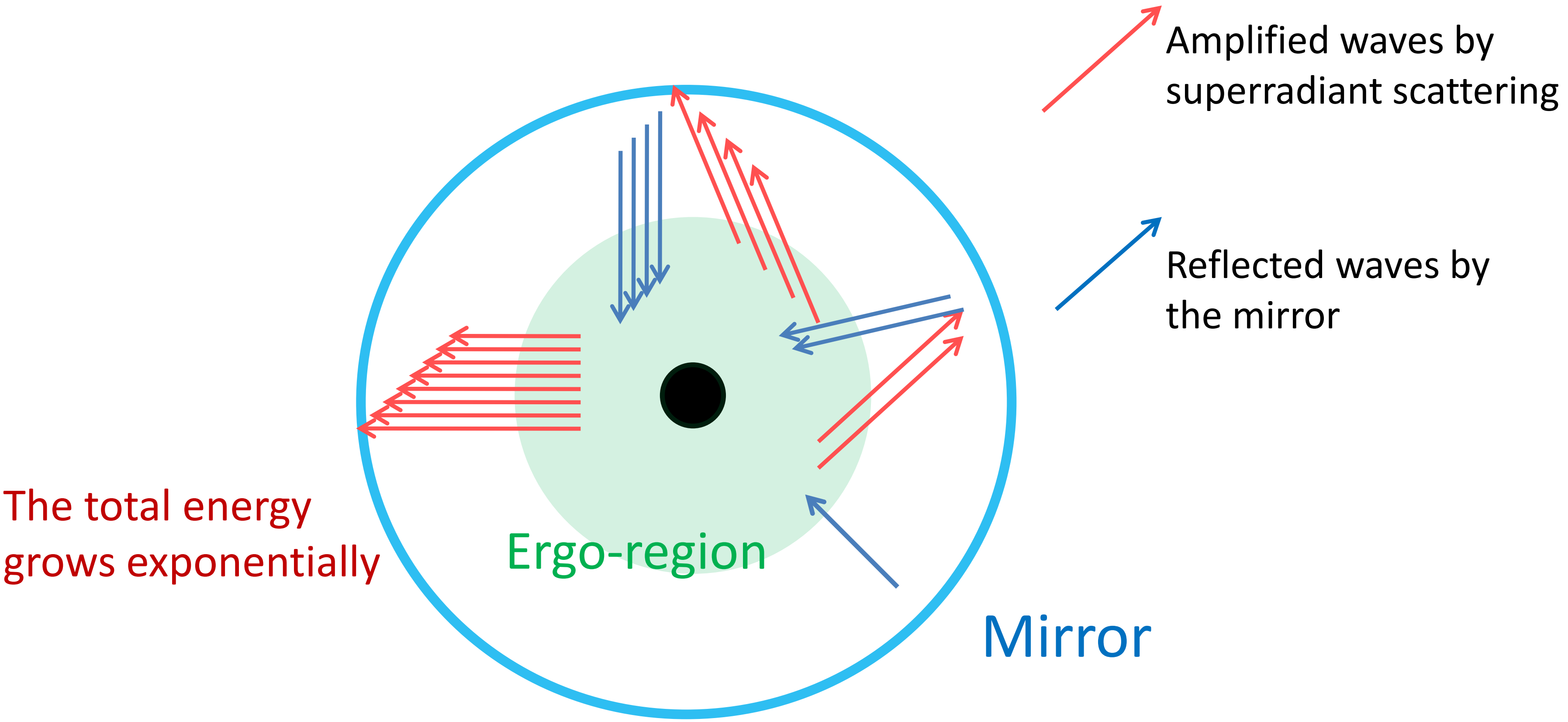
Superradiant scattering

- "Flux" can be negative due to the presence of **Ergo-region** just outside the horizon where time-translation symmetry becomes spacelike



Superradiant instability

- Amplified scattered waves bounce back and forth between the mirror and the BH



Massless scalar field and mirror

- Boundary condition: Black hole bomb

  Press-Teukolsky Nature 238,211 (1972)

  Cardoso-Dias-Lemos-Yoshida PRD70,044039

- Inner edge of accretion disk

  Van Putten Science 284, 115 (1999)

  Aguirre, APJ 529 L9 (2000)

