

Off-center CMB polarization anisotropy in the local void model

GOTO Hajime

@Graduate University for Advanced Studies
(GUAS)

Collaborator: Hideo Kodama @GUAS&KEK

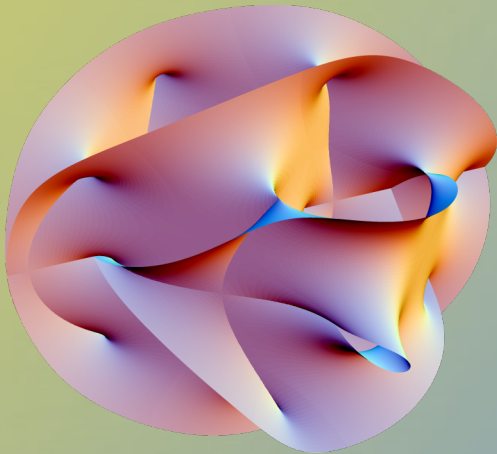
Introduction

- In this talk, we consider only 4 dimensions (3+1 dimensions)

extra
dimensions

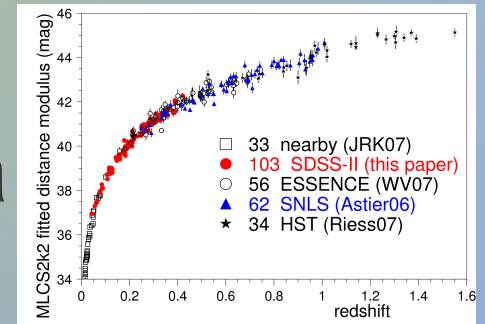
4 (= 3 + 1)
dimensions

3 (= 2 + 1)
dimensions

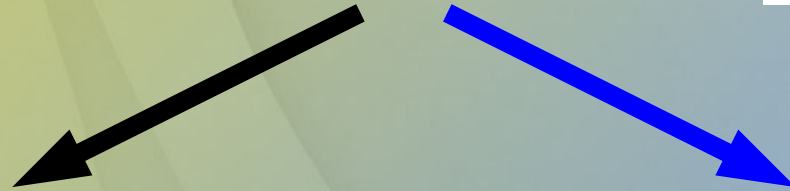


ピカチュウ

Type-Ia supernova observations



Kessler+'09



Assumption

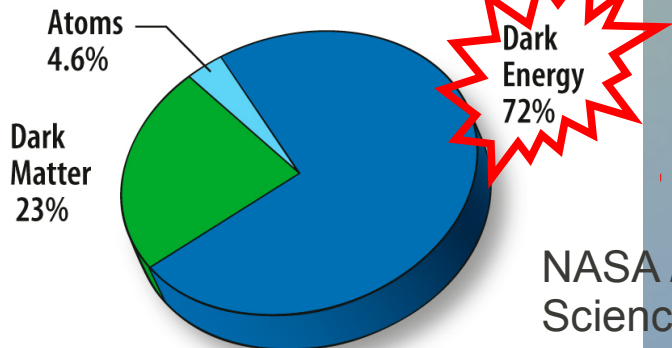
Cosmological Principle

Conclusion

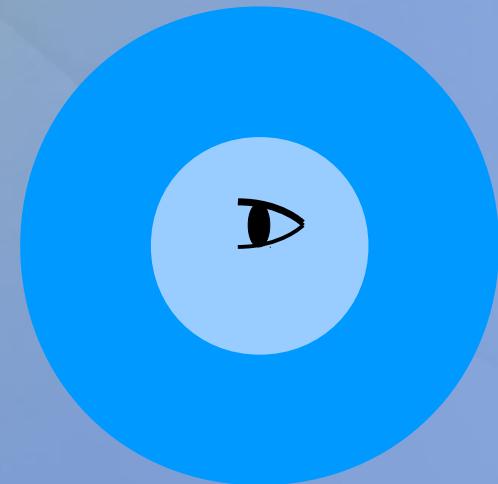
cosmic acceleration

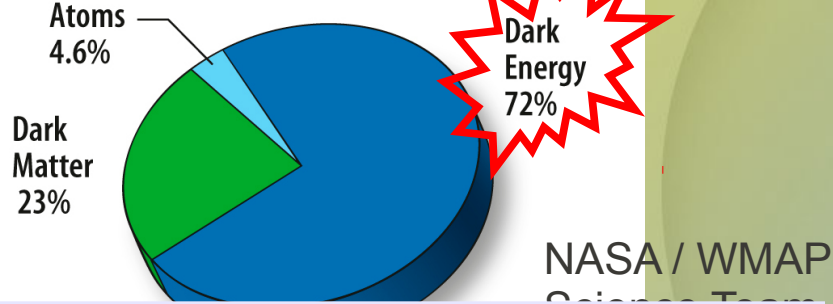
Other explanations

e.g.) local void model
(LTB model)(Tomita'00)



NASA / WMAP
Science Team





(or modified gravity)

How can they be distinguished?

- CMB temperature & polarization anisotropies
- Gravitational lensing effect → B-mode & EB(&TB) correlations, for an off-center observer in the local void model
- Expected to be characteristic observables

Contents

- Gravitational lensing effect in the Lemaitre-Tolman-Bondi(LTB) model
 - Null geodesic in the LTB model
 - Shift vector $\delta\theta_{\text{obs}}$
- CMB polarization in the LTB model
 - Flux intensity tensor
 - Initial condition at the last scattering surface
 - Propagation after last scattering
 - Change in the polarization distribution

Gravitational lensing effect in the LTB model

Null geodesic in the LTB model

- Lemaitre-Tolman-Bondi(LTB) metric

$$ds^2 = -dt^2 + S^2 d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$S = \frac{r'}{\xi} \quad r = r(t, \chi) \quad ' = \frac{\partial}{\partial \chi} \quad \cdot = \frac{\partial}{\partial t}$$

$$\xi = \sqrt{1 - k(\chi)\chi^2} \quad k(\chi): \text{curvature function}$$

- Geodesic equation $dp^\mu / d\lambda = -\Gamma_{\nu\rho}^\mu p^\nu p^\rho$

- Spherical symmetry \rightarrow equations of ω, μ, p_\perp

$$p^t = \omega \quad p^x = \mu\omega/S \quad p_\perp := r \sqrt{(p^\theta)^2 + (p^\varphi)^2 \sin^2 \theta}$$

- 2-dim. problem \rightarrow use the plane of $\varphi = 0, \pi$

Null geodesic in the LTB model

→ Set of ordinary differential equations for

$$\omega(t), \chi(t), \theta(t), \mu(t)$$

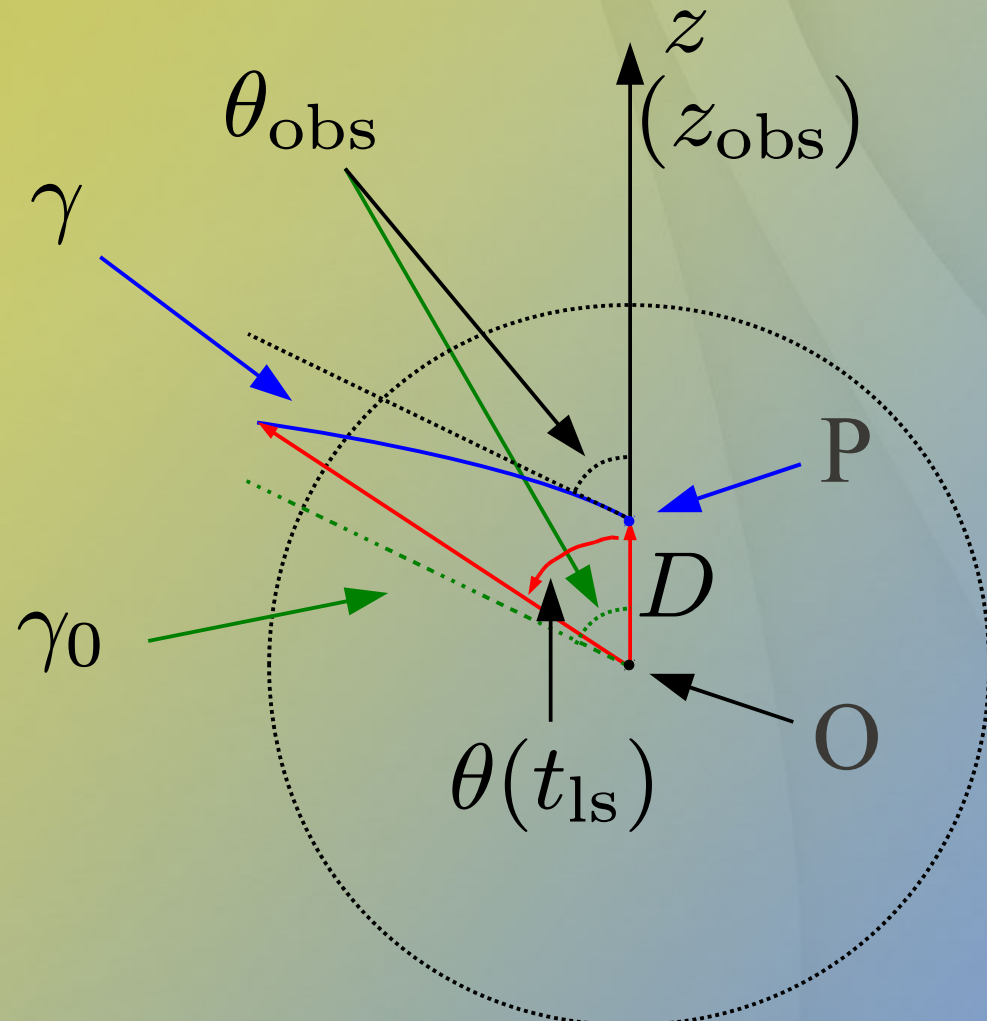
$$\frac{d}{dt} \ln \omega = -\frac{\dot{S}}{S} \mu^2 - (1 - \mu^2) \frac{\dot{r}}{r}$$

$$\frac{d\chi}{dt} = \frac{\mu}{S}$$

$$\frac{d\theta}{dt} = \pm \frac{\sqrt{1 - \mu^2}}{r}$$

$$\frac{1}{1 - \mu^2} \frac{d\mu}{dt} = \frac{\xi}{r} + \mu \left(\frac{\dot{r}}{r} - \frac{\dot{S}}{S} \right)$$

Null geodesic in the LTB model



- Determine the geodesic γ perturbatively with respect to γ_0
- However μ cannot be treated perturbatively
- \rightarrow Instead, we use

$$b := \chi \sqrt{1 - \mu^2}$$

$$c := \chi \mu$$
- Solve the perturbation equations

Null geodesic in the LTB model

$$\theta(t_{\text{ls}}) = - \int_{t_0}^{t_{\text{ls}}} dt \frac{b}{\chi^r} = \dots\dots$$