- AdS curvature

  Hawking-Reall 99

  Cardoso-Dias 04 Cardoso-Dias-Yoshida 06

  Kodama 07 Uchikata-Yoshida-Futamase 09

Massive scalar field case

μ : Scalar field mass

- Numerical work: e.g. Furuhashi-Namb 04 Dolan 07

- Analytic method $\alpha := \mu G M = \text{BH size/Compton wavelength}$

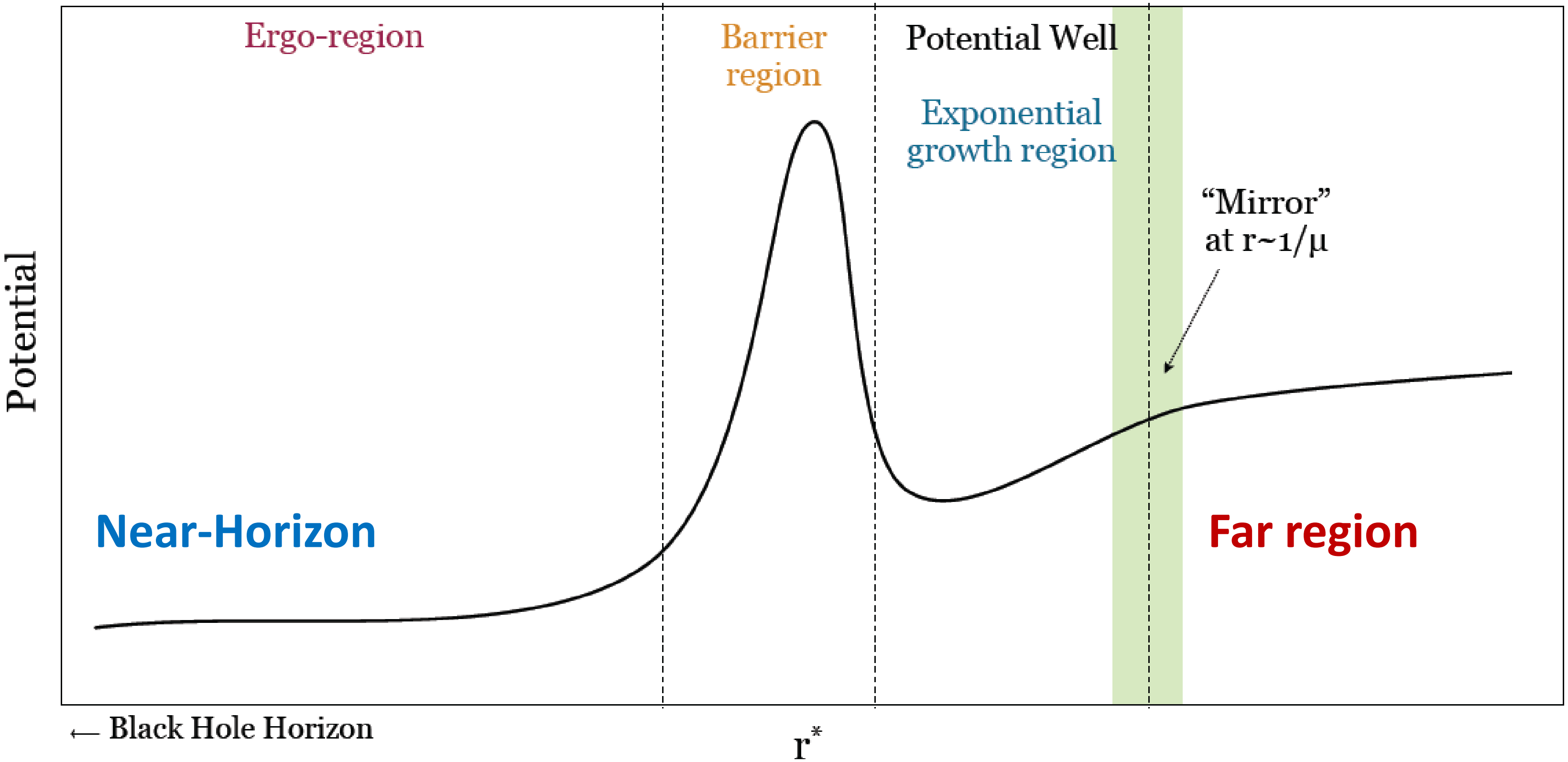
-- Lowmass limit ($\alpha \ll 1$) Detweiler 80

-- Highmass limit ($\alpha \gg 1$) Zouros-Eardley 79

-- Intermid mass (~ 1) Hod-Hod 2010 Rosa 2010

Wave equation: $$\frac{d^2\Psi}{dr_*^2} - V\Psi = 0$$

$$V = -\omega^2 + \frac{4r_g ram\omega - a^2m^2}{(r^2+a^2)^2} + \frac{\Delta}{r^2+a^2}\left(\mu_a^2 + \frac{l(l+1)+k^2a^2}{r^2+a^2} + \frac{3r^2-4r_gr+a^2}{(r^2+a^2)^2} - \frac{3\Delta r^2}{(r^2+a^2)^3}\right)$$



Analytic computation of superradiant instability

- Region I (Near-Horizon) $r \ll \max\left(\frac{l}{\omega}, \frac{l}{\mu}\right)$

- Region II (Far region) $GM \ll r$

$$\left(\frac{d^2}{dr^2} + \omega^2 - \mu^2 + \frac{2M\mu^2}{r} - \frac{l(l+1)}{r^2}\right)(rR) = 0$$

- Boundary conditions: $R \sim e^{-i(\omega - m\Omega_H)r_*}$ near Horizon

  $R \sim \frac{1}{r} e^{-\sqrt{\mu - \omega^2} r_*}$ infinity

- Matching the functional form in Overlapping region

$$\omega GM \ll 1\,, \quad \mu GM \ll 1$$

Growth rate of superradiant instability

- $\alpha = \mu G M \ll 1$ Functional matching:

Detweiler PRD22, 2323 (1980)

$$\tau^{-1} \sim 5 \times 10^{-2} \left(\frac{a}{M}\right) \frac{\alpha^9}{M}$$

The fastest instability is in the sector: $l = m = 1$

- $\alpha = \mu G M \gg 1$ WKB method:

Zouros-Eardley Ann Phys 118,139 (1979)

$$\tau^{-1} \sim 10^{-7} e^{-3.7\alpha} (GM)^{-1}$$

Gaina Sov. Astron. Lett 15, 243 (1989)

The fastest instability is in the sector: $m \sim l \sim \frac{\mu}{\Omega_H} \gg 1$