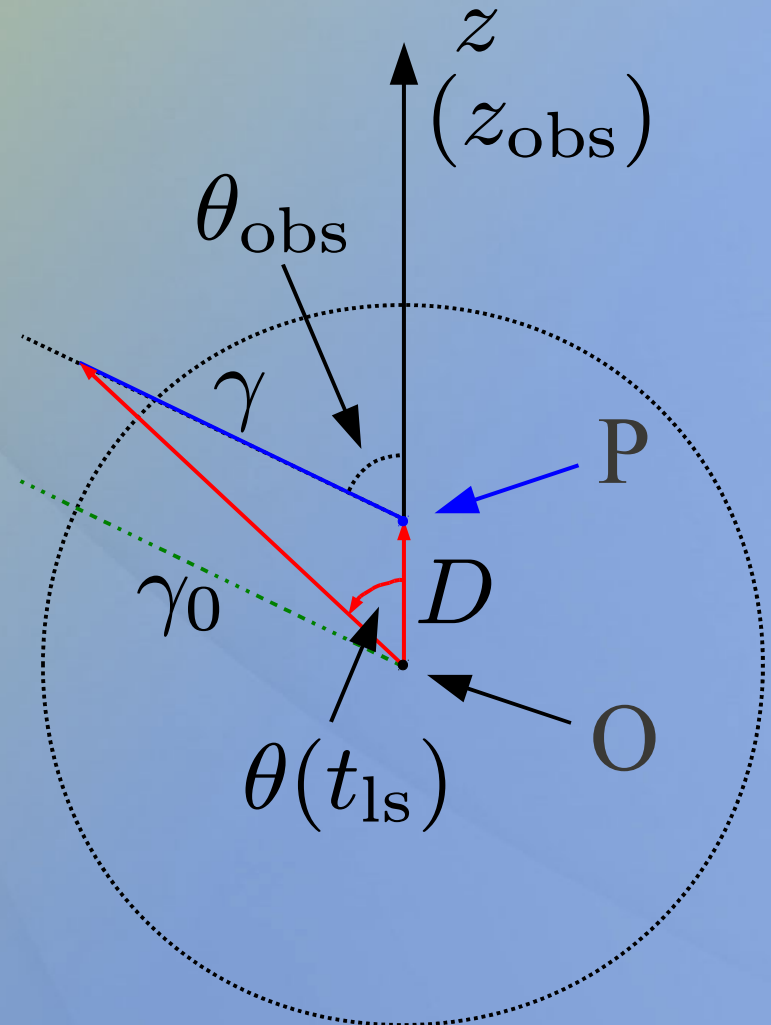
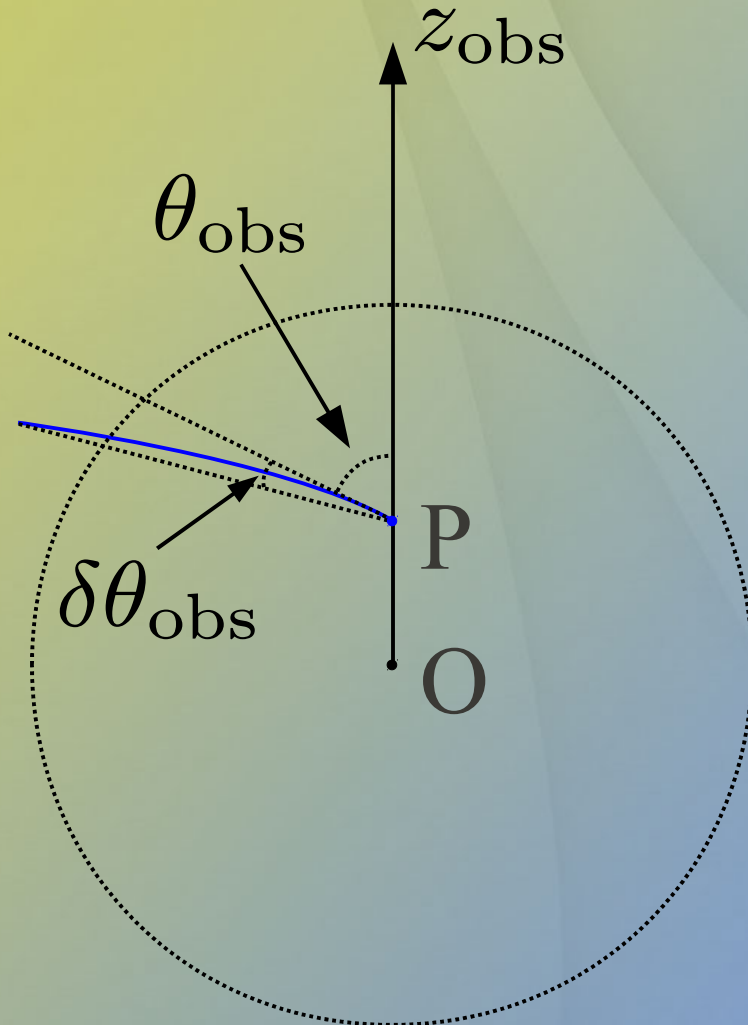
$$= \theta_{\text{obs}} - \frac{b(t_0)}{\chi_{\text{ls}}}$$

$$- b(t_0) \int_{t_{\text{ls}}}^{t_0} \frac{dt}{\chi^2} \left(\frac{1}{S} - \frac{\chi}{r} e^{Y(t,t_0)} \right)$$

where $Y(a, b) = \int_a^b dt \left\{ \frac{\xi}{\chi} \left(\frac{1}{r'} - \frac{\chi}{r} \right) + \frac{\dot{r}}{r} - \frac{\dot{S}}{S} \right\}$

Shift vector $\delta\theta_{\text{obs}}$

Homogeneous limit



Shift vector $\delta\theta_{\text{obs}}$

- In the homogeneous limit, $\delta\theta_{\text{obs}} = 0$
- By eliminating the part that survives in the homogeneous limit,

$$\delta\theta_{\text{obs}} = -D \sin \theta_{\text{obs}} \cdot \int_{t_{\text{ls}}}^{t_0} \frac{dt}{\chi^2} \left(\frac{\chi}{r} (1 - \xi) + \frac{1}{S} - \frac{\chi}{r} e^{Y(t, t_0)} \right)$$

$$\delta\varphi_{\text{obs}} = 0$$

CMB polarization in the LTB model

Flux intensity tensor

- Electric field

$$\mathbf{E} = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{1}{2} \sum_p (\boldsymbol{\epsilon}_p(\mathbf{k}) a_p(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + \boldsymbol{\epsilon}_p^*(\mathbf{k}) a_p(\mathbf{k})^\dagger e^{-i\mathbf{k} \cdot \mathbf{x}})$$

Polarization basis

- Measured components of the electric field

$$\mathcal{E}_p = \int d^3 \mathbf{x} W(\mathbf{x}) \boldsymbol{\epsilon}_p^o \cdot \mathbf{E}(t_0, \mathbf{x})$$

Sensitivity function

Detector polarization basis

Flux intensity tensor

- $\langle a_p(\mathbf{k})^\dagger a_q(\mathbf{k}') \rangle = 2(2\pi)^3 \rho_{pq}(\mathbf{k}) \delta^3(\mathbf{k} - \mathbf{k}')$

ρ_{pq} : flux polarization matrix

- Flux density tensor

$$\rho_{ij}(\mathbf{k}) = \sum_{p,q} \epsilon_{pi}^*(\mathbf{k}) \epsilon_{qj}(\mathbf{k}) \rho_{pq}(\mathbf{k})$$

- Observed correlation of the electric fields

$$\langle : \mathcal{E}_p \mathcal{E}_q : \rangle = \epsilon_p^{oi} \epsilon_q^{oj} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} |\hat{W}(\mathbf{k})|^2 \rho_{(ij)}(\mathbf{k})$$

($\hat{W}(\mathbf{k})$ is the Fourier transform of $W(\mathbf{x})$)

Initial condition at the last scattering surface

- Boltzmann equation ρ_{ij} generalized

$$\left(\frac{k^\mu}{k^0} \nabla_\mu + f^i \partial_{k^i} \right) \rho^{\mu\nu}(x, \mathbf{k}) = C^{\mu\nu}(\rho)$$

- Up to the linear order in perturbations, $\delta\rho_{\mu\nu}$ on the last scattering surface can be Fourier decomposed into the contribution of each perturbation mode with the wave vector \mathbf{K}

$$\delta\rho_{\mu\nu}(t_{\text{ls}}, \mathbf{x}, \mathbf{k}) = \int d^3\mathbf{K} e^{i\mathbf{K}\cdot\mathbf{x}} \rho_{\mu\nu}^{(1)}(\mathbf{K}; \mathbf{k})$$

Initial condition at the last scattering surface

- For an appropriate, customary choice of $\epsilon_p(\mathbf{k})$, $\rho_{pq}^{(1)} = \epsilon_p^{\mu*} \epsilon_q^\nu \rho_{\mu\nu}^{(1)}(\mathbf{K}; \mathbf{k})$ depends only on ω and $\mu = \cos \theta = \hat{\mathbf{K}} \cdot \hat{\mathbf{k}}$
- Solve Boltzmann equation \rightarrow initial condition at the last scattering surface

Propagation after last scattering

- LTB spacetime $\tilde{g}_{\mu\nu}$
- Let us work in the synchronous gauge

$$ds^2 = -dt^2 + \tilde{g}_{ij}(t, x)dx^i dx^j$$

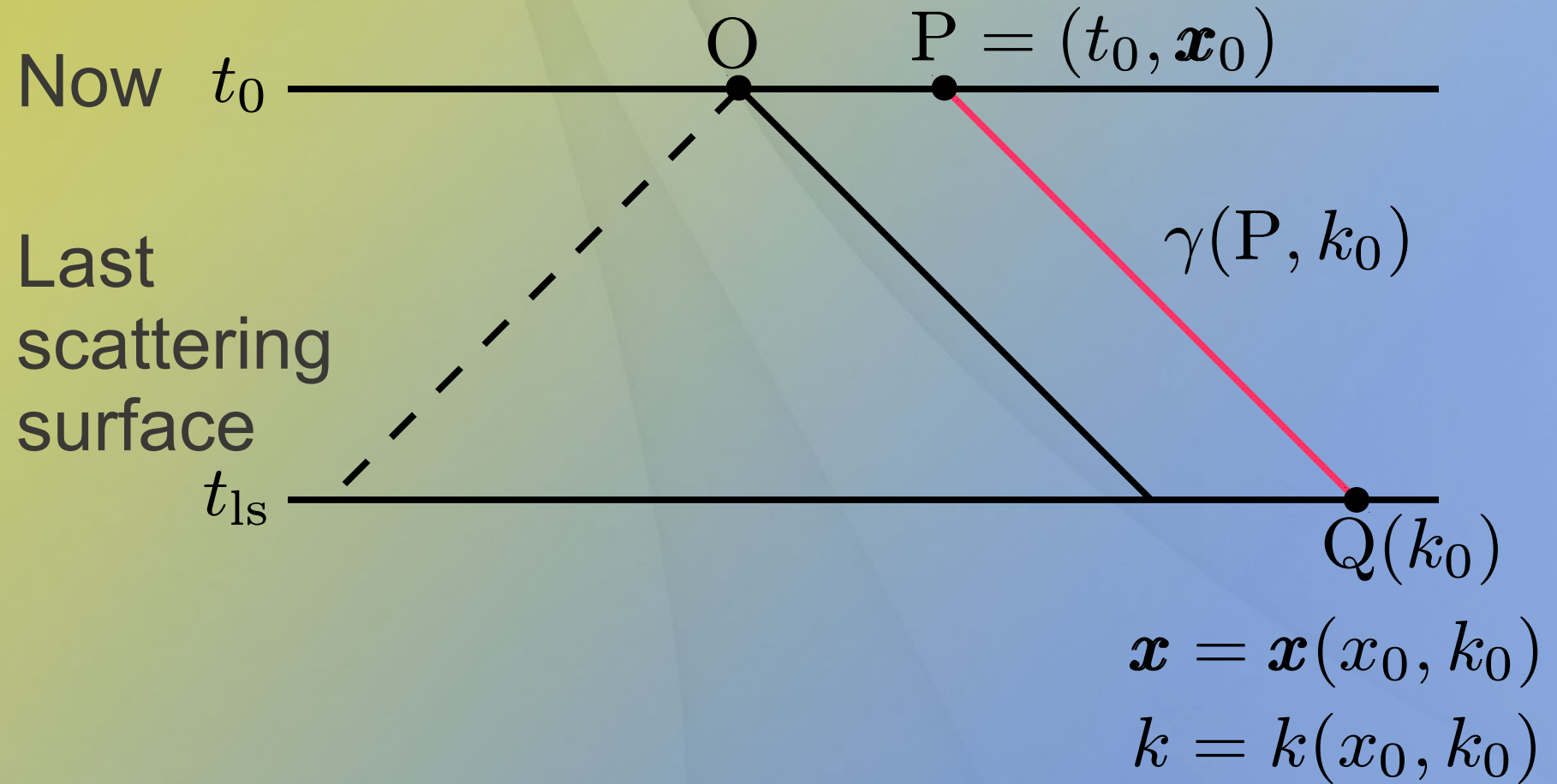
We take the constant-time surfaces so that the last scattering surface is represented by $t = t_{\text{ls}}$