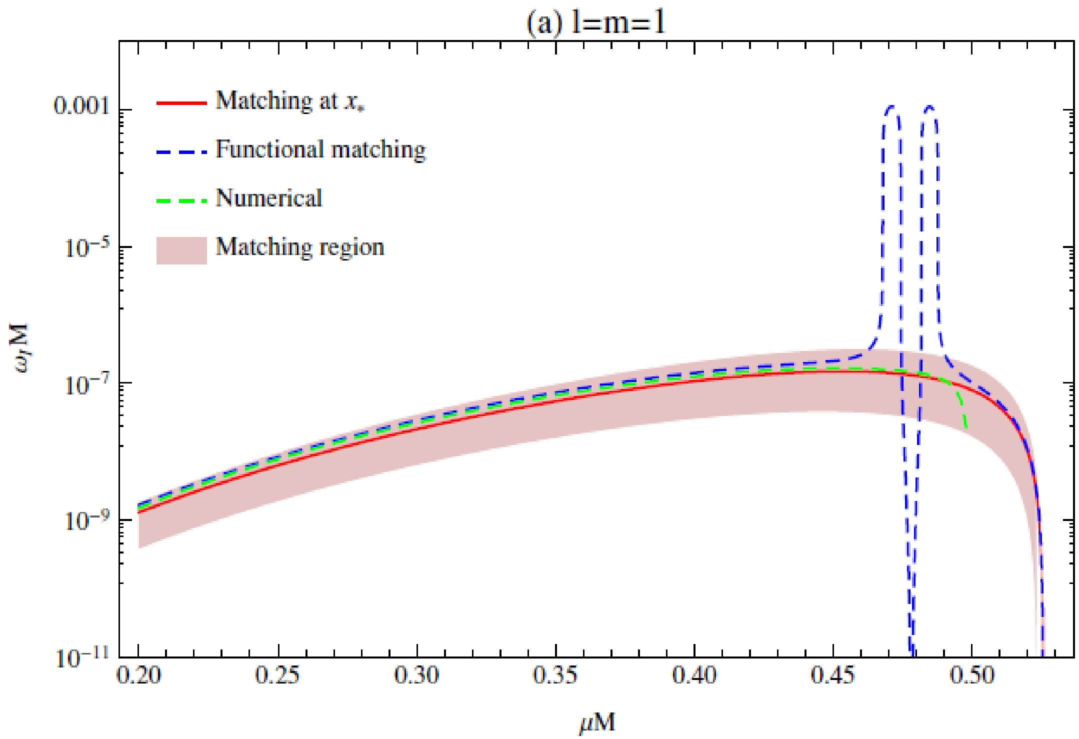
Rosa (2010) Point Matching Method

Maximum growth rate at

$$\alpha = \mu M \sim 0.454$$

$$\omega_I \simeq 1.49 \times 10^{-7} M^{-1}$$

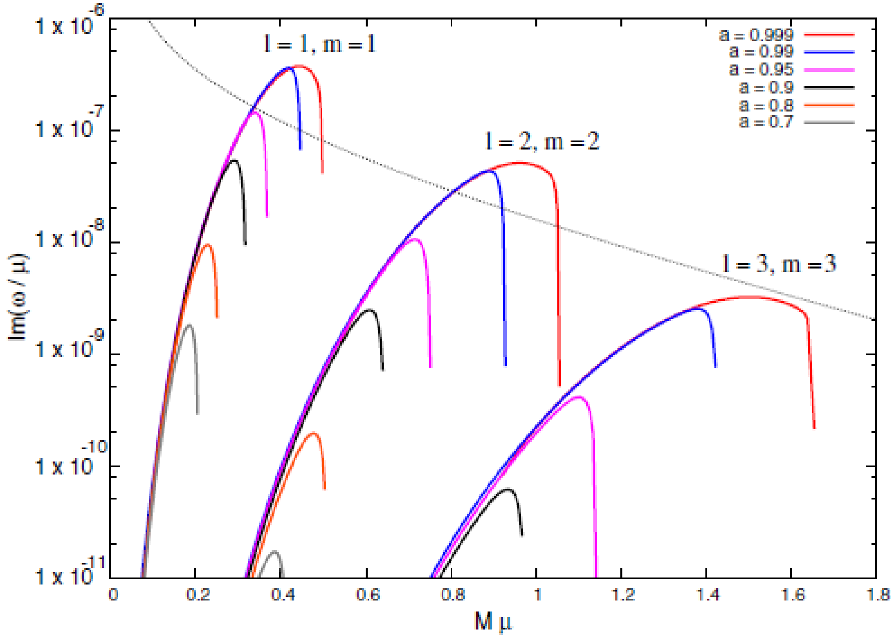
for $l = m = 1$



Good agreement w/ previous numerical results

Numerical work: Dolan PRD76 084001 (2007)

- Continued-fraction method



Most unstable for $\mu GM \leq 1/2$, $l = m = 1$ $a = 0.99$
w/ maximum growth rate