- \tilde{g}_{ij} is close to a spatially homogeneous and isotropic metric g_{ij} in the early universe

$$\tilde{g}_{ij} = g_{ij} + \delta_{\text{LTB}}g_{ij}$$

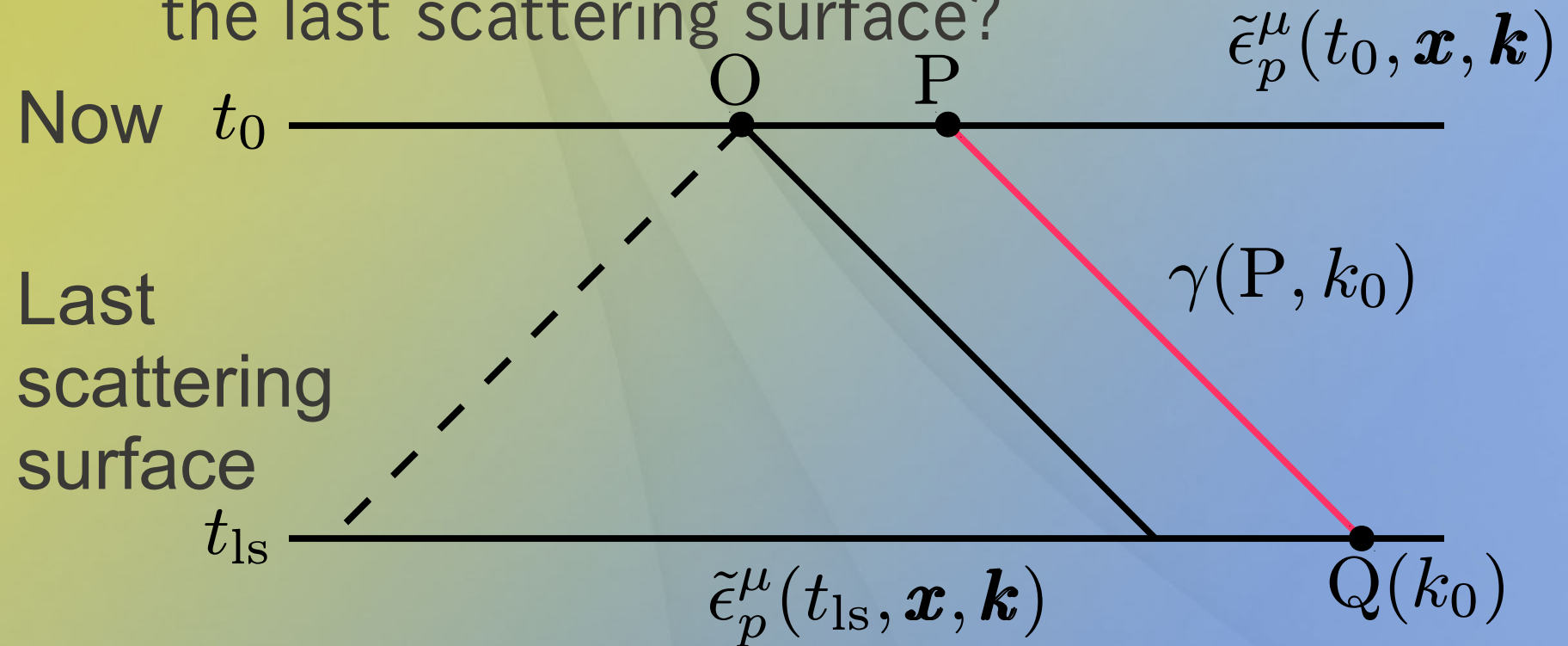
Propagation after last scattering

$$k_0 = (\omega(\mathbf{k}_0), \mathbf{k}_0)$$



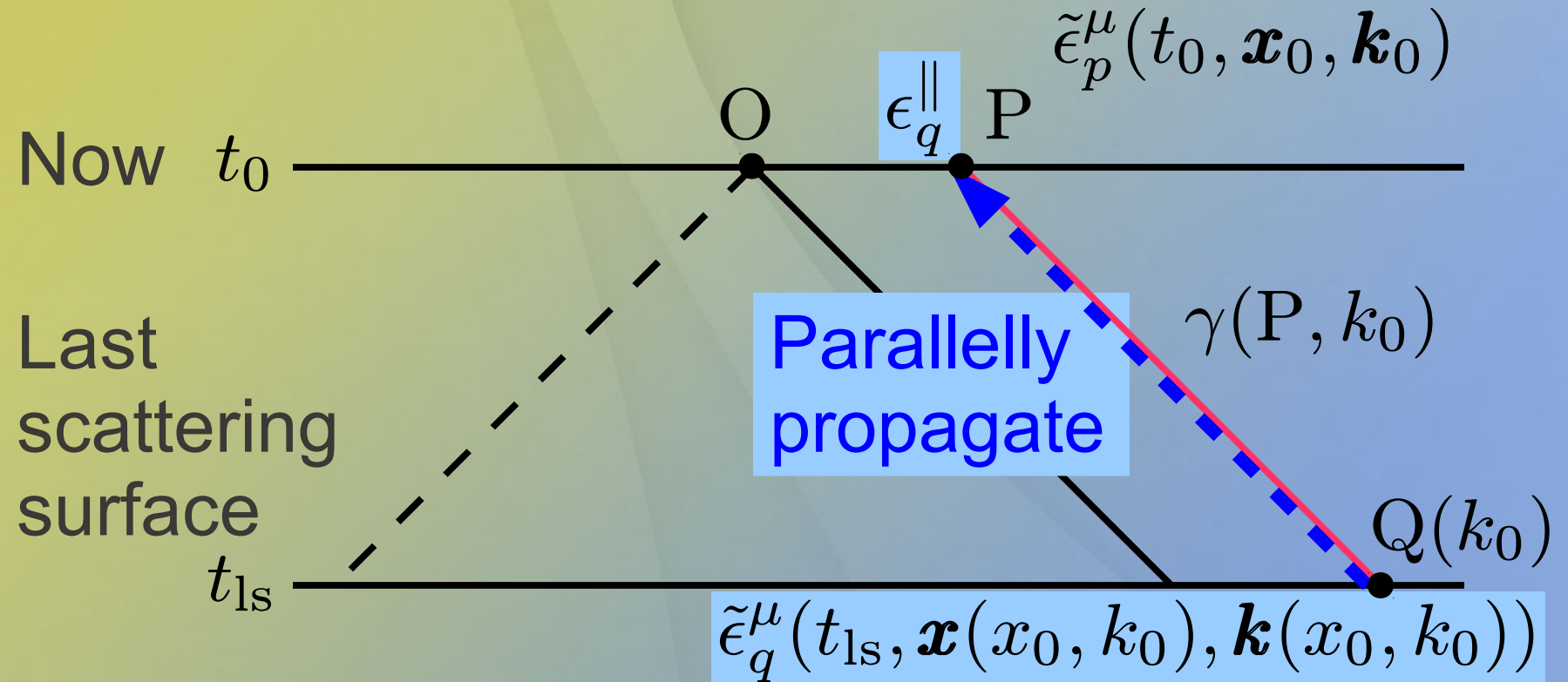
Propagation after last scattering

- How is the polarization now related to that at the last scattering surface?



$$\tilde{\rho}(P, \mathbf{k}_0) = \tilde{C} \tilde{\rho}(t_{ls}, \mathbf{x}, \mathbf{k}) \tilde{C}^\dagger$$

Propagation after last scattering



$$\tilde{\rho}(P, \mathbf{k}_0) = \tilde{C} \tilde{\rho}(t_{ls}, \mathbf{x}, \mathbf{k}) \tilde{C}^\dagger$$

$$\tilde{C}_{pq} = \tilde{\epsilon}_p(t_0, \mathbf{x}_0, \mathbf{k}_0) \cdot \epsilon_q^{\parallel*}$$

Propagation after last scattering

- When the universe is well-described by the FLRW model until last scattering,

$$\tilde{\rho}_{pq}(t_{\text{ls}}, \mathbf{x}, \mathbf{k}) = \frac{1}{2} \delta_{pq} I \left(\frac{\omega}{T_{\text{ls}}} \right) + \delta\rho_{pq}(t_{\text{ls}}, \mathbf{x}, \mathbf{k})$$

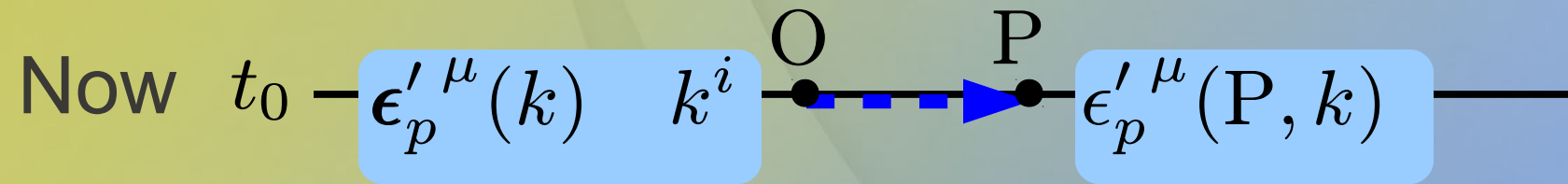
- Using $\sum_p \tilde{C}_{pq} \tilde{C}_{pr}^* = \delta_{qr}$,

$$\tilde{\rho}_{pq}(\mathbf{P}, \mathbf{k}_0) = \frac{1}{2} \delta_{pq} I \left(\frac{\omega}{T_{\text{ls}}} \right) + \tilde{C}_{pr} \tilde{C}_{qs}^* \delta\rho_{rs}(t_{\text{ls}}, \mathbf{x}, \mathbf{k})$$

- If we express $\delta\rho_{rs}(t_{\text{ls}}, \mathbf{x}, \mathbf{k})$ in terms of the following polarization basis that does not depend on the mode wave number(\mathbf{K}), we can set $\tilde{C}_{pq} \approx 1$

Propagation after last scattering

Transport along the radial line $\Omega = \text{const.}$ parallelly with respect to $g_{ij}(t_0, x)$