$$\tau^{-1} \sim 1.5 \times 10^{-7} (GM)^{-1}$$

Summary of superradiant instability

The instability is most effective when

(i) $a \approx M$: Extremal black hole

(ii) $\alpha := \mu GM \sim 1/2$

(iii) $\omega \sim m\Omega_H \quad \omega \sim \mu$

Nearly saturating

superradiance condition $0 < \omega < m\Omega_H$

and bound-state condition $\omega < \mu$

Maximum growth rate: $\tau^{-1} \sim 1.5 \times 10^{-7}(GM)^{-1}$

$\mathrm{Re}\,\omega \simeq 0.98\mu \qquad l = m = 1$

- In order for suprradiant instability to make sense

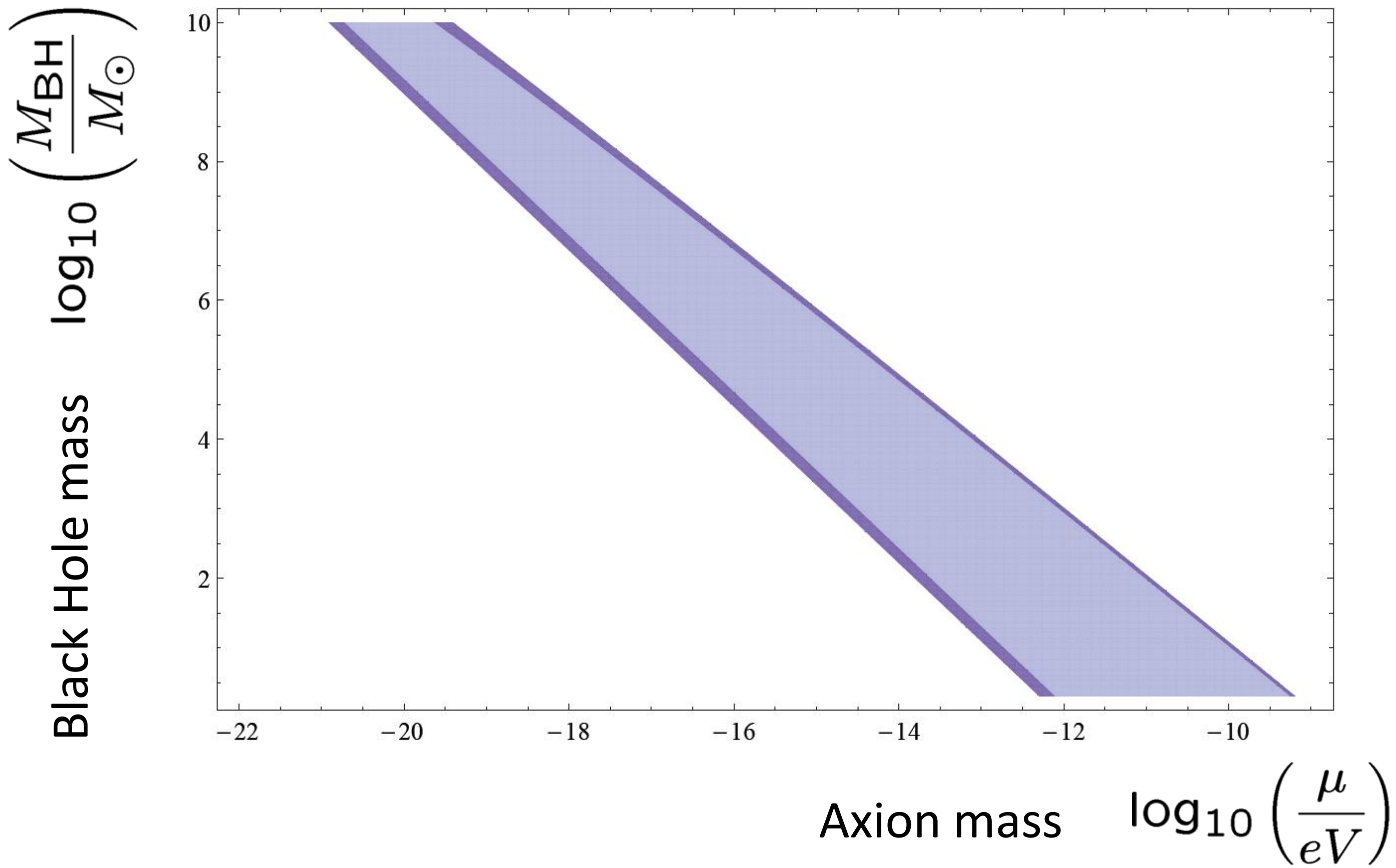
Timescale of the instability should be

$\ll$ The age of the Universe $\sim 10^{17} h^{-1}\text{sec}$

- Pion + solar mass BH $\alpha \sim 10^{18}$ $\tau \sim 10^{7} e^{+3.7\alpha} GM$
- Pion + primordial BH $\alpha \sim O(1)$ $\tau \sim 1.5 \times 10^{-17}\text{sec}$
  $M \sim 10^{15} g$ $\sim$ lifetime
- QCD Axion + Solar mass BH $\alpha \sim O(1)$ $\tau \sim 0.6 \times 10^{7} GM$

- Superradiance timescale must be shorter than the age of Universe (-- to have enough time to build up a superradiant Axion cloud)

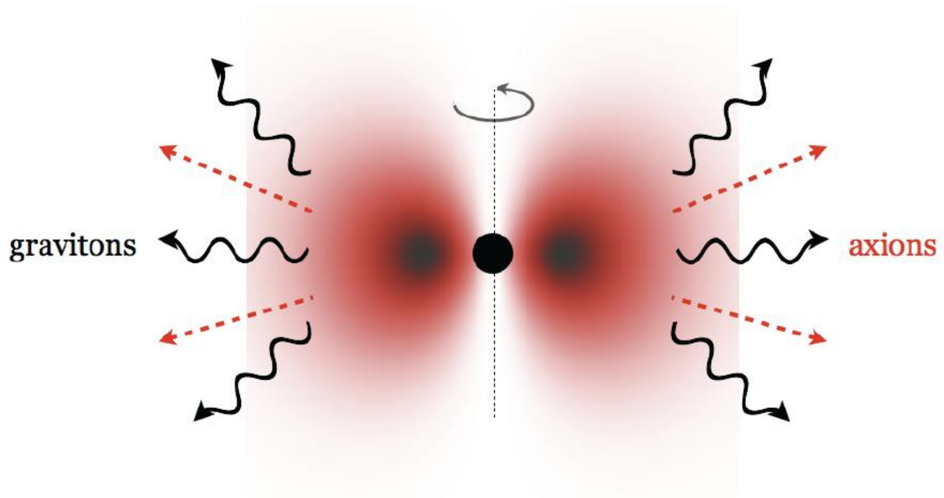
- Superradiance spin-down rate is faster than the spin-up rate by Eddington accretion



OBSERVATIONAL SIGNATURES

A black hole surrounded by an axionic cloud

- Looks like a huge quantum mechanical system
  similar to a hydrogen atom



with each level occupied by bosons rather than electrons

- Axion cloud may behave like an Bose-Einstein Condensate (BEC)

Gravity waves emission

- Processes:

(I) Transitions between levels

-- Analogous to photon emission from atoms

(II) Axion annihilations:

e.g., 2 axions decay into 1 graviton

(III) Bosenova collapse

Axion cloud around BH as an Atom

- Far region $GM \ll r$

$$\left(\frac{d^2}{dr^2} + \omega^2 - \mu^2 + \frac{2M\mu^2}{r} - \frac{l(l+1)}{r^2}\right)(rR) = 0$$

$\mu GM \ll 1 \qquad \omega GM \ll 1$

just like **an electron in the hydrogen atom**

- Solution

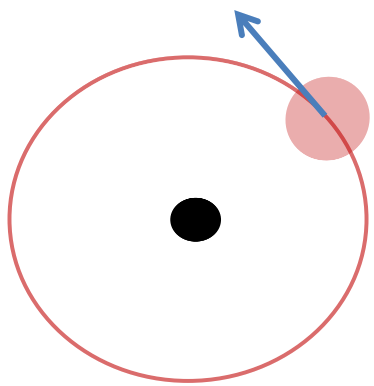
$$R = x^l e^{-x/2} U(n, 2l+2, 2\sqrt{\mu^2 - \omega^2} r) \qquad n = 0, 1, 2, \cdots$$

- Energy level

$$\omega_{\bar{n}} \simeq \mu \left(1 - \frac{\alpha^2}{\bar{n}^2}\right) \qquad \bar{n} := n + l + 1$$

- Axion velocity $$v \simeq \frac{\alpha}{\bar{n}}$$

-- Non-relativisitc for $\alpha \ll 1$



Orbit of level $\bar{n}$

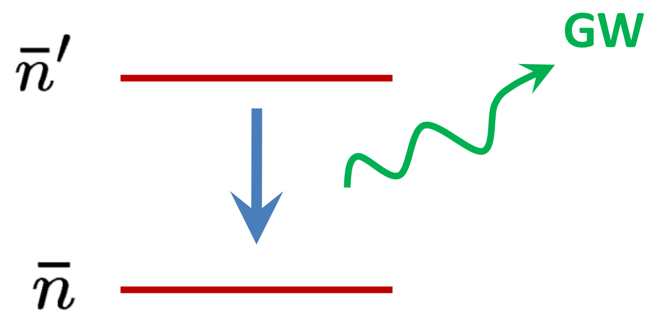
- Size of axion cloud $$r_c \simeq \frac{GM}{v^2}$$

- Quadrupole moment $$I_{ij} \sim \mu r_c^2$$

Gravity wave emission by level transitions

- Level transitions

$$\Delta\omega(\bar{n}' \to \bar{n}) \simeq \frac{\mu\alpha^2}{2}\left(\frac{1}{\bar{n}^2} - \frac{1}{\bar{n}'^2}\right)$$



- Emission rate (Quadrupole formula)

$$\Gamma(\bar{n}' \to \bar{n}) = \frac{2G\Delta\omega^5}{5} N_{\bar{n}'} N_{\bar{n}} \left(I_{ij}I_{ij} - \frac{1}{3}|I^i{}_i|^2\right)(\bar{n}' \to \bar{n})$$