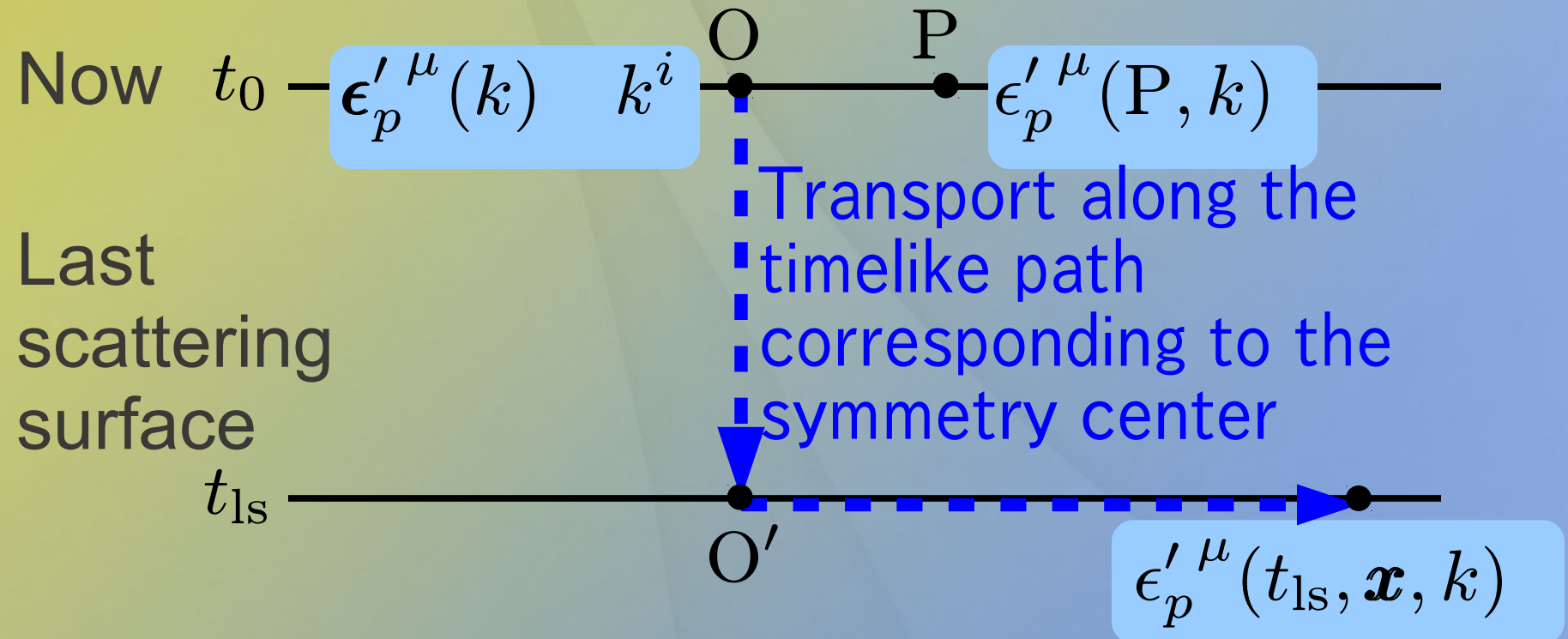


Last scattering surface

t_{1s} —————

In this way, we can define a polarization basis everywhere on the hypersurface $t = t_0$

Propagation after last scattering



Extend to an arbitrary point by the parallel transport with respect to $g_{ij}(t_{1s}, \mathbf{x})$

Propagation after last scattering

- Relation between this basis and the basis that depends on the mode(\mathbf{K}) is easily given by the rotation matrix

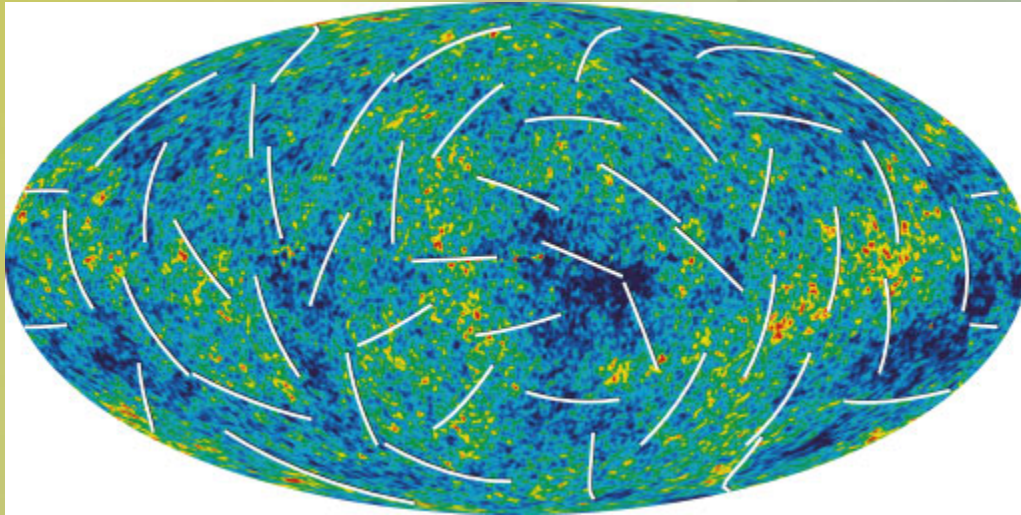
$$A_{pq} := \boldsymbol{\epsilon}'_p(t_{\text{ls}}, \mathbf{x}, \mathbf{k}) \cdot \boldsymbol{\epsilon}_q(\mathbf{K}, \mathbf{k}) = R_{pq}(\theta_{\text{ls}})$$

- \rightarrow Initial condition at the last scattering surface is given by

$$\delta\rho_{pq}(t = t_{\text{ls}}, \mathbf{x}, \mathbf{k}) = \int d^3\mathbf{K} e^{i\mathbf{K}\cdot\mathbf{x}} A(\hat{\mathbf{K}}, \hat{\mathbf{k}}) \rho^{(1)}(\mathbf{K}, \omega, \mu) A(\hat{\mathbf{K}}, \hat{\mathbf{k}})^\dagger$$

- It turns out that $\tilde{C} = 1$ (without going into detail here)

Change in the polarization distribution

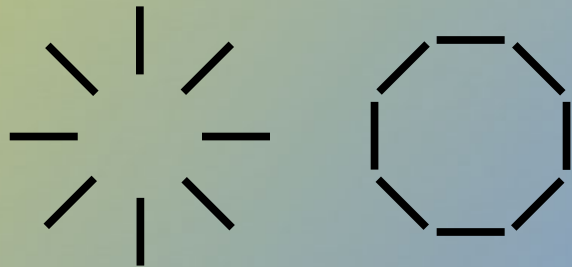


NASA/WMAP Science Team

Polarization distribution on the celestial sphere consists of two patterns

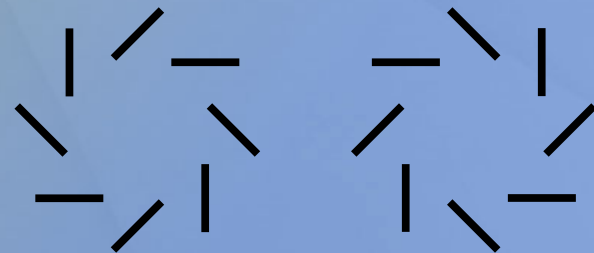
'gradient'

E-mode



'curl'

B-mode



Change in the polarization distribution

- CMB polarization distribution (omitting “obs”)

$$\rho_{ab}(\hat{\mathbf{n}}) = {}_{+2}A(\hat{\mathbf{n}})\bar{m}_a\bar{m}_b + {}_{-2}A(\hat{\mathbf{n}})m_a m_b$$

$$\hat{\mathbf{n}} = (\theta, \varphi) \quad a, b \in \{\theta \equiv 1, \varphi \equiv 2\}$$

$$\mathbf{m} := \frac{1}{\sqrt{2}}(\mathbf{e}_\theta + i\mathbf{e}_\varphi) \quad \bar{\mathbf{m}} := \frac{1}{\sqrt{2}}(\mathbf{e}_\theta - i\mathbf{e}_\varphi)$$

- ${}_{\pm 2}A(\hat{\mathbf{n}})$, spin-2 functions, are expanded with spin-weighted spherical harmonics

$${}_{\pm 2}A(\hat{\mathbf{n}}) = \sum_{\ell m} {}_{\pm 2}A_\ell^m {}_{\pm 2}Y_\ell^m(\hat{\mathbf{n}})$$

Change in the polarization distribution

- Rotationally invariant combination of $\pm_2 A_\ell^m$

$$E_\ell^m := \frac{1}{2} (+_2 A_\ell^m + -_2 A_\ell^m) \quad \text{curl-free (E-mode)}$$

$$B_\ell^m := \frac{1}{2i} (+_2 A_\ell^m - -_2 A_\ell^m) \quad \text{gradient-free (B-mode)}$$

- Power spectra $X_1, X_2 \in \{T, E, B\}$

$$\langle X_{1\ell}^{m*} X_{2\ell'}^{m'} \rangle = C_\ell^{X_1 X_2} \delta_{\ell\ell'} \delta_{mm'} \quad \text{temperature}$$

- If physics and the ensemble for averaging are invariant under a parity inversion,

$$C_\ell^{\text{TB}} = C_\ell^{\text{EB}} = 0$$

Change in the polarization distribution

- ρ_{ab} : polarization distribution without lensing
- ρ'_{ab} : polarization distribution with lensing
- $\hat{\mathbf{n}} := \hat{\mathbf{n}}' + \delta\boldsymbol{\theta}$
 $\hat{\mathbf{n}}'$: direction in which the observer looks
- Gravitational lensing effect $\rightarrow \rho'_{ab}(\hat{\mathbf{n}}') = \rho_{ab}(\hat{\mathbf{n}})$

- Finally,

$$\Theta_{\ell}^{\prime m} = \Theta_{\ell}^m - D\Gamma(\ell + 2) \sqrt{\frac{(\ell + 1)^2 - m^2}{4(\ell + 1)^2 - 1}} \Theta_{\ell+1}^m + D\Gamma(\ell - 1) \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}} \Theta_{\ell-1}^m + \mathcal{O}(D^2)$$

Change in the polarization distribution

$$E'_\ell{}^m = E_\ell{}^m$$

$$-D\Gamma \sqrt{\frac{((\ell + 1)^2 - m^2)((\ell + 1)^2 - 4)}{(\ell + 1)^2(4(\ell + 1)^2 - 1)}} (\ell + 2) E_{\ell+1}^m$$

$$+D\Gamma \sqrt{\frac{(\ell^2 - m^2)(\ell^2 - 4)}{\ell^2(4\ell^2 - 1)}} (\ell - 1) E_{\ell-1}^m$$

$$+iD\Gamma \frac{2m}{\ell(\ell + 1)} B_\ell{}^m + \mathcal{O}(D^2)$$

Change in the polarization distribution

$$B'_\ell{}^m = B_\ell{}^m$$

$$-D\Gamma \sqrt{\frac{((\ell + 1)^2 - m^2)((\ell + 1)^2 - 4)}{(\ell + 1)^2(4(\ell + 1)^2 - 1)}} (\ell + 2) B_{\ell+1}{}^m$$

$$+D\Gamma \sqrt{\frac{(\ell^2 - m^2)(\ell^2 - 4)}{\ell^2(4\ell^2 - 1)}} (\ell - 1) B_{\ell-1}{}^m$$

$$-iD\Gamma \frac{2m}{\ell(\ell + 1)} E_\ell{}^m + \mathcal{O}(D^2)$$

Change in the polarization distribution

- If $B = 0$ in the homogeneous case (the case without lensing),

$$B'_\ell{}^m \approx D\Gamma \frac{2m}{i\ell(\ell+1)} E_\ell{}^m$$

$$\Gamma := - \int_{t_{1s}}^{t_0} \frac{dt}{\chi^2} \left[\frac{\chi}{r} (1 - \xi(\chi)) + \frac{1}{S} - \frac{\chi}{r} \exp \int_t^{t_0} dt_1 \left\{ \frac{\xi(\chi)}{\chi} \left(\frac{1}{r'} - \frac{\chi}{r} \right) + \frac{\dot{r}}{r} - \frac{\dot{S}}{S} \right\}_{t_1} \right]$$

EB correlation (non-zero part only)

$$\langle E_\ell{}^{m*} B'_\ell{}^m \rangle_{\text{CMB}} \approx D\Gamma \frac{2m}{i\ell(\ell+1)} C_\ell^{\text{EE}}$$

(TB similarly)

Change in the polarization distribution

- If $B=0$ in the homogeneous case, correlations in the first order in D become nonzero only if

θ dependence of the shift vector	TT, TE, EE correlations	TB, EB correlations
$\sin \theta$	$\ell' = \ell \pm 1$	$\ell' = \ell$
Cf. $\sin^2 \theta$	$\ell' = \ell \pm 2$	$\ell' = \ell \pm 1$

Cf. anisotropic inflation (Professor Soda's talk)

Summary and outlook

- In this work, we developed a formulation to calculate the gravitational lensing effect on the CMB temperature and polarization for an off-center observer in a spherically symmetric void described by the LTB model
- Next, we are going to numerically estimate this effect
- In future, we will limit the distance from us to the center by the results of B-mode observations