N_a : Occupation number for the level a

- The occupation numbers N_a need be large enough
  Axion-cloud is Bose-Einstein Condensate (BEC)

e.g. transition: $6g(l=m=4, n=1) \to 5g(l=m=4, n=0)$

- Emission rate:

$$\Gamma(6g \to 5g) \sim 5 \times 10^{-7} N_1 N_0 \frac{G\alpha^9}{(GM)^3}$$

- Amplitude:

$$h \equiv \left(\frac{4GP}{r^2 \Delta\omega}\right)^{1/2} \approx 10^{-22} \left(\frac{10\mathrm{Mpc}}{r}\right) \left(\frac{M}{2M_\odot}\right) \sqrt{\epsilon_0 \epsilon_1} \alpha^2$$

$$\epsilon_a \equiv \frac{\mu N_a}{M}$$

- Frequency:

$$\nu \approx 100\,\mathrm{Hz} \left(\frac{2M_\odot}{M}\right) \alpha^3$$

Gravity wave by Level transitions

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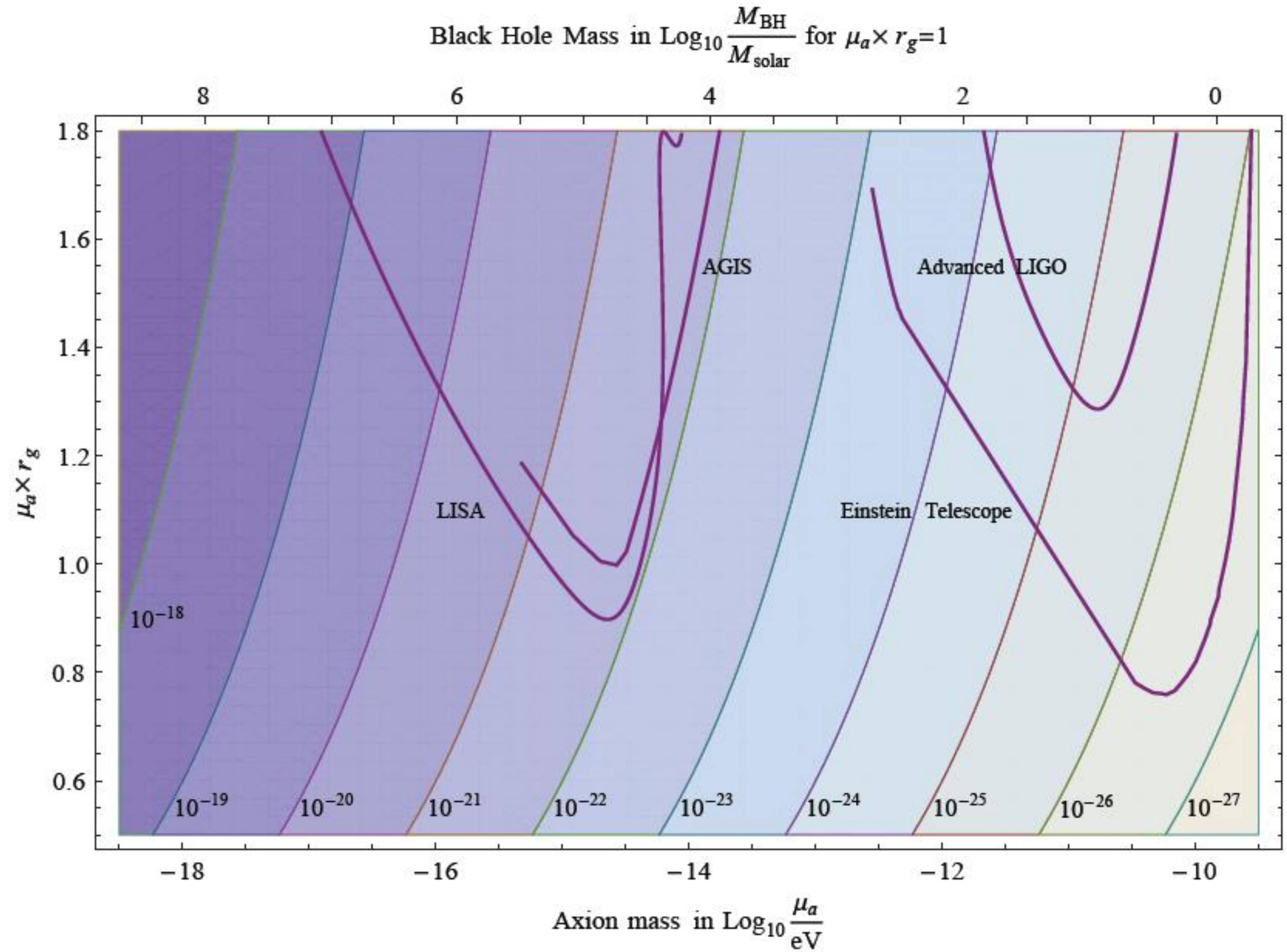
$$h \approx 10^{-22}\left(\frac{10M_{pc}}{r}\right)\frac{M_{\rm BH}}{2M_\odot}\sqrt{\epsilon_0\epsilon_1}\alpha^2 \qquad \nu \sim 10^2{\rm H_z}\times\alpha^3\left(\frac{2M_\odot}{M_{\rm BH}}\right)$$



black hole located at 20 Mpc away from the Earth.

Another process: Axion annihilations

- Two axions annihilate into a single graviton
  - Emission rate:

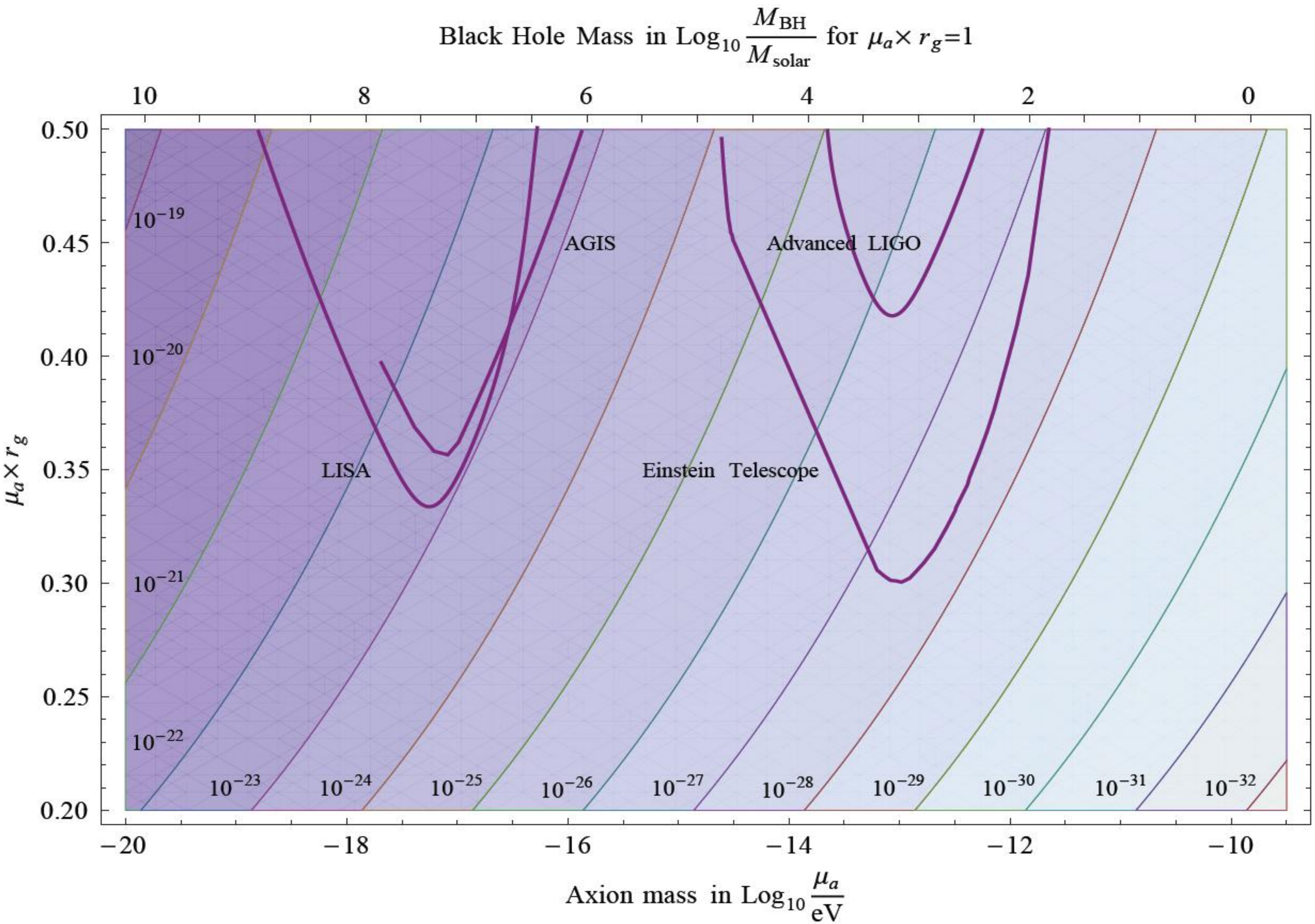
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega}(2 \times 2p \to \text{graviton}) \sim N^2 \times 10^{-7} \frac{G\alpha^{18}}{(GM)^4}$$

  - Amplitude: $h \approx 10^{-22} \left(\frac{10M_{\text{pc}}}{r}\right) \left(\frac{M_{\text{BH}}}{2M_\odot}\right) \epsilon\alpha^7$

  - Frequency: $\nu \sim 30\text{kH}_\text{z} \times \alpha \left(\frac{2M_\odot}{M_{\text{BH}}}\right)$

Gravity wave by Axion annihilations

$$h \approx 10^{-22}\left(\frac{10M_{pc}}{r}\right)\left(\frac{M_{\rm BH}}{2M_\odot}\right)\epsilon\alpha^7 \qquad \nu \sim 30\text{kHz} \times \alpha\left(\frac{2M_\odot}{M_{\rm BH}}\right)$$



Linear approximation

- Level transitions:
- Annihilations:

To make the GW emission effective, it is important that the occupation number of superradiant atomic levels grows exponentially.

Axion self-interactions

$$L = -\frac{1}{2} g^{\mu\nu}(\partial_\mu a)(\partial_\nu a) - \mu^2 f_a^2 \left[1 - \cos\left(\frac{a}{f_a}\right)\right]$$

- Leading term of self-interaction: $(a/f_a)^4$
- Non-relativistic limit : $a(t, \mathbf{x}) = \frac{1}{\sqrt{2\omega}} \left\{ e^{-i\omega t} \psi(t, \mathbf{x}) + e^{i\omega t} \psi^*(t, \mathbf{x}) \right\}$

  $\psi(t, \mathbf{x})$: Slowly varying compared w/ the scale $\omega \sim \mu$

$$i \frac{\partial}{\partial t} \psi = -\frac{1}{2\mu} \Delta \psi + \mu \Phi(r) \psi - \frac{(\psi \psi^*)^2}{12 f_a^2}$$

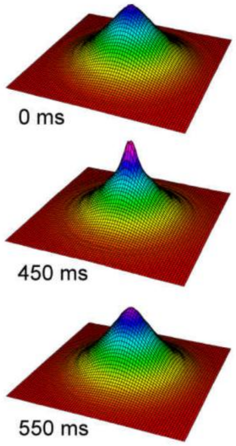
Gross-Pitaevskii equation

$\Phi(r)$: Newtonian potential plays a role of BEC-trap

Dynamics of self-interacting Axions would be similar to the dynamics of trapped BEC w/ attractive interactions

Dynamics of trapped BEC w/ attractive force

- Repeat growth and collapse

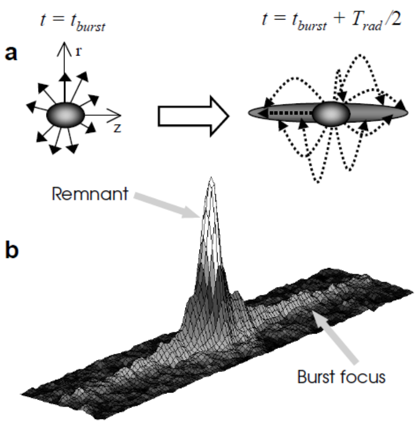


Gerton-Strekalov-Prodan-Hulet 00

^{7}Li

- "Bosenova"

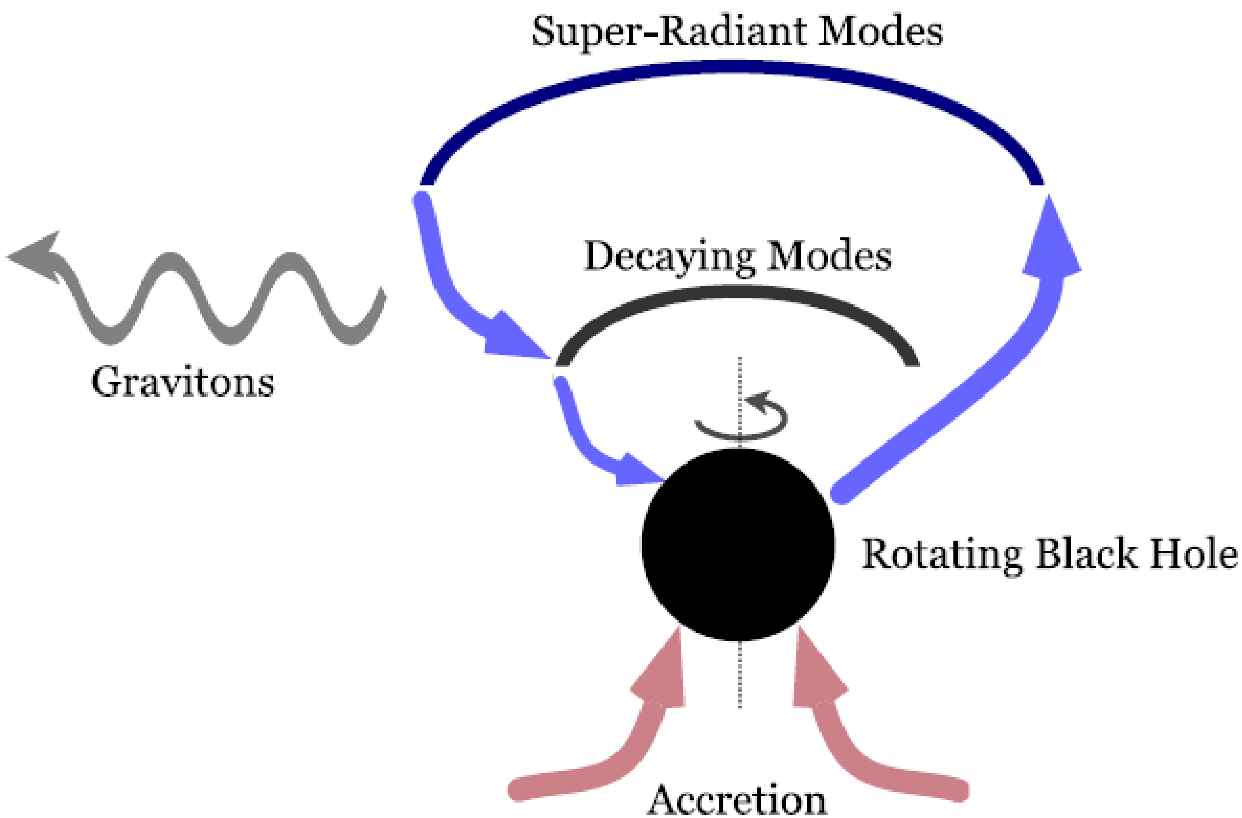
  BEC collapses and gives rise to a burst of high-energy atoms



Cornish-Claussen-Roberts-Cornell-Wieman 00

^{85}Rb

Above some critical mass, Axion-cloud collapses and emits a burst of gravitational waves and axions



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DISCUSSIONS

- The story has been based on the analogy with a quantum mechanical system:

Axion-cloud as Bose-Einstein Condensate

- How far can we push this analogy?

To make the GW emission effective the occupation number N of superradiant atomic levels needs grow exponentially

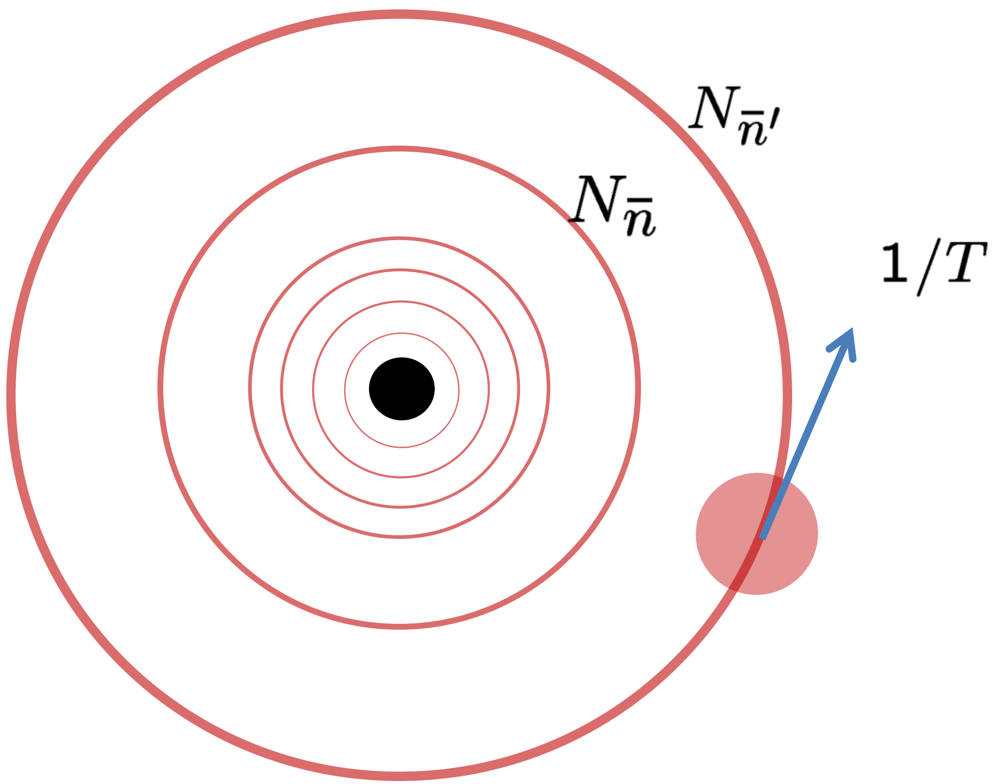
To have large occupation numbers, Axions should be *coherent* so that N states combine together to look like a single large particle of the mass μN

Coherent Axion cloud as a quantum atom

- To have large N Axion-cloud needs be *coherent:*

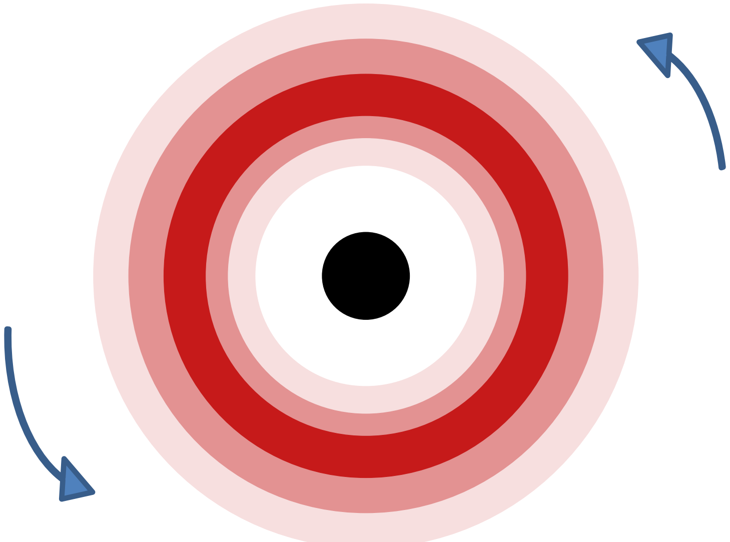
$$\tau \ll T \quad \Rightarrow \quad 1 \ll \frac{\pi}{6}\left(\frac{a}{M}\right)\bar{n}^3\alpha^6$$

-- high energy levels $\bar{n} \sim \alpha^{-2}$ need be excited



Incoherent Axion cloud

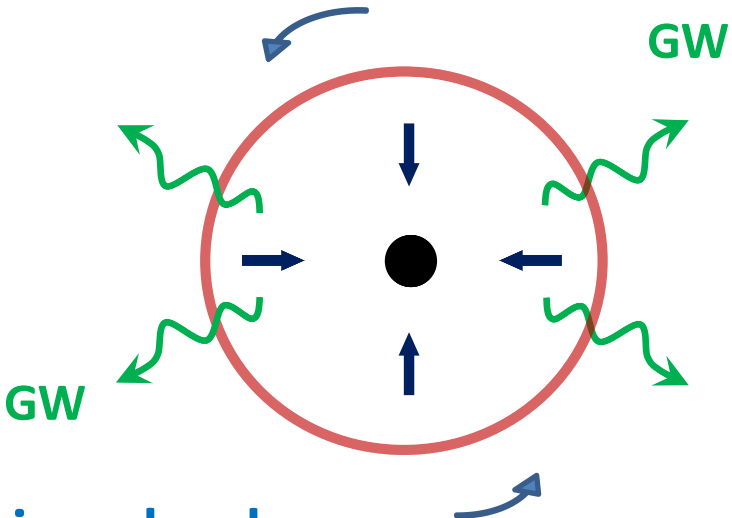
- *Incoherent* superposition of Axion modes form a rotating ring/disk



-- A uniform rotating ring does not emit gravitational waves –

To generate gravitational waves
Need Level transitions, Axion annihilations, or Collapse

A rotating ring falling into a black hole



A uniform ring made of particles with total mass μN falling into Schwarzschild black hole emit gravity waves of energy:

$$\Delta E = \frac{P}{\omega} \simeq 2 \times 10^{-3} \left(\frac{\mu N}{M}\right) \mu N$$

Nakamura-Oohara-Kojima 87

Axion-cloud case:

This may be similar to what would happen when Axion level transitions occur in a coherent Axion-cloud.

For incoherent Axion-cloud, however, there is no reason that a large number of Axions have to make level transitions at the same time.

-- seems less effective to generate gravitational waves

Axion cloud as a classical fluid system

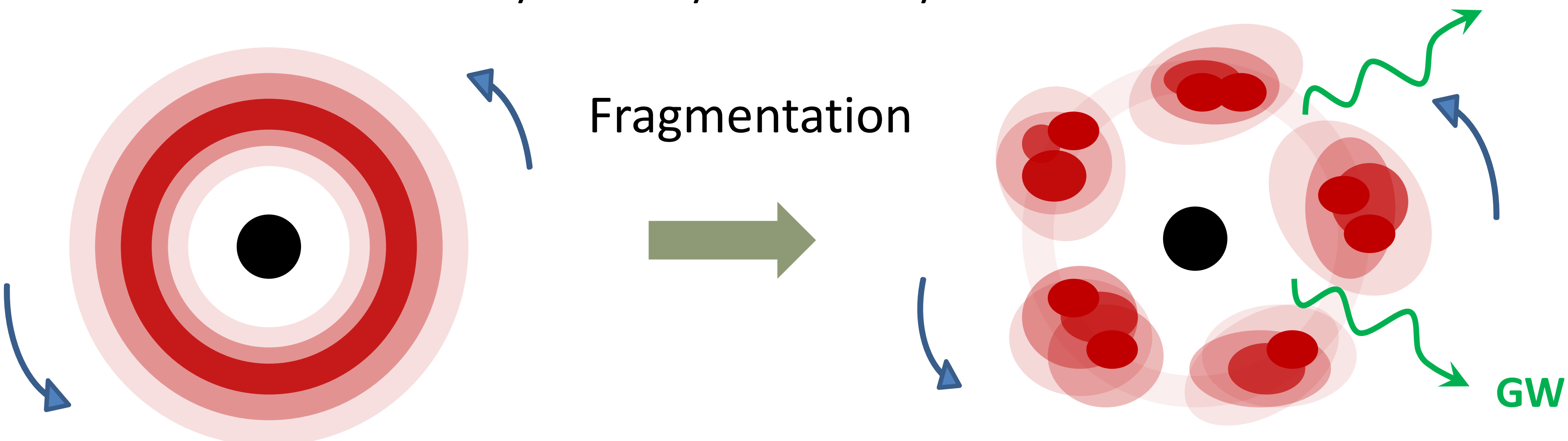
- Uniform Axion cloud may break into pieces
- Toomre's stability criterion for disks: Toomre 64

$$\frac{2v_s\Omega}{\pi^2 G\sigma_0} \sim \frac{1}{2\pi^2}\frac{\alpha}{\bar{n}^2 NGM} > 1$$

Ω : Angular velocity of disk

σ_0 : Surface energy density

-- unlikely to satisfy the stability criterion



$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_X = Ae^{-BX}$$

X : number of clumps

The smaller X the more effective to generate gravitational radiation

e.g. Nakamura-Oohara 83

Open issues - Tasks

- Need to understand the dynamics of Axion cloud

  -- How far can we push the analogy w/ Quantum Atom?

- Obtain accurate quantitative description of superradiance development

- Identify most dominant/effective process for generating gravitational waves

  -- Level transitions, Annihilations vs Bosenova?

  -- Incoherent axion ring and fragmentation?

- Compute gravity wave emission rates and precise waveforms on the Kerr background.

- Templates for the near-horizon geometry

  -- distortion of near-horizon geometry from the Kerr metric due to self-gravity of axion cloud w/ large occupation number